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CP violation in the cubic coupling of neutral gauge bosons

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Abstract

We investigate the *CP* violating form factor of the ZZZ and $ZZ\gamma$ vertices in the pair production of Z^0 bosons. Useful observables in azimuthal distributions are constructed to probe *CP* nonconservation which may originate from these vertices. A simple Two Higgs Model of *CP* violation is used as an illustration.

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In the near future, with the availability of experimental data at energy around the electroweak breaking scale, one expects to learn about the structure of the cubic and quartic self-interactions of gauge bosons. So far, these interactions have not been directly tested in any experiments.

One exciting possibility is that such interactions will give new insights on CP violation, whose physical origin has not been understood with satisfaction yet. The observation of CP violation in the kaon system can be explained in various ways within the framework of gauge theories, and choosing between them requires additional observation of CP violation. With this in mind, it is interesting to look for CP violating signals which may be induced by the self-interactions of gauge bosons. We discuss here one such possibility, where the coupling of three neutral gauge bosons has a CP violating term in it. We first did a model-independent discussion based on the most general form factors. Then a simple model, the two Higgs doublet model, is used as an illustration of how the form factors may arise in a realistic CP violating theory.

I. HELICITY AMPLITUDES.

Such CP -odd term is indeed allowed in general on fundamental grounds, as is obvious from the general parametrization of the cubic coupling of gauge bosons [1–3]. Most theoretical studies along this direction have been done [4–6] only for the process $e^-e^+ \rightarrow W^-W^+$. The effect of CP violation in $e^-e^+ \rightarrow Z^0Z^0$ has not been thoroughly carried through [7] and there is a need of detailed analysis. This motivates us to perform a careful model-independent study. In Section IV, a simple Two Higgs Model is used as an illustration. We follow the helicity formalism for the Z^0 pair production, $e^-(\sigma)e^+(\bar{\sigma}) \rightarrow Z^0(\lambda)Z^0(\lambda')$, outlined in Appendix D of Ref. [3]. Here we include explicitly effects from the form factors f_4 and f_5 which describe the vertex $V(P) \rightarrow Z(q)Z(q')$ for out-going on-shell Z^0 bosons, where the incoming particle V is either another Z -boson or a photon:

$$ie\Gamma_{V \rightarrow ZZ}^{\mu\alpha\beta} = ie \frac{s - m_V^2}{M_Z^2} \left[if_4^V (P^\alpha g^{\mu\beta} + P^\beta g^{\mu\alpha}) + if_5^V \epsilon^{\mu\alpha\beta\rho} (q - q')_\rho \right] \quad (V = Z, \gamma), \quad (1)$$

where $s = P^2$. Note that f_4 term is CP -odd. The f_5 term, although CP -even, is included for completeness. The helicity amplitudes are given by

$$\mathcal{M}_{\sigma, \bar{\sigma}; \lambda, \lambda'}(\Theta) = 4\sqrt{2} e^2 d_{\Delta\sigma, \Delta\lambda}^{\max(|\Delta\sigma|, |\Delta\lambda|)}(\Theta) \left[\frac{(g_{\Delta\sigma})^2 A_{\lambda, \lambda'}(\Theta)}{4\beta^2 \sin^2 \Theta + \gamma^{-4}} + \sum_{i=4,5} \gamma^2 (g_{\Delta\sigma} f_i^Z - f_i^\gamma) A_{\lambda, \lambda'}^{(i)} \right]. \quad (2)$$

The kinematic variables are defined as usual, $\gamma^{-2} = 1 - \beta^2 = 4M_Z^2/s$. The amplitude for the initial helicity configuration $\bar{\sigma} = \sigma$ is highly suppressed due to helicity argument in the high energy limit $\sqrt{s} \gg m_e$. Therefore we are only interested in the cases for which $\Delta\sigma \equiv \frac{1}{2}(\sigma - \bar{\sigma}) = \pm 1$. The relevant Wigner d functions appearing in Eq.(2) are listed below:

$$d_{1, \pm 2}^2(\Theta) = -d_{-1, \mp 2}^2(\Theta) = \pm \frac{1}{2}(1 \pm \cos \Theta) \sin \Theta,$$

$$\begin{aligned}
d_{1,\pm 1}^1(\Theta) &= d_{-1,\mp 1}^1(\Theta) = \frac{1}{2}(1 \pm \cos \Theta), \\
d_{1,0}^1(\Theta) &= -d_{-1,0}^1(\Theta) = -\frac{1}{\sqrt{2}} \sin \Theta.
\end{aligned} \tag{3}$$

In the standard electroweak model at the tree level, the elements $A_{\lambda,\lambda'}(\Theta)$ come from the t -channel exchange diagram. The electron couplings $g_{\Delta\sigma}$ to the Z^0 boson are specified by

$$\begin{aligned}
g_- = g_L &= \left(\frac{1}{\sin \theta_{W'} \cos \theta_{W'}} \right) \left(-\frac{1}{2} + \sin^2 \theta_{W'} \right), \\
g_+ = g_R &= \left(\frac{1}{\sin \theta_{W'} \cos \theta_{W'}} \right) (\sin^2 \theta_{W'}).
\end{aligned} \tag{4}$$

After simplification, we summarize the result for various cases $\Delta\lambda = \lambda - \lambda'$ as follows,

$\Delta\lambda$	λ	λ'	$A_{\lambda\lambda'}(\Theta)$	$A_{\lambda\lambda'}^{(4)}$	$A_{\lambda\lambda'}^{(5)}$
± 2	$\pm \mp$		$-\sqrt{2}(1 + \beta^2)$	0	0
± 1	± 0		$(1/\gamma)[\Delta\sigma\Delta\lambda(1 + \beta^2) - 2 \cos \Theta]$	$+i\gamma\beta$	$-\Delta\lambda\gamma\beta^2$
± 1	$0 \pm$		$(1/\gamma)[\Delta\sigma\Delta\lambda(1 + \beta^2) - 2 \cos \Theta]$	$-i\gamma\beta$	$-\Delta\lambda\gamma\beta^2$
0	$\pm \pm$		$-(1/\gamma^2) \cos \Theta$	0	0
0	0 0		$-(2/\gamma^2) \cos \Theta$	0	0

(5)

When the kinematic variables of the two identical Z^0 boson are interchanged, i.e.,

$$(\lambda, \lambda') \leftrightarrow (\lambda', \lambda), \quad \Theta \leftrightarrow \pi - \Theta, \quad \Phi \leftrightarrow \pi + \Phi, \tag{6}$$

the amplitude is unchanged because of the Bose symmetry, if one includes a negative sign coming from the azimuthal Φ rotation $\exp(i\Delta\sigma\pi)$.

The usual CP transformation is

$$(\lambda, \lambda') \rightarrow (-\lambda, -\lambda'), \quad \Theta \rightarrow \pi - \Theta, \quad \Phi \rightarrow \pi + \Phi. \tag{7}$$

However, we can simplify this CP transformation by incorporating the Bose symmetry in Eq.(6). The resulting CP transformation becomes

$$(\lambda, \lambda') \rightarrow (-\lambda', -\lambda), \quad \Theta, \Phi \text{ unchanged.} \tag{8}$$

The situation now becomes very similar to our previous analysis [6] in the process $e^-e^+ \rightarrow W^-W^+$.

If CP is conserved (when f_i 's are turned off), we have the following relation for the amplitudes in our phase convention:

$$\mathcal{M}_{\sigma,\bar{\sigma};\lambda,\lambda'}(\Theta) = \mathcal{M}_{\sigma,\bar{\sigma};-\lambda',-\lambda}(\Theta). \tag{9}$$

This equality will be destroyed by the presence of CP violating form factors f_i in channels $(\lambda, \lambda') = (0, \pm)$ or $(\pm, 0)$.

II. SPIN-DENSITY MATRICES.

To avoid studying the complicated event topology in the 4-fermion final configuration from the decays of the Z^0 pair, we concentrate our attention to the decay of a single Z^0 . This strategy is equivalent to the study of the density matrix for one of the Z^0 bosons.

We only look at the Z^0 boson at the scattering angle Θ and temporarily ignore the recoiling one, which is considered as produced at the scattering angle $\pi - \Theta$. The polar angle ψ and the azimuthal angle ϕ are defined in the Z^0 rest frame for the lepton ℓ^- in the decay $Z^0 \rightarrow \ell^- \ell^+$. We define the axes of the rest frame of Z^0 as follows. The z -axis is along the direction of motion of Z^0 in the $e^- e^+$ c.m. frame. The x -axis lies on the reaction plane and toward the direction where Θ increases. The y -axis is given by the right-hand rule.

The angular distribution of ℓ^- from the $Z^0 \rightarrow \ell^- \ell^+$ decay is specified by the spin density matrix $\rho_{i,j}$ of the Z^0 boson.

$$\rho(\Theta)_{i,j} = \mathcal{N}(\Theta)^{-1} \sum_{\sigma, \bar{\sigma}, \lambda'} \mathcal{M}_{\sigma, \bar{\sigma}; i, \lambda'}(\Theta) \mathcal{M}_{\sigma, \bar{\sigma}; j, \lambda'}^*(\Theta). \quad (10)$$

Here \mathcal{N} is the normalization such that $\text{Tr} \rho = 1$. ρ is hermitian by definition. The normalized distribution for ℓ^- is given by

$$\begin{aligned} \frac{dN(\ell^-, \Theta)}{d\phi d\cos\psi} = & \frac{1}{4\pi} \frac{3}{4} \sum_{h=\pm} w_h \left[(1 + h \cos\psi)^2 \rho(\Theta)_{++} + (1 - h \cos\psi)^2 \rho(\Theta)_{--} + 2\rho(\Theta)_{00} \sin^2\psi \right. \\ & - 2\sqrt{2} \text{Re} \rho(\Theta)_{+,0} (1 + h \cos\psi) \sin\psi \cos\phi + 2\sqrt{2} \text{Im} \rho(\Theta)_{+,0} (1 + h \cos\psi) \sin\psi \sin\phi \\ & - 2\sqrt{2} \text{Re} \rho(\Theta)_{-,0} (1 - h \cos\psi) \sin\psi \cos\phi - 2\sqrt{2} \text{Im} \rho(\Theta)_{-,0} (1 - h \cos\psi) \sin\psi \sin\phi \\ & \left. + 2\text{Re} \rho(\Theta)_{+,-} (1 - \cos^2\psi) \cos 2\phi - 2\text{Im} \rho(\Theta)_{+,-} (1 - \cos^2\psi) \sin 2\phi \right]. \end{aligned} \quad (11)$$

The two contributions come from helicity configurations $\ell_R^-(h = 1)$ and $\ell_L^-(h = -1)$, with different weights,

$$w_- = g_L^2 / (g_L^2 + g_R^2), \quad w_+ = g_R^2 / (g_L^2 + g_R^2), \quad w_- + w_+ = 1. \quad (12)$$

In our present phase convention, if CP were conserved (*i.e.* when $f_i = 0$), we would have the following identities.

$$\rho(\Theta)_{\lambda, \lambda'} = \rho(\pi - \Theta)_{-\lambda, -\lambda'}, \quad (13)$$

based on the transformation in Eq.(7). Similar expressions were first noticed in Ref. [4] on the process $e^- e^+ \rightarrow W^- W^+$ and in Ref. [8] on the process $e^- e^+ \rightarrow t\bar{t}$.

III. CP VIOLATING OBSERVABLES

Under CP conjugation, we change variables $\Theta \rightarrow \pi - \Theta$, $\psi \rightarrow \pi - \psi$, and $\phi \rightarrow -\phi$. The distribution in Eq.(11) is transformed into itself if we assume CP conservation as in

Eq.(13). In the presence of the CP -violating term f_4 , our analysis of CP -violating observables in Ref. [6] can be easily applied here.

However, as the coupling of $\ell\bar{\ell}Z^0$ is almost purely axial-vectorial, there is approximate charge symmetry C , which assigns this vertex even C -parity, with the f_4 term also even as well. Any C -odd observable will be suppressed.

We find out that the most prominent effect of CP nonconservation resides in the elements $(+, -)$ or $(-, +)$ of the spin-density matrix,

$$\text{Im } \rho(\Theta)_{+,-} - \text{Im } \rho(\pi - \Theta)_{-,+} = \frac{32e^4}{\mathcal{N}(\Theta)} \sum_{\Delta\sigma=\pm} (g_{\Delta\sigma})^2 (\Delta\sigma) \gamma^2 (\beta + \beta^3) \sin^2 \Theta \frac{\text{Re}(f_4^\gamma - g_{\Delta\sigma} f_1^Z)}{4\beta^2 \sin^2 \Theta + \gamma^{-4}}. \quad (14)$$

This particular location in the density matrix produces the azimuthal dependence in the form of $\sin 2\phi$. If we integrate ψ and ϕ over quadrants, we expect that CP nonconservation appears in the folded asymmetry, $\mathcal{A}''(\Theta)$, which is

$$\frac{[dN(\ell, \Theta, \text{I+III}) + dN(\ell, \pi - \Theta, \text{I+III})] - [dN(\ell, \Theta, \text{II+IV}) + dN(\ell, \pi - \Theta, \text{II+IV})]}{[dN(\ell, \Theta, \text{I+II+III+IV}) + dN(\ell, \pi - \Theta, \text{I+II+III+IV})]}. \quad (15)$$

Here the range of the azimuthal angle has been divided into four usual quadrants I,II,III and IV. It turns out that this observable \mathcal{A}'' is C -even and thus it is not subjected to the suppression from the approximate C symmetry.

We can show that

$$\mathcal{A}''(\Theta) = -\frac{1}{\pi} (\text{Im } \rho(\Theta)_{+,-} - \text{Im } \rho(\pi - \Theta)_{-,+}). \quad (16)$$

In Fig. 1, we show the CP -odd asymmetry in the density matrix versus the scattering angle Θ per unit of small $\text{Re } f_4^Z$ at various energies, $\sqrt{s} = 200, 250,$ and 300 GeV. Observation of this asymmetry is a genuine signal CP violation, as it is not faked by the final state interaction.

It is interesting to note that we do not need to know the charge of ℓ as the events are collected over quadrants I+III or II+IV. We can use this fact to apply our formula even to the larger sample of jet events from the Z^0 pair without tagging the charges of the primary partons. Our formalism can be easily translated for the process $q\bar{q} \rightarrow Z^0 Z^0$ in the hadron collider.

IV. TWO HIGGS MODEL

Cubic couplings among neutral gauge bosons do not appear at the tree level in the standard model gauge group of $SU(2)_L \times U(1)_Y$. But they can be induced at the loop level. In the minimal standard model with just one Higgs doublet, such amplitudes do not have any CP violation even at the one-loop level, as will be clear from our analysis below. We therefore perform the calculation of CP violating effects in these trilinear couplings when there are two Higgs doublets [10] present in the model, which is a popular model in its own right. Among the possibilities which open up with the two doublets are: spontaneous

CP violation [11], incorporation of the Peccei-Quinn symmetry [12] to solve the strong CP problem, and incorporation of supersymmetry.

At the one-loop level, cubic coupling obviously comes from triangle diagrams. If the internal lines are fermions, no CP violating effect is generated at the one-loop level, because the Z or photon couplings with fermions are flavor diagonal and CP conserving. There are also triangle diagrams with internal W lines. In the Feynman-t'Hooft gauge, it can be shown that they do not contribute to the form factors as shown in Eq. (1). Thus, for our purpose, we need to calculate only the diagrams involving Higgs bosons in the loop. Obviously, such diagrams can never involve the antisymmetric ε -symbol, so one can only obtain a non-zero f_4^Z . This term has been shown to be non-zero for WWZ coupling at the one-loop level for the model at hand [13]. We want to extend their calculation for the case of V^*ZZ couplings, where V^* can be either an off-shell Z -boson or photon, and the other two Z -bosons are assumed to be on-shell.

To set up the notation, we call the two Higgs multiplets to be φ_1 and φ_2 . Usually, they are assumed to have special transformation properties with respect to some discrete symmetries in order to avoid flavor changing neutral currents at the tree level. We assume that such discrete symmetries are not imposed on the soft terms in the Higgs potential, otherwise CP violation would be eliminated in the Higgs sector of the model. Without any loss of generality, we can take the vacuum expectation values (VEVs) of φ_1 and φ_2 to be $v_1 \exp(i\vartheta)$ and v_2 . One can then define a linear combination φ of the two multiplets which has a VEV $v = \sqrt{v_1^2 + v_2^2}$, and the orthogonal one, φ' , has a vanishing VEV. The components of these doublets can then be written as

$$\varphi = \begin{pmatrix} w^+ \\ \frac{1}{\sqrt{2}}(v + \phi_1 + iz) \end{pmatrix}, \quad \varphi' = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(\phi_2 + i\phi_3) \end{pmatrix}. \quad (17)$$

The fields shown here are complex combinations of the fields in the φ_1 - φ_2 basis. The components w^\pm and z are eaten up by the gauge bosons and disappear from the physical spectrum. There are four physical spinless bosons in the model. One of them is the complex field H^+ . The other three are, in general, superpositions of the fields ϕ_1 , ϕ_2 , and ϕ_3 . We define the eigenstates by H_A , where

$$\phi_a = \sum_{A=1}^3 O_{aA} H_A, \quad (18)$$

O being an orthogonal mixing matrix.

The coupling of these neutral Higgs bosons with the Z -boson looks very simple in the ϕ -basis:

Vertex	Feynman rule	
$\phi_1(p) \xrightarrow{Z_\mu} z(q)$	$\frac{g}{2 \cos \theta_W} (p+q)_\mu$	(19)
$\phi_2(p) \xrightarrow{Z_\mu} \phi_3(q)$	$\frac{g}{2 \cos \theta_W} (p+q)_\mu$	

Using Eq. (18), it is trivial to rewrite these Feynman rules in terms of the mass eigenstates of neutral Higgs bosons:

Vertex	Feynman rule
$H_A(p) \xrightarrow{Z_\mu} z(q)$	$\frac{g}{2 \cos \theta_{11'}} O_{1A}(p+q)_\mu$
$H_A(p) \xrightarrow{Z_\mu} H_B(q)$	$\frac{g}{2 \cos \theta_{11'}} (O_{2A} O_{3B} - O_{2B} O_{3A})(p+q)_\mu$

(20)

Using the orthogonality of the mixing matrix O , we can write

$$O_{2A} O_{3B} - O_{2B} O_{3A} = \sum_C \epsilon_{ABC} O_{1C}, \quad (21)$$

which simplifies the form of the Z -coupling with two physical Higgs bosons. Notice that the Z -coupling between two physical Higgs bosons is necessarily flavor-changing, which opens up the possibility for CP violation at one-loop level. For the reason that the photon field preserves flavors at the tree level, there is no f_1^γ form factor at the one-loop calculation in the Two Higgs Model.

These cubic couplings appear in the triangle diagrams shown in Fig. 2. Notice that, in the figure, the Higgs boson lines have been denoted with subscripts i, j, k , which run from 0 to 3, where H_0 is identified with the unphysical Higgs z which appears as intermediate lines since we adopt the Feynman-t'Hooft gauge. A straightforward calculation now shows that the form-factor f_4^Z from these diagrams can be written in the form

$$ef_4^Z = -\frac{1}{128\pi^2} \left(\frac{e}{\sin \theta_{11'} \cos \theta_{11'}} \right)^3 \frac{M_Z^2}{P^2 - M_Z^2} \sum_{i,j,k} \lambda_{ijk} I(M_i, M_j, M_k). \quad (22)$$

Here, λ_{ijk} is a factor coming from vertices which will be discussed below, and the loop integral $I(M_i, M_j, M_k)$ is equal to:

$$2! \int \int (x-y) \ln \frac{\Lambda^2}{xM_i^2 + yM_j^2 + wM_k^2 - w(1-w)M_Z^2 - xyP^2 - i0^+} dx dy, \quad (23)$$

where the positive Feynman parameters x and y are restricted within the integration domain $x+y \leq 1$ and also $w = 1-x-y$. Λ is a cut-off which disappears in the expression for f_4^Z , as we will show below. When one of the particles denoted by i, j or k is the unphysical Higgs boson, the corresponding mass should be interpreted to be M_Z , because the propagator of the unphysical Higgs boson has a pole for this value of mass in the gauge we use. For future purposes, notice that

$$I(M_i, M_j, M_k) = -I(M_j, M_i, M_k), \quad (24)$$

which follows from the definition in Eq. (23).

Let us now discuss the factor λ_{ijk} . First, consider the case when all the Higgs bosons in the loop are physical ones. Due to the antisymmetry of the coupling of $H_A H_B Z_\mu$ from Eq. (20), all the Higgs bosons in the loop must be different. If, following the direction of the momentum arrow in Fig. 2, we encounter the mass eigenstates H_1, H_2 and H_3 in that order, it is easy to see that the factor coming from the vertices is

$$\lambda_{123} = O_{11} O_{12} O_{13} \equiv \lambda. \quad (25)$$

Obviously, there are three such diagrams, and their total contribution is

$$\lambda \{I(M_1, M_2, M_3) + I(M_2, M_3, M_1) + I(M_3, M_1, M_2)\} . \quad (26)$$

On the other hand, if we encounter the eigenstates in the reverse order, we obtain a factor $-\lambda$ from the vertices. However, this term will be multiplied by

$$\{I(M_2, M_1, M_3) + I(M_3, M_2, M_1) + I(M_1, M_3, M_2)\} .$$

By virtue of Eq. (24), the product of the two is the same as the contribution of Eq. (26).

Next we consider diagrams where one of the internal lines is the unphysical neutral Higgs boson z . Note that since there is no coupling of the Z -boson with two unphysical Higgs bosons, at most one internal line can be the unphysical Higgs boson. In this case, one can derive that

$$\lambda_{120} = \lambda_{230} = \lambda_{310} = -\lambda , \quad (27)$$

and the same value for any even permutation of subscripts, but opposite sign for an odd permutation. Therefore, the last factor of summation in Eq. (22) becomes

$$\begin{aligned} \sum_{i,j,k} \lambda_{ijk} I(M_i, M_j, M_k) = 2\lambda \{ & + I(M_1, M_2, M_3) + I(M_2, M_3, M_1) + I(M_3, M_1, M_2) \\ & - I(M_1, M_2, M_Z) - I(M_2, M_3, M_Z) - I(M_3, M_1, M_Z) \\ & - I(M_Z, M_1, M_2) - I(M_Z, M_2, M_3) - I(M_Z, M_3, M_1) \\ & + I(M_Z, M_1, M_3) + I(M_Z, M_2, M_1) + I(M_Z, M_3, M_2)\} . \quad (28) \end{aligned}$$

One can see that the cutoff Λ dependence is cancelled by pairs in Eq. (28). We also note that f_j^Z remains finite when $P^2 = M_Z^2$ as noted in Ref. [3].

Fig. 3 shows the extremely tiny size ($\sim 10^{-6}$) of f_j^Z for typical choices of parameters. We only use this Two Higgs Model as an illustration how CP violation occurs even in a purely bosonic sector.

V. CONCLUSION

At LEP II, the $Z^0 Z^0$ production cross-section is about 1 pb (See Fig. 4) for $\sqrt{s} = 200$ GeV which can provide about 500 $Z^0 Z^0$ pairs per year for the design luminosity of $5 \cdot 10^{31}$ cm^{-2} s^{-1} . As we have shown in the paper, it is possible to test CP symmetry in purely charged leptonic, purely hadronic or mixed channels of the two Z^0 boson decays. We may require that at least one of the Z^0 decays into the charged leptons in order to avoid backgrounds from the $W^+ W^-$ production. The branching ratio of a single Z^0 decaying into all charged leptonic channels ($e^+ e^- + \mu^+ \mu^- + \tau^+ \tau^-$) is about 10%. While the event statistics probably will not be large enough to test some of the popular alternative gauge models of CP violation, it is nevertheless sufficient to provide nontrivial constraints on the CP -odd form factors in the three gauge boson couplings.

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FIGURE CAPTIONS

- Fig. 1 The CP -odd asymmetry in the density matrix versus the scattering angle Θ per unit of $\text{Re } f_4^Z$ at various energies, $\sqrt{s} = 200, 250, \text{ and } 300 \text{ GeV}$.
- Fig. 2 Triangle diagrams with internal scalar lines which give rise to the Z^*ZZ coupling.
- Fig. 3 The size of $f_4^Z/O_{11}O_{12}O_{13}$ versus the lightest Higgs mass at $\sqrt{s} = 200 \text{ GeV}$, for the case $M_2=150 \text{ GeV}$, and $M_3 = 250 \text{ GeV}$. The real and the imaginary parts are given by the solid and the dashed lines respectively.
- Fig. 4 Differential cross-section $d\sigma/d\cos\Theta$ for $e^-e^+ \rightarrow Z^0Z^0$ at various energies $\sqrt{s}= 200$ (solid), 250 (dashed), and 300 GeV (dashed-dotted), predicted by the Standard Model. The horizontal lines indicate the level of the corresponding total cross-sections.

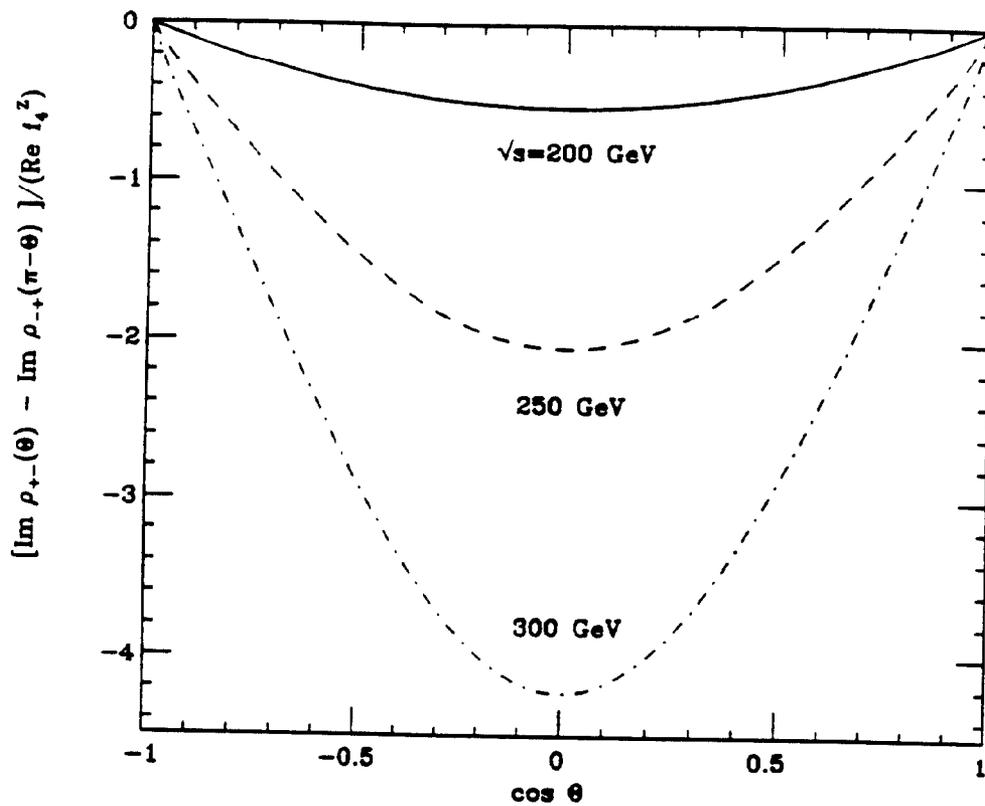


Fig. 1

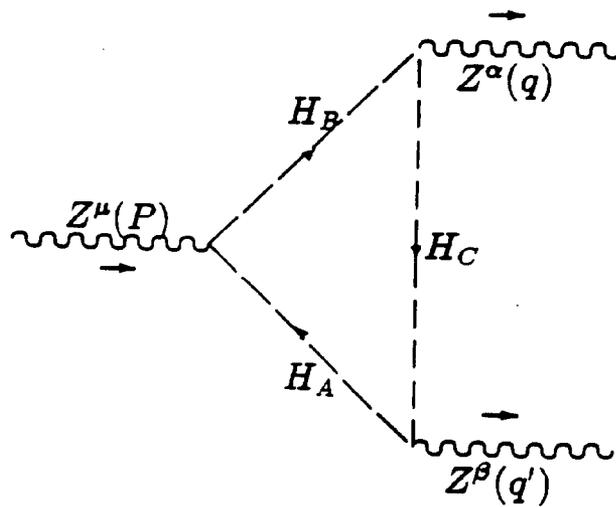


Fig. 2

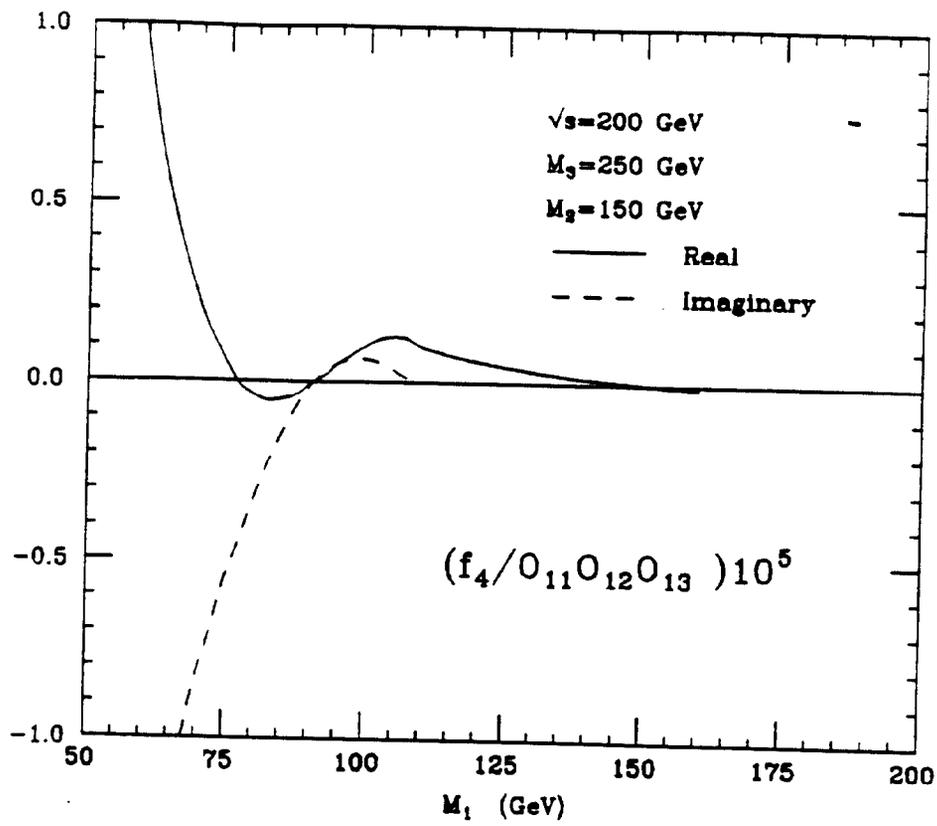


Fig. 3

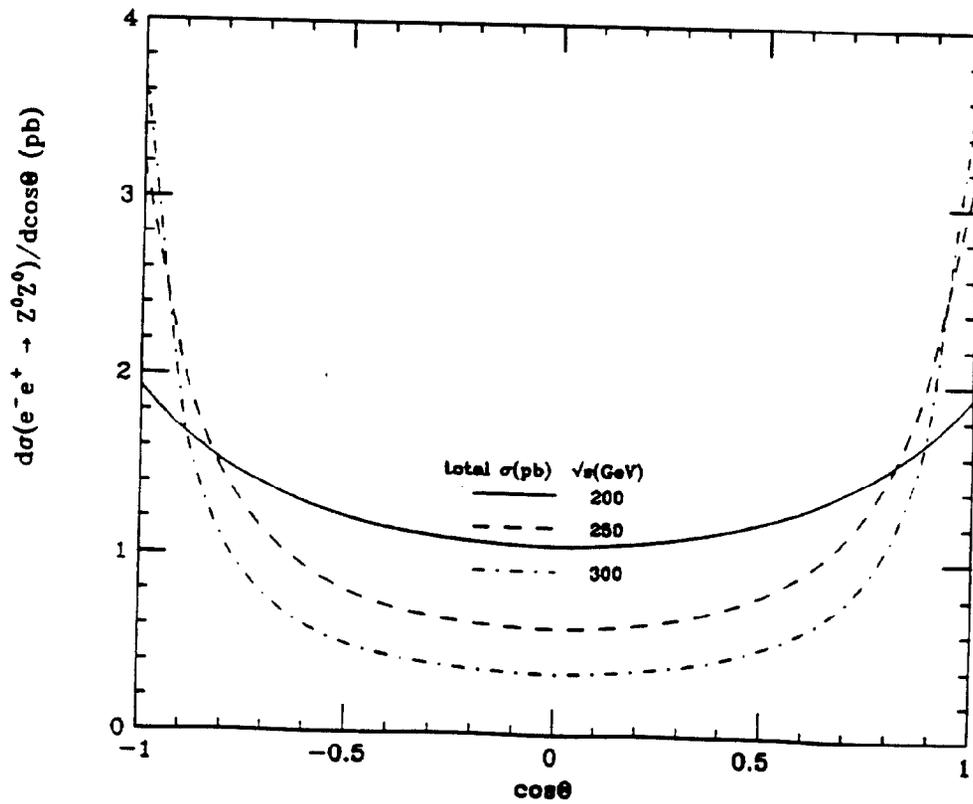


Fig. 4