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from the Gross Llewellyn Smith Sum Rule**

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A MEASUREMENT OF $\alpha_s(Q^2)$ FROM THE GROSS LLEWELLYN SMITH SUM RULE

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Abstract

The Gross Llewellyn Smith sum rule has been measured at different values of four-momentum transfer squared (Q^2) by combining the precise CCFR neutrino data with data from other deep-inelastic scattering experiments at lower values of Q^2 . A comparison with the $\mathcal{O}(\alpha_s^3)$ predictions of perturbative QCD yields a determination of α_s and its dependence on Q^2 in the range $1 \text{ GeV}^2 < Q^2 < 20 \text{ GeV}^2$. Low Q^2 tests have greater sensitivity to $\alpha_s(M_Z^2)$ than high Q^2 tests, since at low Q^2 α_s is large and changing rapidly.

To leading order in perturbative QCD, the structure function xF_3 measured in νN scattering is the difference between the quark and anti-quark momentum distributions. The GLS sum rule predicts that the integral over x of F_3 is simply 3, the number of valence quarks in a nucleon [1]. There are corrections to the sum rule which introduce a dependence of the GLS integral on α_s , the strong coupling constant, in the following way [2]:

$$\int_0^1 xF_3(x, Q^2) \frac{dx}{x} = 3\left(1 - \frac{\alpha_s}{\pi} - a(n_f)\left(\frac{\alpha_s}{\pi}\right)^2 - b(n_f)\left(\frac{\alpha_s}{\pi}\right)^3\right) - \Delta HT \quad (1)$$

where a and b depend on the number of quark flavors, n_f , accessible at a given x and four-momentum transfer squared, Q^2 . ΔHT represents a higher twist contribution, which has been estimated using QCD sum rules, a Vector Meson Dominance Model, and a Non-relativistic Quark Model to be $0.27 \pm 0.14/Q^2 (GeV^2)$ [3]. The Q^2 dependence of α_s is as follows [4]:

$$\frac{\alpha_s(Q^2)}{4\pi} = \frac{1}{\beta_0(n_f) \ln\left(\frac{Q^2}{\Lambda_{MS}^2}\right)} - \frac{\beta_1(n_f)}{\beta_0(n_f)^2} \frac{\ln\left(\ln\left(\frac{Q^2}{\Lambda_{MS}^2}\right)\right)}{\ln^2\left(\frac{Q^2}{\Lambda_{MS}^2}\right)} + \mathcal{O}\left(\frac{1}{\ln^3\left(\frac{Q^2}{\Lambda_{MS}^2}\right)}\right). \quad (2)$$

The challenge in evaluating $\int F_3 dx$ is that for a given Q^2 value, there is a limited x region that is accessible by any one experiment. The incoming neutrino energy imposes a minimum x constraint and detector acceptance imposes a maximum x constraint. CCFR has data at low Q^2 and low x ($10^{-2} < x < 10^{-1}$), and at high Q^2 and high x ($10^{-1} < x < 1$). The CCFR detector and the measurement of xF_3 have been described in detail elsewhere [5]. One way to evaluate $\int F_3 dx$ over all x is to extrapolate xF_3 from all Q^2 regions to a Q_0^2 value where the data is predominantly at low x . A previous CCFR analysis found that for $Q_0^2 = 3 GeV^2$, $\int F_3 dx = 2.50 \pm .018(stat) \pm .078(syst)$ [6]. By using QCD to extrapolate xF_3 to Q_0^2 however, one introduces α_s *a priori* into the problem. Furthermore, higher twist effects are not included in QCD extrapolations.

The goal of this analysis is to evaluate $\int F_3 dx$ without introducing any *ad hoc* Q^2 dependence. By combining the CCFR data with that of several other experiments enough data at different energies are obtained to measure $\int F_3 dx$ without Q^2 extrapolation at values of Q^2 between $1 GeV^2$ and $20 GeV^2$. The xF_3 measurements from experiments WA59, WA25, SKAT, FNAL-E180 [7], and BEBC-Gargamelle [8] were normalized to the CCFR xF_3 measurements in the Q^2 regions of overlap and then were used along with the CCFR xF_3 data. Furthermore, since at high x the structure function $F_2 \approx xF_3$, one can use F_2 data from e^-N scattering at SLAC [9] in this region ($x > 0.5$) by normalizing it to the ratio of xF_3/F_2 as measured in the CCFR data. This is particularly important at low Q^2 where there is no xF_3 data at high x . The published CCFR xF_3 data were modified for new electroweak radiative corrections (Bardin[10]). In addition, the CCFR data were corrected for the contribution from the strange sea [11] of events containing two oppositely charged muons. Finally, by comparing the F_2 values of CCFR to those from SLAC [9], NMC and BCDMS [12], the overall normalization of the CCFR data was determined to be 1.019 ± 0.011 . To integrate over all x , this analysis sums the binned data for $x > 0.02$. For the contribution to the integral at lower x , the data below $x = 0.1$ is fit to a power law and then that function is integrated over $0 < x < 0.02$. Figure 1a shows the combined xF_3 data and the corrected F_2 data for the four lowest Q^2 bins, as well as the power law fit to the low x data and the χ^2 for those fits. To be consistent with theoretical predictions of higher twist effects on the sum rule, the ν -nucleon elastic contribution (described in [8]) was added to the integral, and both the elastic and inelastic contributions were corrected for target mass effects [8]. Figure 1b shows $\int F_3 dx$ as a function of Q^2 and the theoretical prediction (see equations 1 and 2) assuming $\Lambda_{MS}^{(5)} = 150 MeV$.

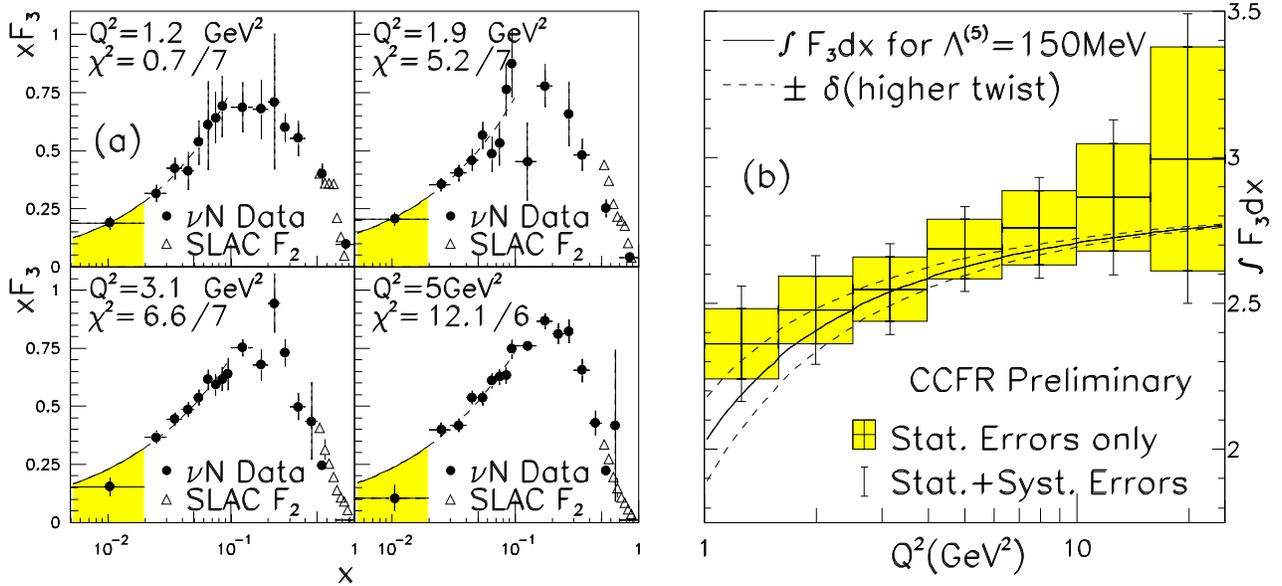


Figure 1: (a) $x F_3$ vs. x for four different low Q^2 values. The function shown is a power law fit to data below $x = 0.1$. (b) $\int F_3 dx$ vs. Q^2 , and the theoretical prediction for the integral for $\Lambda_{\overline{MS}}^{(5)} = 150 \text{ MeV}$. The dashed lines represent the uncertainty in the higher twist correction.

One can determine $\alpha_s(Q^2)$ from $\int F_3 dx$ by using equation 1. The values of $\alpha_s(Q^2)$ determined by this technique are shown in figure 2. The curves plotted in figure 2 show the evolution of α_s as a function of Q^2 (see equation 2), for two different values of $\Lambda_{\overline{MS}}$. From this plot it is clear that low Q^2 measurements have large potential to constrain α_s not only because α_s is large in this kinematic region, but because it is changing rapidly as a function of Q^2 . However, the higher twist uncertainty in $\int F_3 dx$ is also large in this kinematic region and is the largest single systematic error in this analysis. Evolving the four lowest data points for α_s to M_Z^2 , we obtain the following value for $\alpha_s(M_Z^2)$:

$$\alpha_s(M_Z^2) = 0.108 \pm_{.003}^{.003}(\text{stat}) \pm .004(\text{syst}) \pm_{.006}^{.004}(\text{higher twist})$$

For comparison with other low Q^2 α_s measurements, this corresponds to $\alpha_s(Q^2 = 3.0 \text{ GeV}^2) = 0.26 \pm_{.03}^{.02}(\text{stat}) \pm .02(\text{syst}) \pm .03(\text{higher twist})$. Figure 2 puts this result in the context of other measurements by plotting them as a function of Q^2 . In general, the low Q^2 data systematically favor a lower $\Lambda_{\overline{MS}}$ than do the higher Q^2 data. The result from this analysis is consistent with low energy measurements of α_s . In particular, it is consistent with the CCFR determination of α_s from the Q^2 evolution of $x F_3$ and F_2 for $Q^2 > 15 \text{ GeV}^2$ ($\alpha_s(M_Z^2) = 0.111 \pm .004$), and about 2σ lower than that measured from the high Q^2 data [13]. With future experimental improvements (Fermilab NuTeV experiment) and improved theoretical work on higher twist corrections, this fundamental prediction of QCD has promise for being a stringent test of the model.

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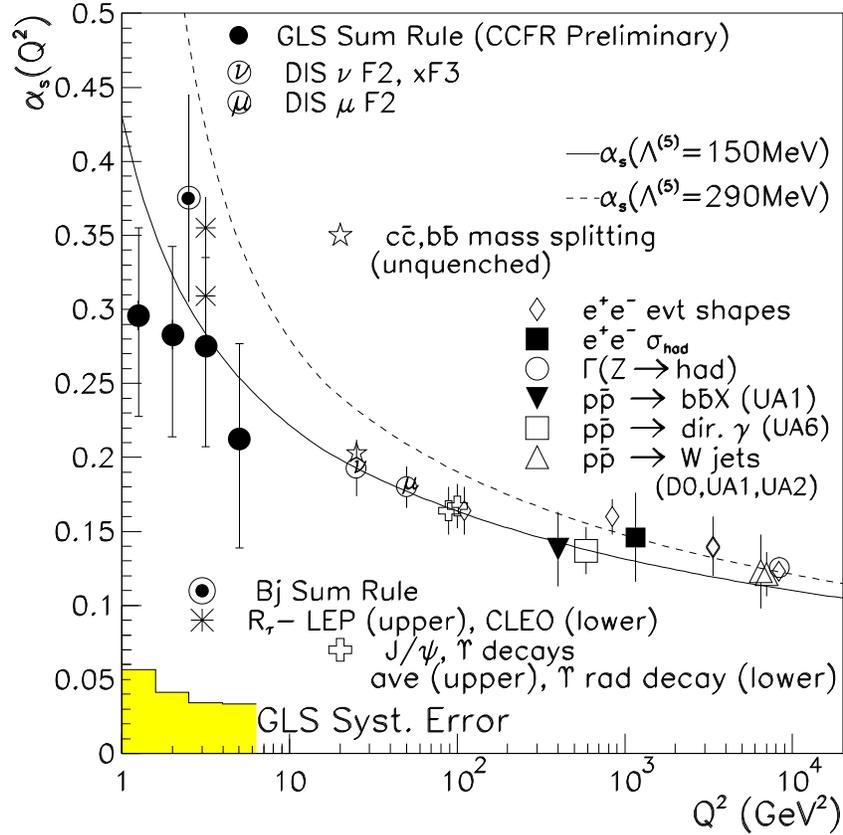


Figure 2: α_s as measured at several different Q^2 values, and the expected Q^2 dependence of the 3-loop (see equation 2) α_s for $\Lambda_{\overline{MS}}^{(5)} = 150 \text{ MeV}$, and $\Lambda_{\overline{MS}}^{(5)} = 290 \text{ MeV}$. Only the statistical errors on the GLS points are plotted at the GLS values, the systematic errors (which are correlated from one Q^2 bin to the next) are shown at the bottom of the plot.

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