Fermi National Accelerator Laboratory

FERMILAB-PUB-94/385-T

Anomalous Cross Section from Perturbation Theory

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Abstract

We present the anomalous cross section for baryon number violation in the standard model from the perturbation of large-order behavior of forward scattering amplitudes to the order $(\epsilon/n)^{8/3}\ln(\epsilon/n)$. An improved high energy behavior of the anomalous cross section is observed. We also argue that the asymptotic form of $F(\epsilon g) \equiv -g \ln \sigma_{ano}$ is given in the form: $F(\epsilon g) \to d + c \cdot \epsilon g$ for $\epsilon g \to \infty$ with c, d constants satisfying $c, d \geq 0$, and $F(\epsilon g) > 0$ for all energies. The constants are not determined.

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

1 Introduction

We have recently shown that the large-order behavior of Green's functions has generally non-trivial energy dependence arising from the Espinosa-Ringwald type anomalous cross section, and in turn the anomalous cross section itself can be deduced from the large-order behavior of forward scattering amplitudes [1]. It was an extension of the observation that in the double-well potential problem in quantum mechanics, the vacuum transition rate determines the large-order behavior of the vacuum to vacuum transition function (here vacuum means one of the perturbative vacua), and in turn the minimum element of the perturbative series of the latter reproduces the exponential part of the former which is the imaginary part of the vacuum transition function. Recall that all the bubble diagrams for the vacuum transition function are real and its imaginary part arises nonperturbatively from the instanton interactions. We also proposed calculating the anomalous cross section by doing perturbation of the Borel transform of the forward scattering amplitudes about its instanton- anti-instanton singularity. We elaborate this proposition further in this paper, and clarify the relation between our proposed method and the energy expansion method in powers of $(E/E_0)^{2/3}$ [2].

In section 2 we review briefly the relation between large-order behavior and the anomalous cross section, and in section 3 we give a new formulation of our proposition in terms of ϵ/n expansion, where n is the order of perturbation. We calculate in section 4 the anomalous cross sections from the ϵ/n expansion, and compare them to those from the energy expansion. Using the formalism in section 3, we argue in section 5 that asymptotically $F(\epsilon g) \equiv -g \ln \sigma_{ano}$

either grows linearly in ϵg or approaches to a nonnegative constant. We also show that $F(\epsilon g) > 0$ for all energies, and make a speculation on the value of d that appears in the asymptotic form.

2 Large-order behavior and anomalous cross section

In the weak coupling limit of the standard model of weak interactions the anomalous cross section for baryon number violation in two-body scattering is given by [2]

$$\sigma_{ano} \sim \left(\frac{1}{g}\right)^{\nu} e^{-\frac{1}{g}F_0(\epsilon g)}, \tag{1}$$

where

$$F_0(\epsilon g) = 1 - U(\epsilon g). \tag{2}$$

$U(x)$ is given in the form [3],

$$U(x) = \frac{1}{2}(3x)^{\frac{4}{3}} - \frac{1}{6}(3x)^2 - \frac{\lambda}{54}(3x)^{\frac{8}{3}}\ln(3x) + O\left((3x)^{\frac{8}{3}}\right), \tag{3}$$

with

$$\lambda = 4 - 3\frac{m_h^2}{m_w^2}, \tag{4}$$

where m_h, m_w are the Higgs and gauge boson mass respectively. The ϵ, g are defined as

$$\epsilon = \frac{E}{m_w}, \qquad \text{and} \qquad g = \frac{\alpha_w}{4\pi}, \tag{5}$$

with E, α_w the c.m. energy and the weak coupling respectively. m_w is assumed to be independent of the weak coupling, and ν is a Green function dependent constant of order one. The weak coupling limit is to be understood as:

$$g \to 0 \quad \text{while} \quad \epsilon g \quad \text{fixed,} \tag{6}$$

and we always assume this limit in this paper.

Crutchfield [4] has shown that the large-order behavior — with renormalon effects not included — of a Green's function can be calculated by doing perturbation of its Borel transform around the instanton-anti-instanton singularity. The energy dependence of the Borel transform of a forward scattering amplitudes arises from instanton- induced amplitudes such as those in which the initial state particles are attached to the anti-instanton and the final states particles to the instanton, or vice versa, in an instanton-anti-instanton background [5]. This kind of amplitudes gives precisely the Espinosa-Ringwald type anomalous cross section [6]. Thus we are interested in the Borel transform of the anomalous cross section

$$\begin{aligned} \tilde{\sigma}(b) &= \frac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} \sigma_{ano}(g)\, e^{\left(\frac{b}{g}\right)}\, d\left(\frac{1}{g}\right) \\ &\sim \int \exp\left(-z(1-b) + zU\left(\frac{\epsilon}{z}\right)\right) z^{\nu} dz \end{aligned} \tag{7}$$

with $z \equiv 1/g$. $\tilde{\sigma}(b)$ has a branch-point type singularity at $b = 1$ [7],

$$\tilde{\sigma}(b) \to (1-b)^{-(\nu+1)} \qquad \text{for} \quad b \to 1. \tag{8}$$

With the perturbative coefficients of forward scattering amplitudes A defined by

$$A(E, g) = \sum a_n g^n, \tag{9}$$

the coefficients induced by the anomalous cross section are given by

$$a_n = (n-1)! c_{n-1} \tag{10}$$

where c_n is defined by

$$\tilde{\sigma}(b) = \sum_n c_n \, b^n. \tag{11}$$

From Eq. (7), (10), and (11), we have

$$a_n \sim \int e^{-z\left(1-U\left(\frac{\epsilon}{z}\right)\right)} z^N dz \tag{12}$$

with $N \equiv n - 1 + \nu$.

For large N, (12) may be evaluated in the saddle point approximation to give

$$a_n \sim \mathcal{F}''\left(z(N), N\right)^{-\frac{1}{2}} \, e^{-\mathcal{F}(z(N),N)} \tag{13}$$

with

$$\mathcal{F}(z, N) = z - K(z) - N \ln z \tag{14}$$

and the saddle point $z(N)$ satisfying

$$\frac{N}{z(N)} - 1 + \frac{d}{dz} K(z)\bigg|_{z=z(N)} = 0, \tag{15}$$

where

$$K(z) = z \, U\left(\frac{\epsilon}{z}\right). \tag{16}$$

The minimum element of the perturbative series is then given by

$$a_{\bar{N}} g^{\bar{N}} \sim e^{-\mathcal{F}\left(z(\bar{N}),\bar{N}\right)+\bar{N}\ln g} \tag{17}$$

with $\bar{N}$ satisfying

$$\frac{d}{dN}\left(-\mathcal{F}\left(z(N), N\right) + N \ln g\right)\bigg|_{N=\bar{N}} = \ln\left(z(\bar{N})g\right) = 0, \tag{18}$$

and using (15). From (18)

$$z(\bar{N}) = \frac{1}{g}. \tag{19}$$

Substituting (19) into (17),

$$-g \ln\left(a_{\bar{N}} g^{\bar{N}}\right) = 1 - U(\epsilon g) = F_0(\epsilon g). \tag{20}$$

The minimum element of the perturbative series correctly produces the exponential part of the anomalous cross section. It is very important to note that in the weak coupling limit , the corrections to the saddle point approximation and the pre-exponential Gaussian determinant in (13) generate negligible terms, and thus Eq.(20) is exact. The details can be found in the appendix.

3 $\frac{\epsilon}{n}$ expansion

Let us now see how we can formulate the problem of the anomalous cross section starting from the following two conditions: that the anomalous cross section F can be deduced by taking the minimum element of the perturbative series of forward scattering amplitudes, and that F is a function of ϵg only. The latter condition was shown to be true for the final state corrections, but not completely for the corrections related to the initial state [8].

Noting from (12) that $(n-1)!c_{n-1}$ depends only on N, we define a function $\tilde{C}(N,\epsilon)$ by

$$(n-1)!c_{n-1} \equiv N!\tilde{C}(N,\epsilon). \tag{21}$$

From the Borel singularity in (8) we find

$$\tilde{C}(\infty, \epsilon) = 1. \tag{22}$$

Now

$$a_n g^n = (n-1)! c_{n-1} g^n \equiv e^{-\frac{1}{g} F(N,g,\epsilon)} \tag{23}$$

where

$$F(N, g, \epsilon) = gN - gN \ln gN - g \ln \tilde{C}(N, \epsilon) \tag{24}$$

in the weak coupling limit and using the Stirling's formula. The maximum of $F(N, g, \epsilon)$ occurs at $N = \bar{N}$ satisfying

$$g\bar{N} = e^{-H(\bar{N},\epsilon)} \tag{25}$$

where

$$H(N, \epsilon) \equiv \frac{\partial}{\partial N} \ln \tilde{C}(N, \epsilon). \tag{26}$$

At $N = \bar{N}$,

$$\begin{aligned} F(\bar{N}, g, \epsilon) &= g\bar{N} - g\bar{N} \ln g\bar{N} - g \int_{\infty}^{\bar{N}} H(N, \epsilon) dN \\ &= g\bar{N} - g\bar{N} \ln g\bar{N} - \int_{\infty}^{g\bar{N}} H(N, \epsilon) d(gN) \end{aligned} \tag{27}$$

For $F(\bar{N}, g, \epsilon)$ to be a function of ϵg only, we see from (25) and (27) that $H(N, \epsilon)$ must be a function of ϵ/N only, that is,

$$H(N, \epsilon) = H\left(\frac{\epsilon}{N}\right). \tag{28}$$

Defining new variables y_0, y

$$y_0 = \frac{\epsilon}{\bar{N}}, \qquad y = \frac{\epsilon}{N} \tag{29}$$

we can write (25), (27) as

$$\epsilon g = y_0\, e^{-H(y_0)} \tag{30}$$

and

$$F(\epsilon g) = e^{-H(y_0)}\,(1 + H(y_0)) + \epsilon g \int_0^{y_0} \frac{H(y)}{y^2} dy. \tag{31}$$

Eq. (30), (31) are our main equations. In our formalism, all the information on the anomalous cross section is contained in $H(y)$, and a natural perturbation scheme emerges, namely, the expansion of $H(y)$ at $y = 0$ in powers of y. Since the integral term in (31) is heavily weighted toward $y = 0$, we expect it to be a good approximation scheme. For the consistency of Eq. (31), we note that

$$\lim_{y \to 0} \frac{H(y)}{y} = 0 \tag{32}$$

should be satisfied. We emphasize that this ϵ/n expansion is different from the energy expansion; In our scheme the latter is an intermediate step to find the function H.

4 $F(\epsilon g)$from $\frac{\epsilon}{n}$ expansion

In this section, we calculate $H(y)$ perturbatively using the anomalous cross section from the energy expansion in one-instanton sector, and then compute F using Eq. (30), (31). Since we find it is instructive to calculate H and F order by order, we present the calculations to each order up to $(\epsilon/n)^{8/3} \ln(\epsilon/n)$.

There are two ways to calculate $H(y)$. The first one is to expand the Borel transform $\tilde{\sigma}(b)$ in (7) around the instanton-anti-instanton singularity. Expanding $\exp(K(z))$ in the integrand in (7) in powers of $1/z$, and then performing the z-integration exactly, $\tilde{\sigma}(b)$ for a given $U(x)$ can be calculated in power series of $(1-b)$. Once we have $\tilde{\sigma}(b)$, it is straightforward to find c_n by expanding $\tilde{\sigma}(b)$ about $b=0$. A simpler way to find $H(y)$ is from the saddle point approximation in (13)—(15). Since as shown in the appendix, the saddle point approximation is exact in the weak coupling limit, we have

$$\begin{aligned}
\ln\{(n-1)!c_{n-1}\} &= \ln N! + \ln\tilde{C}(N,\epsilon) \\
&= N\ln z(N) - z(N) + K(z(N)) - \frac{1}{2}\ln\mathcal{F}''(z(N),N) \\
&= N\ln N - N + \frac{1}{2}\ln N + N\ln\left(\frac{z(N)}{N}\right) + K(z(N)) - z(N)K'(z(N)) \\
&= \ln N! + N\ln\left(\frac{z(N)}{N}\right) + K(z(N)) - z(N)K'(z(N)) \qquad (33)
\end{aligned}$$

using the Stirling's formula and the fact that $\mathcal{F}''=1/N$ in the weak coupling limit. Then

$$\ln\tilde{C}(N,\epsilon) = -N\ln(1-K'(z(N))) + K(z(N)) - z(N)K'(z(N)). \qquad (34)$$

Taking the derivative of (34) in N, we find

$$H(N,\epsilon) = \frac{\partial}{\partial N}\ln\tilde{C}(N,\epsilon) = -\ln(1-K'(z(N))) = \ln\left(\frac{z(N)}{N}\right). \qquad (35)$$

Solving $z(N)$ in (15) perturbatively in $1/N$ with $U(x)$ given in (3), we find

$$\frac{z(N)}{N} = 1 - \frac{1}{6}\left(\frac{3\epsilon}{N}\right)^{\frac{4}{3}} + \frac{1}{6}\left(\frac{3\epsilon}{N}\right)^{2} + \frac{5\lambda}{162}\left(\frac{3\epsilon}{N}\right)^{\frac{8}{3}}\ln\left(\frac{3\epsilon}{N}\right) + O\left(\left(\frac{3\epsilon}{N}\right)^{\frac{8}{3}}\right); \qquad (36)$$

and

$$H(y) = -\frac{1}{6}(3y)^{\frac{4}{3}} + \frac{1}{6}(3y)^{2} + \frac{5\lambda}{162}(3y)^{\frac{8}{3}}\ln(3y) + O\left((3y)^{\frac{8}{3}}\right). \qquad (37)$$

We note that the equivalency of the two methods has been explicitly checked to the order $(3y)^{8/3}$ using the leading term of $U(x)$, i.e.,

$$U(x) = \frac{1}{2}(3x)^{\frac{4}{3}} . \tag{38}$$

Note that $H(y)$ in (37) satisfy Eq. (32). This may explain why the leading term of $U(x)$ has a power larger than unity. Eq. (32), for example, does not allow a term of order $(3x)^{2/3}$. We now compute F order by order up to $(3y)^{\frac{8}{3}} \ln(3y)$.

4.1 Leading order

For this case, $U(x)$ is given by (38) and

$$H(y) = -\frac{1}{6}(3y)^{\frac{4}{3}} . \tag{39}$$

With (39), Eq. (30) is solvable for all energies, and there is a unique solution for a given ϵg. Solving (30) numerically, we plot F in Fig.1. Note that at low energies F and F_0 matches very well, but at high energies there is a sizable difference between them. F gives a better high energy behavior. One may wonder why F and F_0 are different in view of the discussions in section 2. The reason is that $H(y)$ in (39) is only part of the series expansion of that defined in (35). If we had solved $z(N)$ in (15) and expanded $H(y)$ in (35) to an infinite order, the resulting F would have been identical to F_0. In our formalism, there is no point of expanding $H(y)$ to an order higher than that of $U(x)$.

4.2 Second order

$U(x)$ to the second order is given by

$$U(x) = \frac{1}{2}(3x)^{\frac{4}{3}} - \frac{1}{6}(3x)^2, \tag{40}$$

and

$$H(y) = -\frac{1}{6}(3y)^{\frac{4}{3}} + \frac{1}{6}(3y)^2. \tag{41}$$

The function $y\exp(-H(y))$ is plotted in Fig.2, and we see that there is no solution for Eq. (30) for $\epsilon g \geq E_o$, where $E_o = 0.53$. E_o is the upper limit for the applicability of our formalism up to the second order. At energies below E_o there are two solutions for a given ϵg. By integrating $H(y)$ to obtain $\ln \tilde{C}(N,\epsilon)$, it is easy to check that the solution close to the origin is for the true maximum of $F(N,g,\epsilon)$ in (24), and the other solution is an artifact of low order expansion of H. The F, F_0 are given in Fig.3.

4.3 Third order

$U(x)$ and $H(y)$ are given in (3), (37) respectively. The functions $H(y), y\exp(-H(y))$ are plotted in Fig.4 for $\lambda = -1$ and -0.6, and we see that Eq. (30) is solvable for all energies. We plot $g\bar{N}$ and F, F_0 in Fig.5. Note the large difference between F and F_0 at high energies for $\lambda = -0.6$. For $\lambda = -1$, $g\bar{N}$ oscillates around unity. It can be checked that for $0 < \lambda \leq 0.5$, $g\bar{N}(\epsilon g)$ is discontinuous, and F not analytic— though continuous—at the discontinuity. For example, with $\lambda = -0.4$ $g\bar{N}$ is discontinuous at $\epsilon g \approx 0.53$, and F is not analytic at the energy. For the reasons discussed in next section, this discontinuity is believed to be an artifact of the expansion of $H(y)$ to this particular order.

5 Constraints on $U(x)$, $H(y)$

In this section we study various constraints on the functions $U(x)$, $H(y)$. From Eq. (35), we see that for a given $U(x)$, $H(y)$ can be written as

$$H(y) = \ln y - \ln x(y) \tag{42}$$

with $x(y)$ defined implicitly through the relation

$$y = \frac{x}{1 - U(x) + xU'(x)}. \tag{43}$$

Since $\tilde{C}(N, \epsilon)$ is believed to be a smooth, well-defined function, we also expect $H(y)$ to be analytic over the positive real axis. For $H(y)$ to be smooth and well-defined for $y > 0$, the r.h.s. of (43) should be a monotonically increasing function in x, and

$$\lim_{x\to\infty} \frac{x}{1 - U(x) + xU'(x)} = \infty. \tag{44}$$

Since (43) should be invertible, we also have a constraint on $U(x)$,

$$1 - U(x) + xU'(x) > 0 \qquad \text{for} \qquad x > 0. \tag{45}$$

With the transformation rule given in (42) and (43), $H(y)$ may be thought as a dual function of $U(x)$. It can be shown without difficulty that Eq. (31) with (30) is indeed the inverse transform of that defined in (42), (43).

Now we note that Eq. (30), (31) are very suggestive of the following asymptotic form for$F$:

$$F = const. + c \cdot \epsilon g \tag{46}$$

with

$$c = \int_0^\infty \frac{H(y)}{y^2} dy. \tag{47}$$

Using unitarity, we show that this is indeed the case if $H(y)$ has a smooth asymptotic limit, either finite or infinite. This means that $g\bar{N}$ also has a smooth asymptotic limit, as can be seen from (25). Now unitarity applied to Eq. (31) prohibits $H(y)$ from approaching to $-\infty$ asymptotically, and thus the asymptotic limit of $g\bar{N}$ should be finite. Now note that $x(y)$ defined in (43) is a monotonically increasing function and thus so is

$$\ln y - H(y). \tag{48}$$

This, combined with the unitary condition on the asymptotic limit of $H(y)$ mentioned above, implies that the integral in (47) is rapidly convergent in the asymptotic region and thus c is finite.

Let us now consider the integral term in (31),

$$I(\epsilon g) \equiv \int_0^{y_0(\epsilon g)} \frac{H(y)}{y^2} dy \tag{49}$$

with $y_0(\epsilon g)$ defined through (30). Expanding I at $\epsilon g = \infty$, we have

$$I(\epsilon g) = c + d' \cdot \frac{1}{\epsilon g} + \cdots \tag{50}$$

where

$$d' = -\lim_{y\to\infty} \frac{e^{-H(y)}H(y)}{1 - yH'(y)}. \tag{51}$$

Substituting (50) into (31), we find the asymptotic form for F:

$$F \to d + c \cdot \epsilon g \tag{52}$$

with

$$d = \lim_{y\to\infty} \left(e^{-H(y)} (1 + H(y)) - \frac{e^{-H(y)}H(y)}{1 - yH'(y)} \right). \tag{53}$$

Note that unitarity requires

$$c \geq 0. \tag{54}$$

When $c = 0$, F approaches to a constant. Substituting the asymptotic form of $U(x)$,

$$U(x) \to 1 - d - cx \tag{55}$$

into (45), we find

$$d \geq 0. \tag{56}$$

Let us now show that

$$F(x) > 0 \quad \text{for} \quad x > 0. \tag{57}$$

First note that

$$F(0) = 1, \qquad F(\infty) \geq 0. \tag{58}$$

To prove (57), let us suppose that at some finite value of x, $F(x) \leq 0$. Then $F(x)$ must have a minimum $F(x_0)$ at a finite x_0 satisfying

$$F(x_0) \leq 0. \tag{59}$$

However this would contradict (45), because

$$U'(x_0) = 0. \tag{60}$$

Thus the conclusion in (57) follows. Of course, this conclusion does not exclude the possibility of observing baryon number violation in high energy scatterings, because the physical value of g is finite and $F(\epsilon g)$ could have a value close to zero.

Now a speculation on the asymptotic behavior of $H(y)$. We see in Fig.4 that $H(y)$ oscillates around zero. It is tempting to assume that $H(y)$ converges to zero asymptotically. One may doubt this on the observation that the amplitude between the origin and the first node of $H(y)$ is much smaller than that between the first and the second node. However this kind of behavior is expected to satisfy the unitarity condition (54); Since the former is negative, and the integral in (47) is heavily weighted toward the origin, there must be a large positive region for $H(y)$. If $H(y)$ indeed converges to zero, the constant d becomes unity and the anomalous cross section is exponentially suppressed at asymptotic energies. Note that then $g\bar{N}$ also converges to its vacuum value that is unity. If $H(y)$ begins to converge at not too large y, higher order terms of $H(y)$ could eventually reveal the symptom. Thus it is a very interesting problem to calculate higher order terms of $H(y)$ and see the functional behavior.

6 Conclusion

We gave a new formulation of the anomalous cross section from the viewpoint of perturbation theory, and showed that order by order our method consistently gives better high energy behavior for the anomalous cross section. In our formalism the energy expansion of the anomalous cross section is an intermediate step toward the ϵ/n expansion. Using unitarity, we argued that under a plausible condition the asymptotic form of F is either linear in energy or a nonnegative constant, and that $F(\epsilon g) > 0$ for all energies. A speculation on the asymptotic behavior of $H(y)$ was made.

Appendix

Let us consider the integral in (12),

$$e^{W(N)} \equiv \int_{a-i\infty}^{a+i\infty} dz\, e^{-z+K(z)+N\ln z} = \int dz\, e^{-\mathcal{F}}. \tag{61}$$

We would like to show that in the weak coupling limit W is exactly given by the saddle point approximation in (13). This can be shown conveniently in the Feynman diagram technique as employed in the proof that the leading Borel singularity of instanton-induced amplitudes is determined by the saddle point approximation [7]. To simplify the argument, let us assume $K(z)$ is given by the leading term in (3),

$$K(z) = \frac{1}{2}(3\epsilon)^{\frac{4}{3}} z^{-\frac{1}{3}} \equiv C_o z^{-\frac{1}{3}}. \tag{62}$$

Adding higher order terms should be trivial. Expanding $\mathcal{F}$ about the saddle point $z(N)$,

$$\begin{aligned} e^W &= \int_{-\infty}^{\infty} d\eta \exp\left\{-\mathcal{F}(z(N)) - \frac{1}{2}\mathcal{F}''(z(N))\eta^2 - \sum_{n=3} \frac{\mathcal{F}^{(n)}(z(N))}{n!}\eta^n\right\} \\ &= \exp\left\{-\mathcal{F}(z(N)) - \frac{1}{2}\ln|\mathcal{F}''(z(N))| + \sum(\text{ bubble diagrams})\right\}, \end{aligned} \tag{63}$$

where η-integration is over the real axis. Let us call the vertex with i number of legs the i-th vertex. Now consider a vacuum bubble diagram B with n_i number of the i-th vertex, and I number of internal lines. Then

$$\sum_i i \cdot n_i = 2I \tag{64}$$

and

$$B \sim \frac{\Pi_i \left[\mathcal{F}^{(i)}(z(N))\right]^{n_i}}{[\mathcal{F}^{(2)}(z(N))]^I}$$

$$= N^{-\sum_{i=3}\left(\frac{i}{2}-1\right)n_i} \frac{\prod_i \left((i-1)!\right)^{n_i} \left(1 - \frac{C_o \Gamma(i+1/3)}{N\Gamma(1/3)\Gamma(i)} z(N)^{-\frac{1}{3}}\right)^{n_i}}{\left(1 - \frac{4C_o}{9N} z(N)^{-\frac{1}{3}}\right)^{I}}. \tag{65}$$

Since at the saddle point $N = \bar{N}$,

$$\frac{C_o}{N} z(\bar{N})^{-\frac{1}{3}} \sim \left(\frac{\epsilon}{\bar{N}}\right)^{\frac{4}{3}} \sim O(1), \tag{66}$$

we have

$$B \sim O\left(N^{-\sum_{i=3}\left(\frac{i}{2}-1\right)n_i}\right). \tag{67}$$

However, to survive the weak coupling limit B must be at least of $O(N)$, which is impossible. Therefore, the corrections to the saddle point approximation generates negligible terms in the weak coupling limit. Similarly from (66)

$$\ln \mathcal{F}''(z(N)) + \ln N = O(1) \tag{68}$$

in the weak coupling limit.

References

[1] T. Lee, Large-order behavior and nonperturbative effects, FERMILAB-PUB-94/066-T,(to appear in Nucl. Phys. B).

[2] M. Mattis, Phys. Rep. 214 (1992) 159.

[3] I. Balitsky and A. Schaefer, Nucl. Phys. B 404 (1993) 639; P.G. Silvestrov, Phys. Lett. B 323 (1994) 24.

[4] W.Y. Crutchfield II, Phys. Rev. D 19 (1979) 2370.

[5] T. Lee, in Proceedings of the DPF'94 meeting, Albuquerque, NM, 1994(World Scientific, Singapore, to be published).

[6] V.V. Khoze and A. Ringwald, Nucl. Phys. B 356 (1991) 351

[7] T. Lee, Phys. Lett. B 331 (1994) 395.

[8] S. Yu. Khlebnikov, V.A. Rubakov, and P.G. Tinyakov, Nucl. Phys. B 350 (1991) 441; A. H. Mueller, Nucl. Phys. B 353 (1991) 44.

Figure Captions

Fig. 1: F and F_0 versus $3\epsilon g$ to the leading order. Solid and dashed lines are for F and F_0 respectively.

Fig. 2 : The function $3y\exp(-H(y))$ versus $3y$ to the second order. Dashed line is for $3y\exp(-H(y))$.

Fig. 3: F and F_0 versus $3\epsilon g$ to the second order. Solid and dashed lines are for F and F_0 respectively.

Fig. 4 a, 4 b: The functions $H(y), 3y\exp(-H(y))$ versus $3y$ for $\lambda = -1$ and -0.6 respectively. Dashed and dot-dashed lines are for $H(y)$, $3y\exp(-H(y))$ respectively.

Fig. 5 a, 5 b: $g\bar{N}, F$ and F_0 versus $3\epsilon g$ to the third order for $\lambda = -1$ and -0.6 respectively. Note the large difference between F and F_0 at high energies. For $\lambda = -1$, $g\bar{N}$ oscillates around unity. Dot-dashed lines are for $g\bar{N}$, the solid and dotted lines are for F and F_0 respectively.

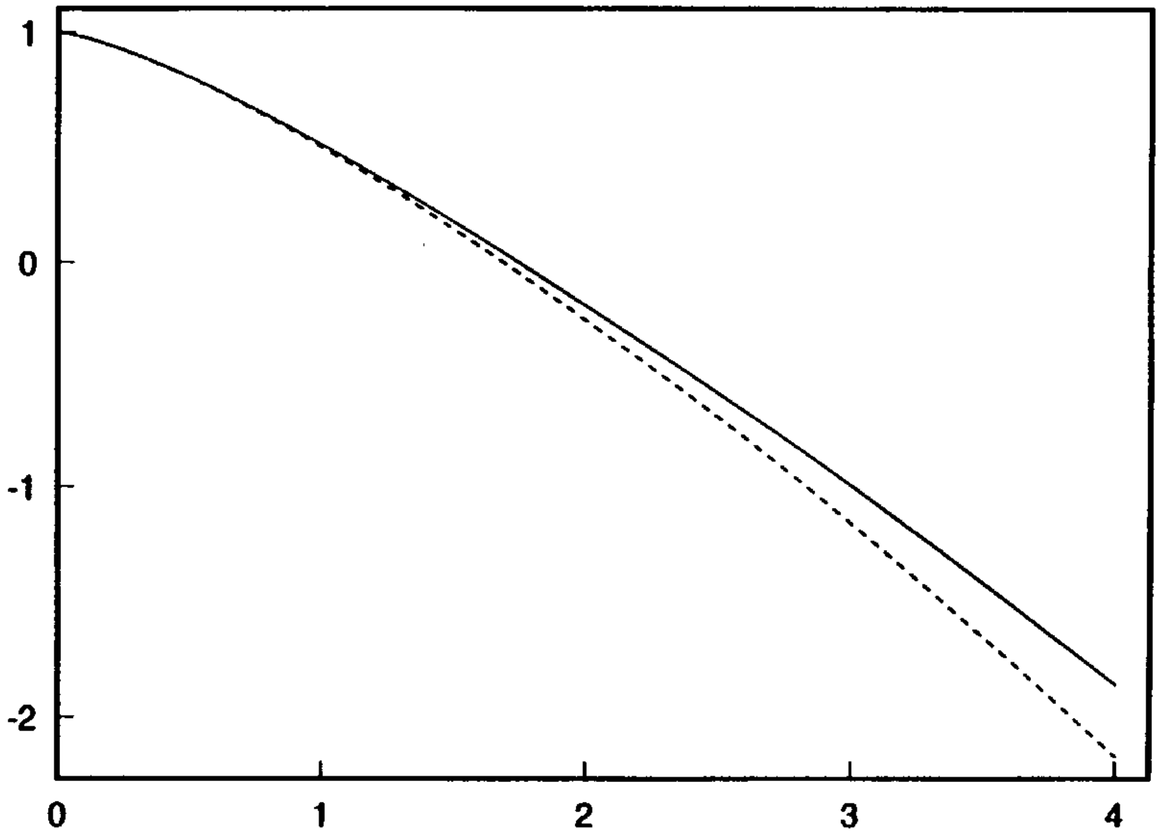

Fig.1

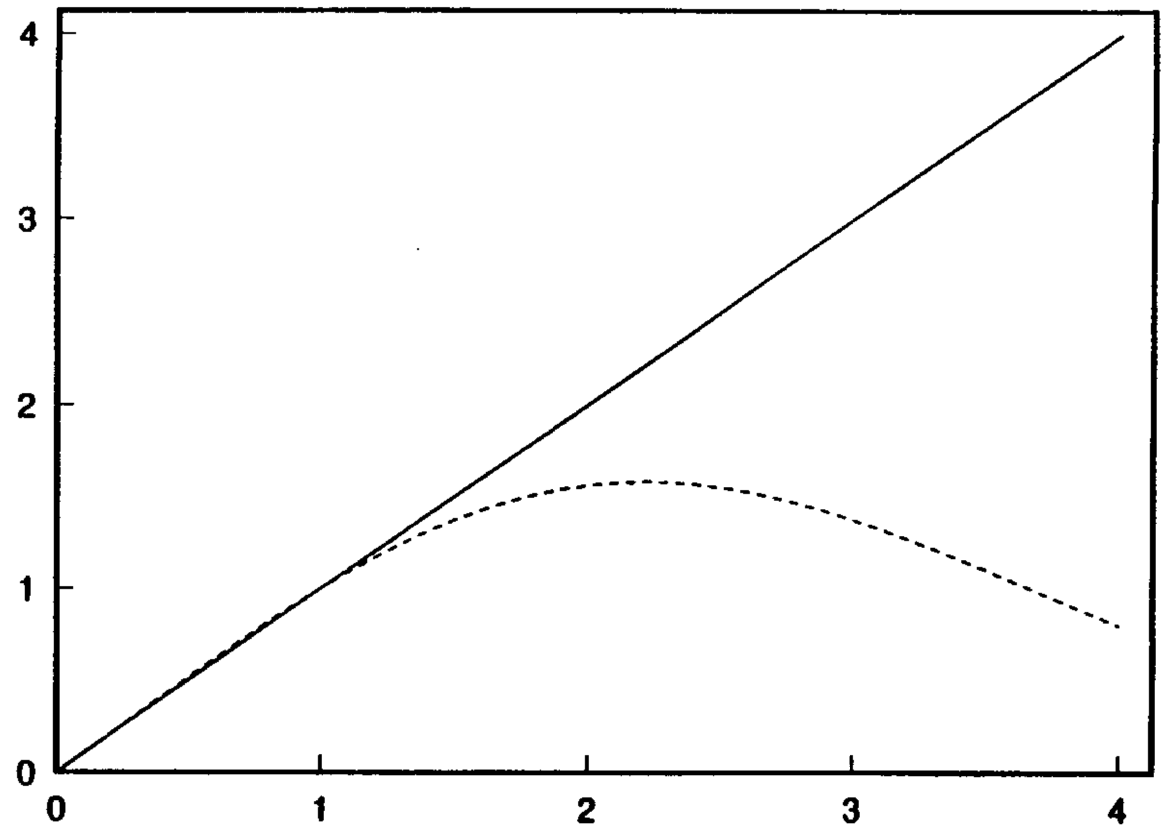

Fig. 2

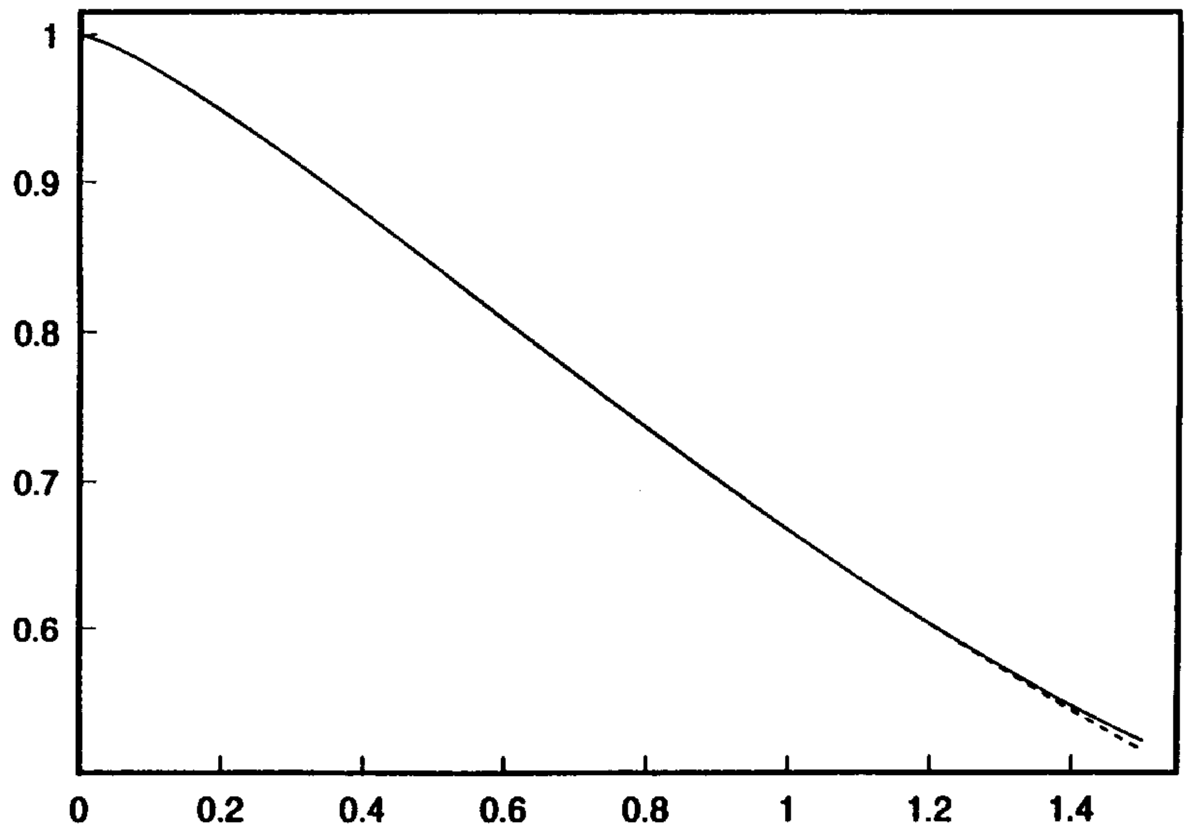

Fig. 3

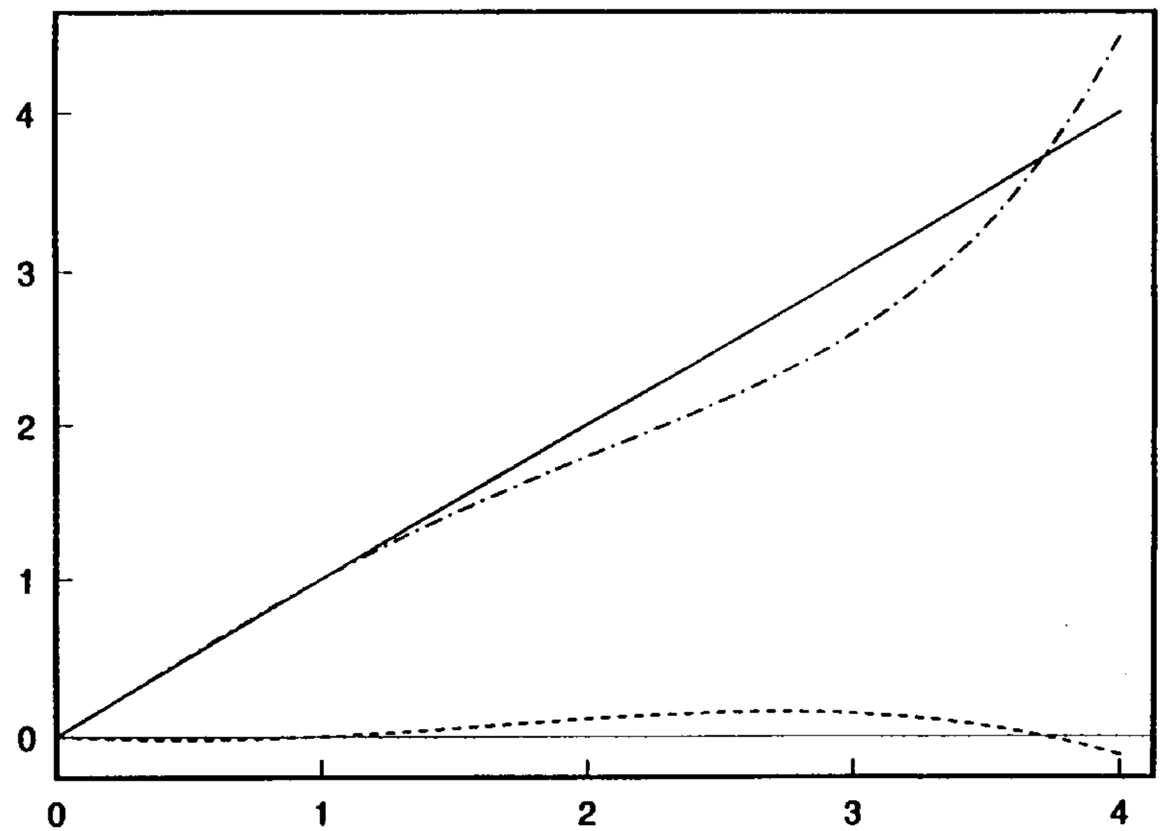

Fig. 4 a

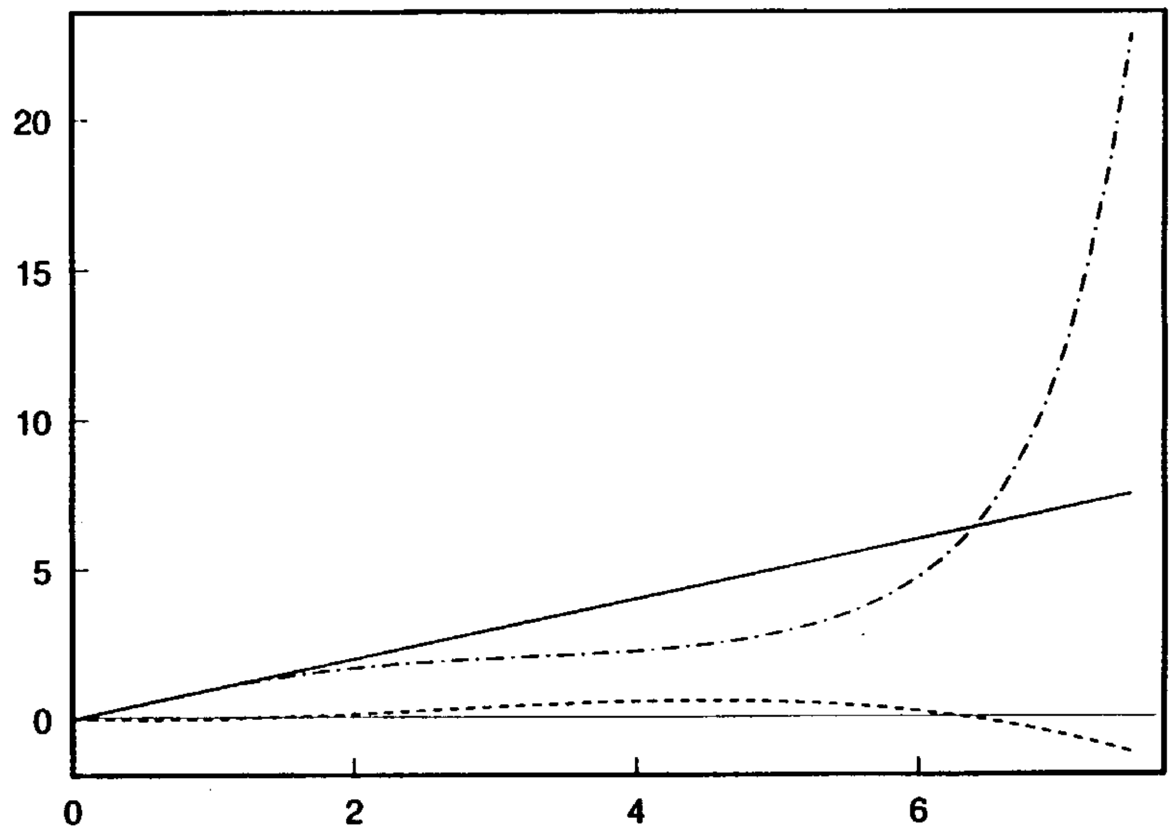

Fig. 4 b

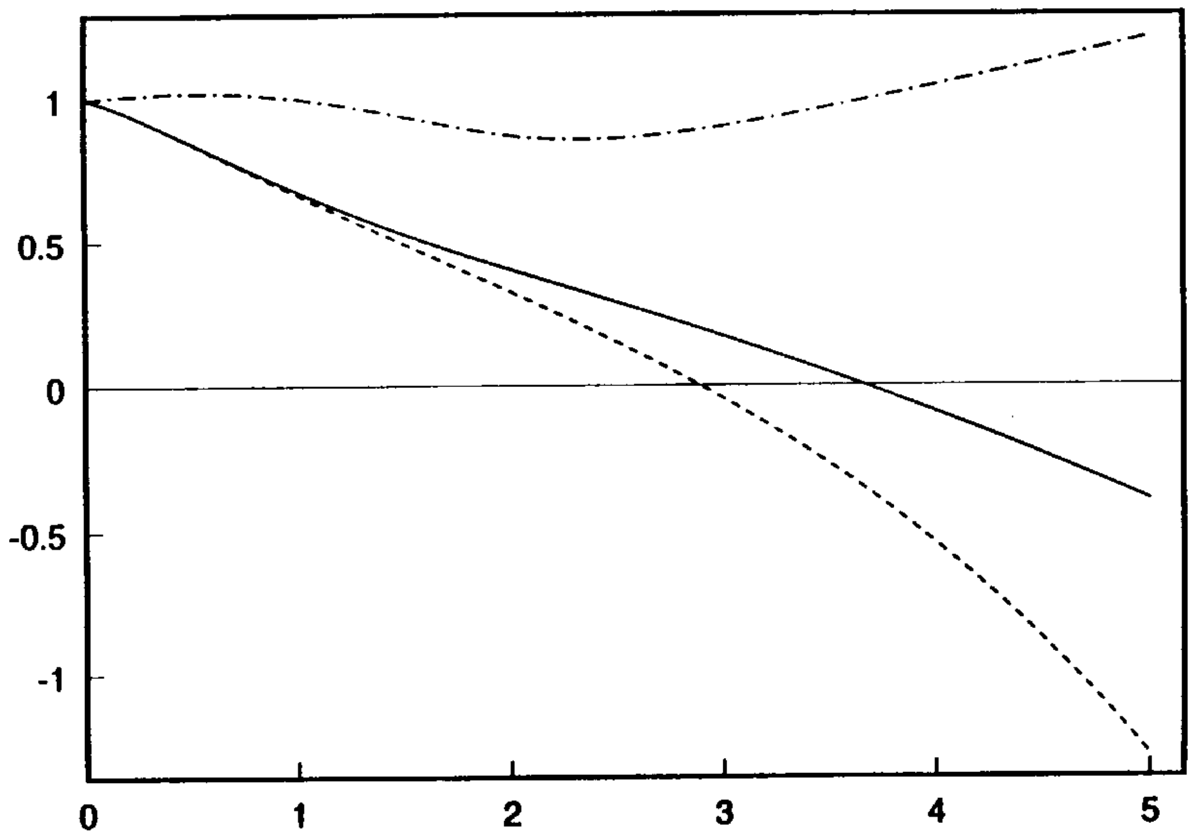

Fig. 5 a

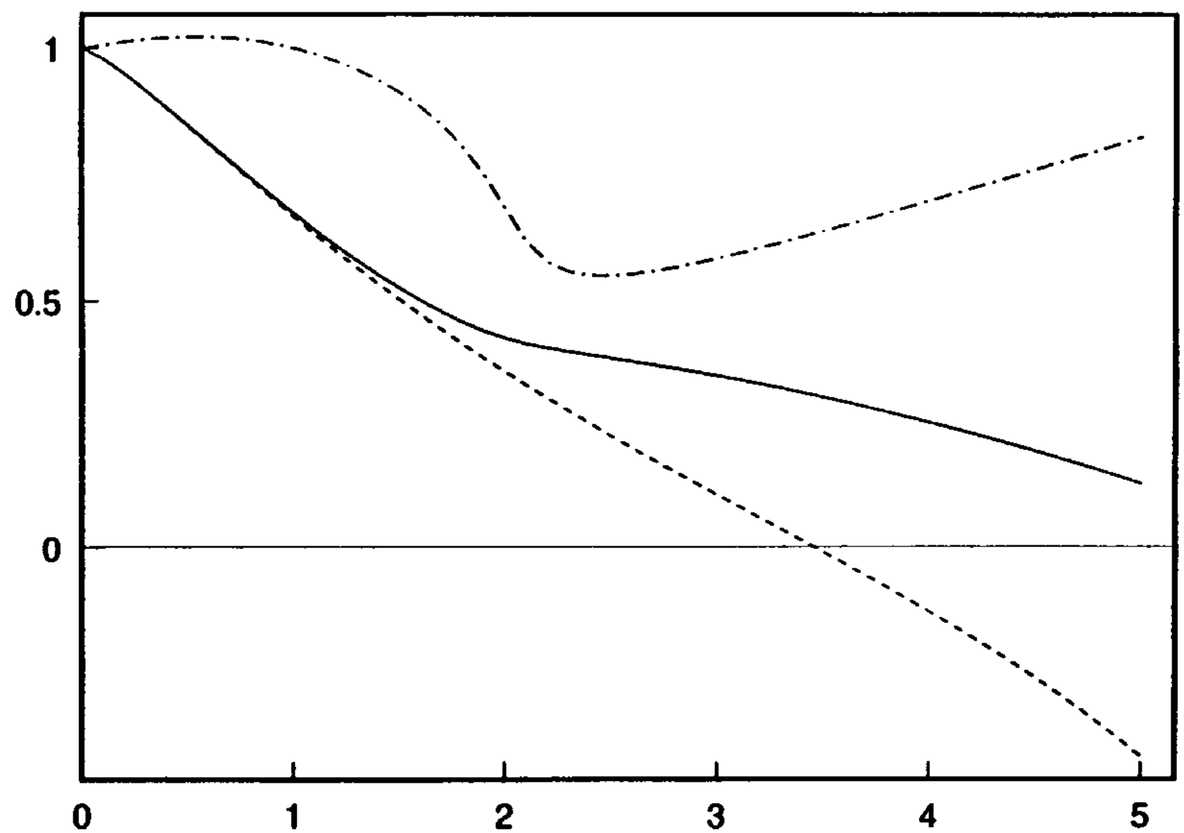

Fig. 5 b