

# Fermi National Accelerator Laboratory

Fermilab-Pub-94/406-T  
hep-ph/9412250

## Finite Width Effects in Top Quark Decays

Gregory Mahlon\* and Stephen Parke†

*Fermi National Accelerator Laboratory*

*P.O. Box 500*

*Batavia, IL 60510*

(December 1994)

### Abstract

Motivated by evidence that the top quark mass lies near the  $bWZ$  threshold, we compute the decay rate for  $t \rightarrow bWZ$  in the Standard Model, including the effects of the finite widths of the  $W$  and  $Z$  bosons. In the limit where the width effects are negligible, our results disagree with previously published calculations. We also examine the decay  $t \rightarrow bWH$ . Although the widths induce a sizable enhancement near threshold for both decays, we find that the rates are still too small to be observed in the present generation of experiments. This means that detection of either mode in one of these experiments would be a signal of new physics.

Typeset using REVTeX



## I. INTRODUCTION

Recent experimental evidence from CDF [1] as well as the global fit to LEP data [2] suggests that the top quark mass lies near the threshold for decay into a  $bWZ$  final state. Decker, Nowakowski, and Pilaftsis (DNP) [3] have presented numerical results for the decay  $t \rightarrow bW^+Z$ . The DNP calculation treats the  $W$  and  $Z$  bosons in the stable particle approximation. In light of the recent experimental data, it is prudent to reexamine this decay, including the subsequent decays of the  $W$  and  $Z$ . One expects, and indeed we find, that there are significant corrections to the stable particle result for the range of interesting top masses. Unfortunately, because of strong phase space suppression, the rates are too small to be seen in the near future, even when the enhancement from the non-zero  $W$  and  $Z$  widths is taken into account. Thus, should this decay mode of the top be observed, it would signal physics beyond the Standard Model.

Another process discussed by DNP [3,4] is  $t \rightarrow bW^+H$ . We take this opportunity to reexamine this decay as well, including the width of the  $W$ . As before, the rate is significantly enhanced near threshold, but not enough to render the mode visible.

## II. THE DECAY $t \rightarrow bWZ$

We begin with a discussion of the decay chain

$$t \rightarrow bW^+Z; \quad W^+ \rightarrow \mu^+\nu_\mu, \quad Z \rightarrow e^+e^-. \quad (1)$$

There are a total of nine Feynman diagrams contributing to the desired final state in the Standard Model. The first three are the usual diagrams for  $t \rightarrow bW^+Z$  where the  $W^+$  and  $Z$  decay to the indicated final states. The next two diagrams involve radiation of the  $Z$  from one of the decay products of the  $W^+$ . These diagrams are expected to be small in comparison to the first three, as all of the  $W$  propagators they contain are far from mass shell. The final four diagrams involve replacing the  $Z$  by a photon. To reduce the contamination from the photon, we impose a cut on the invariant mass of the  $e^+e^-$  pair. This is a reasonable

approach because experimentally such a cut would also be implemented. If we choose a cut of  $0.8 m_Z$ , we find that the contribution from the photon diagrams is typically less than 2%. Nevertheless, the numerical results we present below include all nine contributions.

Because the  $t$  mass is likely to lie near the threshold for  $b W^+ Z$  production, it is important to include the finite widths of the  $W^+$  and  $Z$  in the calculation. We take the propagator of a  $W$ -boson of momentum  $W$ , mass  $m_W$ , and width  $\Gamma_W$  in the unitary gauge to be

$$\frac{-i}{W^2 - \hat{m}_W^2} \left( g^{\mu\nu} - \frac{W^\mu W^\nu}{\hat{m}_W^2} \right) \quad (2)$$

where  $\hat{m}_W \equiv m_W + \frac{1}{2}i\Gamma_W$ . We use a similar expression for the  $Z$  propagator. Note the presence of the complex mass in the longitudinal part of the propagator: it is required to preserve gauge invariance in the process  $t \rightarrow b W^+ \gamma$ . Similarly, it is inconsistent to put a complex mass in the  $t$ -quark propagator unless the production process is included. We have found that a 1 GeV shift in  $m_t$  produces a shift of 10%–15% in the rate for top masses just above the  $b W^+ Z$  threshold. We have convolved our curve with a Breit-Wigner distribution in  $m_t$  to simulate what one might find were production properly included, and see a change of only a few percent. Hence, we exclude the width of the  $t$  from our discussion.

In the case where both the  $W$  and  $Z$  decay to the same lepton family, there are additional diagrams resulting from the interchange of the identical antifermions in the final state. We have checked the size of the interference term from these contributions and have found that it is generally small, ranging from about 6% for a top mass of 160 GeV, to a percent or less above the  $b W Z$  threshold.

In addition to the full calculation described above, we have also obtained a “narrow width” approximation to the result. We define this approximation to consist of constraining the  $W$  and  $Z$  invariant masses to their central values by introducing a delta function into the phase space integral. The result, when normalized to the appropriate branching ratios, should be the same at the stable particle result, except for (small) correction terms proportional to  $\Gamma_W/m_W$  or  $\Gamma_Z/m_Z$ . Near threshold, this approximation differs from the full result as the contributions from that portion of phase space where the  $W$  and  $Z$  are off mass shell

become important. Below threshold in this approximation, the rate is identically zero. Note that the cut on the invariant mass of the  $e^+e^-$  pair is irrelevant in this approximation. By examining the ratio of the full calculation to the narrow width result, we get a measure of the size of the enhancement for masses just above threshold.

In the discussion and plots that follow, we define

$$\Gamma(t \rightarrow bW^+Z) \equiv \frac{\Gamma(t \rightarrow b\mu^+\nu_\mu e^+e^-)}{\mathcal{B}(W^+ \rightarrow \mu^+\nu_\mu)\mathcal{B}(Z \rightarrow e^+e^-)}, \quad (3)$$

where  $\mathcal{B}(W^+ \rightarrow \mu^+\nu_\mu)$  and  $\mathcal{B}(Z \rightarrow e^+e^-)$  are the experimental  $W$  and  $Z$  branching ratios respectively [5]. We compare this quantity to the dominant decay mode of a heavy top quark in the Standard Model, which is  $t \rightarrow bW^+$ . The rate for this decay is given by [6]

$$\begin{aligned} \Gamma(t \rightarrow bW^+) = \frac{G_F}{\sqrt{2}} \frac{m_W^2}{8\pi m_t} |V_{tb}|^2 & \left[ \frac{(m_t^2 - m_b^2)^2}{m_t^2 m_W^2} + \frac{m_t^2 + m_b^2 - 2m_W^2}{m_t^2} \right] \\ & \times \sqrt{[m_t^2 - (m_W + m_b)^2][m_t^2 - (m_W - m_b)^2]}, \end{aligned} \quad (4)$$

where  $G_F$  is the Fermi coupling constant, and  $V_{tb}$  is the CKM matrix element. The values we have used for the various masses, widths, coupling constants, and branching ratios are collected in Table I.

In Figure 1, we have plotted the ratio  $\Gamma(t \rightarrow bW^+Z)/\Gamma(t \rightarrow bW^+)$  for a top mass range of 140–260 GeV. Note that  $V_{tb}$  cancels in this ratio. The solid (dotted) curve is the result of the full (narrow width) calculation. The large enhancement of the decay rate near threshold is clearly evident. Some representative values for this ratio appear in Table II.

As has already been stated, the narrow width approximation normalized as indicated by Eq. 3 should reproduce the stable particle result, except for small corrections. We find, however, that our result is in disagreement with the DNP curve [3]. In fact, the two curves do not even have the same shape. Our result is larger by a factor of between 1.6 and 2 over the range  $200 \text{ GeV} \leq m_t \leq 300 \text{ GeV}$ . Nevertheless, because we are in agreement with a concurrent, independent calculation by Ellis [7] of  $t \rightarrow bW^+Z$  in the stable particle approximation, we believe that our curve is indeed correct.

Figure 2 explores the effect of varying the value of the cut on the invariant mass of the  $Z$  decay products, for selected values of the top quark mass. As would be expected from simple phase space considerations, the effect of the photon is most visible for low top masses, especially below the  $bWZ$  threshold. In all cases, the choice  $E_{cut} = 0.8 m_Z$ , which corresponds to just over seven widths below the  $Z$  peak, captures the vast majority of the  $Z$  events, while avoiding excessive background events from the photon.

### III. THE PROCESS $t \rightarrow bWH$

The second decay chain which we wish to consider is

$$t \rightarrow bW^+H; \quad W^+ \rightarrow \mu^+\nu_\mu. \quad (5)$$

Since the width of the Standard Model Higgs is only about 5 MeV for masses below the  $W$ -pair threshold, we do not include its width in our calculation. This is a much simpler process than (1), as there are just four Feynman diagrams in the Standard Model: the Higgs may be attached to the  $t$ ,  $b$ ,  $W$ , or  $\mu^+$ . Since we work in the approximation  $m_\mu = 0$ , only the first three of these diagrams actually contribute. Although small, we retain the contribution from the  $bbH$  vertex.

Defining

$$\Gamma(t \rightarrow bW^+H) \equiv \frac{\Gamma(t \rightarrow b\mu^+\nu_\mu H)}{\mathcal{B}(W^+ \rightarrow \mu^+\nu_\mu)} \quad (6)$$

and proceeding as before, we plot as Fig. 3 the ratio  $\Gamma(t \rightarrow bW^+H)/\Gamma(t \rightarrow bW^+)$  in both the full and narrow width approximation, as a function of  $m_t$  for selected Higgs masses. The narrow width curves plotted here do indeed agree with the literature [3,4]. Some selected values for this ratio are listed in Table III.

Although the rates are somewhat more favorable in this case than for  $bWZ$ , provided  $m_H < m_Z$ , the difficulty in actually seeing the Higgs associated with this decay makes observation of this mode in the near future unlikely.

#### IV. CONCLUSIONS

We have performed a calculation of the decays  $t \rightarrow bW^+Z$  and  $t \rightarrow bW^+H$  including the subsequent decays of the gauge bosons. For the  $bWZ$  final state, we disagree with the existing stable particle calculation [3] well above threshold. We agree with the previously published results [3,4] for the  $bWH$  final state in this limit. In both decays, we find a significant enhancement in the rate for interesting values of the top quark mass. This enhancement is not sufficient, however, to render either decay visible to the current generation of experiments. Thus, we conclude that the observation of either decay would signal the existence of physics beyond the Standard Model.

#### ACKNOWLEDGMENTS

We would like to thank Keith Ellis for useful discussions. This work was performed at the Fermi National Accelerator Laboratory, which is operated by Universities Research Association, Inc., under contract DE-AC02-76CHO3000 with the U.S. Department of Energy.

## REFERENCES

\* Electronic address: gdm@fnth03.fnal.gov

† Electronic address: parke@fnalv.fnal.gov

- [1] F. Abe, *et. al.*, Phys. Rev. **D50**, 2966 (1994); Phys. Rev. Lett. **73**, 225 (1994).
- [2] Aguilar-Benitez, *et. al.*, Phys. Rev. **D50**, 1173 (1994).
- [3] R. Decker, M. Nowakowski, and A. Pilaftsis, Z. Phys. **C57**, 339 (1993).
- [4] R. Decker, M. Nowakowski, and A. Pilaftsis, Mod. Phys. Lett. **A6**, 3491 (1991); Erratum **A7**, 819 (1992).
- [5] Strictly speaking, we should use an appropriately weighted sum composed of results from all possible decay modes of the  $W$  and  $Z$ . Provided that we choose the  $e^+e^-$  invariant mass cut so as to minimize the contamination from the photon, our definition is accurate up to errors of  $O(\alpha_s)$ . We have verified that the results obtained by substituting other decay modes for the  $W$  or  $Z$  are indeed consistent at this level.
- [6] I. Bigi, Y. Dokshitzer, V. Khoze, J. Kühn, and P. Zerwas, Phys. Lett. **181B**, 157 (1986).
- [7] R.K. Ellis, private communication.

## FIGURES

FIG. 1. The ratio  $\Gamma(t \rightarrow bW^+Z)/\Gamma(t \rightarrow bW^+)$  versus  $m_t$ , with an  $e^+e^-$  invariant mass cut of  $0.8m_Z$ . The solid line is the full calculation including the  $W$  and  $Z$  width effects, while the dotted line is the narrow width approximation.

FIG. 2. The ratio  $\Gamma(t \rightarrow bW^+Z)/\Gamma(t \rightarrow bW^+)$  versus the  $e^+e^-$  invariant mass cut described in the text, for top quark masses of 160, 176, 200, and 250 GeV.

FIG. 3. The ratio  $\Gamma(t \rightarrow bW^+H)/\Gamma(t \rightarrow bW^+)$  versus  $m_t$ , for Higgs masses of 70, 90, 110, and 130 GeV. In each case, the solid line is the full calculation including the  $W$  width effects, while the dotted line is the narrow width approximation.

TABLES

TABLE I. Input values used in calculation of  $t \rightarrow bW^+Z(H)$ .

$m_W$	80 GeV
$m_Z$	91 GeV
$m_b$	5 GeV
$\Gamma_W$	2.11 GeV
$\Gamma_Z$	2.49 GeV
$G_F$	$1.166 \times 10^{-5} \text{ GeV}^{-2}$
$\sin^2 \theta_W$	0.2319
$B(W^+ \rightarrow \mu^+ \nu_\mu)$	0.107
$B(Z \rightarrow e^+ e^-)$	0.03367

TABLE II. The ratio  $\Gamma(t \rightarrow bW^+Z)/\Gamma(t \rightarrow bW^+)$  for the full calculation (F), the narrow width approximation (NW), and their ratio (F/NW).

$m_t$ (GeV)	F	NW	F/NW
150	$6.68 \times 10^{-8}$	-	-
160	$1.60 \times 10^{-7}$	-	-
177	$7.47 \times 10^{-7}$	$1.45 \times 10^{-9}$	516
185	$2.14 \times 10^{-6}$	$5.57 \times 10^{-7}$	3.84
200	$1.36 \times 10^{-5}$	$1.00 \times 10^{-5}$	1.35
250	$2.56 \times 10^{-4}$	$2.54 \times 10^{-4}$	1.01

TABLE III. The ratio  $\Gamma(t \rightarrow b W^+ H)/\Gamma(t \rightarrow b W^+)$  with a 90 GeV Higgs for the full calculation (F), the narrow width approximation (NW), and their ratio (F/NW).

$m_t$ (GeV)	F	NW	F/NW
160	$5.75 \times 10^{-8}$	-	-
176	$3.06 \times 10^{-7}$	$2.70 \times 10^{-9}$	113
185	$1.57 \times 10^{-6}$	$8.67 \times 10^{-7}$	1.81
200	$1.34 \times 10^{-5}$	$1.19 \times 10^{-5}$	1.13
250	$3.32 \times 10^{-4}$	$3.28 \times 10^{-4}$	1.01

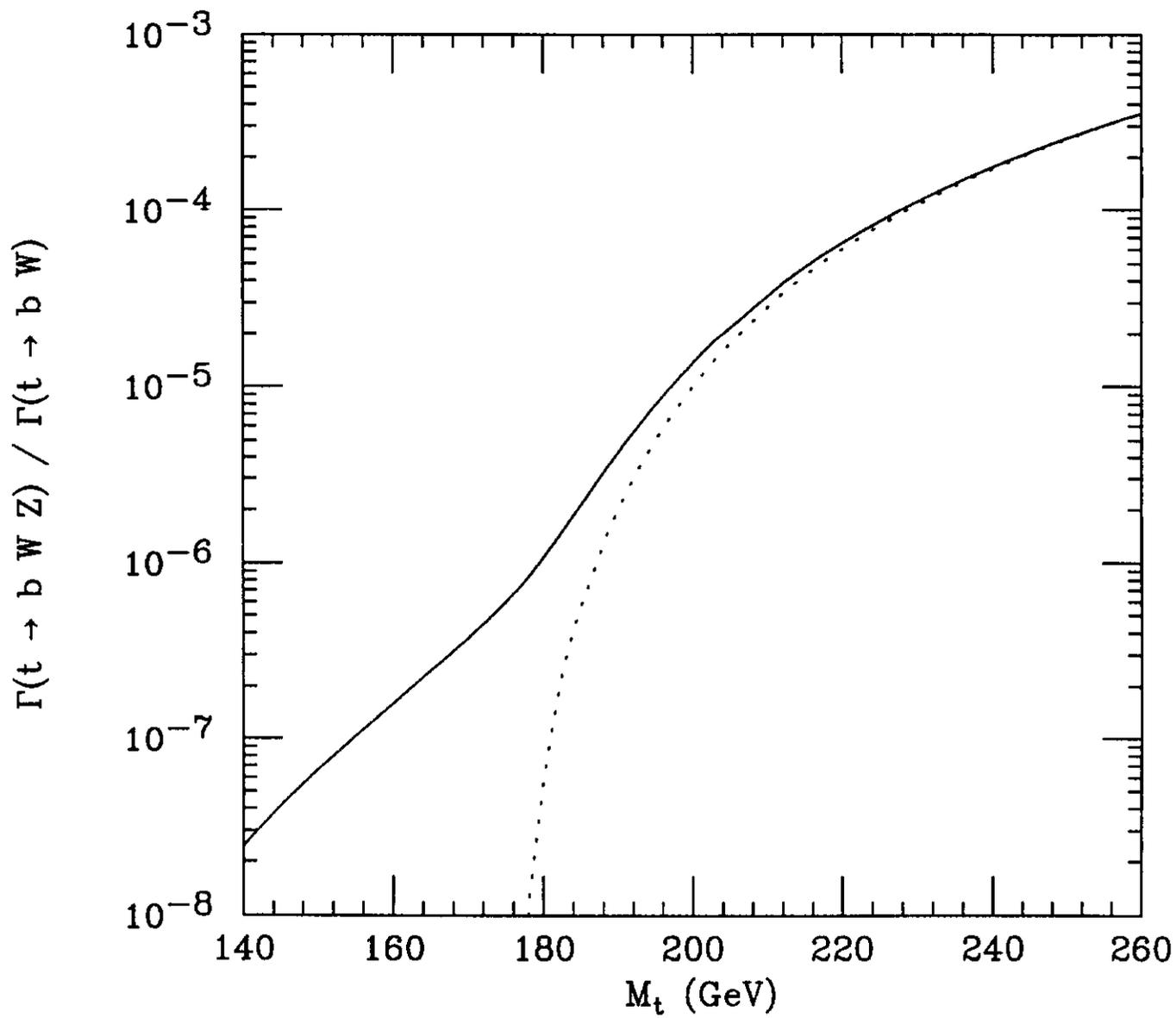


fig. 1

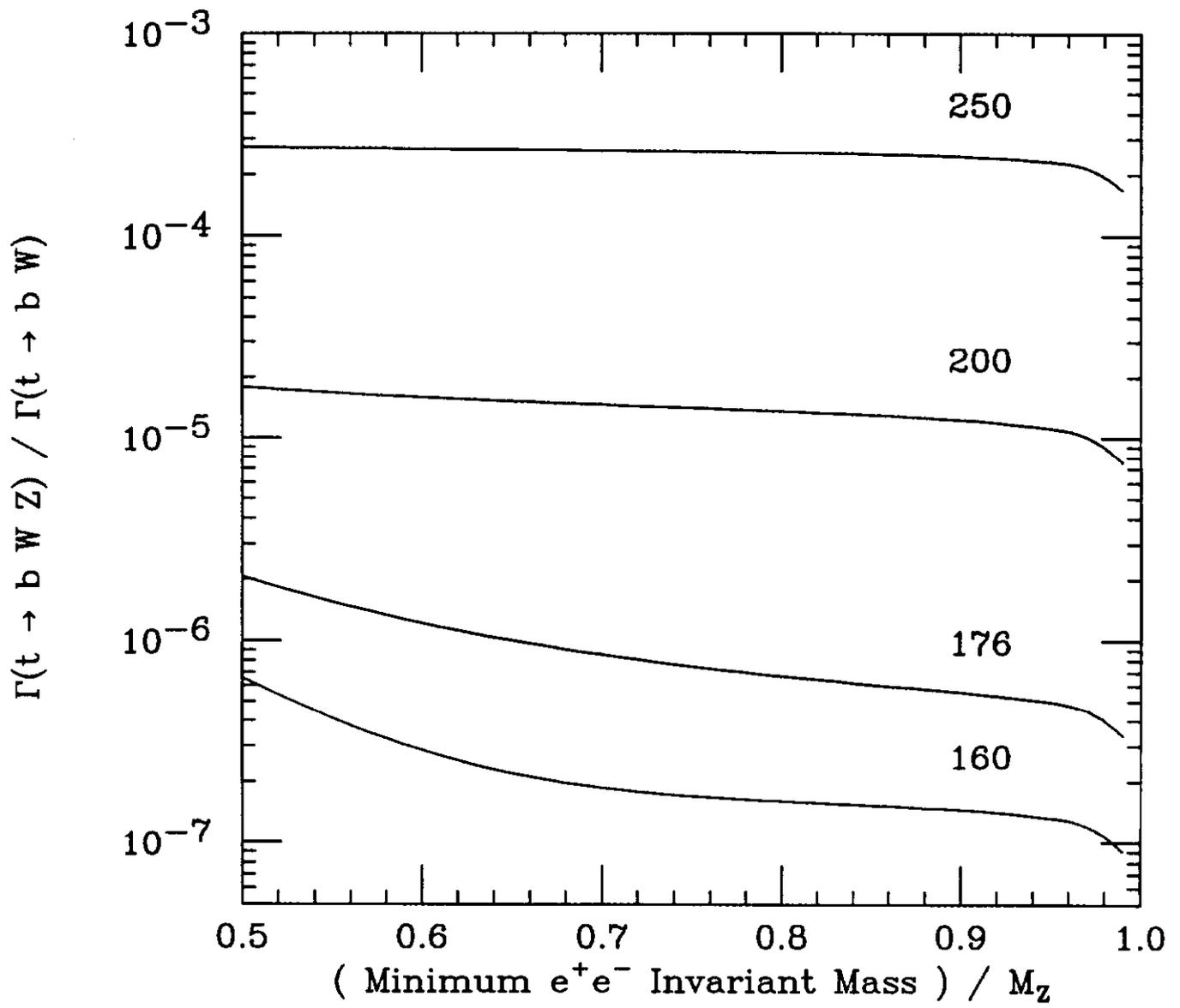


fig. 2

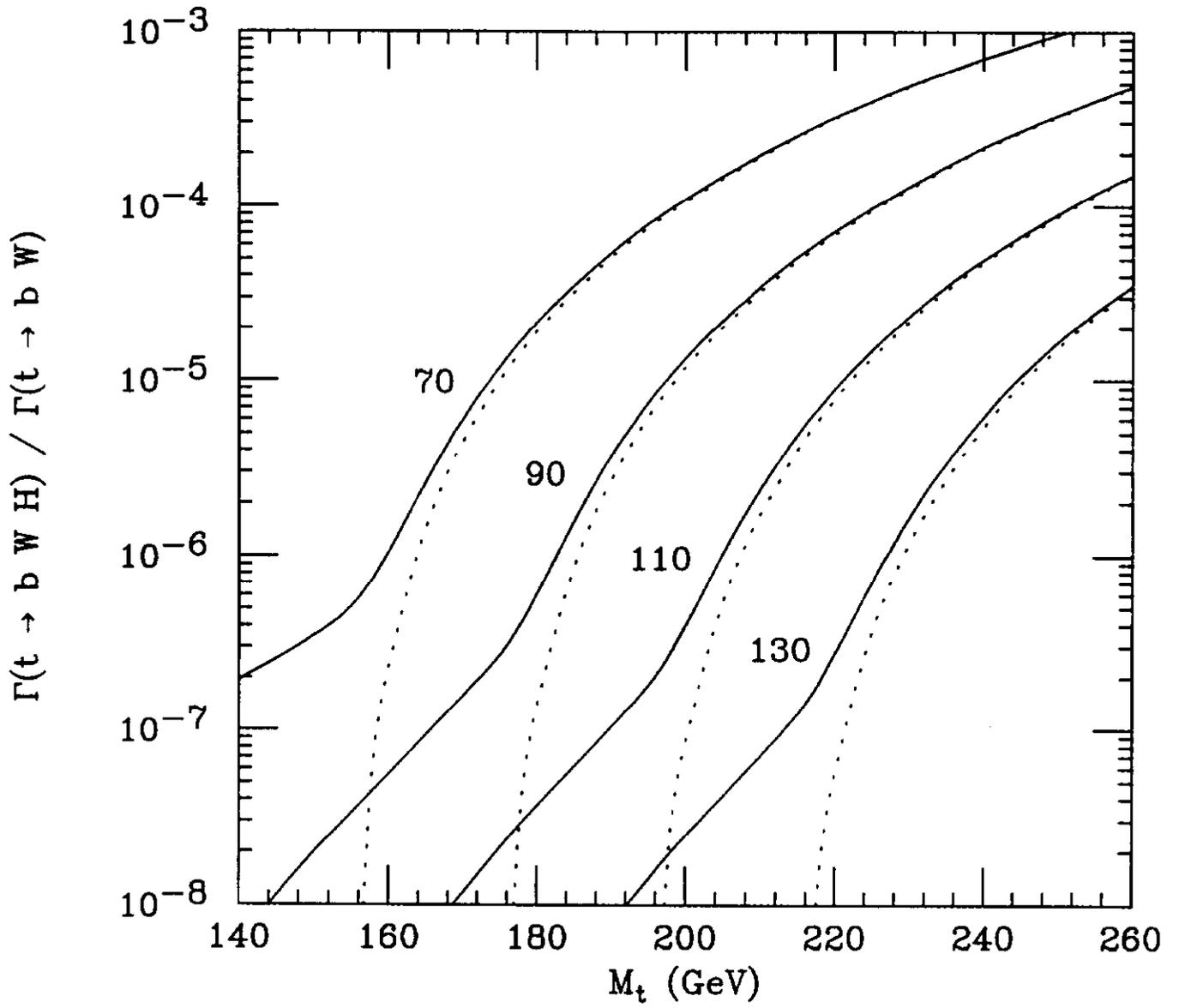


fig. 3