Fermi National Accelerator Laboratory

Inflation: From Theory To Observation and Back

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OVERVIEW

Alan Guth introduced cosmologists to inflation at the 1980 Texas Symposium. Since, inflation has had almost as much impact on cosmology as the big-bang model itself. However, unlike the big-bang model, it has little observational support. Hopefully, that situation is about to change as a variety and abundance of data begin to test inflation in a significant way. The observations that are putting inflation to test involve the formation of structure in the Universe, especially measurements of the anisotropy of the cosmic background radiation. The cold dark matter models of structure formation motivated by inflation are holding up well as the observational tests become sharper. In the next decade inflation will be tested even more significantly, with more precise measurements of CBR anisotropy, the mean density of the Universe, the Hubble constant, and the distribution of matter, as well as sensitive searches for the nonbaryonic dark matter predicted to exist by inflation. As an optimist I believe that we may be well on our way to a standard cosmology that includes inflation and extends back to around 10^{-32} sec, providing an important window on the earliest moments and fundamental physics.

1 BEYOND THE BIG BANG MODEL

The hot big-bang cosmology is a remarkable achievement. It provides a reliable account of the Universe from about 10^{-2} sec to the present. Further, it together with modern ideas in particle physics—the Standard Model, supersymmetry, grand unification, and superstring theory—provides a sound framework for sensible speculation all the way back to the Planck epoch and perhaps even earlier.[1]

These speculations have allowed cosmologists to address a deeper set of questions: What is the nature of the ubiquitous dark matter that is the dominant component of the mass density? Why does the Universe contain only matter? What is the origin of the tiny inhomogeneities that seeded the formation of structure, and

[1]Before the advent of the Standard Model (point-like quarks and leptons with "weak interactions" at short distances) cosmology hit a wall at about 10^{-5} sec. At this time the Universe was a strongly interacting gas of overlapping hadrons with the number of "fundamental particles" increasing exponentially with mass.

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how did that structure evolve? Why is the portion of the Universe that we can see so flat and smooth? What is the value of the vexing cosmological constant? How did the expansion begin—or was there a beginning?

In the past fifteen years much progress has been made, and many believe that the answers to all these questions involve events that took place during the earliest moments and involved physics beyond the Standard Model [1]. For example, the matter-antimatter asymmetry, quantified as a net baryon number of about 10^{-10} per photon, is believed to have developed through interactions that do not conserve baryon number and C, CP (matter-antimatter symmetry) and occurred out of thermal equilibrium. Until recently it was believed that "baryogenesis" involved unification-scale physics and occurred around 10^{-34} sec; recent work suggests that baryogenesis might have occurred at the weak scale ($T \sim 300$ GeV and $t \sim 10^{-11}$ sec) and involved baryon-number and C, CP violation within the Standard Model [2].

The most optimistic early-Universe cosmologists (of which I am one) believe that we are on the verge of solving all of the above problems and extending our knowledge of the Universe back to around 10^{-32} sec after "the bang." The key to this is inflation. Among other things, inflation has led to the cold dark matter models of structure formation, which are characterized by scale-invariant density perturbations and dark matter whose composition is primarily slowly moving elementary particles (e.g., axions or neutralinos). The cold dark matter theory is crucial to testing inflation, and if it proves correct, would complete the standard cosmology by connecting the theorist's early Universe which is smooth and formless to the astronomer's Universe which is inhomogeneous and abounds with structure.

1.1 Evidence

Four pillars provide the observational support on which the hot big-bang model rests: (1) The uniform distribution of matter on large scales and the isotropic expansion that maintains this uniformity; (2) The existence of a nearly uniform and accurately thermal cosmic background radiation (CBR); (3) The abundances (relative to hydrogen) of the light elements D, ^{3}He, ^{4}He, and ^{7}Li; and (4) The existence of small fluctuations in the temperature of the CBR across the sky at the level about 10^{-5}. The Hubble expansion supports the general notion of an expanding Universe; the CBR provides almost indisputable evidence of a hot, dense beginning. The agreement between the light-element abundances predicted by primordial nucleosynthesis and those observed tests the model back to about 10^{-2} sec and leads to the most accurate determination of the baryon density, $\Omega_B \simeq 0.009h^{-2} - 0.022h^{-2}$ [3]. The small fluctuations in the temperature of CBR across the sky indicate the existence of primeval density perturbations of a similar size which amplified by gravity over the age of the Universe has led to the abundance of structure seen today (galaxies, clusters of galaxies, superclusters, voids, and great walls).

At present, observational support for inflation is fragmentary at best. However, a wealth of diverse observations are beginning to seriously test inflation, and even someone who is not an early-Universe optimist would have to concede that inflation is likely to be tested in a significant way within the next five years or so. The crucial tests include measurements of CBR anisotropy, the present degree of inhomogeneity as probed by redshift surveys and peculiar-velocity measurements, x-ray studies of

clusters of galaxies, increasingly accurate measurements of the Hubble constant, the study of the Universe at high redshift by large ground-based telescopes and the Hubble Space Telescope, the mapping of dark matter through gravitational lensing, the search for baryonic dark matter through microlensing and particle dark matter through both direct and indirect techniques, and on and on.

While the observational support for inflation is not overwhelming—yet!—there are a number of observations which are very encouraging. The evidence that the mass density of the Universe is significantly larger than that which baryons can account for continues to grow: measurements based upon peculiar velocities of the Milky Way and other galaxies indicate that $\Omega_{\rm matter} \gtrsim 0.3$ [4, 5] and x-ray and weak-gravitational lensing measurements of the masses of rich clusters continue to indicate mass to light ratios consistent with $\Omega_{\rm matter} \gtrsim 0.2$ or greater. (For a Hubble constant of greater than $60\,{\rm km\,s^{-1}\,Mpc^{-1}}$, baryons can contribute at most 5% of the critical density.) Further, the mass fraction of rich clusters that can be readily identified as baryonic (mostly hot gas) is only about $0.04h^{-3/2} - 0.1h^{-3/2}$ [6]. Likewise, the fraction of the dark halo of our galaxy that can be accounted for by baryons (faint stars and MACHOs) is only about $0.05 - 0.3$ [7]. CBR anisotropy has now been measured on angular scales from about 0.5° to 90° [8, 9], probing the spectrum of metric perturbations on scales from about $30h^{-1}$ Mpc to almost 10^4h^{-1} Mpc. The measurements are consistent with the scale-invariant prediction of inflation; see Fig. 1. Likewise, measurements of the spectrum of inhomogeneity on smaller scales from redshift surveys, say less than about $100h^{-1}$ Mpc, are generally consistent with the predictions of cold dark matter model (see Fig. 2). The success of cold dark matter is all the more striking given that the only other models (Peebles' baryons only PBI model and topological defects + nonbaryonic dark matter) are on the verge of being ruled out.

2 INFLATIONARY THEORY

2.1 Generalities

As successful as the big-bang cosmology it suffers from a dilemma involving initial data. Extrapolating back, one finds that the Universe apparently began from a very special state: A slightly inhomogeneous and very flat Robertson-Walker spacetime. Collins and Hawking showed that the set of initial data that evolve to a spacetime that is as smooth and flat as ours is today of measure zero [10]. (In the context of simple grand unified theories, the hot big bang suffers from another serious problem: the extreme overproduction of superheavy magnetic monopoles; in fact, it was an attempt to solve the monopole problem which led Guth to inflation.)

The cosmological appeal of inflation is its ability to lessen the dependence of the present state of the Universe upon the initial state. Two elements are essential to doing this: (1) accelerated ("superluminal") expansion and the concomitant tremendous growth of the scale factor; and (2) massive entropy production [11]. Together, these two features allow a small, smooth subhorizon-sized patch of the early Universe to grow to a large enough size and contain enough heat (entropy in excess of 10^{88}) to easily encompass our present Hubble volume. Provided that

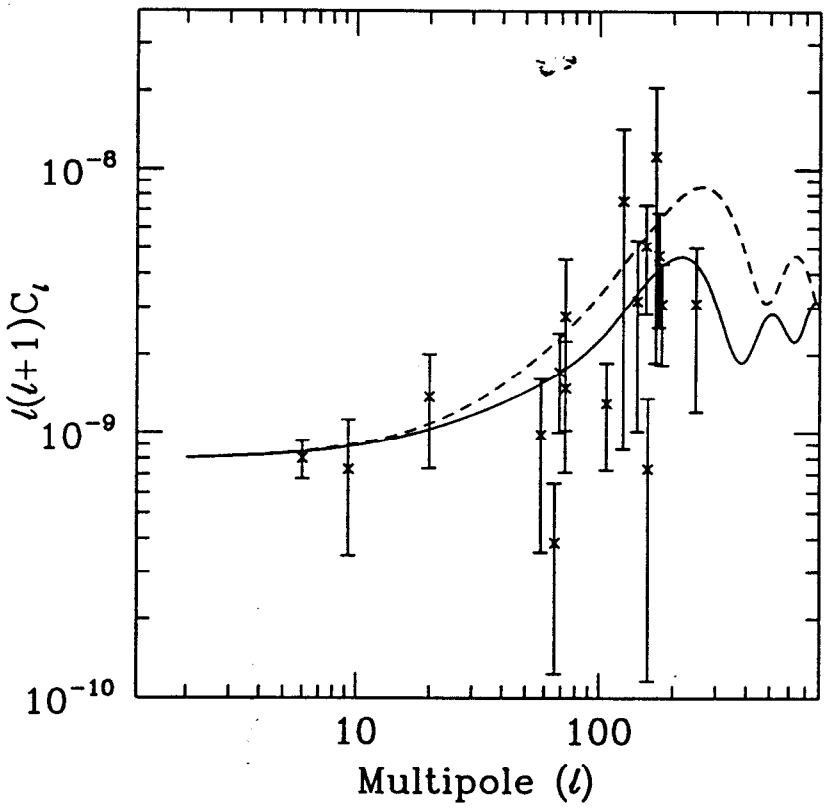

Figure 1: Summary of current measurements of CBR anisotropy in terms of a spherical-harmonic decomposition, $C_l \equiv \langle |a_{lm}|^2 \rangle$. The rms temperature fluctuation measured between two points separated by an angle θ is roughly given by: $(\delta T/T)_\theta \simeq \sqrt{l(l+1)C_l}$ with $l \simeq 200°/\theta$. The curves are the cold dark matter predictions, normalized to the COBE detection, for Hubble constants of $50\,\mathrm{km\,s^{-1}\,Mpc}$ (solid) and $35\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (broken). (Figure courtesy of M. White.)

the region was originally small compared to the curvature radius of the Universe it would appear flat then and today (just as any small portion of the surface of a sphere appears flat).

While there is presently no standard model of inflation—just as there is no standard model for physics at these energies (typically 10^{15} GeV or so)—viable models have much in common. They are based upon well posed, albeit highly speculative, microphysics involving the classical evolution of a scalar field. The superluminal expansion is driven by the potential energy ("vacuum energy") that arises when the scalar field is displaced from its potential-energy minimum, which results in nearly exponential expansion. Provided the potential is flat, during the time it takes for the field to roll to the minimum of its potential the Universe undergoes many e-foldings of expansion (more than around 60 or so are required to realize the beneficial features of inflation). As the scalar field nears the minimum, the vacuum energy has been converted to coherent oscillations of the scalar field, which correspond to nonrelativistic scalar-field particles. The eventual decay of these particles into lighter particles and their thermalization results in the "reheating" of the Universe and

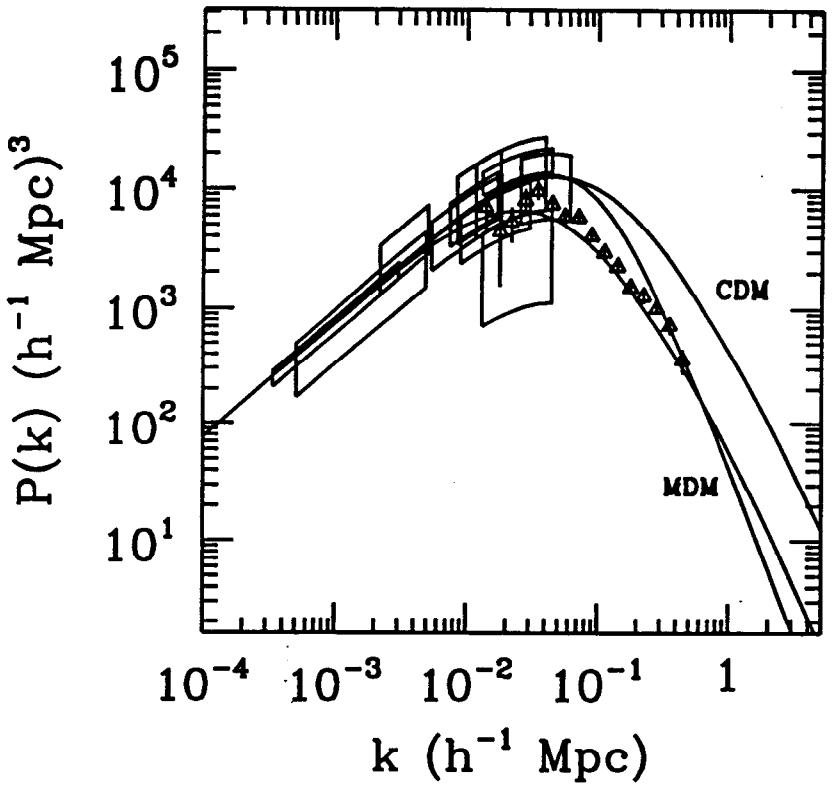

Figure 2: Comparison of the cold dark matter perturbation spectrum with CBR anisotropy measurements (boxes) and the distribution of galaxies today (triangles). Wavenumber k is related to length scale, $k = 2\pi/\lambda$; error flags are not shown for the galaxy distribution. The curve labeled MDM is hot + cold dark matter ("5 eV" worth of neutrinos); the other two curves are cold dark matter models with Hubble constants of 50 km s^{-1} Mpc (labeled CDM) and 35 km s^{-1} Mpc. (Figure courtesy of M. White.)

accounts for all the heat in the Universe today (the entropy production event).

Superluminal expansion and the tremendous growth of the scale factor (by a factor greater than that since the end of inflation) allow quantum fluctuations on very small scales ($\lesssim 10^{-23}$ cm) to be stretched to astrophysical scales ($\gtrsim 10^{25}$ cm). Quantum fluctuations in the scalar field responsible for inflation ultimately lead to an almost scale-invariant spectrum of density perturbations [12], and quantum fluctuations in the metric itself lead to an almost scale-invariant spectrum of gravity-waves [13]. Scale invariance for density perturbations means scale-independent fluctuations in the gravitational potential (equivalently, density perturbations of different wavelength cross the horizon with the same amplitude); scale invariance for gravity waves means that gravity waves of all wavelengths cross the horizon with the same amplitude. Because of subsequent evolution, neither the scalar nor the tensor perturbations are scale invariant today.

2.2 Metaphysical implications

Inflation alleviates the "specialness" problem greatly, but does not eliminate all dependence upon the initial state [14]. All open FRW models will inflate and become flat; however, many closed FRW models will recollapse before they can inflate. If one imagines the most general initial spacetime as being comprised of negatively and positively curved FRW (or Bianchi) models that are stitched together, the failure of the positively curved regions to inflate is of little consequence: because of exponential expansion during inflation the negatively curved regions will occupy most of the space today. Nor does inflation solve the smoothness problem forever; it just postpones the problem into the exponentially distant future: We will be able to see outside our smooth inflationary patch and Ω will start to deviate significantly from unity at a time $t \sim t_0 \exp[3(N - N_{\rm min}]$, where N is the actual number of e-foldings of inflation and $N_{\rm min} \sim 60$ is the minimum required to solve the horizon/flatness problems.

Linde has emphasized that inflation has changed our view of the Universe in a very fundamental way [15]. While cosmologists have long used the Copernician principle to argue that the Universe must be smooth because of the smoothness of our Hubble volume, in the post-inflation view, our Hubble volume is smooth because it is a small part of a region that underwent inflation. On the largest scales the structure of the Universe is likely to be very rich: Different regions may have undergone different amounts of inflation, may have different laws of physics because they evolved into different vacuum states (of equivalent energy), and may even have different numbers of spatial dimensions. Since it is likely that most of the volume of the Universe is still undergoing inflation and that inflationary patches are being constantly produced (eternal inflation), the age of the Universe is a meaningless concept and our expansion age merely measures the time back to the end of our inflationary event!

2.3 Specifics

In Guth's seminal paper [16] he introduced the idea of inflation, sung its praises, and showed that the model that he based the idea upon did not work! Thanks to very important contributions by Linde [17] and Albrecht and Steinhardt [18] that was quickly remedied, and today there are many viable models of inflation. That of course is both good news and bad news; it means that there is no standard model of inflation. Again, the absence of a standard model of inflation should be viewed in the light of our general ignorance about fundamental physics at these energies.

Many different approaches have taken in constructing particle-physics models for inflation. Some have focussed on very simple scalar potentials, e.g., $V(\phi) = \lambda\phi^4$ or $= m^2\phi^2/2$, without regard to connecting the model to any underlying theory [19, 20]. Others have proposed more complicated models that attempt to make contact with speculations about physics at very high energies, e.g., grand unification [21], supersymmetry [22, 23, 24], preonic physics [25], or supergravity [26]. Several authors have attempted to link inflation with superstring theory [27] or "generic predictions" of superstring theory such as pseudo-Nambu-Goldstone boson fields [28]. While the scale of the vacuum energy that drives inflation is typically of

order $(10^{15}\,\mathrm{GeV})^4$, a model of inflation at the electroweak scale, vacuum energy $\approx (1\,\mathrm{TeV})^4$, has been proposed [29]. There are also models in which there are multiple epochs of inflation [30].

In all of the models above gravity is described by general relativity. A qualitatively different approach is to consider inflation in the context of alternative theories of gravity. (After all, inflation probably involves physics at energy scales not too different from the Planck scale and the effective theory of gravity at these energies could well be very different from general relativity; in fact, there are some indications from superstring theory that gravity in these circumstances might be described by a Brans-Dicke like theory.) Perhaps the most successful of these models is first-order inflation [31, 32]. First-order inflation returns to Guth's original idea of a strongly first-order phase transition; in the context of general relativity Guth's model failed because the phase transition, if inflationary, never completed. In theories where the effective strength of gravity evolves, like Brans-Dicke theory, the weakening of gravity during inflation allows the transition to complete. In other models based upon nonstandard gravitation theory, the scalar field responsible for inflation is itself related to the size of additional spatial dimensions, and inflation then also explains why our three spatial dimensions are so big, while the other spatial dimensions are so small.

All models of inflation have one feature in common: the scalar field responsible for inflation has a very flat potential-energy curve and is very weakly coupled. This typically leads to a very small dimensionless number, usually a dimensionless coupling of the order of 10^{-14}. Such a small number, like other small numbers in physics (e.g., the ratio of the weak to Planck scales $\approx 10^{-17}$ or the ratio of the mass of the electron to the W/Z boson masses $\approx 10^{-5}$), runs counter to one's belief that a truly fundamental theory should have no tiny parameters, and cries out for an explanation. At the very least, this small number must be stabilized against quantum corrections—which it is in all of the previously mentioned models.[2] In some models, the small number in the inflationary potential is related to other small numbers in particle physics: for example, the ratio of the electron mass to the weak scale or the ratio of the unification scale to the Planck scale. Explaining the origin of the small number that seems to be associated with inflation is both a challenge and an opportunity.

Because of the growing base of observations that bear on inflation, another approach to model building is emerging: the use of observations to constrain the underlying inflationary potential—and hence the title of this paper. In the next Section I focus on just this. Before I do, I want to emphasize that while there are many varieties of inflation, there are robust predictions which are crucial to sharply testing inflation.

[2] It is sometimes stated that inflation is unnatural because of the small coupling of the scalar field responsible for inflation; while the small coupling certainly begs explanation, these inflationary models are not unnatural in the rigorous technical sense as the small number is stable against quantum fluctuations.

2.4 Three robust predictions

Inflation makes three robust[3] predictions:

1. **Flat universe.** Because solving the "horizon" problem (large-scale smoothness in spite of small particle horizons at early times) and solving the "flatness" problem (maintaining Ω very close to unity until the present epoch) are linked geometrically [1, 11], this is the most robust prediction of inflation. Said another way, it is the prediction that most inflationists would be least willing to give up. (Even so, models of inflation have been constructed where the amount of inflation is tuned just to give Ω_0 less than one today [33].) Through the Friedmann equation for the scale factor, flat implies that the total energy density (matter, radiation, vacuum energy, ...) is equal to the critical density.

2. **Nearly scale-invariant spectrum of gaussian density perturbations.** Essentially all inflation models predict a nearly scale-invariant spectrum of gaussian density perturbations. Described in terms of a power spectrum, $P(k) \equiv \langle|\delta_k|^2\rangle = Ak^n$, where δ_k is the Fourier transform of the primeval density perturbations, and the spectral index $n \approx 1$ is equal to unity in the scale-invariant limit. The overall amplitude A is model dependent. Density perturbations give rise to CBR anisotropy as well as seeding structure formation. Requiring that the density perturbations are consistent with the observed level of anisotropy of the CBR (and large enough to produce the observed structure formation) is the most severe constraint on inflationary models and leads to the small dimensionless number that all inflationary models have.

3. **Nearly scale-invariant spectrum of gravitational waves.** These gravitational waves have wavelengths from $\mathcal{O}(1\,\mathrm{km})$ to the size of the present Hubble radius and beyond. Described in terms of a power spectrum for the dimensionless gravity-wave amplitude at early times, $P_T(k) \equiv \langle|h_k|^2\rangle = A_T k^{n_T-3}$, where the spectral index $n_T \approx 0$ in the scale-invariant limit. Once again, the overall amplitude A_T is model dependent (varying as the value of the inflationary vacuum energy). Unlike density perturbations, which are required to initiate structure formation, there is no cosmological lower bound to the amplitude of the gravity-wave perturbations. Tensor perturbations also give rise to CBR anisotropy; requiring that they do not lead to excessive anisotropy implies that the energy density that drove inflation must be less than about $(10^{16}\,\mathrm{GeV})^4$. This indicates that if inflation took place, it did so at an energy well below the Planck scale.[4]

There are other interesting consequences of inflation that are less generic. For example, in models of first-order inflation, in which reheating occurs through the nucleation and collision of vacuum bubbles, there is an additional, larger amplitude,

[3] Because theorists are so clever, it is not possible nor prudent to use the word immutable. Models that violate any or all of these "robust predications" can and have been constructed.

[4] To be more precise, the part of inflation that led to perturbations on scales within the present horizon involved subPlanckian energy densities. In some models of inflation, the earliest stages, which do not influence scales that we are privy to, involve energies as large as the Planck scale.

but narrow-band, spectrum of gravitational waves ($\Omega_{\rm GW}h^2 \sim 10^{-6}$) [34]. In other models large-scale primeval magnetic fields of interesting size are seeded during inflation [35].

3 STRUCTURE FORMATION: A WINDOW TO THE EARLY UNIVERSE

The key to testing inflation is to focus on its robust predictions and their implications. Earlier I discussed the prediction of a flat Universe and its bold implication that most of the matter in Universe exists in the form of particle dark matter. Much effort is being directed at determining the mean density of the Universe and detecting particle dark matter.

The scale-invariant scalar metric perturbations lead to CBR anisotropy on angular scales from less than 1° to 90° and seed the formation of structure in the Universe. Together with the nucleosynthesis determination of Ω_B and the inflationary prediction of a flat Universe, scale-invariant density perturbations lead to a very specific scenario for structure formation; it is known as cold dark matter because the bulk of the particle dark matter is comprised of slowly moving particles (e.g., axions or neutralinos) [36].[5] A large and rapidly growing number of observations are being brought to bear in the testing of cold dark matter, making it the centerpiece of efforts to test inflation.

Finally, there are the scale-invariant tensor perturbations. They lead to CBR anisotropy on angular scales from a few degrees to 90° and a spectrum of gravitational waves. The CBR anisotropy arising from the tensor perturbations can in principle be separated from that arising from scalar perturbations. However, because the sky is finite, sampling variance sets a fundamental limit: the tensor contribution to CBR anisotropy can only be separated from that of the scalar if T/S is greater than about 0.14 [37] (T is the contribution of tensor perturbations to the variance of the CBR quadrupole and S is the same for scalar perturbations). It is also possible that the stochastic background of gravitational waves itself can be directly detected, though it appears that the LIGO facilities being built will lack the sensitivity and even space-based interferometery (e.g., LISA) is not a sure bet [38].

Before going on to discuss how cold dark matter models are testing inflation I want to emphasize the importance of the tensor perturbations. The attractiveness of a flat Universe with scale-invariant density perturbations was appreciated long before inflation. Verifying these two predictions of inflation, while important, will not provide a "smoking gun." The tensor perturbations are a unique feature of inflation. Further, they are crucial to obtaining information about the scalar potential responsible for inflation.

[5] The simpler possibility, that the particle dark matter exists in the form of 30 eV or so neutrinos which is known as hot dark matter, was falsified almost a decade ago. Because neutrinos move rapidly, they can diffuse from high density to low density regions damping perturbations on small scales. In hot dark matter large, supercluster-size objects must form before galaxies, and thus hot dark matter cannot account for the abundance of galaxies, damped Lyman-α clouds, etc. that is observed at high redshift.

3.1 Vanilla Cold Dark Matter: almost, but not quite?

The simplest version of cold dark matter, vanilla cold dark matter if you will, is characterized by: (1) $\Omega_B \sim 0.5$ and $\Omega_{\rm CDM} \sim 0.95$; (2) Hubble constant of $50\,{\rm km\,s^{-1}\,Mpc^{-1}}$; (3) Precisely scale-invariant density perturbations ($n = 1$); and (4) No contribution of tensor perturbations to CBR anisotropy. In cold dark matter models structure forms hierarchically, with small objects forming first and merging to form larger objects. Galaxies form at redshifts of order a few, and rarer objects like QSOs form from higher than average density peaks earlier. In general, cold dark matter predicts a Universe that is still evolving at recent epochs. N-body simulations are crucial to bridging the gap between theory and observation, and several groups have carried out large numerical studies of vanilla cold dark matter [39].

There are a diversity of observations that test cold dark matter; they include CBR anisotropy and spectral distortions, redshift surveys, pairwise velocities of galaxies, peculiar velocities, redshift space distortions, x-ray background, QSO absorption line systems, cluster studies of all kinds, studies of evolution (clusters, galaxies, and so on), measurements of the Hubble constant, and on and on. I will focus on how these measurements probe the power spectrum of density perturbations, emphasizing the role of CBR-anisotropy measurements and redshift surveys.

Density perturbations on a (comoving) length scale λ give rise to CBR anisotropy on an angular scale $\theta \sim \lambda/H_0^{-1} \sim 1^\circ(\lambda/100h^{-1}\,{\rm Mpc})$.[6] CBR anisotropy has now been detected by more than ten experiments on angular scales from about 0.5° to 90°, thereby probing length scales from $30h^{-1}\,{\rm Mpc}$ to $10^4h^{-1}\,{\rm Mpc}$. The very accurate measurements made by the COBE DMR can be used to normalize the cold dark matter spectrum (the normalization scale corresponds to about 20°). When this is done, the other ten or so measurements are in agreement with the predictions of cold dark matter (see Fig. 1).

The COBE-normalized cold dark matter spectrum can be extrapolated to the much smaller scales probed by redshift surveys, from about $1h^{-1}\,{\rm Mpc}$ to $100h^{-1}\,{\rm Mpc}$. When this is done, there is general agreement. However, on closer inspection the COBE-normalized spectrum seems to predict excess power on these scales (about a factor of four in the power spectrum; see Fig. 2). This conclusion is supported by other observations, e.g., the abundance of rich clusters and the pairwise velocities of galaxies. It suggests that cold dark matter has much of the truth, but perhaps not all of it [40], and has led to the suggestion that something needs to be added to the simplest cold dark matter theory.

There is another important challenge facing cold dark matter. X-ray observations of rich clusters are able to determine the ratio of hot gas (baryons) to total cluster mass (baryons + CDM) (by a wide margin, most of the baryons "seen" in clusters are in the hot gas). To be sure there are assumptions and uncertainties; the data at the moment indicate that this ratio is $0.04h^{-3/2} - 0.1h^{-3/2}$ [6]. If clusters provide a fair sample of the universal mix of matter, then this ratio should equal $\Omega_B/(\Omega_B + \Omega_{\rm CDM}) \simeq (0.009 - 0.022)h^{-2}/(\Omega_B + \Omega_{\rm CDM})$. Since clusters are large objects they should provide a pretty fair sample. Taking the numbers at face

[6] For reference, perturbations on a length scale of about 1 Mpc give rise to galaxies, on about 10 Mpc to clusters, on about 30 Mpc to large voids, and on about 100 Mpc to the great walls.

value, cold dark matter is consistent with the cluster gas fraction provided either: $\Omega_B + \Omega_{\rm CDM} = 1$ and $h \sim 0.3$ or $\Omega_B + \Omega_{\rm CDM} \sim 0.3$ and $h \sim 0.7$. The cluster baryon problem has yet to be settled, and is clearly an important test of cold dark matter.

Finally, before going on to discuss the variants of cold dark matter now under consideration, let me add a note of caution. The comparison of predictions for structure formation with present-day observations of the distribution of galaxies is fraught with difficulties. Theory most accurately predicts "where the mass is" (in a statistical sense) and the observations determine where the light is. Redshift surveys probe present-day inhomogeneity on scales from around one Mpc to a few hundred Mpc, scales where the Universe is nonlinear ($\delta n_{\rm GAL}/n_{\rm GAL} \gtrsim 1$ on scales $\lesssim 8h^{-1}$ Mpc) and where astrophysical processes undoubtedly play an important role (e.g., star formation determines where and when "mass lights up," the explosive release of energy in supernovae can move matter around and influence subsequent star formation, and so on). The distance to a galaxy is determined through Hubble's law ($d = H_0^{-1} z$) by measuring a redshift; peculiar velocities induced by the lumpy distribution of matter are significant and prevent a direct determination of the actual distance. There are the intrinsic limitations of the surveys themselves: they are flux not volume limited (brighter objects are seen to greater distances and vice versa) and relatively small (e.g., the CfA slices of the Universe survey contains only about 10^4 galaxies and extends to a redshift of about $z \sim 0.03$). Last but not least are the numerical simulations which link theory and observation; they are limited in dynamical range (about a factor of 100 in length scale) and in microphysics (in the largest simulations only gravity, and in others only a gross approximation to the effects of hydrodynamics/thermodynamics). Perhaps it would be prudent to withhold judgment on vanilla cold dark matter for the moment and resist the urge to modify it—but that wouldn't be as much fun!

3.2 The many flavors of cold dark matter

The spectrum of density perturbations today depends not only upon the primeval spectrum (and the normalization on large scales provided by COBE), but also upon the energy content of the Universe. While the fluctuations in the gravitational potential were initially (approximately) scale invariant, the Universe evolved from an early radiation-dominated phase to a matter-dominated phase which imposes a characteristic scale on the spectrum of density perturbations seen today; that scale is determined by the energy content of the Universe, $k_{\rm EQ} \sim 10^{-1} h\,{\rm Mpc}^{-1}\,(\Omega_{\rm matter} h/\sqrt{g_*})$ (g_* counts the relativistic degrees of freedom, $\Omega_{\rm matter} = \Omega_B + \Omega_{\rm CDM}$). In addition, if some of the nonbaryonic dark matter is neutrinos, they reduce power on small scales somewhat through freestreaming (see Fig. 2). With this in mind, let me discuss the variants of cold dark matter that have been proposed to improve its agreement with observations.

1. **Low Hubble Constant + cold dark matter (LHC CDM) [41].** Remarkably, simply lowering the Hubble constant to around $30\,{\rm km\,s^{-1}\,Mpc^{-1}}$ solves all the problems of cold dark matter. Recall, the critical density $\rho_{\rm crit} \propto H_0^2$; lowering H_0 lowers the matter density and has precisely the desired effect. It has two other added benefits: the expansion age of the Universe is comfortably

consistent with the ages of the oldest stars and the baryon fraction is raised to a value that is consistent with that measured in x-ray clusters. Needless to say, such a small value for the Hubble constant flies in the face of current observations [42]; further, it illustrates that the problems of cold dark matter get even worse for the larger values of H_0 that are favored by recent observations.

2. **Hot + cold dark matter (νCDM) [43].** Adding a small amount of hot dark matter can suppress density perturbations on small scales; adding too much leads back to the longstanding problems of hot dark matter. Retaining enough power on very small scales to produce damped Lyman-α systems at high redshift limits Ω_ν to less than about 20%, corresponding to about "5 eV worth of neutrinos" (i.e., one species of mass 5 eV, or two species of mass 2.5 eV, and so on). This admixture of hot dark matter rejuvenates cold dark matter provided the Hubble constant is not too large, $H_0 \lesssim 55\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$; in fact, a Hubble constant of closer to $45\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is preferred.

3. **Cosmological constant + cold dark matter (ΛCDM) [44].** (A cosmological constant corresponds to a uniform energy density, or vacuum energy.) Shifting 50% to 80% of the critical density to a cosmological constant lowers the matter density and has the same beneficial effect as a low Hubble constant. In fact, a Hubble constant as large as $80\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ can be accommodated. In addition, the cosmological constant allows the age problem to solved even if the Hubble constant is large, addresses the fact that few measurements of the mean mass density give a value as large as the critical density (most measurements of the mass density are insensitive to a uniform component), and allows the baryon fraction of matter to be larger, which alleviates the cluster baryon problem. Not everything is rosy; cosmologists have invoked a cosmological constant twice before to solve their problems (Einstein to obtain a static universe and Bondi, Gold, and Hoyle to solve the earlier age crisis when H_0 was thought to be $250\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$). Further, particle physicists can still not explain why the energy of the vacuum is not at least 50 (if not 120) orders of magnitude larger than the present critical density, and expect that when the problem is solved the answer will be zero.

4. **Extra relativistic particles + cold dark matter (τCDM) [45].** Raising the level of radiation has the same beneficial effect as lowering the matter density. In the standard cosmology the radiation content consists of photons + three (undetected) cosmic seas of neutrinos (corresponding to $g_* \simeq 3.36$). While we have no direct determination of the radiation beyond that in the CBR, there are at least two problems: What are the additional relativistic particles? and Can additional radiation be added without upsetting the successful predictions of primordial nucleosynthesis which depend critically upon the energy density of relativistic particles? The simplest way around these problems is an unstable tau neutrino (mass anywhere between a few keV and a few MeV) whose decays produce the radiation. This fix can tolerate a larger Hubble constant, though at the expense of more radiation.

5. **Tilted cold dark matter (TCDM) [46].** While the spectrum of density perturbations in most models of inflation is very nearly scale invariant, there are models where the deviations are significant ($n \approx 0.8$) which leads to smaller fluctuations on small scales. Further, if gravity waves account for a significant part of the CBR anisotropy, the level of density perturbations can be lowered even more. A combination of tilt and gravity waves can solve the problem of too much power on small scales, but seems to lead to too little power on intermediate and very small scales.

In evaluating these better fit models, one should keep the words of Francis Crick in mind (loosely paraphrased): A model that fits all the data at a given time is necessarily wrong, because at any given time not all the data are correct(!). ΛCDM provides an interesting/confusing example. When I discussed it in 1990, I called it the best-fit Universe, and quoting Crick, I said that ΛCDM was certain to fall by the wayside [47]. In 1995, it is still the best-fit model [48].

Let me end by defending the other point of view, namely, that to add something to cold dark matter is not unreasonable, or even as some have said, a last gasp effort to saving a dying theory. Standard cold dark matter was a starting point, similar to early calculations of big-bang nucleosynthesis. It was always appreciated that the inflationary spectrum of density perturbations was not exactly scale invariant [20] and that the Hubble constant was unlikely to be exactly $50\,\mathrm{km\,s^{-1}\,Mpc}$. As the quality and quantity of data improve, it is only sensible to refine the model, just as has been done with big-bang nucleosynthesis. Cold dark matter seems to embody much of the "truth." The modifications suggested all seem quite reasonable (as opposed to contrived). Neutrinos exist; they are expected to have mass; there is even some experimental data that indicates they do have mass. It is still within the realm of possibility that the Hubble constant is less than $50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, and if it is as large as $70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ to $80\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ a cosmological constant seems inescapable based upon the age problem alone. There is no data that can preclude more radiation than in the standard cosmology and deviations from scale invariance were always expected.

3.3 Reconstruction

If inflation and the cold dark matter theory is shown to be correct, then a window to the very early Universe ($t \sim 10^{-34}\,\mathrm{sec}$) will have been opened. While it is certainly premature to jump to this conclusion, I would like to illustrate one example of what one could hope to learn. As mentioned earlier, the spectra and amplitudes of the the tensor and scalar metric perturbations predicted by inflation depend upon the underlying model, to be specific, the shape of the inflationary scalar-field potential. If one can measure the power-law index of the scalar spectrum and the amplitudes of the scalar and tensor spectra, one can recover the value of the potential and its first two derivatives around the point on the potential where inflation took place [49]:

$$V = 1.65 T\, m_{\mathrm{Pl}}{}^4, \tag{1}$$

$$V' = \pm\sqrt{\frac{8\pi r}{7}}\, V/m_{\mathrm{Pl}}, \tag{2}$$

$$V'' = 4\pi\left[(n-1)+\frac{3}{7}r\right] V/{m_{\rm Pl}}^2, \tag{3}$$

where $r \equiv T/S$, a prime indicates derivative with respect to ϕ, $m_{\rm Pl} = 1.22\times10^{19}\,{\rm GeV}$ is the Planck energy, and the sign of V' is indeterminate. In addition, if the tensor spectral index can be measured a consistency relation, $n_T = -r/7$, can be used to further test inflation. Reconstruction of the inflationary scalar potential would shed light both on inflation as well as physics at energies of the order of 10^{15} GeV.

4 The Future

The stakes for cosmology are high: if correct, inflation/cold dark matter represents a major extension of the big bang and our understanding of the Universe, which can't help but shed light on the fundamental physics at energies of order 10^{15} GeV.

What are the crucial tests and when will they be carried out? Because of the many measurements/observations that can have significant impact, I believe the answer to when is sooner rather than later. The list of pivotal observations is long: CBR anisotropy, large redshift surveys (e.g., the Sloan Digital Sky Survey will have 10^6 redshifts), direct searches for nonbaryonic in our neighborhood (both for axions and neutralinos) and baryonic dark matter (microlensing), x-ray studies of galaxy clusters, the use of back-lit gas clouds (quasar absorption line systems) to study the Universe at high redshift, evolution (as revealed by deep images of the sky taken by the Hubble Space Telescope and the Keck 10 meter telescope), measurements of both H_0 and q_0, mapping of the peculiar velocity field at large redshifts through the Sunyaev-Zel'dovich effect, dynamical estimates of the mass density (using weak gravitational lensing, large-scale velocity fields, and so on), age determinations, gravitational lensing, searches for supersymmetric particles (at accelerators) and neutrino oscillations (at accelerators, solar-neutrino detectors, and other large underground detectors), searches for high-energy neutrinos from neutralino annihilations in the sun using large underground detectors, and on and on. Let me end by illustrating the interesting consequences of several possible measurements.

A definitive determination that H_0 is greater than $55\,{\rm km\,s^{-1}\,Mpc^{-1}}$ would falsify LHC CDM and νCDM. A definitive determination that H_0 is $75\,{\rm km\,s^{-1}\,Mpc^{-1}}$ or larger would necessitate a cosmological constant. A flat Universe with a cosmological constant has a very different deceleration parameter than one dominated by matter, $q_0 = -1.5\Omega_\Lambda + 0.5 \sim -(0.4-0.7)$ compared to $q_0 = 0.5$, and this could be settled by galaxy number counts or numbers of lensed quasars. The level of CBR anisotropy in τCDM and LHC CDM on the 0.5° scale is about 50% larger than the other models, which should be easily discernible. If neutrino-oscillation experiments were to provide evidence for a neutrino of mass 5 eV (or two of mass 2.5 eV) νCDM would seem almost inescapable.

Many more CBR measurements are in progress and there should many interesting results in the next few years. In the wake of the success of COBE there are proposals, both in the US and Europe, for a satellite-borne instrument to map the CBR sky with a factor of ten better resolution. A map of the CBR with $0.5^\circ - 1^\circ$ resolution could separate the gravity-wave contribution to CBR anisotropy and provide evidence for the third robust prediction of inflation, as well as determining

other important parameters [50], e.g., the scalar and tensor indices, Ω_Λ, and even Ω_0 (the position of the "Doppler" peak scales as $0.5°/\sqrt{\Omega_0}$) [51]).

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References

[1] See e.g., E.W. Kolb and M.S. Turner, *The Early Universe* (Addison-Wesley, Redwood City, CA, 1990).

[2] A. Cohen, D. Kaplan, and A. Nelson, *Annu. Rev. Nucl. Part. Sci.* **43**, 27 (1992).

[3] C. Copi, D.N. Schramm, and M.S. Turner, *Science* **267**, 192 (1995).

[4] A. Dekel, *Annu. Rev. Astron. Astrophys.* **32**, 371 (1994).

[5] M.A. Strauss et al., *Astrophys. J.* **397**, 395 (1992); N. Kaiser et al., *Mon. Not. R. astron. Soc.* **252**, 1 (1990).

[6] S.D.M. White et al., *Nature* **366**, 429 (1993); U.G. Briel et al., *Astron. Astrophys.* **259**, L31 (1992); D.A. White and A.C. Fabian, *Mon. Not. R. astron. Soc.*, in press (1995).

[7] E. Gates, G. Gyuk, and M.S. Turner, *Phys. Rev. Lett.* **74**, 3724 (1995).

[8] M. White, D. Scott, and J. Silk, *Ann. Rev. Astron. Astrophys.* **32**, 319 (1994).

[9] D. Wilkinson, in these proceedings (1995).

[10] C.B. Collins and S.W. Hawking, *Astrophys. J.* **180**, 317 (1973).

[11] Y. Hu, M.S. Turner, and E.J. Weinberg, *Phys. Rev. D* **49**, 3830 (1994).

[12] A. H. Guth and S.-Y. Pi, *Phys. Rev. Lett.* **49**, 1110 (1982); S. W. Hawking, *Phys. Lett. B* **115**, 295 (1982); A. A. Starobinskii, *ibid* **117**, 175 (1982); J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, *Phys. Rev. D* **28**, 697 (1983).

[13] V.A. Rubakov, M. Sazhin, and A. Veryaskin, *Phys. Lett. B* **115**, 189 (1982); R. Fabbri and M. Pollock, *ibid* **125**, 445 (1983); A.A. Starobinskii *Sov. Astron. Lett.* **9**, 302 (1983); L. Abbott and M. Wise, *Nucl. Phys. B* **244**, 541 (1984).

[14] M.S. Turner and L.M. Widrow, *Phys. Rev. Lett.* **57**, 2237 (1986); L. Jensen and J. Stein-Schabes, *Phys. Rev. D* **35**, 1146 (1987); A.A. Starobinskii, *JETP Lett.* **37**, 66 (1983).

[15] A.D. Linde, *Inflation and Quantum Cosmology* (Academic Press, San Diego, CA, 1990).

[16] A.H. Guth, *Phys. Rev. D* **23**, 347 (1981).

[17] A.D. Linde, *Phys. Lett. B* **108**, 389 (1982).

[18] A. Albrecht and P.J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982).

[19] A.D. Linde, *Phys. Lett. B* **129**, 177 (1983).

[20] P.J. Steinhardt and M.S. Turner, *Phys. Rev. D* **29**, 2162 (1984).

[21] Q. Shafi and A. Vilenkin, *Phys. Rev. Lett.* **52**, 691 (1984); S.-Y. Pi, *ibid* **52**, 1725 (1984).

[22] R. Holman, P. Ramond, and G.G. Ross, *Phys. Lett. B* **137**, 343 (1984).

[23] K. Olive, *Phys. Repts.* **190**, 309 (1990).

[24] H. Murayama et al., *Phys. Rev. D(RC)* **50**, R2356 (1994).

[25] M. Cvetic, T. Hubsch, J. Pati, and H. Stremnitzer, *Phys. Rev. D* **40**, 1311 (1990).

[26] E.J. Copeland et al., *Phys. Rev. D* **49**, 6410 (1994).

[27] See e.g., M. Gasperini and G. Veneziano, *Phys. Rev. D* **50**, 2519 (1994); R. Brustein and G. Veneziano, *Phys. Lett. B* **329**, 429 (1994); T. Banks et al., hep-th/9503114.

[28] K. Freese, J.A. Frieman, and A. Olinto, *Phys. Rev. Lett.* **65**, 3233 (1990).

[29] L. Knox and M.S. Turner, *Phys. Rev. Lett.* **70**, 371 (1993).

[30] J. Silk and M.S. Turner, *Phys. Rev. D* **35**, 419 (1986); L.A. Kofman, A.D. Linde, and J. Einsato, *Nature* **326**, 48 (1987).

[31] D. La and P.J. Steinhardt, *Phys. Rev. Lett.* **62**, 376 (1991).

[32] E.W. Kolb, *Physica Scripta* **T36**, 199 (1991).

[33] M. Bucher A.S. Goldhaber, and N. Turok, hep-ph/9411206 (1994).

[34] M.S. Turner and F. Wilczek, *Phys. Rev. Lett.* **65**, 3080 (1990); A. Kosowsky, M.S. Turner, and R. Watkins, *ibid* **69**, 2026 (1992).

[35] M.S. Turner and L.M. Widrow, *Phys. Rev. D* **37**, 2743 (1988); B. Ratra, *Astrophys. J.* **391**, L1 (1992).

[36] For an overview of the cold dark matter scenario of structure formation see e.g., G. Blumenthal et al., *Nature* **311**, 517 (1984).

[37] L. Knox and M.S. Turner, *Phys. Rev. Lett.* **73**, 3347 (1994).

[38] M.S. Turner, J. Lidsey, and M. White, *Phys. Rev. D* **48**, 4613 (1993).

[39] See e.g., C. Frenk, *Physica Scripta* **T36**, 70 (1991).

[40] See e.g., J.P. Ostriker, *Ann. Rev. Astron. Astrophys.* **31**, 689 (1993); A. Liddle and D. Lyth, *Phys. Rep.* **231**, 1 (1993).

[41] J. Bartlett et al., *Science* **267**, 980 (1995).

[42] W. Freedman et al., *Nature* **371**, 757 (1994); M. Fukugita, C.J. Hogan, and P.J.E. Peebles, *ibid* **366**, 309 (1993).

[43] Q. Shafi and F. Stecker, *Phys. Rev. Lett.* **53**, 1292 (1984); S. Achilli, F. Occhionero, and R. Scaramella, *Astrophys. J.* **299**, 577 (1985); S. Ikeuchi, C. Norman, and Y. Zahn, *Astrophys. J.* **324**, 33 (1988); A. van Dalen and R.K. Schaefer, *Astrophys. J.* **398**, 33 (1992); M. Davis, F. Summers, and D. Schlegel, *Nature* **359**, 393 (1992); J. Primack et al., *ibid* **74**, 2160 (1995); D. Pogosyan and A.A. Starobinskii, astro-ph/9502019.

[44] M.S. Turner, G. Steigman, and L. Krauss, *Phys. Rev. Lett.* **52**, 2090 (1984); M.S. Turner, *Physica Scripta* **T36**, 167 (1991); P.J.E. Peebles, *Astrophys. J.* **284**, 439 (1984); G. Efstathiou et al., *Nature* **348**, 705 (1990); L. Kofman and A.A. Starobinskii, *Sov. Astron. Lett.* **11**, 271 (1985).

[45] S. Dodelson, G. Gyuk, and M.S. Turner, *Phys. Rev. Lett* **72**, 3578 (1994); J.R. Bond and G. Efstathiou, *Phys. Lett. B* **265**, 245 (1991).

[46] R. Cen, N. Gnedin, L. Kofman, and J.P. Ostriker, *ibid* **399**, L11 (1992); R. Davis et al., *Phys. Rev. Lett.* **69**, 1856 (1992); F. Lucchin, S. Mattarese, and S. Mollerach, *Astrophys. J.* **401**, L49 (1992); D. Salopek, *Phys. Rev. Lett.* **69**, 3602 (1992); A. Liddle and D. Lyth, *Phys. Lett. B* **291**, 391 (1992); J.E. Lidsey and P. Coles, *Mon. Not. R. astron. Soc.* (1993); T. Souradeep and V. Sahni, *Mod. Phys. Lett. A* **7**, 3541 (1992).

[47] M.S. Turner, *Physica Scripta* **T36**, 167 (1991).

[48] L. Krauss and M.S. Turner, astro-ph/9504003.

[49] E.J. Copeland, E.W. Kolb, A.R. Liddle, and J.E. Lidsey, *Phys. Rev. Lett.* **71**, 219 (1993); *Phys. Rev. D* **48**, 2529 (1993); M.S. Turner, *ibid*, 3502 (1993); *ibid* **48**, 5539 (1993).

[50] L. Knox, astro-ph/950454 (submitted to *Phys. Rev. D*).

[51] M. Kamionkowski et al., *Astrophys. J.* **426**, L57 (1994).