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Radiative Leptonic Decays of Heavy Mesons

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Abstract

We compute the photon spectrum and the rate for the decays $B(D) \rightarrow l\nu\gamma$. These photonic modes constitute a potentially large background for the purely leptonic decays which are used to extract the heavy meson decay constants. While the rate for $D \rightarrow l\nu\gamma$ is small, the radiative decay in the B meson case could be of comparable magnitude or even larger than $B \rightarrow \mu\nu$. This would affect the determination of f_B if the τ channel cannot be identified. We obtain theoretical estimates for the photonic rates and discuss their possible experimental implications.



I. INTRODUCTION

The leptonic decays of heavy mesons are of great interest both theoretically and experimentally. The purely leptonic decay $B^- \rightarrow l^- \nu_l$ can be used to extract $|V_{ub}|$ by using predicted values for the B meson decay constant f_B from lattice calculations. Or conversely, if $|V_{ub}|$ is measured in semileptonic decays, the leptonic decay is in principle the only mode to access f_B . On the other hand, the situation in the charmed mesons is clearer given our knowledge of the CKM mixing angles involved ($|V_{cs}|$ and $|V_{cd}|$). Recently the observation of the decay $D_s \rightarrow \mu \nu_\mu$ has been reported [1]. Although the results are still preliminary it is a first step towards understanding the behavior of heavy meson decay constants. The experimental difficulty in the measurement of purely leptonic decays of heavy pseudoscalars is due mostly to the well known effect of helicity suppression: back-to-back leptons must make a spin 0 final state, but the anti-neutrino is right-handed and forces the charged lepton to this helicity which introduces a factor of the lepton mass in the amplitude. In the end the decay rate is suppressed by the factor $(m_l/m_H)^2$, where m_H is the heavy meson mass. For instance for the B meson

$$\Gamma(B \rightarrow l \bar{\nu}_l) = \frac{G_F^2}{8\pi} |V_{ub}|^2 f_B^2 \left(\frac{m_l}{m_B}\right)^2 m_B^3 \left(1 - \frac{m_l^2}{m_B^2}\right) \quad (1)$$

where the pseudoscalar meson decay constant is defined by

$$\langle 0 | \bar{q} \gamma_\mu (1 - \gamma_5) b | B(P) \rangle = i f_B P_\mu \quad (2)$$

and analogous expressions can be written for D mesons. Thus, when the charged lepton is an electron the purely leptonic decay is practically inaccessible. At the other extreme, when the charged lepton is a tau there is no suppression. However the observation of this decay is experimentally difficult (only one visible particle in the final state). Muons seem more promising to allow the observation of the decay constants of D and B mesons. However in $B \rightarrow \mu \nu$ the suppression factor is still 4×10^{-4} giving a branching fraction of $10^{-7} - 10^{-6}$.

There are other decays that indirectly involve the heavy meson decay constants. For instance, the decay $B \rightarrow \pi l \nu_l$ is expected to be largely dominated by a B^* pole diagram at very low recoiling pion energies [2–4]. This implies the presence of the vector-meson decay constant f_{B^*} which can be related to f_B by Heavy Quark Spin Symmetry [8]. However this region of phase space is difficult to access due to kinematic suppression.

In this letter we investigate the decay modes $B^-(D^-) \rightarrow l^- \nu_l \gamma$. There are two types of contributions: Internal Bremsstrahlung (IB) and Structure Dependent (SD) photon emission [6]. As is known the IB contributions are still helicity suppressed. On the other hand, the SD contributions are reduced by the electromagnetic coupling constant α but they are *not* suppressed by the charged lepton mass. Therefore, what in principle could be regarded as a mere radiative correction to the purely leptonic decays has the potential to be of comparable magnitude and in some cases even much larger. In what follows we analyze the situation in charmed and beauty mesons. In Sec. II we establish the phenomenological relevance of these decays and in Sec. III we discuss theoretical estimates of the unknown constants involved. Conclusions and comments are presented in Sec. IV.

II. PHENOMENOLOGY OF $H \rightarrow \mu\nu\gamma$

As mentioned in the previous section, the μ modes are the most interesting from the point of view of the extraction of heavy meson decay constants. We will concentrate on the case $l = \mu$ but the treatment for $l = e$ is analogous and the numerical differences between these two cases will be stressed when relevant.

The emission of a real photon in leptonic decays of heavy mesons can proceed via the two mechanisms mentioned in Sec. I. The possible IB diagrams are shown in Fig. 1. The corresponding amplitude is given by

$$\mathcal{M}_{PB} = m_\mu f_B \times \left\{ \left(\frac{\epsilon \cdot p_l}{p_l \cdot k} - F \frac{\epsilon \cdot P}{P \cdot k} \right) \bar{\mu}(1 - \gamma_5)\nu + \frac{1}{2p_l \cdot k} \bar{\mu} \not{\epsilon} \not{k} (1 - \gamma_5)\nu \right\} \quad (3)$$

where P , p_l and k are the four-momenta of the decaying meson, the charged lepton and the emitted photon, respectively, ϵ is the polarization of the photon and F is the electromagnetic form-factor of the B meson. (This is not simply related to the Isgur-Wise function [7] given that the matrix element of the electromagnetic current between two B mesons receives contributions from both the heavy and light quarks). The important feature of (3) is that it is suppressed by a factor of the lepton mass, as one would expect of bremsstrahlung photons.

On the other hand, the SD diagrams in Fig. 2 involve the contributions from heavy intermediate states coupling to the initial heavy pseudoscalar and the photon. In Fig. 2 we show the contributions from vector and axial-vector mesons. The helicity suppression is avoided because the meson directly coupling to the lepton pair has spin one. These types of diagrams were previously considered in the context of light pseudoscalar decays, in particular in $\pi \rightarrow l\nu\gamma$ and $K \rightarrow l\nu\gamma$ [6]. It was also shown there that this phenomenological picture in fact correctly accounts for the SD contributions.

The general form of the amplitude corresponding to Fig. 2(a) requires the knowledge of the $B^*B\gamma$ coupling. This is defined in the process $B^* \rightarrow B\gamma$ with the amplitude given by

$$\mathcal{A}(B^* \rightarrow B\gamma) = e\mu_V \epsilon^{\alpha\beta\gamma\delta} \epsilon_\alpha^* v_\beta k_\gamma \lambda_\delta \quad (4)$$

where v is the velocity four-vector of the decaying particle, k is the photon four-momentum, λ is the polarization four-vector of the B^* and μ_V is a constant characterizing the strength of the M1 transition. Eqn. (4) can be used to write the amplitude of Fig. 2(a) by having first the B meson decaying into a real photon and an off-shell vector meson, which after propagating decays weakly through

$$\langle 0 | \bar{q} \gamma_\mu (1 - \gamma_5) b | B^*(v, \lambda) \rangle = i f_{B^*} \epsilon_\mu \quad (5)$$

Eqn. (5) defines the vector-meson decay constant. Inserting the heavy vector-meson propagator $i(-g_{\mu\nu} + v_\mu v_\nu)/2(v \cdot k + \Delta)$, where $\Delta = m_{B^*} - m_B$, we obtain its contribution to the SD diagram

$$\mathcal{A}_V(B \rightarrow \mu\nu\gamma) = e\mu_V \frac{1}{2(v \cdot k + \Delta)} f_{B^*} \epsilon^{\alpha\beta\gamma\delta} \epsilon_\alpha^* v_\beta k_\gamma \mathcal{L}_\delta \quad (6)$$

with $\mathcal{L}_\delta = \bar{u}_l \gamma_\delta (1 - \gamma_5) v_\nu$ the lepton current.

There will also be contributions from heavy axial-vector meson states ($J^P = 1^+$). The Heavy Quark Symmetries [8] of the strong interactions and their consequences in the hadron spectrum will help us to identify the relevant contributions. In the Heavy Quark Limit the spin of the heavy quark decouples from the light degrees of freedom. Thus their total angular momentum j_l can be used as a good label. The meson angular momentum is $J = 1/2 \pm j_l$. For the ground state $j_l = s_l$, the spin carried by the light degrees of freedom and therefore a $(0^-, 1^-)$ spin doublet is predicted. This can be identified with the (B, B^*) . If we allow for orbital angular momentum $L = 1$, then there will be two additional spin doublets as excited states. They correspond to $j_l = 1/2$ and $j_l = 3/2$ and their parity is even: $(0^+, 1^+)$ and $(1^+, 2^+)$. Thus in principle there could be two different axial-vector mesons contributing to $B \rightarrow \mu\nu\gamma$. We write down the amplitude for the process in Fig. 2(b), which is the generic contribution of an axial-vector meson intermediate state. The coupling is of the form

$$\mathcal{A}_A^{(i)}(B_1 \rightarrow B\gamma) = e\mu_A^i (v \cdot k \lambda \cdot \epsilon - v \cdot \epsilon \lambda \cdot k) \quad (7)$$

where the superscript $i = 1/2, 3/2$ identifies the spin-parity doublet to which the axial-vector meson belongs. The axial-vector decay constants $f_A^{1/2}$ and $f_A^{3/2}$ are defined analogously to f_B in (5). Putting these together the contributions of Fig. 2(b) take the form

$$\mathcal{A}_A^{(i)}(B \rightarrow \mu\nu\gamma) = \frac{e\mu_A^{(i)} f_A^i}{2(v \cdot k + \Delta_i)} [v \cdot \epsilon k_\mu - v \cdot k \epsilon_\mu] \mathcal{L}^\mu \quad (8)$$

where $\Delta_{1/2} = m_{B'_1} - m_B$ and $\Delta_{3/2} = m_{B_1} - m_B$ are the excitation energies of the $j_l = 1/2$ and $j_l = 3/2$ even parity doublets and we again neglected pieces proportional to the charged lepton mass. It is worth noticing that the axial-vector pseudoscalar mass differences $\Delta_{1/2}$ and $\Delta_{3/2}$ are quantities that remain constant in the limit $m_Q \rightarrow \infty$ whereas Δ goes to zero in the same limit. In the Heavy Quark Effective Theory (HQET) [10] a term that goes as Δ^2 can be neglected in the B^* propagator on the basis of the heavy mass suppression. In the case of the B_1 or B'_1 propagators in Fig. 2(b) the quadratic terms in $\Delta_{1/2}$ and $\Delta_{3/2}$ can still be considered numerically small even when they are quantities of order 1 in the heavy masses, when compared with $2m_B$.

Most importantly, Heavy Quark Spin Symmetry can be used to obtain relations between the decay constants of particles in the same parity multiplet. For instance

$$f_{B^*} = m_B f_B \quad (9)$$

and analogous relations can be written between the decay constants of the members of the $j_l^P = 1/2^+$ and the $j_l^P = 3/2^+$ doublets. However HQS does not predict relations among decay constants of members of different spin-parity doublets. Then the contributions from excited states will imply the presence of new unknowns other than the pseudoscalar decay constant. Defining $x = 2E_\gamma/m_B$ and $y = 2E_l/m_B$ as the rescaled photon and charged lepton energies in the B rest frame the double differential decay rate is given by

$$\begin{aligned} \frac{d^2\Gamma}{dx dy} &= \frac{G_F^2 |V_{ub}|^2}{32\pi^2} \alpha m_B^3 f_{B^*}^2 \mu_V^2 \times \left\{ \frac{1}{(x + \frac{2\Delta}{m_B})^2} + \left(\sum_i \frac{\gamma_i}{x + \frac{2\Delta_i}{m_B}} \right)^2 \right\} \\ &\times \left[y^2(1-x) + y(3x - x^2 - 2) + \frac{1}{2}(3x^2 - 4x - x^3 + 2) \right] \quad (10) \end{aligned}$$

where we defined

$$\gamma_i = \frac{\mu_A^{(i)} f_A^i}{\mu_V f_B} \quad (11)$$

as the relative axial-vector to vector meson coupling strength. The fact that there are no relations among decay constants of states of different spin-parity doublets will be reflected in the persistence of the unknown γ_i 's when we normalize to $\Gamma(B \rightarrow \mu\nu)$. We will analyze the γ_i 's and μ_V from the theoretical point of view in the next section. For now let us assume that μ_V and $\mu_A^{(i)}$ do not depend on the photon energy so we can integrate and compare with the purely leptonic decay. The result is

$$\begin{aligned} R_B^\mu &= \frac{\Gamma(B \rightarrow \mu\nu\gamma)}{\Gamma(B \rightarrow \mu\nu)} \\ &= \frac{1}{6\pi} \alpha \mu_V^2 m_B^2 \left(\frac{m_B}{m_\mu}\right)^2 \times \int_0^1 dx x^3 (1-x) \left\{ \frac{1}{\left(x + \frac{2\Delta}{m_B}\right)^2} + \left(\sum_i \frac{\gamma_i}{x + \frac{2\Delta_i}{m_B}}\right)^2 \right\} \end{aligned} \quad (12)$$

To have an idea of the potential importance of the photonic mode we take a definite value for the mass differences $\Delta_i = 600$ MeV as suggested by the charmed meson system [9]. This gives approximately

$$R_B^\mu \approx 2\mu_V^2 \left(1 + \frac{(\gamma_{1/2} + \gamma_{3/2})^2}{2}\right) \text{GeV}^2 \quad (13)$$

which shows that unless there are unnaturally small B photon couplings (see next section) the photonic decay, which in principle could have been considered a small radiative correction, will dominate the leptonic decay or will be at least of comparable magnitude. Therefore it is important to have a good theoretical understanding of the couplings of all the relevant intermediate states in order to subtract these events as a background for $B \rightarrow \mu\nu$. The fact that, to this order in $(m_\mu/m_B)^2$, the result of Eqn. (10) is independent of the lepton mass implies that

$$\Gamma(B \rightarrow e\nu\gamma) = \Gamma(B \rightarrow \mu\nu\gamma) \quad (14)$$

and that

$$\Gamma(B \rightarrow e\nu\gamma) \gg \Gamma(B \rightarrow e\nu) \quad (15)$$

allows for the separation of both effects by using the electronic modes as well. Thus integrating (10) over the photon energy and integrating the resulting μ spectrum around the end point over a region the size of the experimental μ energy resolution in $B \rightarrow \mu\nu$ will eliminate the background. Perhaps even more interesting, given the richness of the physics that enters in them, is the possibility of observing the photonic modes at branching ratios that will soon be accessible ($\approx 10^{-5} - 10^{-6}$).

The treatment of charmed mesons is entirely analogous to B mesons. With the obvious replacements in (12) we obtain

$$R_D^\mu \approx 4 \times 10^{-2} \mu_V^2 \left(1 + \frac{(\gamma_{1/2} + \gamma_{3/2})^2}{2} \right) \text{GeV}^2 \quad (16)$$

which is suppressed by the factor $(m_D/m_B)^4$ in rescaling (12). Therefore the effect is, as expected, less spectacular in the D mesons although of a branching fraction comparable to that of the effect in the B system.

In the following section we discuss various theoretical estimates of μ_V and the γ_i 's.

III. THEORETICAL ESTIMATES

The radiative leptonic decays depend crucially on the vector and axial-vector couplings to the heavy pseudoscalar, μ_V and $\mu_A^{(i)}$ as well as the ratio of the axial-vector to vector meson decay constants. The photon couples to both the heavy and light quark pieces of the electromagnetic current. Heavy Quark Symmetry (HQS) fixes the heavy quark contribution. In the Heavy Quark Effective Theory [10] the coupling of the heavy quark Q to the electromagnetic field responsible for the B^* to B transition is given by the operator

$$\frac{Q_Q}{2m_Q} \bar{h}_v \sigma_{\mu\nu} h_v F^{\mu\nu} \quad (17)$$

where h_v is the heavy quark field characterized by its four-velocity v and Q_Q is its electromagnetic charge. This leads to

$$\mu^{(h)} = \frac{Q_Q}{m_Q} \quad (18)$$

where the superscript h indicates a contribution from the heavy quark.

The piece of the $B^*B\gamma$ coming from the coupling to the light degrees of freedom is not given by HQS. In the $SU(3)$ limit can be written as [13]

$$\mu^{(l)} = Q_q \beta \quad (19)$$

where Q_q is the charge of the light degrees of freedom and β is an unknown quantity. In the non-relativistic quark model this is predicted to be

$$\beta_{NRQM} = 1/m_q \quad (20)$$

with m_q the constituent light quark mass ($m_u \approx m_d \approx 330$ MeV and $m_s \approx 450$ MeV). On more general grounds β is expected to be of the order $1/\Lambda_{QCD}$. If we take the HQS and the NRQM predictions for $\mu^{(h)}$ and $\mu^{(l)}$ respectively in order to have an estimate of the magnitude of the coupling we have the simple expression

$$\mu_V = \frac{Q_Q}{m_Q} + \frac{Q_q}{m_q} \quad (21)$$

which is the naive quark model result but now supplemented by HQS. In the charmed mesons, it is sufficient to explain the ratio $\Gamma(D^{*0} \rightarrow D^0\gamma)/\Gamma(D^{*+} \rightarrow D^+\gamma)$ [11]. Therefore

we will rely on (21) for estimates in the B meson decays. For $m_b = 5$ GeV and taking $\beta = 3\text{GeV}^{-1}$, we see that the value for the coupling

$$\mu_V \approx -2\text{GeV}^{-1} \quad (22)$$

is such that the ratio (12) could be sizeable.

The axial-vector meson couplings are expected to be of the same order as μ_V . In fact the NRQM gives $\mu_A^{(1/2)} = \mu_V/\sqrt{3}$ and $\mu_A^{(3/2)} = \sqrt{2/3}\mu_V$, where the superscripts indicate an axial-vector meson belonging to the $(0^+, 1^+)$ and $(1^+, 2^+)$ doublets respectively. The contributions of Fig. 2(b) are also proportional to f_A^i , the axial-vector meson decay constants. But in the NRQM these are zero given that the wave function at the origin for an orbitally excited state vanishes due to the presence of a centrifugal barrier. Although this seems to imply that $\gamma_i \ll 1$, relativistic effects could be important and modify this prediction drastically. Unfortunately there is no conclusive experimental indication from other hadronic systems about the size of the axial-vector meson contributions. In experiments involving $\pi \rightarrow \mu\nu\gamma$ and $K \rightarrow \mu\nu\gamma$ [9] the ratio of the axial-vector to vector contributions is consistent with zero and also with being large.

In order to address corrections to this simple picture it is useful to consider an effective theory coupling heavy hadrons (in this case mesons) to goldstone bosons and also low energy photons. This theory incorporates both Heavy Quark Symmetry and Chiral Symmetry by introducing the spin-parity doublet in the form of 4×4 matrices as follows [3-5,12]

$$H_a = \frac{(1 + \not{p})}{2} \{\not{B}^* - B\gamma_5\} \quad (23a)$$

$$S_a = \frac{(1 + \not{p})}{2} \{\not{B}'_1\gamma_5 - B'_0\} \quad (23b)$$

$$T_a^\mu = \frac{(1 + \not{p})}{2} \left\{ B_2^{*\mu\nu}\gamma_\nu - \sqrt{\frac{2}{3}}B_1^\nu\gamma_5(g_\nu^\mu - \frac{1}{3}\gamma_\nu(g^\mu - v^\mu)) \right\} \quad (23c)$$

where $a = 1, 2, 3$ is the $SU(3)$ index of the light degrees of freedom in the hadron. H_a corresponds to the $(0^-, 1^-)$ ground state doublet; S_a is the excited state doublet $(0^+, 1^+)$ corresponding to the spin of the light degrees of freedom being $j_l = 1/2$ and T_a^μ is the excited state doublet $(1^+, 2^+)$ corresponding to $j_l = 3/2$.

An important set of corrections are those arising from the loop diagrams of Fig. 3, where the photon couples to a goldstone boson in the loop. This type of correction therefore is not suppressed by the heavy mass. They were first considered in Ref. [13] for the $D^*D\gamma$ coupling in the context of Heavy Hadron Chiral Perturbation Theory (HHChPT) and introduce a non-analytic dependence on the quark masses, $m_q^{1/2}$ as well as on the mass differences Δ and Δ_i that break $SU(3)$ in (19). To calculate these diagrams we need the coupling of two heavy hadrons to one goldstone boson. These are given by [12]

$$\begin{aligned} \mathcal{L}_{1\pi} = & gTr [\bar{H}_a H_b A_{ba} \gamma_5] + g'Tr [\bar{S}_a S_b A_{ba} \gamma_5] + g''Tr [\bar{T}_a^\mu T_{\mu b} A_{ba} \gamma_5] \\ & + f'Tr [\bar{S}_a T_b^\mu A_{\mu ba} \gamma_5] + f'''Tr [\bar{H}_a S_b A_{ba} \gamma_5] \end{aligned} \quad (24)$$

with the traces taken over Dirac indices and where the first three terms correspond to the axial couplings between members of the same spin-parity doublet and the last two terms give the transitions between doublets. As it was pointed out in [12] the axial couplings between the $(1^+, 2^+)$ doublet and the ground state vanish to this order in the chiral expansion, that is $\text{Tr} [\bar{H}_a T_b^\mu A_{\mu ba} \gamma_5]$ vanishes. This means that the diagrams in Fig. 3 corresponding to having the (B_1, B_2^*) inside the loop can be neglected. On the other hand we can also see from (24) that to this order there will be no $B_1' B \pi$ coupling which prevents a contribution from the axial-vector meson to the loop. There are four diagrams remaining which correspond to the heavy mesons in Fig. 3(a) being (from left to right) (B, B^*, B^*) ; (B, B^*, B_1') ; (B, B_0^*, B_1') and (B, B_0^*, B_1) . In terms of the couplings, the first one corresponds to a correction to μ_V , the two following to a correction to $\mu_A^{(1/2)}$ coming from $B_1' \rightarrow B\gamma$ and the last one to a correction to $\mu_A^{(3/2)}$ from $B_1 \rightarrow B\gamma$. Finally the diagrams of Fig. 3(b)-(c) vanish in the limit $m_l \rightarrow 0$ and will be strongly suppressed. Therefore the only diagrams contributing are the ones represented in Fig. 3(a), with pion and kaon loops giving

$$\mu_V = \frac{-1}{3m_b} - \frac{2}{3m_q} + \frac{g^2}{4\pi^2 f_\pi^2} I(m_\pi, \Delta) + \frac{g^2}{4\pi^2 f_K^2} I(m_K, \Delta) \quad (25)$$

where

$$I(m, \Delta) = \Delta \left(\ln \frac{m^2}{\mu^2} + 2F(m/\Delta) \right) \quad (26)$$

with μ the renormalization scale and

$$F(x) = \begin{cases} \sqrt{x^2 - 1} \tanh^{-1} \sqrt{x^2 - 1} & ; x \leq 1 \\ -\sqrt{x^2 - 1} \tan^{-1} \sqrt{x^2 - 1} & ; x \geq 1 \end{cases} \quad (27)$$

Analogously, the corrections to $\mu_A^{(i)}$ can be computed. With the corrections mentioned above, the $B_1' B \gamma$ coupling is now

$$\mu_A^{(1/2)} = \frac{1}{\sqrt{3}} \left(\frac{-1}{3m_b} - \frac{2}{3m_q} \right) + f''(g' - g/2) \left\{ \frac{1}{4\pi^2 f_\pi^2} I(m_\pi, \Delta_{1/2}) + \frac{1}{4\pi^2 f_K^2} I(m_K, \Delta_{1/2}) \right\} \quad (28)$$

whereas the $B_1 B \gamma$ coupling now is

$$\mu_A^{(3/2)} = \frac{2}{\sqrt{3}} \left(\frac{-1}{3m_b} - \frac{2}{3m_q} \right) + f' f'' \left\{ \frac{1}{4\pi^2 f_\pi^2} I(m_\pi, \Delta_{3/2}) + \frac{1}{4\pi^2 f_K^2} I(m_K, \Delta_{3/2}) \right\} \quad (29)$$

The quantities g , g' , f' and f'' are expected to be of order one on dimensional grounds. In fact the NRQM predicts $g = 1$. Unfortunately, the D^* lifetime has not been measured yet, preventing the extraction of g from $D^* \rightarrow D\pi$. The experimental upper limit [15] is $g < 0.7$. The model of Ref. [14] predicts $g = 0.32$. Taking g to be somewhere in between we see that the magnetic coupling in (25) still remains a quantity of order one or possibly larger. The same can be said about the corrections to the contributions from axial-vector mesons, where the corrections do not necessarily reduce the value of the couplings given the different signs in (28) and (29) as well as the potential relative phases between these strong couplings.

So far we have not made a distinction between the contributions coming from the B'_1 axial-vector meson, corresponding to $j_l = 1/2$, and the B_1 corresponding to $j_l = 3/2$. In general one can think that their relative contribution is only governed by the size of $\mu_A^{(1/2)}$ and $\mu_A^{(3/2)}$ and their respective decay constants. There is already evidence from the charm meson system that the $(0^+, 1^+)$ is very broad relative to the $(1^+, 2^+)$. This is mostly due to the fact that the latter does not couple to the ground state to leading order but only through a D -wave amplitude [12]. Therefore the narrow width approximation implicitly used in the calculation of (10) should be reconsidered. In principle, we expect a modest suppression of the B'_1 contribution. We have also neglected the possibility of (B'_1, B_1) mixing which could be sizable. It has been pointed out in Ref. [12] that in the D meson system this indeed may occur.

There will be additional corrections coming from terms suppressed by m_B and by the chiral symmetry breaking scale Λ_χ , which are subleading terms in the heavy mass and chiral expansions respectively. However we believe that one should not expect these or the one loop corrections to reduce the value of the couplings significantly, which therefore will remain of order one or larger. In the case of μ_V , the $B^*B\gamma$ coupling, the one loop corrections are always of the opposite sign. However this is not always true for the μ_A 's, where the corrections could even enhance the value of the coupling considerably.

Perhaps the most important question in terms of the phenomenological impact of these decays is the approximation made in Sec. II, namely the neglect of the form-factor suppression that affects μ_V and $\mu_A^{(i)}$ and that would soften the spectrum. This peaks at a photon energy of $\approx 1.2\text{GeV}$ when the couplings are approximated to be constant. When an energy suppression is taken into account the average photon energy would drop typically to a few hundred MeV. This suppression would result in a smaller value of R_B^μ by a factor that, as for the shape of the photon spectrum, strongly depends on the energy dependence chosen. Typical energy dependences would reduce R_B^μ in (12) by a factor of 2 to 4. The first estimate is obtained with a monopole type suppression whereas the second case corresponds to an exponential suppression. In both cases the energy scale was chosen to be 1GeV.

On the other hand, we lack an understanding of the axial-vector meson decay constants and there is a lot of room for theoretical improvements. For the purpose of this letter, however, it was sufficient to show that a rough estimate of the plausible values of the relevant couplings suggests that the branching ratio for the photonic decays $B \rightarrow \mu\nu\gamma$ is comparable to $B \rightarrow \mu\nu$ and it could be even greater.

IV. CONCLUSIONS

Radiative leptonic decays of heavy mesons are very interesting, not only as a possible background to purely leptonic decays, but also because they yield new information about the strong and electromagnetic interactions of heavy hadrons. As expected, they compensate for their suppression by a factor of α by avoiding helicity suppression. In the charmed mesons, D and D_s , the effect is small for the muon but sizeable for the electron. Allowing for a typical form-factor suppression in μ_V and $\mu_A^{(i)}$ and assuming a large range for the axial-vector decay constants we have

$$R_D^\mu \approx (1 - 8) \times 10^{-2} \mu_V^2 \text{GeV}^2 \quad (30)$$

which would translate into $Br(D \rightarrow \mu\nu\gamma) \approx 10^{-5}$.

The situation is very different for the B meson decays. Since the helicity suppression is so large, we benefit much more by avoiding it; we get

$$R_B^\mu \approx (1 - 4)\mu_V^2 \left(1 + \frac{1}{2}(\gamma_{1/2} + \gamma_{3/2})^2\right) \text{ GeV}^2 \quad (31)$$

We have seen in Sec. III that the value of μ_V in the B mesons is likely to remain a quantity of order one even after considering corrections that might reduce it. This indicates that a measurement of f_B can only be achieved in the decays to muons if the radiative decays are correctly subtracted.

This class of decays clearly deserves further study. In particular it requires knowledge of the strong couplings g , g' , f' and f'' mentioned above. In any case it seems reasonable to expect values in the range $R_B^\mu = (1 - 20)$. This would translate into

$$Br(B \rightarrow l\nu\gamma) \approx (10^{-7} - 10^{-6}) \quad (32)$$

Therefore the observation of these decays would be of great interest, adding to our understanding of μ_V , $\mu_A^{(i)}$ and the decay constants of orbitally excited heavy mesons.

There are in principle several other decays of heavy mesons that proceed via the SD mechanism. For instance, the decay $B_s \rightarrow \mu^+\mu^-\gamma$ will be enhanced over the helicity suppressed $B_s \rightarrow \mu^+\mu^-$ by an expression similar to (31). In the Standard Model, the latter has a branching ratio of $\approx 10^{-9}$ whereas the radiative decays could be an order of magnitude greater. In some theories beyond the SM these decays are expected to have larger branching ratios. For instance, it is argued in Ref. [16] that Extended Technicolor scenarios enhance $Br(B_s \rightarrow \mu^+\mu^-)$ by up to two orders of magnitude, which would imply, together with (31), $Br(B_s \rightarrow \mu^+\mu^-\gamma) \approx 10^{-7} - 10^{-6}$. This puts this radiative decay within experimental reach and, together with $B \rightarrow \mu^+\mu^-X$ [17], can impose severe constraints on physics beyond the standard model.

On the other hand, $B \rightarrow D^{(*)}\gamma l\nu$ and any other exclusive semileptonic decay with a photon added to the final state can be considered. Similar results may hold for non-leptonic decays. Although these decays are indeed suppressed by α relative to the corresponding non-photon decays, some of them are of interest in their own right and might become observable in the near future.

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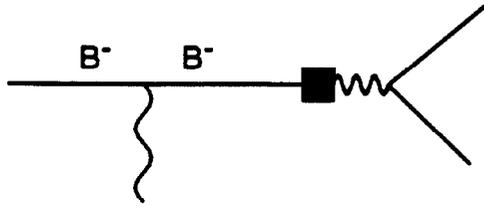
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FIGURES

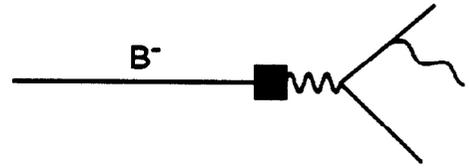
Fig. 1: Bremsstrahlung diagrams entering in $B \rightarrow \mu\nu\gamma$. The black squares denote the action of the weak current.

Fig. 2: Structure Dependent diagrams entering in $B \rightarrow \mu\nu\gamma$. The black circles denote the action of the couplings μ_V and $\mu_A^{(i)}$. (a): Vector meson contribution. (b): Axial-Vector meson contribution.

Fig. 3: One Loop corrections to μ_V and $\mu_A^{(i)}$. Dashed lines denote goldstone bosons while solid lines denote heavy mesons.

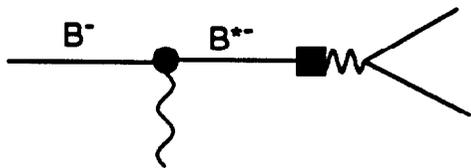


(a)

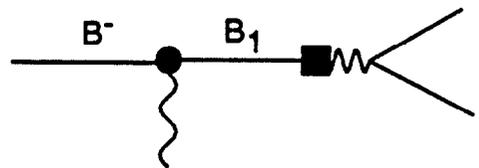


(b)

Fig. 1

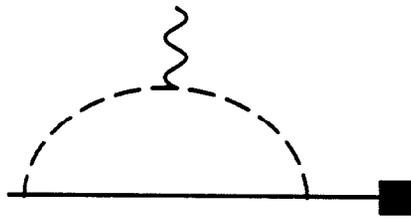


(a)

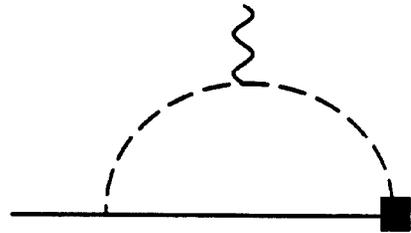


(b)

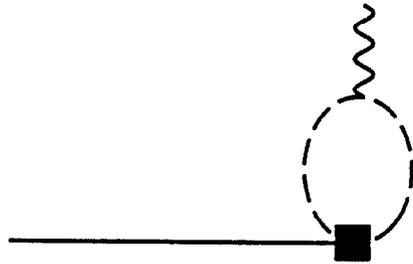
Fig. 2



(a)



(b)



(c)

Fig. 3