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Fermion Masses from Superstring Models with Adjoint Scalars [1]

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Abstract

We explore the possibility of embedding supersymmetric GUT texture ideas into superstring models. We discuss the construction of GUT models using free fermionic strings. We find $SO(10)$ models with adjoint scalars, three generations of chiral fermions, and a fundamental Higgs with a Yukawa coupling to only the third generation.

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1 Introduction

Supersymmetric grand unified (GUT) texture models have had considerable success reproducing the existing data for fermion masses and mixings, starting from a restricted set of effective operators at the GUT scale. Two generic features of such schemes suggest that it may be desirable, or even necessary, to eventually embed these structures into full-fledged superstring models.

The first feature has to do with how SUSY GUT texture models generate small numbers. Fermion masses and mixings exhibit a number of distinct hierarchies, characterized by ratios in the range roughly 1/10 - 1/100 or even smaller, depending on how one (arbitrarily) chooses to parametrize them. A popular idea in texture models[1, 2] is that most of these small numbers arise, not from small Yukawa couplings, but rather from replacing Yukawas with higher dimension operators suppressed by powers of

$$\frac{\langle \Phi_{\mathbf{adjoint}} \rangle}{M_X} \quad , \tag{1}$$

where the numerator is the vev of an adjoint scalar, assumed to be the GUT scale of about 10^{16} GeV, while M_X is an even higher mass scale, assumed to be roughly 10^{17} GeV. Thus, for example, in one of the models of Anderson et al[3], the following $SO(10)$ invariant dimension 6 operator resides in the 2-3 component of the charged fermion mass matrices:

$$O_{23} = 16_2\, 10\, \frac{45^2_{B-L}}{45^2_1}\, 16_3 \quad . \tag{2}$$

The superheavy scale M_X is verging on the string scale, which is estimated as $M_{\mathbf{string}} \sim 5 \times 10^{17}\mathrm{GeV} \times g_{\mathbf{string}}$. At any rate, the use of nonrenormalizable operators sensitive to such high scales entails the risk of being overwhelmed by "Planck slop". Since superstring theory is the only known method of controlling Planck slop, it may be necessary to invoke strings in order to make valid statements about SUSY GUT textures.

The second feature of texture models which strongly suggests a string interpretation are the textures themselves. The usefulness of these schemes requires that the number of effective GUT scale operators which make the dominant contributions to the fermion mass matrices is rather few. In particular, the number of effective operators up to, say, dimension 6, should

be considerably less than the full set of effective operators allowed by the unbroken gauge symmetries at the GUT scale. This requires that there are nontrivial flavor-sensitive selection rules, which provide the extra contraints and thus the desired texture. Superstrings are an obvious source for such selection rules.

In string models, couplings correspond to correlators of vertex operators on the worldsheet, and their vanishing depends on worldsheet symmetries. Since many of these symmetries do not correspond to unbroken gauge symmetries in spacetime, the effective spacetime field theory below the string scale is generically subject to a number of selection rules. Of course, it is possible to avoid strings and obtain selection rules from extra broken $U(1)$'s, discrete symmetries, R symmetries, and the like. However superstring theory not only provides similar symmetries and selection rules, but also provides a more fundamental understanding of them. This is true even though we at present have no clue how nonperturbative string dynamics selects among the vast number of perturbatively degenerate string vacua.

To see why, observe that concrete phenomenological inputs can greatly narrow the range of viable string vacua. Given the SUSY GUT texture framework, as well as improved low energy data, one can obtain very specific guidance for string model building. What is required is that one translate the phenomenological constraints on the GUT scale effective Lagrangian into constraints on the world sheet symmetries of the string models. Then, for each particular string model, which fixes a choice of the string vacuum, one can hope to extract relationships between, say, the worldsheet structure that guarantees three light generations of chiral fermions, and the worldsheet structure that guarantees one of the observed hierarchies in the fermion mass matrices. Any such relationships (if valid) would be a profound new insight into particle physics. It is also important to note that such relationships depend only on order-of-magnitude computation, and thus do not (necessarily) require the ability to make precise determinations of string moduli.

2 Superstring GUT's

Historically, superstring model builders have made very little contact with conventional GUT's. One of the reasons for this is that, in string theory, gauge coupling unification occurs at the string scale independently of whether

or not matter fields assemble into GUT multiplets[4]. Thus the agreement between LEP data and minimal SUSY gauge coupling unification does not signify the existence of GUT's in a string context[5, 6, 7]. Many quasi-realistic string models thus attempt to identify the gauge coupling unification scale with the string scale, either by raising the former or by effectively lowering the latter, and do not obviously present two independent superheavy scales.

A technically important reason why so little effort has been put into superstring GUT's is the fact that most string model constructions simply do not allow the appearance of adjoint GUT scalars in the massless spectrum at the string scale. Thus conventional GUT symmetry-breaking vevs are not available. This is because in most string constructions the GUT gauge group is realized at Kac-Moody level one. The Kac-Moody level is a positive integer label needed to specify unitary irreducible representations (irreps) when Lie algebras are combined with worldsheet conformal symmetry to produce Kac-Moody algebras. For fixed level there is a constraint on the allowed irreps for massless matter fields. Thus at level one the allowed irreps of $SU(5)$ and $SO(10)$ are given by:

$$\begin{aligned} \mathrm{SU}(5) &: \ 1, 5, \bar{5}, 10, \bar{10} \\ \mathrm{SO}(10) &: \ 1, 10, 16, \bar{16} \quad . \end{aligned}$$

A more precise statement is that massless adjoint scalars are incompatible with the presence of massless chiral fermions in level one string models[8].

It is possible to build level one string models with unconventional structures which are similar to GUT's. The flipped $SU(5)$ model[9] is the most developed example of this; there the breaking of $SU(5)$ is accomplished by vevs of a Higgs 10, rather than the adjoint. Since the $SU(3)\times SU(2)$ singlet in the 10 has nonzero hypercharge, flipped $SU(5)$ requires an extra $U(1)$ and a nonstandard treatment of hypercharge. While it is not a pure GUT, flipped $SU(5)$ is a useful prototype for quasi-realistic models in the free fermionic superstring construction. Indeed we have borrowed some of its worldsheet structure in building the superstring GUT's described below.

Yet another reason why superstring GUT's have been neglected is the fear of exotics. A conventional superstring GUT requires a string construction with Kac-Moody level at least two. Higher levels are required if scalars in other non-fundamental irreps are desired. For example, to obtain a massless 126 of $SO(10)$ would require[10] a string model with $SO(10)$ at level $\geq$

five. Higher levels introduce the possibility that modular invariance of the string model will require the presence of other exotic scalars whenever, say, the adjoint or the 126 are present. Indeed the simplest Kac-Moody modular invariants, the left-right symmetric diagonal invariants, require that all *allowed* irreps do in fact appear in the spectrum. Thus one might worry that for $SO(10)$ at level two the 54 would appear in addition to the 45, while at level five the 54, 120, 144, etc would appear in addition to the 45 and 126. We will show below that this expectation is false for fermionic string models, and thus that there is no problem with unwanted exotics.

There are two basic examples of superstring GUT constructions in the literature. The first, due to Lewellen[8], is a free fermionic string model based on the minimal embedding of $SO(10)$ at Kac-Moody level two, with adjoint scalars and chiral fermions. The second, due to Font, Ibanez, and Quevedo[10], is an orbifold construction that realizes the GUT group G at level n starting with n copies of G at level one. We do not know of any particular reason to prefer one of these constructions over the other. However, we have chosen to base our exploration of superstring GUT's on Lewellen's free fermionic string model.

We have not addressed the question of how a GUT scale of 10^{16} GeV gets generated in our models, separate from the string scale. Understanding the hierarchy of scales in string theory presumably requires an understanding of strong dynamics.

3 Model Building

Four dimensional closed free fermionic string models are heterotic superstring vacua described by a worldsheet lagrangian for 64 real (Majorana-Weyl) free fermions, together with the bosons that embed the 4-d spacetime, and ghosts. This construction is described in detail in refs [11, 12, 13, 14]; we will, for the most part, follow the notation and conventions of refs [14, 8]. Models are conveniently specified by their one-loop partition functions; these involve a sum over spin structures:

$$Z_{\text{fermion}} = \sum_{\alpha,\beta} C^{\alpha}_{\beta}\, Z^{\alpha}_{\beta} \quad , \tag{3}$$

where the C^{α}_{β}'s are numerical coefficients, while α and β are 64-dimensional vectors labelling different choices of boundary conditions for the fermions

around the two independent cycles of the worldsheet torus. For each real fermion there are two possible choices of boundary conditions around a given cycle: either periodic (Ramond) or antiperiodic (Neveu-Schwarz). However for fixed α and β the real fermions always pair up into either Majorana or Weyl fermions; if a particular Weyl pairing occurs consistently across *all* α and β, then this pair can be regarded as a single *complex* fermion. For such complex fermions more general boundary conditions -any rational "twists"- are then allowed[11, 13, 15]. A useful notation denotes a pair of Ramond fermions as a $-1/2$ twist, while a general m/n twist indicates the complex fermion boundary condition

$$\Psi \rightarrow \exp\left[2\pi i \frac{m}{n}\right] \Psi \quad . \tag{4}$$

It is convenient to regard the partition function as a sum over physical "sectors" labelled by the α's. The contribution of any sector α to the partition function contains a generalized GSO projection operator. Up to an overall constant, this is given by:

$$\sum_{\beta} C^{\alpha}_{\beta} \exp\left[-2\pi i \beta \cdot \hat{N}(\alpha)\right] \quad , \tag{5}$$

where $\hat{N}(\alpha)$ is the fermion number operator defined in the sector α. There are subtleties in the proper definition of $\hat{N}(\alpha)$ for real Ramond fermions; these are discussed in ref [14].

Thus building a fermionic string model amounts to choosing an appropriate set of α's, β's, and C^{α}_{β}'s, then performing the GSO projections to find the physical spectrum. These choices are greatly constrained by the requirement of modular invariance of the one-loop partition function; in addition, higher loop modular invariance imposes a factorization condition on the C^{α}_{β}'s. Together these requirements imply that the $\{\beta\}$ are the same set of vectors as the $\{\alpha\}$, and that, if two sectors α_1 and α_2 appear in the partition function, then the sector $\alpha_1 + \alpha_2$ must also appear. These facts allow one to specify the full set of α's and β's by a list of "basis vectors", denoted V_i.

Of the 64 real fermions, 20 are right-moving and 44 are left-moving. The first pair of right-movers are spacetime fermions (corresponding to the two transverse directions in 4-d), while the other 18 right-movers are "internal". The requirement of a worldsheet supercurrent contructed out of the right-movers and the spacetime bosons is a consistency constraint on model building. As a result there is always a sector -denoted V_1- that contains massless

gravitinos, corresponding to an N=4 spacetime supersymmetry before the GSO projection. After the projection one may have N=4, N=2, N=1, or no spacetime SUSY at all. We will only consider models with N=1 spacetime SUSY.

Fermionic string models always contain an "untwisted sector", with 32 Neveu-Schwarz Weyl fermions. The untwisted sector contains the graviton, dilaton, and antisymmetric tensor field. It generally also contains gauge bosons, and massless scalars including gauge-singlet moduli.

The first step in building a fermionic string model for a SUSY GUT is to find an embedding of the GUT root lattice in the left-moving fermions. As discussed in ref [8], this means identifying the root vectors with vectors of "fermionic charges" for n complex left-movers, where n is $\geq$ the rank of the gauge group. The fermionic charge of the Neveu-Schwarz vacuum is 0; it is ± 1 for an excited Neveu-Schwarz fermion/antifermion. For the Ramond vacuum the charge is $\pm 1/2$, corresponding to the two degenerate vacuum states. Thus using complex Neveu-Schwarz and Ramond fermions, root vectors have components taking only the values 0, $\pm 1/2$, and ± 1. The length-squared of each root vector is equal to $2/k$, where k is the Kac-Moody level.

Consider a particular example. A level two embedding of $SO(10)$ using, let's say, only Neveu-Schwarz and Ramond fermions, requires that we find 5 simple roots, each with length-squared one, whose inner products reproduce the Cartan matrix of $SO(10)$, and whose components take only the values 0, $\pm 1/2$, and ± 1. Such embeddings exist for any number of complex fermions ≥ 6 (although $SO(10)$ has rank five, there is no solution with only 5 complex fermions).

The minimal embedding, using 6 complex fermions, has simple roots given by[8]:

$$\begin{gathered}(0,0,0,1,0,0), \quad (\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2},0,0),\\ (0,0,1,0,0,0), \quad (0,\tfrac{1}{2},-\tfrac{1}{2},0,-\tfrac{1}{2},\tfrac{1}{2}),\\ (0,\tfrac{1}{2},-\tfrac{1}{2},0,\tfrac{1}{2},-\tfrac{1}{2}).\end{gathered}$$

Since there is an additional vector orthogonal to the space spanned by these roots, these 6 fermions actually embed $SO(10) \times U(1)$.

The maximal nontrivial embedding, using 10 complex fermions, has sim-

ple roots given by:

$$
\begin{gathered}
(\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2},0,0,0,0,0,0),\\
(0,0,\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2},0,0,0,0),\\
(0,0,0,0,\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2},0,0),\\
(0,0,0,0,0,0,\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2},\tfrac{1}{2}),\\
(0,0,0,0,0,0,\tfrac{1}{2},\tfrac{1}{2},-\tfrac{1}{2},-\tfrac{1}{2}),
\end{gathered}
$$

We adopt the convention that the first $2n$ real left-movers of our string models will correspond to the n complex fermions that define an embedding. Having chosen a particular embedding, the next challenge is to find a set of basis vectors, consistent with modular invariance and the GSO projections, such that we generate sectors with massless vector states whose fermionic charges reproduce the root vectors of the embedding. Assuming that we can in fact produce all the gauge bosons of some GUT group, we are then guaranteed that all physical states will assemble into irreps of the GUT group. Note that it is a simple matter to translate from the Dynkin basis to the basis defined by the simple roots written as fermionic charge vectors. Thus we can determine which irreps physical states belong to by simply mapping their weights back to the Dynkin basis.

4 Three Generations

We have begun an exploration of level two $SO(10)$ and $SU(5)$ fermionic string GUT models which employ Lewellen's minimal embedding of $SO(10)$. So far this search has been limited to models whose fermions are Neveu-Schwarz, Ramond, or complex with $\pm 1/4$ twists. Since the introduction of other rational twists *loosens* the modular invariance constraints, we expect that we have merely scratched the surface of possible models with this embedding.

We find that the requirement of three light chiral generations (three 16_L's in $SO(10)$) is very restrictive in our models, much more so than, e.g., the requirement of adjoint scalars. This is not surprising, since other fermionic string model builders have encountered the same difficulty[9, 16, 17]. From an exhaustive search, we find that there exist *no* examples of three generation level two $SO(10)$ models, with the minimal embedding, using *only* Neveu-Schwarz and Ramond fermions. Thus three generations requires fermions

with other twists.

A useful starting point for constructing three generation models is the $Z_2 \times Z_2$ symmetric orbifold structure described by Faraggi in ref [18]. As pointed out in [18, 19], this structure is present in all known level one fermionic string models that have three generations. The orbifold structure consists of two Z_2 twists θ_1, θ_2, acting symmetrically on left and right-moving $[SO(4)]^3$ lattices. From now on it will be very convenient to write 0, 1 for Neveu-Schwarz and Ramond fermions, respectively. In this notation:

$$\begin{aligned} \theta_1 &= (1100)(1100)(0000) \\ \theta_2 &= (0000)(0011)(1100) \quad . \end{aligned}$$

When this structure is realized in a full fermionic string model, the partition function can obviously be decomposed into a sum of four pieces with respect to this orbifold: an untwisted sector, and the three twisted sectors θ_1, θ_2, and $\theta_1\theta_2$. For level one models, there is a straightforward way -the "NAHE" set- of constructing basis vectors such that one chiral generation resides in each of the three twisted sectors. It is quite possible that there are many unrelated ways of obtaining three generation fermionic string GUT's, but we have so far found it convenient to adapt structures similar to the NAHE set in our GUT models.

This orbifold trick for obtaining three generations is not, strictly speaking, compatible with realizing $SO(10)$ at Kac-Moody level two. The left-moving $[SO(4)]^3$ lattice cannot, of course, overlap with the 12 left-mover slots reserved for the $SO(10)$ embedding. In addition, for a sector containing a massless chiral 16_L fermion *all* of the left-moving structure is rather severely constrained by the requirements of $SO(10)$ and modular invariance. Furthermore, even if one realizes the orbifold structure in three twisted sectors (which one might as well take to be three basis vectors), the additional basis vectors which produce $SO(10)$ gauge bosons will not respect the left-right symmetry of the $Z_2 \times Z_2$ orbifold.

In spite of these difficulties, we have found ways to simultaneously realize both $SO(10)$ level two and obtain three generations. This is demonstrated in the next section with a specific model.

5 Features of Superstring GUT's

Since we have only a sampling of models, and have made particular choices of embedding, level, gauge group, and string construction, it would be imprudent to try to make any general statements about the properties of superstring GUT's. Instead, we will be content making a few observations about features of our models.

As an example, we list below the basis vectors of a particular level two $SO(10)$ fermionic string model. Here 0, 1 denote real Neveu-Schwarz or Ramond fermions, and $\pm$ denotes a real fermion which pairs with another $\pm$ real fermion to make a complex fermion with $\pm 1/4$ twist. The 20 right-movers are separated from the 44 left-movers by a double vertical line. A vertical line separates out the 12 left-movers that embed the $SO(10)$ weights. The first two right-movers are the spacetime fermions.

V_0 is required in all fermionic string models by modular invariance. The V_1 sector contains the gravitino, as already discussed. Superpartners of states in some sector α will be found in the sector $V_1 + \alpha$. The 45 massless gauge bosons of $SO(10)$ level two are contained in the untwisted sector, V_2, V_3, V_4, V_2+V_3, V_2+V_4, V_3+V_4, and $V_2+V_3+V_4$.

$$
\begin{aligned}
V_0 &= (1111111111111111111||111111111111|111111111111111111111111111111111111) \\
V_1 &= (11100100100100100100||000000000000|000000000000000000000000000000000000) \\
V_2 &= (00000000000000000000||111111110000|111111110000000000000000000000000000) \\
V_3 &= (00000000000000000000||000000000000|000011111111000000000000000000000000) \\
V_4 &= (00000000000000000000||110000111111|110011001100110000000000000000000000) \\
V_5 &= (11100100010010010010||111100001100|101010101010101110000000000000000000) \\
V_6 &= (11010010100100001001||111100001100|101001011010010000010000_{++--++++}) \\
V_7 &= (11001001001001100100||111100001100|111100001111000000000110000000000000) \\
V_8 &= (00110110110110000000||000000000000|010101010101010000001000000000000000) \\
V_9 &= (00_{+-}0_{+-}0_{++}1_{++}11_{++}1_{++}||00000000_{++++}|00001111000011001001_{++++++++}0000)
\end{aligned}
$$

This type of model can always be transformed into a similar level two $SU(5)$ model, either by adding a basis vector or altering existing ones. This is because the 24 roots of $SU(5)$ contained in $SO(10)$ appear precisely in the gauge boson sectors listed above that do not contain V_3. It is tempting

to suppose that such $SU(5)$ models may retain the fermion mass texture of their $SO(10)$ parent models virtually intact.

Three generations of 16_L fermions are contained in V_5, V_6, V_7, plus 3×6 additional sectors obtained by adding V_2, V_4, V_2+V_3, V_2+V_4, V_3+V_4, or $V_2+V_3+V_4$ to these (the massless states in $V_3+V_{5,6,7}$ are GSO projected out). To describe the $[SO(4)]^3$ lattice of the $Z_2 \times Z_2$ orbifold structure, let $\{r_1 \ldots r_{20}\}$ denote the right-movers, and $\{l_1 \ldots l_{44}\}$ the left-movers. Then the right and left-moving $[SO(4)]^3$ lattices consists of the fermions

$$\{r_4, r_5, r_7, r_8, r_{10}, r_{11}, r_{13}, r_{14}, r_{16}, r_{17}, r_{19}, r_{20}\}$$
$$\{l_{14}, l_{16}, l_{17}, l_{18}, l_{19}, l_{20}, l_{22}, l_{24}, l_{25}, l_{26}, l_{27}, l_{32}\}$$

If we were to truncate this model to include only V_0, V_1, and $V_{5,6,7}$, then it would be apparent that three sets of chiral fermions reside in the three twisted sectors of a $Z_2 \times Z_2$ orbifold. However this structure *does not* guarantee three generations in the full model. In fact it is difficult to avoid generating extra chiral fermions in other twisted sectors. This has proven so far to be the most severe constraint on model building.

The observable gauge group of this model is $SO(10) \times [U(1)]^4$. The hidden sector gauge group is $[SU(2)]^3 \times [U(1)]^2$. The hidden sector gauge group, hidden sector matter content, and the number of extra $U(1)$'s, are very model dependent. However it appears that the rank of the hidden sector gauge group is always ≤ 5.

This model contains a 45 of adjoint scalars contained in sector V_8 and the seven other sectors obtained by adding the gauge boson sectors to V_8. Although this model has 246 different sectors that contain massless particles (before the GSO projections), there are no additional 45's and no 54's. This seems to be a robust feature: singlets, 10's, and 16's proliferate in these models, but 45's and 54's appear once, twice, or not at all. This is easily explained by writing the weights of various irreps in our fermionic charge basis. One finds that the 45's and 54's have weights which require Neveu-Schwarz excited fermions and antifermions. These make large contributions to the mass formula, and it then becomes difficult to satisfy all the constraints while keeping these irreps massless.

The above argument should generalize for (at least) most higher level fermionic string models. Thus we conclude that these models do not have a generic problem with exotics -i.e., large massless irreps tend *not* to occur.

This fact may also imply that fermionic string models cannot embed $SO(10)$ GUT models which, e.g., solve the doublet-triplet splitting problem[20] by introducing several 45's and 54's.

The most interesting feature of the fermionic string GUT's which we have looked at so far, is that they naturally possess one of the key properties of most SUSY GUT texture schemes. Unlike level one models, our models quite often have only a single set of Higgs in the fundamental which is allowed by fermionic charge conservation[11, 21] to have a Yukawa coupling to chiral fermions. One then observes that this Higgs generates *at most* one Yukawa coupling, that of the (by definition) third generation:

$$\phi_{10}\Psi_{16}^{(3)}\Psi_{16}^{(3)} \tag{6}$$

For example, in the model above, there is a single 10 with weights distributed among the untwisted sector, V_3, V_4, V_2+V_4, V_3+V_4, and $V_2+V_3+V_4$. All of these states have a Neveu-Schwarz fermion/antifermion excitation in r_9/r_{12}. Conservation of fermionic charge then tells us that the only allowed Yukawa coupling to this 10 is the diagonal coupling to the fermion generation in V_6. In this particular model that is not the end of the story, because the 16_L in V_6 also carries an extra $U(1)$ charge not carried by the Higgs 10, and so the third generation Yukawa is also killed. However we have found that the question of whether the third generation Yukawa survives or not is very model dependent.

It is very gratifying to see the key feature of most texture schemes appear so naturally in fermionic string GUT's. Furthermore we see that this feature is intimately related to the structure of the chiral fermion sectors which was needed to produce three generations!

Level one models with three generations have similar properties, although they typically contain several Higgs, each with only one allowed Yukawa. One must also be extremely cautious about attaching too much significance to Yukawas. String models generate operators which are higher dimension but contain scalars that can get Planck scale vevs. Such terms are then unsuppressed and contribute to the GUT scale effective action just like an ordinary Yukawa. Clearly we need a detailed analysis of higher dimension operators in fermionic string GUT models before we can draw any definite conclusions. Further work is also needed to understand fermion mixings and the sources of masses for the first and second generations.

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