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The Hot Big Bang and Beyond

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1 Overview

The hot big-bang cosmology provides a reliable account of the Universe from 10^{-2} sec after the bang until the present, as well as a robust framework for speculating back to times as early as 10^{-43} sec. Cosmology faces a number of important challenges; foremost among them are determining the quantity and composition of dark matter in the Universe and developing a detailed and coherent picture of how structure (galaxies, clusters of galaxies, superclusters, voids, great walls, and so on) developed. At present there is a working hypothesis—cold dark matter—which squarely addresses both issues. According to the cold dark matter theory, which is motivated by inflation, the Universe is flat, the density perturbations are almost scale invariant, and the bulk of the dark matter is in the form of slowly moving particles left over from the earliest moments (e.g., neutralinos or axions). If correct, cold dark matter would extend the big-bang model back to 10^{-32} sec and shed light on the unification of the forces. Many experiments and observations, from CBR anisotropy measurements to Hubble Space Telescope observations to experiments at Fermilab and CERN, are now putting the cold dark matter theory to the test. At present it appears that the theory is viable only if the Hubble constant is smaller than current measurements indicate (around $35\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$), or if the theory is modified slightly, e.g., by the addition of a cosmological constant, a small admixture of hot dark matter (5 eV "worth of neutrinos"), more relativistic particles, or a tilted spectrum of density perturbations.

2 Successes

The success of the hot big-bang cosmology (or standard cosmology as it is known) is simple to describe: It provides a reliable and tested account of the Universe from a fraction of a second after the bang (temperatures of order a few MeV) until the present 15 Billion years later (temperature 2.726 K). When supplemented by the Standard Model of particle physics and various ideas about physics at higher

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energies (e.g., supersymmetry, grand unification, and superstrings) it provides a sound foundation for speculations about the Universe back to 10^{-43} sec after the bang (temperatures of 10^{19} GeV) and perhaps even earlier [1].

The fundamental observational data that support the standard cosmology are: the universal expansion (Hubble flow of galaxies); the cosmic background radiation (CBR); and the abundance of the light elements D, ^{3}He, ^{4}He, and ^{7}Li. The Hubble law ($z \simeq v/c \simeq H_0 d$) has been tested to a redshift $z \sim 0.05$ [2] and the highest redshift object is a QSO with $z = 4.90$. (One plus redshift is the size of the Universe today relative to its size at the time of emission, $1 + z = R_0/R_E$; R is the cosmic scale factor).

The surface of last scattering for the CBR is the Universe at an age of a few hundred thousand years ($T \sim 0.3$ eV and redshift $z \sim 1100$). COBE has determined its temperature to be 2.726 ± 0.005 K and constrains any deviations from a black-body spectrum to be less than 0.03% [3]. The CBR temperature is very uniform: the difference between two points separated by angles from arcminutes to 90° is less than $300\mu K$, indicating that the Universe had a very smooth beginning.

According to the big-bang model the temperature of the CBR decreases as the Universe expands, and a recent measurement has confirmed this [4]. The relative populations of hyperfine states in neutral Carbon atoms seen in a gas cloud at redshift $z = 1.776$ indicated a thermodynamic temperature, 7.4 ± 0.8 K, which is consistent with the big-bang prediction for the CBR temperature at this earlier time $T(z) = (1 + z)2.726\,\mathrm{K} = 7.58\,\mathrm{K}$.

There is a dipole anisotropy in the CBR temperature of about 3 mK, due to our motion with respect to the cosmic rest frame (the "peculiar velocity" of the Local Group is $620\,\mathrm{km\,s^{-1}}$ toward the constellation Leo), and temperature differences on angular scales from 0.5° to 90° have been detected by about ten experiments at the level of about 30μK [5].

The abundance of the light elements, which range from about 24% for ^{4}He to 10^{-5} for D and ^{3}He and 10^{-10} for ^{7}Li are consistent with the predictions of the hot big-bang model. The comparison between the predicted abundances and the light-element abundances measured today is not a simple matter; it is complicated by 15 Gyr of "chemical evolution" (astrophysical processes destroy D, produce ^{4}He, and destroy or produce ^{3}He and ^{7}Li). However, three decades of careful theoretical and observational work has put the comparison on a firm footing, and there is good agreement provided that the ratio of baryons to photons is between 2.5×10^{-10} and 6×10^{-10} [6]; see Fig. 1. Since the synthesis of the light elements occurred when the Universe was of order seconds old and the temperature was of order MeV, big-bang nucleosynthesis is the earliest and perhaps most impressive test of the standard cosmology.

Finally, the standard cosmology provides a general framework for understanding how the very smooth early Universe evolved to the highly structured Universe today—galaxies, clusters of galaxies, superclusters, voids, great walls and so on. Small (primeval) variations in the matter density ($\delta\rho/\rho \sim 10^{-5}$) were amplified by gravity over the age of the Universe (the Jeans' instability in the ex-

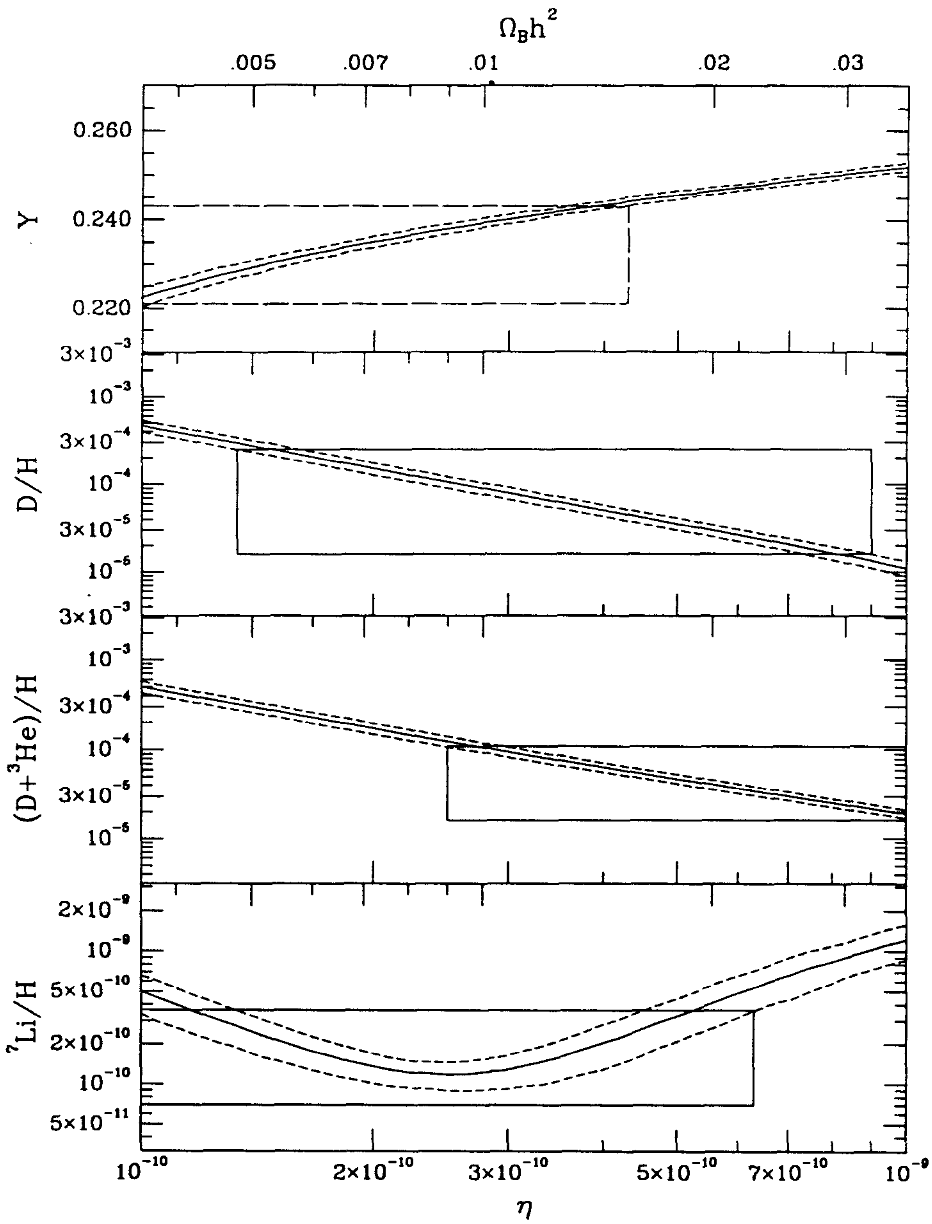

Fig. 1. The predicted light-element abundances as a function of the baryon-to-photon ratio η and $\Omega_B h^2$. The broken curves delineate the two-sigma theoretical uncertainties and the boxes delineate the acceptable range for η as allowed by measured abundances. The predictions for all four light elements are consistent with the observations for $\eta \simeq 2.5 \times 10^{-10} - 6 \times 10^{-10}$. (Figure courtesy of C. Copi.)

panding Universe) eventually resulting in the structure seen today [7]. The CBR temperature fluctuations detected on angular scales from 0.5° to 90° are strong evidence for the existence of these primeval density fluctuations; see Fig. 2.

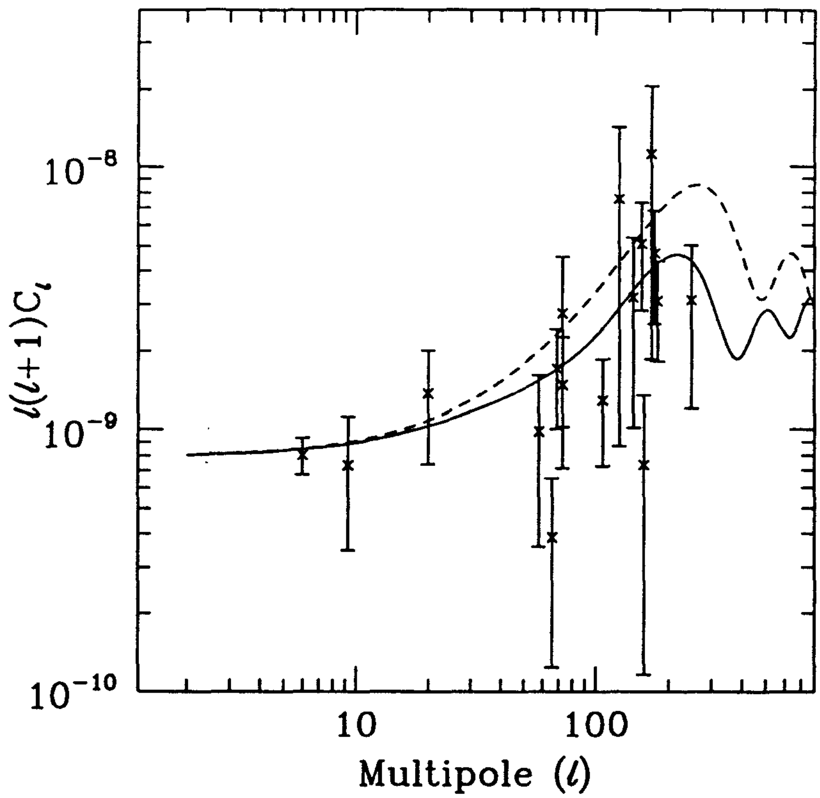

Fig. 2. Summary of current measurements of CBR anisotropy in terms of a spherical-harmonic decomposition, $C_l \equiv \langle |a_{lm}|^2 \rangle$. The rms temperature fluctuation measured between two points separated by an angle θ is roughly given by: $(\delta T/T)_\theta \simeq \sqrt{l(l+1)C_l}$ with $l \simeq 200°/\theta$. The curves are the cold dark matter predictions, normalised to the COBE detection, for Hubble constants of $50\,\mathrm{km\,s^{-1}}$ Mpc (solid) and $35\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (broken). (Figure courtesy of M. White.)

According to the Standard Model of particle physics the fundamental particles are point-like quarks and leptons whose interactions are weak enough to treat perturbatively. The cosmological implications of this are profound: The Universe at temperatures greater than about 150 MeV (times earlier than 10^{-5} sec) consisted of a hot, dilute gas of quarks, leptons, and gauge bosons (photons, gluons, and at high enough temperatures W and Z bosons, the carriers of the electromagnetic, strong and weak forces). The Standard Model of particle physics, which has been tested up to energies of several hundred GeV, supplies the input microphysics needed for times as early as 10^{-11} sec. In addition, it provides a firm platform for speculations about the unification of forces and particles (e.g., supersymmetry and grand unification), and in turn, for extending cosmological speculations back to the Planck epoch. Earlier than the Planck time a quantum description of gravity is needed, and superstring theory is a good candidate for such.

While the hot big-bang cosmology and modern particle theory allow "sensible"—and very interesting—speculations about the early Universe, there is no evidence *yet* that any of these speculations is correct. However, contrast this with the situation before the early 1970s. The count of "elementary particles" (baryons and mesons) had exceeded 100 and was growing exponentially with mass; this, the strength of their interactions and their finite sizes precluded any sensible speculation about the Universe at times earlier than about 10^{-5} sec [8].

3 Challenges

Cosmology is not without its challenges. Because of its success, the hot big-bang model has allowed cosmologists to ask even deeper questions. They include: What is the quantity and composition of the ubiquitous dark matter? What is the nature and origin of the primeval density perturbations that seeded structure and precisely how did the structure arise? What is the origin of the cosmic asymmetry between matter and antimatter? Why is the observed portion of the Universe so smooth and flat? And does this mean that the entire Universe is the same? Are there observational consequences of the phase transitions that the Universe has undergone (transition from quarks to nucleons and related particles, electroweak symmetry breaking, and possibly others) during its earliest moments? Are there observable consequences of the quantum-gravity epoch? Why does the Universe have four dimensions? What caused the expansion in the first place?

The first two of these challenges, the nature of the dark matter and the details of structure formation, are in my opinion the most pressing—and could well be resolved soon. They offer an excellent opportunity for extending the big-bang cosmology back to much earlier times.

That is not to say that the other challenges are not important or do not have potential for advancing our understanding. In addition, there are "important practical problems;" for example, a precise determination of the three traditional parameters used to describe "our world model," the Hubble constant, the deceleration parameter, and the cosmological constant, or an explanation for the primeval magnetic fields required to seed the magnetic fields seen throughout the Universe today.

3.1 Discard the Big Bang?

There are a few who believe the big bang faces challenges of such enormity that they will led to its downfall [9]. One challenge involves the tension between the Hubble constant and stellar ages. If the Hubble constant is as large as some determinations indicate, say around $80\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and the oldest stars are as old as some determinations indicate, say around 16 Gyr, then a real dilemma exists because *without recourse to a cosmological constant* the time back to the bang is less than 12 Gyr [10]. At present, the uncertainties in both the Hubble constant and stellar ages preclude drawing any firm conclusions.

Before the COBE discovery of CBR anisotropy in 1992 [11], some had argued that the absence of anisotropy precluded inhomogeneity of a size large enough to seed the structure seen today. The big-bang has weathered that storm: Fluctuations in the CBR temperature have been detected and are now seen on scales from $0.5°$ to $90°$; see Fig. 2. In fact, careful calculations indicate that if anything the level of temperature fluctuations seen is slightly larger than expected [5, 12].

The only two competitors to the big bang are the quasi steady-state model [13] and the plasma-universe model. At the moment the problems that these models face seem far more daunting: themalization of starlight to produce the 2.726 K black-body background with no spectral distortion (quasi steady-state) and the formulation of a model definite enough to be tested (plasma universe). Until these models (or another model) can account for the cosmological data that have been firmly established (expansion, CBR, light elements, and structure formation), the standard cosmology is without a serious competitor.

3.2 Dark Matters

An accurate inventory of matter in the Universe still eludes cosmologists. What we do know is: (i) luminous matter (i.e., matter closely associated with bright stars) contributes a fraction of the critical density that is about $0.003h^{-1}$ [14]; (ii) based upon big-bang nucleosynthesis baryons contributions a fraction of critical density between $0.009h^{-2}$ and $0.022h^{-2}$ [6], which for a generous range of the Hubble constant corresponds to between about 0.01 and 0.15 of the critical density; (iii) there are indications that the fraction of critical density contributed by all forms of matter is *at least* $0.1-0.3$ [15]—flat rotation curves of spiral galaxies, virial mass determinations of rich clusters—and perhaps around the critical density—the peculiar motions of galaxies, cluster-mass determinations based upon gravitational lensing and x-ray measurements [15, 16]. (Here the Hubble constant $H_0 = 100h\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ and the critical density $\rho_{\rm crit} = 3H_0^2/8\pi G = 1.88h^2 \times 10^{-29}\,\mathrm{g\,cm^{-3}} \simeq 1.05h^2 \times 10^4\,\mathrm{eV\,cm^{-3}}$.)

From this one concludes that: (i) most of the matter in the Universe is dark; (ii) most of the baryons are dark; (iii) the dark matter is not closely associated with bright stars, i.e., it is more diffusely distributed, e.g., in the extended halos of spiral galaxies; and (iv) if the total mass density is greater than about 20% of the critical density, then there must be another form of matter since baryons can at most account for 15% of the critical density (and only for a very low value of the Hubble constant: if $h \geq 0.6$, then $\Omega_B \lesssim 5\%$).

The case for $\Omega_0 \gtrsim 0.2$ and nonbaryonic dark matter receives additional support, albeit indirectly, from other lines of reasoning. First, it is difficult to reconcile all the data concerning the formation of structure in the Universe with a theory without nonbaryonic dark matter (the one model that *may* be able to do so is Peebles' primeval baryon isocurvature model or PBI [17]). Second, the most compelling and comprehensive theory of the early Universe, inflation [18], predicts a flat Universe (total energy density equal to the critical density) and thus requires something other than baryons. Third, since the deviation of Ω from unity grows with time, if Ω_0 is not equal to unity, the epoch when Ω_0 just begins to deviate significantly from one is a special epoch and is today(!) (this is often called the Dicke-Peebles timing argument).

Last but not least, there are three compelling candidates for the nonbaryonic dark matter: an axion of mass between $10^{-6}\,\mathrm{eV}$ and $10^{-4}\,\mathrm{eV}$; a neutralino of mass between 10 GeV and 1000 GeV; and a light neutrino species of mass between 10 eV and about 50 eV [19]. By compelling, I mean these particles arose out of efforts to unify the forces of Nature, and the fact that a particle was predicted whose relic mass density is close to critical is a bonus. This may be the "Great Hint"—or the "Grand Misdirection" it it proves to be wrong.

For the axion, the underlying particle physics is Peccei-Quinn symmetry which is the most attractive solution to the so-called strong-CP problem (the fact that Standard Model of particle physics predicts the electric dipole moment of the neutron to be almost ten orders of magnitude larger than the current upper limit). For the neutralino, it is supersymmetry, the symmetry that relates fermions and bosons and which helps to explain the large discrepancy between the weak scale (300 GeV) and the Planck scale and may hold the key to unifying gravity with the other forces. Unlike the axion or the neutralino, neutrinos are known to exist, come in three varieties, and have a relic abundance known to three significant figures ($113\,\mathrm{cm}^{-3}$ per species). The only issue is their mass. Almost all attempts to unify the forces and particles of Nature lead to the prediction that neutrinos have mass, often in the "eV range" (meaning anywhere from $10^{-6}\,\mathrm{eV}$ or smaller to keV).

The axion and neutralino are referred to as "cold dark matter" because they move very slowly (neutralinos because they are heavy and axions because they were produced coherently in the early Universe with very small momenta). Neutrinos on the other hand are referred to as "hot dark matter" because they move rapidly (due to their small mass). The distinction between the two is crucial for structure formation: at early times neutrinos can "run out" of overdense regions and into underdense regions, damping density perturbations on scales smaller than those corresponding to superclusters. This means that in the absence of additional seed perturbations that don't involve neutrinos (e.g., cosmic string) the sequence of structure formation in a hot dark matter universe proceeds from the "top down:" objects like superclusters form first and then fragment into smaller objects (galaxies and the like). Because there is now much evidence that "small objects" (galaxies, quasars, neutral hydrogen clouds, and clusters) were ubiquitous at redshifts from 1 to 4, and "large objects" are just forming today, hot dark matter is not viable.

To end on a sober note, at present the data can neither prove nor disprove: (i) $\Omega_0 = \Omega_B \simeq 0.15$; (ii) $\Omega_0 = 1$ with $\Omega_B \sim 0.05$ and $\Omega_{\mathrm{CDM}} \sim 0.95$. (In the first case the Hubble constant must be near its lower extreme since the nucleosynthesis constrains $\Omega_B = 0.009h^{-2} - 0.022h^{-2}$.) In any case, I will focus on the second, more radical, possibility.

3.3 Coherent Picture of Structure Formation

Because the energy densities of matter (baryons + CDM?) and radiation (photons, light neutrinos, and at early times all the other particles in the thermal plasma) evolve differently, R^{-3} for matter and R^{-4} for radiation, the energy density in radiation exceeded that in matter earlier at early times, $t \lesssim t_{\rm EQ} \sim 10^4\,$yr ($T \gtrsim T_{\rm EQ} \sim 5\,$eV and $R \lesssim R_{\rm EQ} \sim 3 \times 10^{-5} R_{\rm today}$). Moreover, matter density perturbations do not grow during the radiation-dominated era, and thus the formation of structure did not begin in earnest until the epoch of matter-radiation equality. After that, (linear) perturbations in the matter grow as the scale factor, for a total (linear) growth factor of around 30,000. This factor sets the characteristic amplitude of density perturbations, about $1/30,000 \sim few \times 10^{-5}$ (nonlinear structures have formed by the present) and thus the expected size of temperature fluctuations in the CBR (density perturbations lead to fluctuations in the CBR temperature of comparable size).

The detection of CBR anisotropy at the level of about 10^{-5} validates the gravitational instability picture of structure formation. This success should be viewed in the same way that the evidence for a large primeval mass fraction of ^{4}He validated the basic idea of primordial nucleosynthesis in the late 1960s. From this early success, big-bang nucleosynthesis developed into a coherent and detailed explanation for the abundances of D, ^{3}He, ^{4}He and ^{7}Li, and now provides the earliest test of the big bang, the most reliable determination of the baryon density, and an important probe of particle physics. It is not unreasonable to hope that a detailed and coherent picture of structure formation will develop, and that when it does, it will lead to similar advances in our understanding of the Universe and possibly even fundamental physics.

Two crucial elements underlay any detailed picture: specification of the quantity and composition of the dark matter and the nature of the density perturbations. With regard to the latter, what is wanted is a mathematical description of the spectrum of density perturbations. For example, the Fourier components δ_k of the density field and their statistical properties.

Presently there are three viable theories: cold dark matter models; topological-defect models [20]; and the primeval baryon isocurvature model (PBI) [17]. The effort being brought to bear—both experimental and theoretical—is great, and I am confident that *at least* two of these models, if not all three (!), will be falsified soon. It is my view that only cold dark matter will survive the next cut, but of course others may hold a different opinion.

Topological defect models, where the seeds are cosmic string, monopoles or textures produced in an early Universe phase transition and the dark matter is either neutrinos (cosmic string) or cold dark matter (textures), seem to predict CBR anisotropy on the degree scale that is significantly less than that measured. In addition, when normalized to the COBE measurements of anisotropy, they require a high level of "bias;" bias refers to the "mismatch" between the light and mass distributions, $b \simeq (\delta n_{\rm GAL}/n_{\rm GAL})/(\delta\rho/\rho)$, and b is generally believed to be of order $1-2$. Much of the difficulty in assessing the defect models is on the theoretical side; density perturbations are constantly being produced as the defect network evolves and thus cannot easily be described by Fourier components whose evolution is simple.

The basic philosophy behind the PBI model is to explain the formation of structure by using "what is here," rather then what early-Universe theorists (like myself) hope is here! The parameters for PBI are: $\Omega_0 = \Omega_B \sim 0.2$ and $H_0 \sim 70\,{\rm km\,s^{-1}\,Mpc^{-1}}$. An arbitrary power-law spectrum of fluctuations in the local baryon number (cut off at small scales to avoid difficulties with primordial nucleosynthesis) is postulated and its parameters (slope and normalization) are determined by the data (CBR fluctuations and large-scale structure). PBI has some serious problems: the baryon density violates the nucleosynthesis bound by a wide margin ($\Omega_B h^2 \sim 0.1 \gg 0.02$); it is difficult to make PBI consistent with the measurements of CBR anisotropy [21]. To wit, Peebles has considered variations on the basic theme [22] (e.g., adding a cosmological constant, or even cold dark matter). At the very least PBI provides a useful model against which scenarios that postulate exotic dark matter can be compared; at its best, it may closely represent our Universe.

4 Inflation and Cold Dark Matter

Inflation represents a bold attempt to extend the standard big-bang cosmology to times as early as 10^{-32} sec and to resolve some of the most fundamental questions in cosmology. In particular, inflation addresses squarely both the dark matter and structure formation problems, as well as providing an explanation for the flatness and smoothness of the Universe. That is, it explains the apparent extreme specialness of the initial data required to produce a Universe qualitatively similar to ours. (The set of initial data that lead to a Universe qualitatively similar to ours is of measure zero [23].) If inflation is correct, it would represent a truly remarkable addition to the standard cosmology.

Two elements are essential to inflation: (1) accelerated ("superluminal") expansion and the concomitant tremendous growth of the scale factor; and (2) massive entropy production [24]. Together, these two features allow a small, smooth subhorizon-sized patch of the early Universe to grow to a large enough size and contain enough heat (entropy in excess of 10^{88}) to easily encompass our present Hubble volume. Provided that the region was originally small compared to the curvature radius of the Universe it would appear flat then and today (just as any small portion of the surface of a sphere appears flat).

While there is presently no standard model of inflation—just as there is no standard model for physics at these energies (typically 10^{15} GeV or so)—viable models have much in common. They are based upon well posed, albeit highly speculative, microphysics involving the classical evolution of a scalar field. The superluminal expansion is driven by the potential energy ("vacuum energy") that arises when the scalar field is displaced from its potential-energy minimum, which results in nearly exponential expansion. Provided the potential is flat, during the time it takes for the field to roll to the minimum of its potential the Universe undergoes many e-foldings of expansion (more than around 60 or so are required to realize the beneficial features of inflation). As the scalar field nears the minimum, the vacuum energy has been converted to coherent oscillations of the scalar field, which correspond to nonrelativistic scalar-field particles. The eventual decay of these particles into lighter particles and their thermalization results in the "reheating" of the Universe and accounts for all the heat in the Universe today (the entropy production event).

Superluminal expansion and the tremendous growth of the scale factor (by a factor greater than that since the end of inflation) allow quantum fluctuations on very small scales ($\lesssim 10^{-23}$ cm) to be stretched to astrophysical scales ($\gtrsim 10^{25}$ cm). Quantum fluctuations in the scalar field responsible for inflation ultimately lead to an almost scale-invariant spectrum of density perturbations [25], and quantum fluctuations in the metric itself lead to an almost scale-invariant spectrum of gravity-waves [26]. Scale invariance for density perturbations means scale-independent fluctuations in the gravitational potential (equivalently, density perturbations of different wavelength cross the horizon with the same amplitude); scale invariance for gravity waves means that gravity waves of all wavelengths cross the horizon with the same amplitude. Because of subsequent evolution, neither the scalar nor the tensor perturbations are scale invariant today.

Although there is no standard model of inflation, there are three predictions that almost all viable models make: (1) spatially flat Universe [27]; (2) nearly scale-invariant spectrum of gaussian density (scalar metric) perturbations [25]; (3) nearly scale-invariant spectrum of gravity waves (tensor metric perturbations) [26].

The prediction of a flat Universe means the total energy density (including matter, radiation, and the vacuum energy density associated with a cosmological constant) is equal to the critical density, that is $\Omega_0 = 1$. Coupled with our knowledge of the baryon density, this implies that the bulk of matter in the Universe (95% or so) must be nonbaryonic. The two simplest possibilities are hot dark matter and cold dark matter. Structure formation with hot dark matter has been studied, and, sadly, does not work; thus we are led to cold dark matter.

While inflation predicts that the density perturbations are nearly scale invariant and are described by gaussian statistics, the overall amplitude of the spectrum is model dependent. In that regard the COBE measurement of CBR anisotropy was crucial as it allows the spectrum to be normalized. The density perturbations and their statistical properties are described by the power spectrum, $P(k) \equiv \langle|\delta_k|^2\rangle$, which at early times is $P(k) = Ak^n$ ($n = 1$ corresponds to scale invariant). The *rms* density perturbation on a given scale is related to $P(k)$ by $(\delta\rho/\rho)^2 \sim k^3 P(k)/2\pi^2$.

When the Universe became matter dominated, the sizes of density perturbations were largest on small scales. For cold dark matter there is no damping of perturbations on small scales, and so small structures formed first ("bottom up"). Clumps of dark matter and baryons continuously merge to form larger objects. "Typical galaxies" are formed at redshifts $z \sim 1-2$; "rare objects" such as quasars and radio galaxies can form earlier from regions where the density perturbations have larger than average amplitude. Clusters form in the very recent past (redshifts less than order unity), and superclusters are just forming today. Voids naturally arise as regions of space are evacuated to form objects [28].

4.1 Metaphysical Implications

Inflation alleviates the "specialness" problem greatly, but does not eliminate all dependence upon the initial state [29]. All open FRW models will inflate and become flat; however, many closed FRW models will recollapse before they can inflate. If one imagines the most general initial space-time as being comprised of negatively and positively curved FRW (or Bianchi) models that are stitched together, the failure of the positively curved regions to inflate is of little consequence: because of exponential expansion during inflation the negatively curved regions will occupy most of the space today. Nor does inflation solve the smoothness problem forever; it just postpones the problem into the exponentially distant future: We will be able to see outside our smooth inflationary patch and Ω will start to deviate significantly from unity at a time $t \sim t_0 \exp[3(N - N_{\rm min}]$, where N is the actual number of e-foldings of inflation and $N_{\rm min} \sim 60$ is the minimum required to solve the horizon/flatness problems.

Linde has emphasized that inflation has changed our view of the Universe in a very fundamental way [30]. While cosmologists have long used the Copernician principle to argue that the Universe must be smooth because of the smoothness of our Hubble volume, in the post-inflation view, our Hubble volume is smooth because it is a small part of a region that underwent inflation. On the largest scales the structure of the Universe is likely to be very rich: Different regions may have undergone different amounts of inflation, may have different laws of physics because they evolved into different vacuum states (of equivalent energy), and may even have different numbers of spatial dimensions. Since it is likely that most of the volume of the Universe is still undergoing inflation and that inflationary patches are being constantly produced (eternal inflation), the age of the Universe is a meaningless concept and our expansion age merely measures the time back to the end of our inflationary event!

4.2 Almost, But Is Something Missing?

The cold dark matter scenario is the most well motivated, most specific, and most successful scenario for structure formation yet proposed. It is so attractive that it has inspired a generation of observers to go out and prove it wrong! As I will describe, thus far it receives general support from a diversity of observations; however, there are indications that it does not have "all the truth" and requires some minor adjusting.

Broadly speaking, testing the cold dark matter scenario involves measuring the quantity, composition, and distribution of dark matter and determining the spectrum of density perturbations. I have already discussed the current state of our knowledge of dark matter. While a host of observations provide information about the primeval spectrum of density perturbations, measurements of the anisotropy of the CBR and mapping the distribution of matter today (as traced by bright galaxies) are perhaps most crucial. (For reference, perturbations on scales of about 1 Mpc correspond to galactic size perturbations, on 10 Mpc to cluster size perturbations, on 30 Mpc to the large voids, and 100 Mpc to the great walls. Fourier wavenumber is related to wavelength by $k = 2\pi/\lambda$)

CBR anisotropy probes the power spectrum on large scales. The CBR temperature difference measured on a given angular scale is related to the power spectrum on length scales $\lambda \sim (\theta/\text{deg})100h^{-1}\,\text{Mpc}$. Since the COBE detection, a host of ground-based and balloon-borne experiments have also detected CBR anisotropy, on scales from about 0.5° to 90°, at the level of around 30μK, corresponding to $\delta T/T \sim 10^{-5}$. The measurements are consistent with the predictions of cold dark matter (see Fig. 2), though there are still large statistical uncertainties as well as concerns about contamination by foreground sources [5]. There is a great deal of experimental activity (more than ten groups), and measurements in the near future should improve the present situation significantly. The CBR contains important information on angular scales down to about 0.1 deg (anisotropy on smaller angular scales is washed out due to the finite thickness of the last scattering surface). A satellite-borne experiment with ten times greater resolution than COBE is being studied in both Europe and the US, and a variety of earth-based and balloon-based experiments should hopefully map CBR anisotropy on scales from about 0.1 deg to 90 deg in the next decade.

The COBE detection of CBR anisotropy not only provided the first evidence for the existence of primeval density perturbations, but also an unambiguous way to normalize the spectrum of density perturbations: Given the shape of the power spectrum (for cold dark matter, approximately scale invariant) the COBE measurement (on an angular scale of about 10° which corresponds to a length scale of about $10^3h^{-1}\,\text{Mpc}$) ties down the spectrum on all scales. This leads to definite predictions that can be tested by other CBR measurements and observations of large-scale structure.

The comparison of predictions for structure formation with present-day observations of the distribution of galaxies is very important, but fraught with difficulties. Theory most accurately predicts "where the mass is" (of course, only in a statistical sense) and the observations determine where the light is. Redshift surveys probe present-day inhomogeneity on scales from around one Mpc to a few hundred Mpc, scales where the Universe is nonlinear ($\delta n_{\rm GAL}/n_{\rm GAL} \gtrsim 1$ on scales $\lesssim 8h^{-1}\,\text{Mpc}$) and where astrophysical processes undoubtedly play an important role (e.g., star formation determines where and when "mass lights up," the explosive release of energy in supernovae can move matter around and influence subsequent star formation, and so on). The distance to a galaxy is determined through Hubble's law ($d = H_0^{-1}z$) by measuring a redshift; peculiar velocities induced by the lumpy distribution of matter are significant and prevent a direct determination of the true distance. There are the intrinsic limitations of the surveys themselves: they are flux not volume limited (brighter objects are seen to greater distances and vice versa) and relatively small (e.g., the CfA slices of the Universe survey contains only about 10^4 galaxies and extends to a redshift of about $z \sim 0.03$). Last but not least are the numerical simulations which bridge theory and observation; they are limited in dynamical range (about a factor of 100 in length scale) and in microphysics (in the largest simulations only gravity, and in others only a gross approximation to the effects of hydrodynamics/thermodynamics).

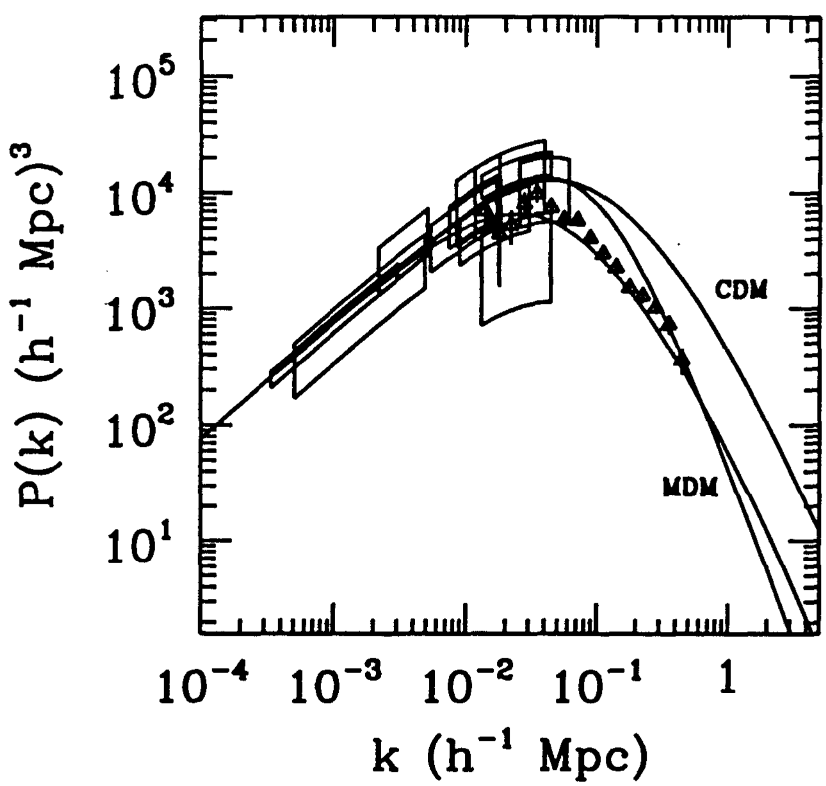

Fig. 3. Comparison of the cold dark matter perturbation spectrum with CBR anisotropy measurements (boxes) and the distribution of galaxies today (triangles). Wavenumber k is related to length scale, $k = 2\pi/\lambda$; error flags are not shown for the galaxy distribution. The curve labeled MDM is hot + cold dark matter ("5 eV" worth of neutrinos); the other two curves are cold dark matter models with Hubble constants of $50\,\mathrm{km\,s^{-1}\,Mpc}$ (labeled CDM) and $35\,\mathrm{km\,s^{-1}\,Mpc}$. (Figure courtesy of M. White.)

This being said, redshift surveys do provide an important probe of the power spectrum on small scales ($\lambda \sim 1 - 300\,\mathrm{Mpc}$). Even with their limitations redshift surveys (as well as other data) indicate that while the simplest version of COBE-normalized cold dark matter is in broad agreement with the data, the shape of the power spectrum as well as its amplitude on small scales is not quite right [12, 31]. At least three possibilities come to mind: (i) the comparison of numerical simulations and the observations is still too primitive to draw firm conclusions; (ii) cold dark matter has much, but not all, of the "truth;" or (iii) cold dark matter has been falsified.

For three reasons I believe that it is worthwhile exploring the second possibility, namely that cold dark matter needs a little tinkering. First, cold dark matter is such an attractive theory and part of a bold attempt to extend greatly the standard cosmology. Second, many observations seem to point to the same problem (e.g., the abundance of x-ray clusters and the cluster-cluster correlation function). Third, there are other reasons to believe that the Universe is more complicated than the simplest model of cold dark matter.

4.3 The Cold Dark Matter Family of Models

Somewhat arbitrarily, standard cold dark matter has come to mean: precisely scale-invariant density perturbations; baryons + CDM only; and Hubble constant of $50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ (to ensure a sufficiently aged Universe with a Hubble constant still within the range of observations). This is the vanilla or default model, which, when normalized to COBE has too much power on small scales and the wrong spectral shape on slightly larger scales.

The spectrum of density perturbations today depends not only upon the primeval spectrum (and the normalization on large scales provided by COBE), but also upon the energy content of the Universe. While the fluctuations in the gravitational potential were initially (approximately) scale invariant, the Universe evolved from an early radiation-dominated phase to a matter-dominated phase which imposes a characteristic scale on the spectrum of density perturbations seen today; that scale is determined by the energy content of the Universe, $k_{\mathrm{EQ}} \sim 10^{-1}h\,\mathrm{Mpc}^{-1}\,(\Omega_{\mathrm{matter}}h/\sqrt{g_*})$ (g_* counts the relativistic degrees of freedom, $\Omega_{\mathrm{matter}} = \Omega_B + \Omega_{\mathrm{CDM}}$). In addition, if some of the nonbaryonic dark matter is neutrinos, they reduce power on small scales somewhat through freestreaming (see Fig. 2). With this in mind, let me discuss the variants of cold dark matter that have been proposed to improve its agreement with observations.

1. **Low Hubble Constant + cold dark matter (LHC CDM) [32].** Remarkably, simply lowering the Hubble constant to around $30\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ solves all the problems of cold dark matter. Recall, the critical density $\rho_{\mathrm{crit}} \propto H_0^2$; lowering H_0 lowers the matter density and has precisely the desired effect. It has two other added benefits: the expansion age of the Universe is comfortably consistent with the ages of the oldest stars and the baryon fraction is raised to a value that is consistent with that measured in x-ray clusters. Needless to say, such a small value for the Hubble constant flies in the face of current observations [10]; further, it illustrates that the problems of cold dark matter get even worse for the larger values of H_0 that are favored by recent observations.

2. **Hot + cold dark matter (νCDM) [33].** Adding a small amount of hot dark matter can suppress density perturbations on small scales; adding too much leads back to the longstanding problems of hot dark matter. Retaining enough power on very small scales to produce damped Lyman-α systems at high redshift limits Ω_ν to less than about 20%, corresponding to about "5 eV worth of neutrinos" (i.e., one species of mass 5 eV, or two species of mass 2.5 eV, and so on). This admixture of hot dark matter rejuvenates cold dark matter provided the Hubble constant is not too large, $H_0 \lesssim 55\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$; in fact, a Hubble constant of closer to $45\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ is preferred.
3. **Cosmological constant + cold dark matter (ΛCDM) [34].** (A cosmological constant corresponds to a uniform energy density, or vacuum energy.) Shifting 50% to 80% of the critical density to a cosmological constant lowers the matter density and has the same beneficial effect as a low Hubble constant. In fact, a Hubble constant as large as $80\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ can be accommodated. In addition, the cosmological constant allows the age problem to solved even if the Hubble constant is large, addresses the fact that few measurements of the mean mass density give a value as large as the critical density (most measurements of the mass density are insensitive to a uniform component), and allows the baryon fraction of matter to be larger, which alleviates the cluster baryon problem. Not everything is rosy; cosmologists have invoked a cosmological constant twice before to solve their problems (Einstein to obtain a static universe and Bondi, Gold, and Hoyle to solve the earlier age crisis when H_0 was thought to be $250\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$). Further, particle physicists can still not explain why the energy of the vacuum is not at least 50 (if not 120) orders of magnitude larger than the present critical density, and expect that when the problem is solved the answer will be zero.
4. **Extra relativistic particles + cold dark matter (τCDM) [35].** Raising the level of radiation has the same beneficial effect as lowering the matter density. In the standard cosmology the radiation content consists of photons + three (undetected) cosmic seas of neutrinos (corresponding to $g_* \simeq 3.36$). While we have no direct determination of the radiation beyond that in the CBR, there are at least two problems: What are the additional relativistic particles? and Can additional radiation be added without upsetting the successful predictions of primordial nucleosynthesis which depend critically upon the energy density of relativistic particles? The simplest way around these problems is an unstable tau neutrino (mass anywhere between a few keV and a few MeV) whose decays produce the radiation. This fix can tolerate a larger Hubble constant, though at the expense of more radiation.
5. **Tilted cold dark matter (TCDM) [36].** While the spectrum of density perturbations in most models of inflation is very nearly scale invariant, there are models where the deviations are significant ($n \approx 0.8$) which leads to smaller fluctuations on small scales. Further, if gravity waves account for a significant part of the CBR anisotropy, the level of density perturbations can be lowered even more. A combination of tilt and gravity waves can solve the

problem of too much power on small scales, but seems to lead to too little power on intermediate and very small scales.

In evaluating these better fit models, one should keep the words of Francis Crick in mind (loosely paraphrased): A model that fits all the data at a given time is necessarily wrong, because at any given time not all the data are correct(!). ΛCDM provides an interesting/confusing example. When I discussed it in 1990, I called it the best-fit Universe, and quoting Crick, I said that ΛCDM was certain to fall by the wayside [37]. In 1995, it is still the best-fit model [38].

Let me end by defending the other point of view, namely, that to add something to cold dark matter is not unreasonable, or even as some have said, a last gasp effort to saving a dying theory. Standard cold dark matter was a starting point, similar to early calculations of big-bang nucleosynthesis. It was always appreciated that the inflationary spectrum of density perturbations was not exactly scale invariant [39] and that the Hubble constant was unlikely to be exactly $50\,\mathrm{km\,s^{-1}\,Mpc}$. As the quality and quantity of data improve, it is only sensible to refine the model, just as has been done with big-bang nucleosynthesis. Cold dark matter seems to embody much of the "truth." The modifications suggested all seem quite reasonable (as opposed to contrived). Neutrinos exist; they are expected to have mass; there is even some experimental data that indicates they do have mass. It is still within the realm of possibility that the Hubble constant is less than $50\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$, and if it is as large as $70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ to $80\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ a cosmological constant seems inescapable based upon the age problem alone. There is no data that can preclude more radiation than in the standard cosmology and deviations from scale invariance were always expected.

5 The Future

5.1 Testing and Discriminating

The stakes for cosmology are high: if correct, inflation/cold dark matter represents a major extension of the big bang and our understanding of the Universe, which can't help but shed light on the fundamental physics at energies of order $10^{15}\,\mathrm{GeV}$ or higher.

What are the crucial tests and when will they be carried out? Because of the many measurements/observations that can have significant impact, I believe the answer to when is sooner rather than later. The list of pivotal observations is long: CBR anisotropy, large redshift surveys (e.g., the Sloan Digital Sky Survey will have 10^6 redshifts), direct searches for nonbaryonic in our neighborhood (both for axions and neutralinos) and baryonic dark matter (microlensing), x-ray studies of galaxy clusters, the use of back-lit gas clouds (quasar absorption line systems) to study the Universe at high redshift, evolution (as revealed by deep images of the sky taken by the Hubble Space Telescope and the Keck 10 meter telescope), measurements of both H_0 and q_0, mapping of the peculiar velocity field at large redshifts through the Sunyaev-Zel'dovich effect, dynamical estimates of the mass density (using weak gravitational lensing, large-scale velocity fields, and so on), age determinations, gravitational lensing, searches for supersymmetric particles (at accelerators) and neutrino oscillations (at accelerators, solar-neutrino detectors, and other large underground detectors), searches for high-energy neutrinos from neutralino annihilations in the sun using large underground detectors, and on and on. Let me end by illustrating the interesting consequences of several possible measurements.

A definitive determination that H_0 is greater than $55\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ would falsify LHC CDM and νCDM. Likewise, if H_0 is shown to be $75\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ or larger a cosmological constant would be mandatory. A flat Universe with a cosmological constant has a very different deceleration parameter than one dominated by matter, $q_0 = -1.5\Omega_\Lambda + 0.5 \sim -(0.4 - 0.7)$ compared to $q_0 = 0.5$, and this could be settled by galaxy number counts or numbers of lensed quasars. The level of CBR anisotropy in τCDM and LHC CDM on the 0.5° scale is about 50% larger than the other models, which should be easily discernible. If neutrino-oscillation experiments were to provide evidence for a neutrino of mass 5 eV (or two of mass 2.5 eV) νCDM would seem almost inescapable.

Many more CBR measurements are in progress and there should many interesting results in the next few years. In the wake of the success of COBE there are proposals, both in the US and Europe, for a satellite-borne instrument to map the CBR sky with a factor of ten better resolution. A map of the CBR with $0.5^\circ - 1^\circ$ resolution could separate the gravity-wave contribution to CBR anisotropy and provide evidence for the third robust prediction of inflation, as well as determining other important parameters [40], e.g., the scalar and tensor indices, Ω_Λ, and even Ω_0 (the position of the "Doppler" peak scales as $0.5^\circ/\sqrt{\Omega_0}$) [41]).

There are key measurements that can shed light on the quantity and composition of dark matter in the Universe. Steady progress is being made in the quest to measure Ω_0 and to detect particle dark matter. At the moment there are two particularly pressing issues involving measurements of the baryonic fraction of dark matter.

X-ray observations of rich clusters are able to determine the ratio of hot gas (baryons) to total cluster mass (baryons + CDM) (by a wide margin, most of the baryons "seen" in cluster are in the hot gas). To be sure there are assumptions and uncertainties; the data at the moment indicate that this ratio is $0.04h^{-3/2} - 0.1h^{-3/2}$ [42]. If clusters provide a fair sample of the universal mix of matter, then this ratio should equal $\Omega_B/(\Omega_B + \Omega_{\rm CDM}) \simeq (0.009 - 0.022)h^{-2}/(\Omega_B + \Omega_{\rm CDM})$. Since clusters are large objects they should provide an approximately fair sample. Taking the numbers at face value, cold dark matter is consistent with the cluster gas fraction provided either: $\Omega_B + \Omega_{\rm CDM} = 1$ and $h \sim 0.3$ or $\Omega_B + \Omega_{\rm CDM} \sim 0.3$ and $h \sim 0.7$, favoring LHC CDM or ΛCDM. The cluster baryon fraction is an important test of and discriminator between cold dark matter models, and we haven't heard the final word yet.

If cold dark matter is correct, then a significant, if not dominant, fraction of the dark halo of our galaxy should be cold dark matter (the halos of spiral galaxies are not large enough to guarantee that they represent a fair sample). Direct searches for faint stars have failed to turn up enough to account for the halo [43]. Over the past few years, microlensing has been used to search for dark stars (stars below the $0.08 M_\odot$ limit for hydrogen burning). Five stars in the LMC have been observed to change brightness in a way consistent with their being microlensed by dark halo objects passing along the line of sight. While the statistics are small, and there are uncertainties concerning the size of dark halo, these results indicate that only a small fraction (5% to 30%) of the dark halo is in the form of dark stars [44]. So far, this one goes in the "plus column" for cold dark matter.

5.2 Reconstruction

If cold dark matter is shown to be correct, then a window to the very early Universe ($t \sim 10^{-34}$ sec) will have been opened. While it is certainly premature to jump to this conclusion, I would like to illustrate one example of what one could hope to learn. As mentioned earlier, the spectra and amplitudes of the the tensor and scalar metric perturbations predicted by inflation depend upon the underlying model, to be specific, the shape of the inflationary scalar-field potential. (Inflation involves the classical evolution of a scalar field ϕ rolling down its potential energy curve $V(\phi)$.) If one can measure the power-law index of the scalar spectrum and the amplitudes of the scalar and tensor spectra, one can recover the value of the potential and its first two derivatives around the point on the potential where inflation took place [45]. (Measuring the power-law index of the tensor perturbations in addition, allows an important consistency check of inflation.) Reconstruction of the inflationary scalar potential would shed light both on inflation as well as physics at energies of the order of 10^{14} GeV.

5.3 Concluding Remarks

We live in exciting times. We have a cosmological model that provides a reliable account of the Universe from 0.01 sec until the present. Together with the Standard Model of particle physics it provides a framework for both asking and addressing deeper questions about the Universe. Much progress has been made in the past fifteen years of intensive study of the earliest history of the Universe. With inflation and cold dark matter we may be on the verge of a very significant extension of the standard cosmology. Most importantly, the data needed to test the cold dark matter theory is coming in at a rapid rate. At the very least we should know soon whether we are on the right track or if it's back to the drawing board.

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