

Polynomial Hybrid Inflation

Yun Wang

NASA/Fermilab Astrophysics Center

Fermi National Accelerator Laboratory, Batavia, IL 60510-0500

Abstract

We study a simple extension of Linde's hybrid inflation model, with the inflaton mass term replaced by the most general renormalizable potential for ϕ . The unprocessed power spectrum of density perturbations can have two minima and one maximum, roughly corresponding to two steep regions separated by a somewhat flat region in $V(\phi)$. In the examples studied here, sufficient amount of inflation and normalization to COBE require a vacuum scale of 10^{16} GeV and a ϕ mass of 10^{13} GeV. Depending on the initial value of ϕ , our model can give either less ($\sim n < 1$) or more ($\sim n > 1$) power on small scales compared to the scale-invariant spectrum ($n = 1$), given the normalization to COBE.

PACS index numbers: 98.80.Cq



I. Introduction

Inflation has remained the most attractive solution to the problems of the standard Cosmology, which are the smoothness problem, the flatness problem, and the formation of structure problem. Up to date, there has been no definitive model of inflation which solves all the old problems without creating new ones. The search for more appealing models of inflation continues.

Recently, Linde proposed "hybrid inflation" [1]. The effective potential of this model is given by

$$V(\sigma, \phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2. \quad (1.1)$$

This model looks like a hybrid of chaotic inflation with an inflaton (ϕ) mass term and the usual theory with spontaneous symmetry breaking with a Higgs field σ . Inflation usually ends by the final stage rapid rolling of the slow-rolling inflaton field ϕ , or by a first order phase transition. The important difference between the model of Eq.(1.1) and other theories of this type [2] [3] [4] is: in hybrid inflation, inflation ends with the very rapid rolling of the scalar field σ , which is triggered by the inflaton field ϕ . The Higgs field σ remains a physical degree of freedom after the Higgs effect in an underlying gauge theory with spontaneous symmetry breaking; hence it acquires positive values only, which removes the possibility of domain wall formation in this theory.

Hybrid inflation is very elegant and simple. It gives the appropriate amplitude of density perturbations with reasonable parameter choices. However, the raw power spectrum has a minimum on a usually large scale, which means that there is likely less power on the large scales relevant to observation. There has always been a lot of interest in obtaining power spectra with more power on large scales [5]. We are interested in finding a model of hybrid inflation which gives a more complex behavior of the power spectrum, with sufficient power on large scales.

In the spirit of simplicity, we consider the following effective potential:

$$V(\sigma, \phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + f_0(\phi) + \frac{g^2}{2}\phi^2\sigma^2, \quad (1.2)$$

where

$$f_0(\phi) = A\phi^4/4 + B\phi^3/3 + C\phi^2/2. \quad (1.3)$$

$f_0(\phi)$ is the most general renormalizable potential for ϕ with $f_0(0) = 0$. Note that $C \equiv m^2 > 0$ for a positive ϕ mass term. Hodges et al. studied this potential in the context of the usual slow-roll inflation [6], where they looked for sets of parameters that can strongly break the scale invariance and give a valley in the usual Zel'dovich spectrum. In this work we study the potential in more details within the context of hybrid inflation, and normalize the power spectrum using the COBE DMR results [7].

II. General properties of the inflaton potential

Consider the effective potential $V(\sigma, \phi)$ of Eq.(1.2). The effective mass of the σ field is

$$m_\sigma^2 = \frac{\partial^2 V(\sigma, \phi)}{\partial \sigma^2} = 3\lambda\sigma^2 + (g^2\phi^2 - M^2). \quad (2.1)$$

For $\phi > \phi_c \equiv M/g$, $\sigma = 0$ is the only minimum of the effective potential in the σ direction.

In hybrid inflation, we are interested in potentials which have a much greater curvature in the σ direction than in the ϕ direction [1]. For sufficiently large ϕ , this implies $A \ll g^2/3$. When this condition is satisfied, the σ field rolls down to $\sigma = 0$ initially, while the ϕ field can remain large for a much longer time.

Define a dimensionless variable

$$y \equiv \frac{\phi}{\phi_0}. \quad (2.2)$$

where ϕ_0 is a constant with the dimension of mass. The potential of the ϕ field can be written as:

$$\begin{aligned} V(0, \phi) &= V_0[1 + bf(y)], \\ f(y) &= \frac{y^2}{2} + \frac{\alpha}{3}y^3 + \frac{\beta}{4}y^4, \end{aligned} \quad (2.3)$$

with

$$\begin{aligned} V_0 &\equiv \frac{M^4}{4\lambda}, & b &\equiv \frac{C\phi_0^2}{V_0} = \frac{m^2\phi_0^2}{V_0}, \\ \alpha &\equiv \frac{B\phi_0}{C}, & \beta &\equiv \frac{A\phi_0^2}{C}. \end{aligned}$$

Note that $f'(y) = y[1 + \alpha y + \beta y^2]$, $f''(y) = 1 + 2\alpha y + 3\beta y^2$. The roots of $f'(y) = 0$ are

$$\begin{aligned} y = 0, & & y_{\pm} &= \frac{1}{2\beta} \left[-\alpha \pm \sqrt{\alpha^2 - 4\beta} \right], \\ & & \left(f''(y_{\pm}) &= \frac{\sqrt{\alpha^2 - 4\beta}}{2\beta} \left[\sqrt{\alpha^2 - 4\beta} \mp \alpha \right] \right). \end{aligned} \quad (2.4)$$

$y = 0$ is always a minimum of $f(y)$. If $\alpha^2 < 4\beta$, $y = 0$ is the only extremum of $f(y)$. If $\alpha^2 = 4\beta$, $y_+ = y_- = -2/\alpha$ is the inflection point of $f(y)$. If $\alpha^2 > 4\beta$, there are three cases: (i) $\beta < 0$, $y_+ < 0$, $y_- > 0$, y_{\pm} are maxima of $f(y)$, y (i.e. ϕ) has to tunnel to reach its vacuum if $y > y_-$ initially; (ii) $\beta > 0$ and $\alpha > 0$, $y_- < y_+ < 0$, y_- is a second vacuum and y_+ is the maximum of $f(y)$, there are no extremum for $y > 0$; (iii) $\beta > 0$ and $\alpha < 0$, $y_+ > y_- > 0$, y_+ is the false vacuum if $\alpha^2 < 9\beta/2$, $y = y_+$ and $y = 0$ are degenerate vacua if $\alpha^2 = 9\beta/2$, otherwise y_+ is the true vacuum and a positive constant term must be added to $f(y)$ to yield zero cosmological constant.

It is desirable that we demand $\phi = 0$ is the only extremum (the global minimum) of the ϕ potential $V(0, \phi)$, which is the property of the original hybrid inflation model. This allows us to avoid complications (such as a false vacuum) not of interest to us here. Hence we want either $\alpha^2 < 4\beta$, or $\beta > 0$ and $\alpha > 0$.

The extrema of $f'(y)$ roughly correspond to the extrema of the power spectrum, although the power spectrum has an additional minimum (the only extremum in Linde's

model [1] for which $f''(y) = 1$). Different parameter choices give power spectra with different features. The roots of $f''(y) = 0$ are

$$y = y_{a,b} \equiv \frac{1}{3\mathcal{J}} \left[-\alpha \mp \sqrt{\alpha^2 - 3\mathcal{J}} \right]. \quad (2.5)$$

Since $f'''(y_{a,b}) = \mp 2\sqrt{\alpha^2 - 3\mathcal{J}}$, for $\alpha^2 > 3\mathcal{J}$, y_a and y_b are the maximum and minimum of $f'(y)$ respectively. $\alpha^2 = 3\mathcal{J}$ gives the inflection point of $f'(y)$. For $\alpha^2 > 3\mathcal{J}$, there are three cases: (i) $\mathcal{J} < 0$, $y_a > 0$, $y_b < 0$, but the conditions for $\phi = 0$ being the only extremum of $V(0, \phi)$ are not satisfied; (ii) $\mathcal{J} > 0$ and $\alpha > 0$, $y_a < y_b < 0$; (iii) $\mathcal{J} > 0$ and $\alpha < 0$, $y_b > y_a > 0$. If $\alpha^2 < 3\mathcal{J}$ or $\alpha = \mathcal{J} = 0$ (which corresponds to Linde's model of hybrid inflation), $f''(y) = 0$ has no roots; this and case (ii) of $\alpha^2 > 3\mathcal{J}$ closely resemble Linde's model, which we will not discuss here.

We are interested in case (iii) of $\alpha^2 > 3\mathcal{J}$, for which

$$\mathcal{J} > 0, \quad \alpha < 0: \quad 3\mathcal{J} < \alpha^2 < 4\mathcal{J}, \quad (2.6)$$

or

$$A > 0, \quad B < 0: \quad 3AC < B^2 < 4AC, \quad (2.7)$$

in terms of the constants A , B , and C from Eq.(1.3).

III. The equations of motion

Setting $\sigma = 0$, the equations of motion are

$$\begin{aligned} H^2 &= \frac{8\pi G}{3} \left[V(\phi) + \frac{\dot{\phi}^2}{2} \right], \\ \ddot{\phi} + 3H\dot{\phi} &= -V'(\phi). \end{aligned} \quad (3.1)$$

where $V(\phi) \equiv V(0, \phi)$ (see Eq.(2.3)). Inflation occurs if $H^2 > 4\pi G\dot{\phi}^2$.

Let us define dimensionless variables

$$\bar{H} \equiv \frac{H}{H_0}, \quad \tau = H_0 t. \quad (3.2)$$

where $H_0^2 \equiv 8\pi G V_0/3$. The equations of motion can be written as

$$\begin{aligned}\bar{H}^2 &= 1 + b f(y) + \frac{1}{2} \left(\frac{dy}{d\tau} \right)^2, \\ \frac{d^2 y}{d\tau^2} + 3 \bar{H} \frac{dy}{d\tau} &= -b f'(y),\end{aligned}\tag{3.3}$$

where $y \equiv o/o_0$ with $o_0 = \sqrt{3/(8\pi)} M_p$. The slow-roll conditions [8] are

$$c_1 \equiv \frac{1}{\sqrt{18}} \left[\frac{b f'(y)}{1 + b f(y)} \right] \ll 1, \quad c_2 \equiv \frac{1}{9} \left[\frac{b f''(y)}{1 + b f(y)} \right], \quad |c_2| \ll 1.\tag{3.4}$$

When both conditions are satisfied, we have

$$\begin{aligned}\bar{H}^2 &\simeq 1 + b f(y), \\ 3 \bar{H} \frac{dy}{d\tau} &\simeq -b f'(y).\end{aligned}\tag{3.5}$$

The number of e -folds from the beginning of inflation is

$$\begin{aligned}N(y) &\equiv \int_0^\tau \bar{H} d\tau \simeq \frac{3}{b} \int_y^{y_i} dy \frac{1 + b f(y)}{f'(y)} \\ &= 3 \left[\frac{1}{b} \int_y^{y_i} \frac{dy}{f'(y)} + \int_y^{y_i} dy \frac{f(y)}{f'(y)} \right].\end{aligned}\tag{3.6}$$

After simple integration we find

$$N(y) = 3 \left[\frac{1}{b} \ln \left(\frac{y_i}{y} \right) + \frac{1}{b} \mu(y; y_i, \alpha, \beta) + \nu(y; y_i, \alpha, \beta) \right],\tag{3.7}$$

where

$$\begin{aligned}\mu(y; y_i, \alpha, \beta) &\equiv \frac{1}{2} \ln \left(\frac{1 + \alpha y + \beta y^2}{1 + \alpha y_i + \beta y_i^2} \right) - \frac{\alpha}{\sqrt{4\beta - \alpha^2}} \left[\arctan \left(\frac{\alpha + 2\beta y_i}{\sqrt{4\beta - \alpha^2}} \right) \right. \\ &\quad \left. - \arctan \left(\frac{\alpha + 2\beta y}{\sqrt{4\beta - \alpha^2}} \right) \right], \\ \nu(y; y_i, \alpha, \beta) &\equiv \frac{(y_i - y)}{24\beta} [3\beta(y_i + y) + 2\alpha] + \frac{(5\beta - \alpha^2)}{12\beta^2} \mu(y; y_i, \alpha) \\ &\quad + \frac{(\alpha^2 - 4\beta)}{12\beta^2} \ln \left(\frac{1 + \alpha y + \beta y^2}{1 + \alpha y_i + \beta y_i^2} \right).\end{aligned}\tag{3.8}$$

At $o \leq o_c \equiv M/g$, the phase transition with symmetry breaking occurs. The minimum of the σ field changes to

$$\sigma_0(o) = \sqrt{\frac{M^2(o)}{\lambda}}, \quad \text{with} \quad M^2(o) \equiv g^2(o_c^2 - o^2). \quad (3.9)$$

The effective mass of σ at $\sigma = \sigma_0$ is $m_\sigma^2(\sigma_0) = \partial^2 V(\sigma_0, o)/\partial \sigma^2 = 2M^2(o)$. When o drops below o_c , σ rolls down to σ_0 within $\Delta t \sim H^{-1}$ if $m_\sigma^2(\sigma_0) \gg H^2$. If the slow-roll conditions are satisfied, this gives

$$c_f \equiv \left(\frac{3}{8}\right) \frac{y_c^3 [1 + b f(y_c)]^2}{b f'(y_c)} \ll \frac{\lambda}{g^2}, \quad (3.10)$$

where $y_c \equiv o_c/o_0 = \sqrt{8\pi/(3g^2)} (M/M_P)$. σ oscillates around $\sigma_0(o)$ and loses energy due to the expansion of the Universe. σ cannot simply relax, because $V(\sigma, o)$ has a non-zero slope in the o direction at $\sigma = \sigma_0(o)$. o rolls down to $o = 0$ within $\Delta t \sim H^{-1}$ if $\partial^2 V(\sigma, o)/\partial o^2 \gg H^2$ near $o = o_c$, which can be written as

$$c_f \ll 1. \quad (3.11)$$

The reheating in our model can proceed via the standard way of coherent oscillations [8].

The amplitude of density perturbations produced during inflation is [9]

$$\frac{\delta\rho}{\rho} \Big|_{RH=k}^{\text{HOR}} = \frac{C_\rho H^2}{2\pi\dot{\phi}} \Big|_{RH=k} [1 + (2\gamma - 1)\epsilon + \gamma\delta], \quad (3.12)$$

where $C_\rho = -6/5, -4/3$ depending on when the density perturbations re-enter the horizon (during matter-dominated or radiation-dominated era). R is the scale factor, k is the wavenumber, $\gamma = 2 - \ln 2 - \gamma' \simeq 0.7296$ with γ' denoting Euler's constant, and

$$\epsilon \equiv -\frac{\dot{H}}{H^2} \simeq 3c_1^2, \quad \delta \equiv \frac{\ddot{\phi}}{H\dot{\phi}} \simeq -3c_2, \quad (3.13)$$

where we have used the slow-roll conditions in relating ϵ and δ to c_1 and c_2 . Eq.(3.12) is a second order expression for the unprocessed power spectrum, with ϵ and δ characterizing corrections to the usual first order expression ($\epsilon = \delta = 0$). Ref.[10] shows that these

corrections may be important in relating spectral indices to tensor and scalar amplitudes in inflation.

In terms of our dimensionless parameters,

$$\frac{\delta\rho}{\rho}\Big|_{RH=k}^{\text{HOR}} = P_0 \left(\frac{\delta\rho}{\rho}\right)_0 \equiv P_0 \left(\frac{\delta\rho}{\rho}\right)_0 [1 + (2\gamma - 1)\epsilon + \gamma\delta]. \quad (3.14)$$

where

$$P_0 \equiv \left(\frac{3|C_\rho|g^2}{4\pi\sqrt{\lambda}}\right) y_c^2. \quad (3.15)$$

roughly sets the amplitude of density fluctuations, while

$$\left(\frac{\delta\rho}{\rho}\right)_0 \equiv \frac{\overline{H}^2}{3\left|\frac{dy}{d\tau}\right|} \simeq \frac{[1 + b f(y)]^{3/2}}{b f'(y)} \equiv F(y), \quad (3.16)$$

determines the shape of the power spectrum.

There are four independent parameters in our model, (M, m, α, β) or (y_c, b, α, β) . Fitting two physical observations, the minimum amount of inflation and the anisotropy on large scales from COBE DMR observations, we are left with two free parameters (see Sec.V). It turns out most convenient for us to choose y_c and b to be the free parameters.

IV. Power spectrum shape function $F(y)$

The function $F(y)$ determines the shape of the raw power spectrum. To study the roots of $F'(y) = 0$ analytically, let us define

$$x \equiv \sqrt{K} y, \quad b_0 \equiv \frac{b}{K}, \quad \alpha_0 \equiv \frac{\alpha}{\sqrt{K}}, \quad \beta_0 \equiv \frac{\beta}{K}. \quad (4.1)$$

We can write

$$\begin{aligned} f(y) &= \frac{w(x)}{K}, & w(x) &\equiv \frac{x^2}{2} + \frac{\alpha_0}{3} x^3 + \frac{\beta_0}{4} x^4, \\ F(y) &= \left(\frac{\sqrt{K}}{b}\right) W(x), & W(x) &\equiv \frac{[1 + b_0 w(x)]^{3/2}}{w'(x)}. \end{aligned} \quad (4.2)$$

Since $F'(y) = W'(x)/b_0$, the roots of $F'(y) = 0$ are related to the roots of $W'(x) = 0$ as follows

$$y_{1,2,3} = x_{1,2,3}/\sqrt{K}. \quad (4.3)$$

We can choose K such that one of the roots ($x_{a,b}$) of $u''(x) = 0$ is 1. This gives us a relation between α_0 and β_0 :

$$3\beta_0 + 2\alpha_0 + 1 = 0. \quad (4.4)$$

Let us choose $x_b = 1$, i.e., we scale the ϕ field with its value at the minimum of $u'(x)$. This gives us (see Sec.II)

$$x_b = \frac{1}{3\beta_0} \left[|\alpha_0| + \sqrt{\alpha_0^2 - 3\beta_0} \right] = 1.$$

i.e.,

$$K = \left[|\alpha| - \sqrt{\alpha^2 - 3\beta} \right]^2 = \left(\frac{|\alpha|}{2 - \Delta} \right)^2, \quad (4.5)$$

where we have defined $\Delta \equiv \alpha_0 + 2$. Hence

$$\beta = \frac{\alpha^2}{3} \left[1 - \left(\frac{1 - \Delta}{2 - \Delta} \right)^2 \right]. \quad (4.6)$$

Now we have

$$0 < \Delta < 1, \quad \frac{1}{3} < \beta_0 < 1, \quad \text{for } x_a = \frac{1}{3 - 2\Delta}, \quad x_b = 1. \quad (4.7)$$

where the upper bound on Δ comes about because $x_a < x_b$, and we have chosen $x_b = 1$. $\Delta = 0$ corresponds to the inflection point of $w(x)$, and $\Delta = 1$ the inflection point of $w'(x)$. $W(x)$ is determined by Δ and b_0 . Fig.1 shows $w(x) = K f(y)$ for $\Delta = 0.1$ and 0.5. Fig.2 shows the corresponding $W(x) = (b/\sqrt{K})F(y)$ for $b_0 = 0.1, 1,$ and 10.

We have chosen $V(\phi) = V_0[1 + b_0 w(x)]$ to increase monotonically with ϕ , i.e., $w(x)$ increases with x . Since $1/w'(x)$ has a minimum at $x_a = 1/(3 - 2\Delta)$ and a maximum at

$x_b = 1$, we expect $W(x)$ to have a minimum at $0 < x_1 \lesssim x_a$, a maximum at $x_2 \gtrsim x_b$, and a second minimum at $x_3 > x_2$. Note that

$$\begin{aligned} W'(x) &= \frac{[1 + b_0 w(x)]^{1/2}}{2[w'(x)]^2} \left\{ 3b_0 [w'(x)]^2 - 2u''(x)[1 + b_0 w(x)] \right\} : \\ W''(x) &= \frac{[1 + b_0 w(x)]^{1/2}}{[w'(x)]^2} \left\{ 2b_0 u'(x)u''(x) - 2u^{(3)}(x)[1 + b_0 w(x)] \right\}, \\ &\text{at } W'(x) = 0. \end{aligned} \quad (4.8)$$

$W'(x) = 0$ can be solved numerically for given Δ and b_0 . Fig.3 shows the three roots $x_1 < x_2 < x_3$ as functions of b_0 for $\Delta = 0.1, 0.5$, and 0.9 .

To understand the behavior of $x_{1,2,3}(b_0)$ qualitatively, let us solve $W'(x) = 0$ in various limits. $W'(x) = 0$ can be written as $3b_0[u'(x)]^2 - 2u''(x)[1 + b_0 w(x)] = 0$, or

$$12(1 + 2\alpha_0 x + 3\beta_0 x^2) = b_0 x^2 [9\beta_0^2 x^4 + 18\alpha_0 \beta_0 x^3 + 5(2\alpha_0^2 + 3\beta_0)x^2 + 20\alpha_0 x + 12]. \quad (4.9)$$

For $b_0 \gg 1$, $x_1 \ll 1$. We can drop all except the last term on the right hand side of Eq.(4.9), which gives

$$x_1 \simeq \frac{\sqrt{b_0 + (1 - \Delta)^2} + \Delta - 2}{b_0 - 3 + 2\Delta}, \quad b_0 \gg 1. \quad (4.10)$$

For $b_0 \ll 1$, we can drop all the terms on the right hand side of Eq.(4.9), which gives

$$x_1 \simeq \frac{1}{3 - 2\Delta} = x_a, \quad x_2 \simeq 1, \quad b_0 \ll 1. \quad (4.11)$$

Let $x_2 = 1 + r$ with $r < 1$, we can expand all terms in Eq.(4.9) to second order in r . We find

$$x_2 \simeq 1 + \frac{2b_0 [\sqrt{1 + \omega \Delta^2} - 1]}{\omega [12 + b_0(1 + 2\Delta)](1 - \Delta)}, \quad \omega \equiv \frac{6b_0 [b_0 + 4(3 - 2\Delta)]}{[12 + b_0(1 + 2\Delta)]^2 (1 - \Delta)^2}. \quad (4.12)$$

Eq.(4.12) applies quite well for all b_0 , with Δ not far from the inflection point $\Delta = 0$ (say, $0 < \Delta \lesssim 0.5$).

$W'(x) = 0$ can be written as

$$\begin{aligned} & (x-1) \left\{ 12[r(3-2\Delta) - 1] - b_0 x^2 [\omega(x) - 2r(6x^2 - 15x + 10)\Delta - 4x^2(x-2)\Delta^2] \right\} \\ & = 2b_0 x^4 (2x-3)^2 \Delta^2, \end{aligned} \quad (4.13)$$

where $\omega(x) \equiv 9x^3 - 27x^2 + 28x - 12$. For $b_0 \rightarrow \infty$ and $\Delta = 0$, the above equation becomes

$$(x-1)\omega(x) = 0, \quad (4.14)$$

which has the exact solution $x = 1$ and $x = x_* \equiv 1.5447884$ [6]. To order Δ , the second minimum of $W(x)$ is

$$x_3 \simeq x_* \left[1 + \frac{2(6x_*^2 - 15x_* + 10)\Delta}{27x_*^2 - 54x_* + 28} \right] \simeq x_* \left(1 + \frac{\Delta}{4} \right), \quad b_0 \rightarrow \infty. \quad (4.15)$$

For b_0 large and to order Δ ,

$$x_3 \simeq x_* \left[1 + \frac{\Delta}{4} + \frac{5.25 - 2.76\Delta}{b_0(4 + 2.315\Delta) - 6.7 + 2.76\Delta} \right], \quad b_0 \gg 1. \quad (4.16)$$

For $b_0 \ll 1$, $x_3 \gg 1$. Keeping the highest power in x terms from each side in Eq.(4.9), we find

$$x_3 \simeq \left[\frac{12}{b_0(3-2\Delta)} \right]^{1/4}, \quad b_0 \ll 1. \quad (4.17)$$

V. Sufficient inflation and normalization to COBE

The minimum amount of inflation required to solve the smoothness problem is [8]

$$N_{\min} = 60.6 + \frac{2}{3} \ln y_c + \ln \bar{H}(y_i) + \frac{1}{3} \ln \left(\frac{T_{RH}}{10^{10} \text{ GeV}} \right). \quad (5.1)$$

y_i and y_c are the scaled ϕ field at the beginning and end of inflation respectively. T_{RH} is the reheating temperature.

During inflation, the comoving scale λ which crosses outside the horizon when $\phi = \phi_0$ is given by

$$\ln \left(\frac{\lambda}{h^{-1} \text{ Mpc}} \right) \simeq 8 - N(y) + (N_{\text{tot}} - N_{\min}). \quad (5.2)$$

where $N(y)$ is the number of e -folds of expansion since the beginning of inflation, and N_{tot} denotes the total amount of inflation.

We can choose $y_i > y_3$, or $y_2 > y_i > y_1$, such that a valley appears in the power spectrum at an interesting scale. At the end of inflation

$$N(y_c; b, \alpha, \beta) = N_{\text{tot}}. \quad (5.3)$$

For given (α, β) and N_{tot} , this gives y_c in terms of b .

The processed power spectrum is

$$P(k, t_0) = \frac{\pi}{(2\pi)^3} \left[\frac{d_H(t_0)}{R(t_0)} \right]^4 \left[\left(\frac{\delta\rho}{\rho} \right)_{\text{HOR}}^k \right]^2 T^2(k) k. \quad (5.4)$$

$T(k)$ is the transfer function. For cold dark matter, we use [11]

$$T(q) = \frac{\ln(1 + 2.34q)}{2.34q} \left[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.72q)^4 \right]^{-1/4}, \quad (5.5)$$

where $q \equiv k/(\Omega_0 h^2 \text{Mpc}^{-1})$. We take $\Omega_0 = 1$, $h = 0.5$, the standard CDM model.

Cosmic anisotropy produces an excess variance σ_{sky}^2 in the ΔT maps produced by the Differential Microwave Radiometer (DMR) on COBE that is over and above the instrument noise. After smoothing to an effective resolution of 10° , this excess, $\sigma_{\text{sky}}(10^\circ)$, provides an estimate for the amplitude of the primordial density perturbation power spectrum with a cosmic uncertainty of only 12% [7]. Following the notation of Ref.[12], we write

$$\sigma_T^2(10^\circ) = \sum_{l \geq 2} \frac{2l+1}{4\pi} \langle |a_{lm}|^2 \rangle \exp[-l(l+1)\theta_0^2]. \quad (5.6)$$

where $\theta_0 = 0.125 \theta_{\text{FWHM}} = 1.27^\circ$ is the Gaussian angle corresponding to the antenna beam and additional smearing of the raw data. $\sigma_T(10^\circ)$ is directly related to the power spectrum $P(k)$. For the anisotropy in the cosmic microwave background radiation on large angular scales, the Sachs-Wolfe effect dominates. For adiabatic perturbations

$$\langle |a_{lm}|^2 \rangle = \frac{H_0^4}{2\pi} \int_0^\infty dk k^{-2} P(k) j_l^2(kr_{\text{rec}}). \quad (5.7)$$

where j_l is a spherical Bessel function and $r_{\text{rec}} \simeq 2H_0^{-1}$ is our comoving distance from the surface of recombination.

Based on the latest results of COBE DMR [7], we take $\sigma_T(10^\circ) = (1.25 \pm 0.2) \times 10^{-5}$ [13]. For given (α, β) , this enables us to normalize the power spectrum $P(k)$, thus determine b .

Let us take $\Delta = 0.1$ to study the properties of our model. Fig.4 shows the number of e -folds $N(y)$ since the beginning of inflation, for (i) $\alpha = -2$, $y_i = y_2 - 0.2$; (ii) $\alpha = -2$, $y_i = y_3 + 5$; (iii) $\alpha = -0.8$, $y_i = y_3 + 5$. The ϕ field rolls slowly if it starts in the not too steep region of the potential, i.e., between the first and second minimum of the power spectrum shape function $F(y)$, $y_1 < y_i \equiv \phi_i/\phi_0 < y_3$ (see Fig.2). For large ϕ_i ($y_i > y_3$), ϕ rolls more slowly in the astrophysically relevant region ($N(y) \lesssim 20$) for smaller $|\alpha|$. Sufficient inflation can be obtained for initial values of $y \equiv \phi/\phi_0$ greater than the first minimum y_1 of the power spectrum shape function $F(y)$ (see Fig.2).

Fig.5 shows $(\delta\rho/\rho)_* = P_0^{-1}(\delta\rho/\rho)_{\text{HOR}}$ (see Eq.(3.14)) corresponding to the parameter choices of Fig.4, to the first (dash line) and second order (solid line) in slow roll approximation. The deviation of the first order curve from the second order curve increases as ϕ rolls faster.

Fig.6 gives four examples of the power spectrum $P(k)$ with $\Delta = 0.1$. For $\alpha = -2$, $y_i = y_2 - 0.2$ (short dash line), $y_2 + 0.1$ (dot line), and $y_3 + 5$ (long dash line) respectively. For $\alpha = -0.8$, $y_i = y_3 + 5$ (dot - long dash line). The solid line is CDM with $n = 1$. We have used the second order formula for $\delta\rho/\rho$ in calculating $P(k)$. If ϕ starts by rolling over the peak in the power spectrum shape function $F(y)$, our model gives somewhat more power on small scales than the standard CDM with $n = 1$. If ϕ starts by rolling down one of the valleys in the power spectrum shape function $F(y)$, our model gives less power on small scales than the standard CDM with $n = 1$. ϕ rolling down the steeper valley in $F(y)$ at $y > y_3$ leads to more dramatic reduction of power on small scales. For

given Δ and $(y_i - y_3) > 0$, larger $|\alpha|$ seems to lead to less power on small scales. But the slow-roll conditions (see Eq.(3.4)) may break down near $y = y_3$, hence it may not be appropriate to use Eq.(3.12) in the calculation of $\delta\rho/\rho$. For $|\alpha| \gtrsim 10$ (with $\Delta = 0.1$ and $y_i = y_3 + 5$), the slow-roll parameter $c_2 > 1$ near $y = y_3$. The proper mapping of the entire allowed parameter range and the discussion of the tensor contribution to the CMBR anisotropy will be presented elsewhere [14].

The σ vacuum scale M and the ϕ mass m can be expressed as

$$\frac{M}{M_P} = \sqrt{\frac{3g^2}{8\pi}} y_c, \quad \frac{m}{M_P} = \sqrt{\frac{2\pi b}{3\lambda}} \left(\frac{M}{M_P}\right)^2. \quad (5.8)$$

For $\Delta = 0.1$ and $\alpha = -2$, $M \sim 10^{16}$ GeV, $m \sim 10^{13}$ GeV. For given α and $(y_i - y_3) > 0$, $P(k)$ is not sensitive to Δ : the scales M and m vary less than an order of magnitude as we change Δ from 0.1 to 0.9 (recall that $0 < \Delta < 1$). Larger $|\alpha|$ seems to lead to smaller M and m ($M \lesssim 10^{15}$ GeV, $m \lesssim 10^{12}$ GeV). We have taken the dimensionless coupling constants λ and g to be of order 1.

Let us now examine the conditions for successful hybrid inflation. For inflation to occur, we want σ to roll down to $\sigma = 0$ initially while ϕ remains large, which requires $A \ll g^2/3$ (see Sec.II). This can be expressed as

$$\alpha \ll g \left(\frac{M_P}{m}\right). \quad (5.9)$$

The above condition is always satisfied since $M_P \gg m$. The fast-roll conditions Eqs.(3.10) and (3.11) are automatically satisfied, because $y_c \ll 1$ in our model. Typically $c_f \lesssim 10^{-5}$. This means that both σ and ϕ roll down to their true vacuum states almost instantly at the time of the phase transition ($\phi = \phi_c \equiv M/g$), thus putting an abrupt end to inflation (see Sec.III).

To sum up, the requirement of sufficient inflation and normalization to COBE give us two constraints on the four parameters (b, y_c, α, β) in our model. (α, β) roughly determine the shape of the unprocessed power spectrum. For given (α, β), we can find

(b, y_c) or (m, M) which satisfy the physical constraints. The same choice of parameters also guarantees successful hybrid inflation. The typical values of the vacuum scale M and the ϕ mass m are: $M \lesssim 10^{16}$ GeV, $m \lesssim 10^{13}$ GeV. The dimensionless constants α and β in the scaled ϕ potential $f(y)$ (see Eq.(2.3)) are usually of order 1. The constants in the original form of the ϕ potential (see Eq.(1.2)) are: $A \sim (m/M_{\text{P}})^2$, $B \sim m^2/M_{\text{P}}$, $C = m^2$; λ and g can be taken to be of order 1.

VI. Remarks

The model we have presented here has a very simple potential. It involves only one extra term, the cubic ϕ self-coupling term, compared to previous theories of its type [1] [5]. Inflation can end abruptly in our model. Sufficient inflation is easily obtained. The power spectrum shape function $F(y) \propto (\delta\rho/\rho)_{\text{HOR}}$ has two valleys and one peak, while the original hybrid inflation model gives only one valley. When normalized to COBE, $\phi \equiv y\phi_0$ starting on a slope in $F(y)$ and going down either of the valleys in $F(y)$ (see Fig.2) leads to less power on small scales ($\sim n < 1$), while ϕ starting between the peak and the second valley in $F(y)$ (see Fig.2) leads to more power on small scales ($\sim n > 1$), compared to the standard CDM with $n = 1$. Our model obtains interesting power spectra quite naturally.

Acknowledgments

I thank Rocky Kolb for inspiring conversation and helpful suggestions. I thank Josh Frieman for helpful comments. I thank Scott Dodelson and Igor Tkachev for useful discussions. This work was supported by the DOE and NASA under Grant NAGW-2381.

References

1. A. Linde, Phys. Rev. D. **49**, 748 (1994).
2. L.A. Kofman and A.D. Linde, Nucl. Phys. **B282**, 555 (1987).
3. L.A. Kofman and D.Yu. Pogosyan, Phys. Lett. B **214**, 508 (1988); D.S. Salopek, J.R. Bond, and J.M. Bardeen, Phys. Rev. D **40**, 1753 (1989); L.A. Kofman, Phys. Scr. **T36**, 108 (1991).
4. F.C. Adams and K. Freese, Phys. Rev. D., **43**, 353 (1991).
5. D.S. Salopek, J.R. Bond, and J.M. Bardeen, Phys. Rev. D. **40**, 1753 (1989).
6. H. Hodges, G. Blumenthal, L. Kofman, and J. Primack, Nucl. Phys., **B335**, 197 (1990).
7. E.L. Wright, G.F. Smoot, A. Kogut, G. Hinshaw, L. Tenorio, C. Lineweaver, C.L. Bennett, P.M. Lubin, Ap.J. **420**, 1 (1994); C.L. Bennett, A. Kogut, G. Hinshaw, *et al.*, astro-ph/9401012.
8. E.W. Kolb and M.S. Turner, The Early Universe (Addison-Wesley Publishing Company, 1990).
9. E.D. Stewart and D.H. Lyth, Phys. Lett. B. **302**, 171 (1993).
10. E.W. Kolb and S.L. Vadas, FERMILAB-Pub-94/046-A, astro-ph/9403001 (1994).
11. J.M. Bardeen, J.R. Bond, N. Kaiser, A.S. Szalay, Ap. J. **304**, 15 (1986).
12. F.C. Adams, J.R. Bond, K. Freese, J.A. Frieman, and A.V. Olinto, Phys. Rev. D. **47**, 426 (1993).
13. P. Peter, D. Polarski, and A.A. Starobinsky, DAMTP-R94/20, (1994).

14. Y. Wang, in preparation (1994).

Figure Captions

Fig.1. $w(x) = K f(y)$ for $\Delta = 0.1$ and 0.5 (dash line). $f(y)$ is the scaled ϕ potential.

Fig.2. $W(x) = (b/\sqrt{K})F(y)$ for $\Delta = 0.1$ and 0.5 (dash line), with $b_0 = 0.1, 1,$ and 10 . $F(y) \propto (\delta\rho/\rho)_{\text{HOR}}$ is the power spectrum shape function.

Fig.3. The three roots of $W'(x) = 0$, $x_{1,2,3} = \sqrt{K} y_{1,2,3}$, as functions of b_0 for $\Delta = 0.1$ (solid line), 0.5 (dash line) and 0.9 (dot line). Note that $x_1 < x_2 < x_3$ for a given Δ .

Fig.4. The number of e -folds $N(y)$ since the beginning of inflation with $\Delta = 0.1$, for (i) $\alpha = -2$, $y_i = y_2 - 0.2$; (ii) $\alpha = -2$, $y_i = y_3 + 5$; (iii) $\alpha = -0.8$, $y_i = y_3 + 5$.

Fig.5. $(\delta\rho/\rho)_-$ with $\Delta = 0.1$, to the first (dash line) and second order (solid line) in slow roll approximation, for (i) $\alpha = -2$, $y_i = y_2 - 0.2$; (ii) $\alpha = -2$, $y_i = y_3 + 5$; (iii) $\alpha = -0.8$, $y_i = y_3 + 5$.

Fig.6. Four examples of the power spectrum $P(k)$ with $\Delta = 0.1$. For $\alpha = -2$, $y_i = y_2 - 0.2$ (short dash line), $y_2 + 0.1$ (dot line), and $y_3 + 5$ (long dash line) respectively. For $\alpha = -0.8$, $y_i = y_3 + 5$ (dot-long dash line). The solid line is CDM with $n = 1$.

Fig. 1

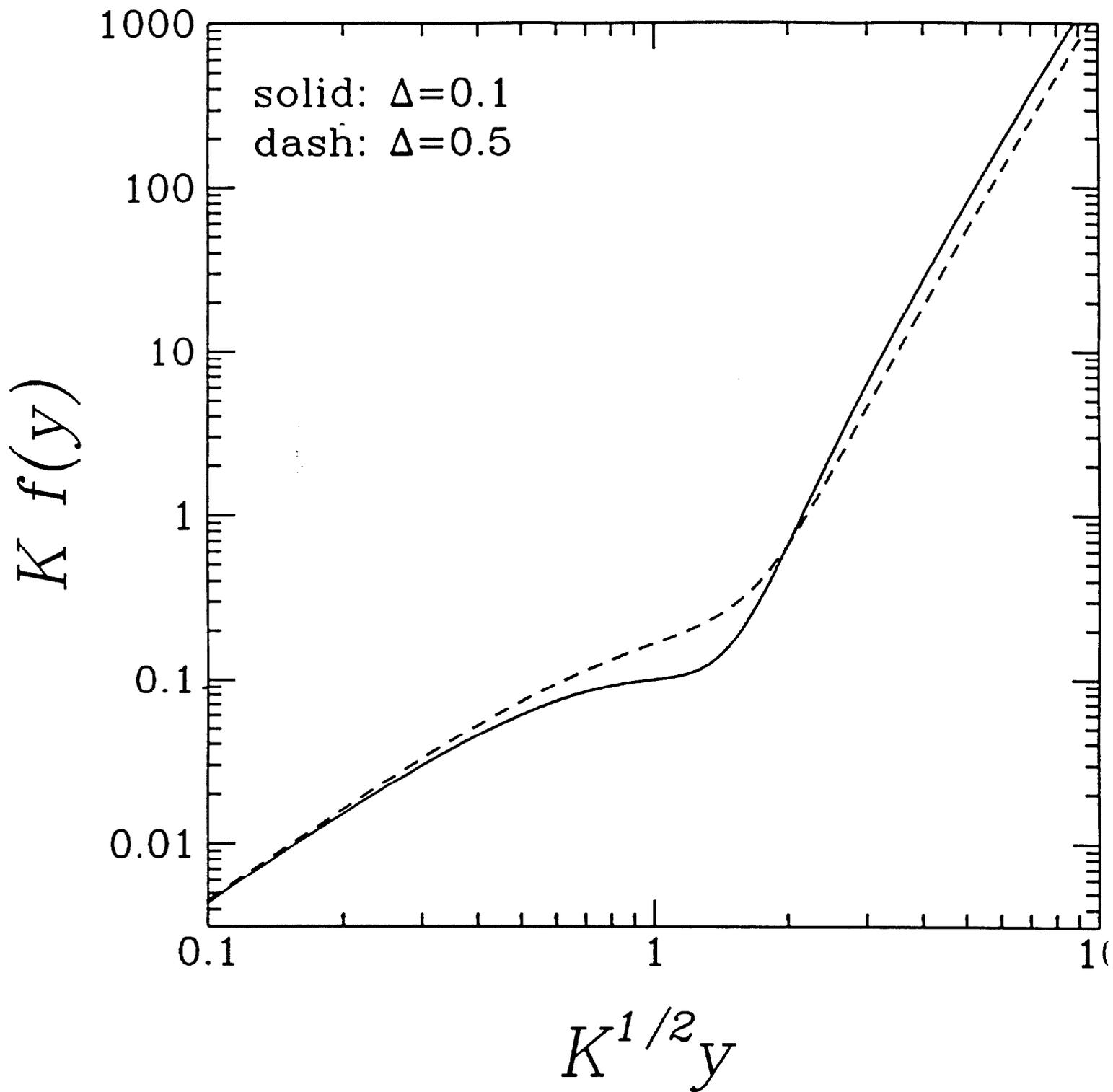


Fig. 2

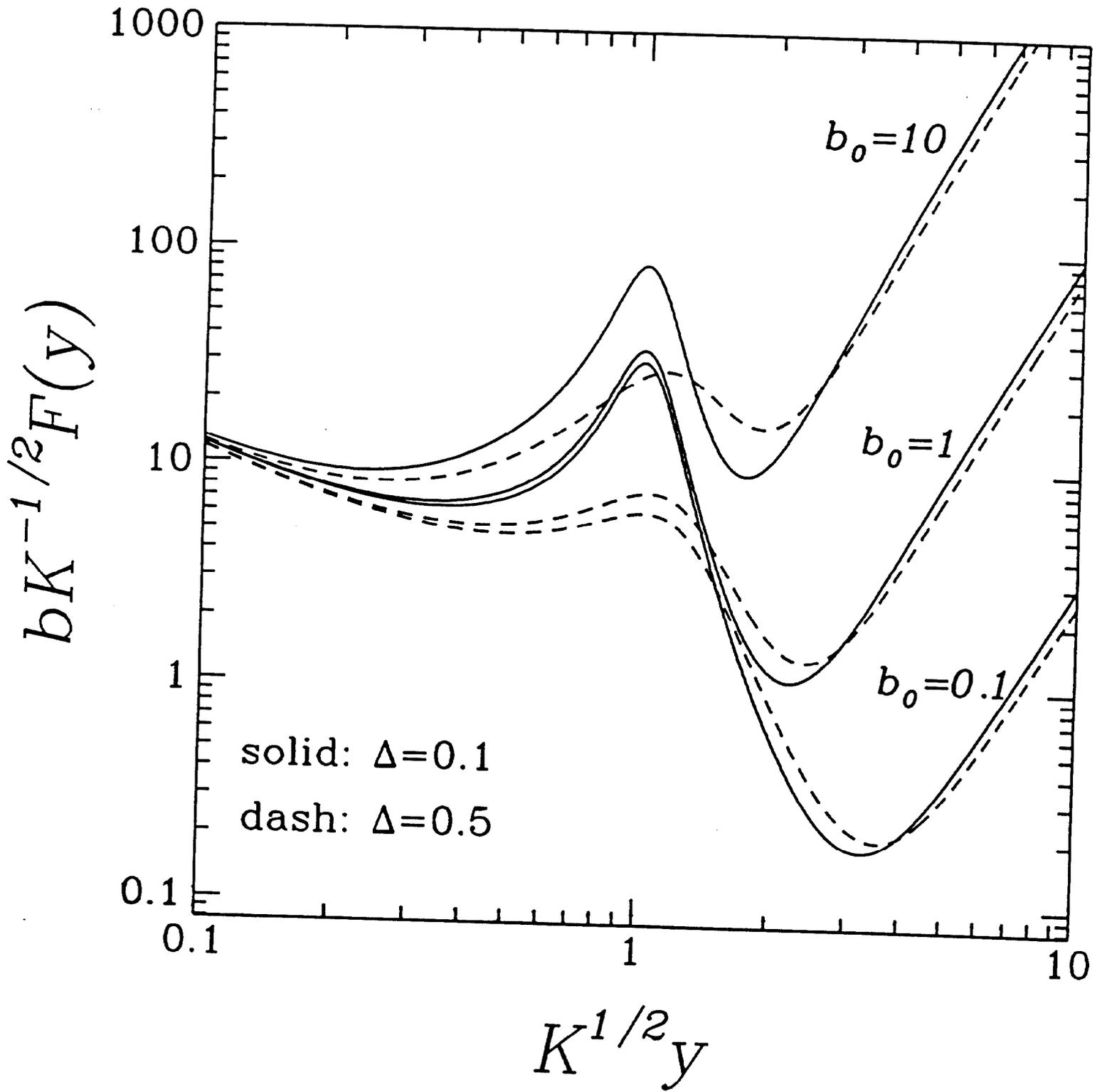


Fig. 3

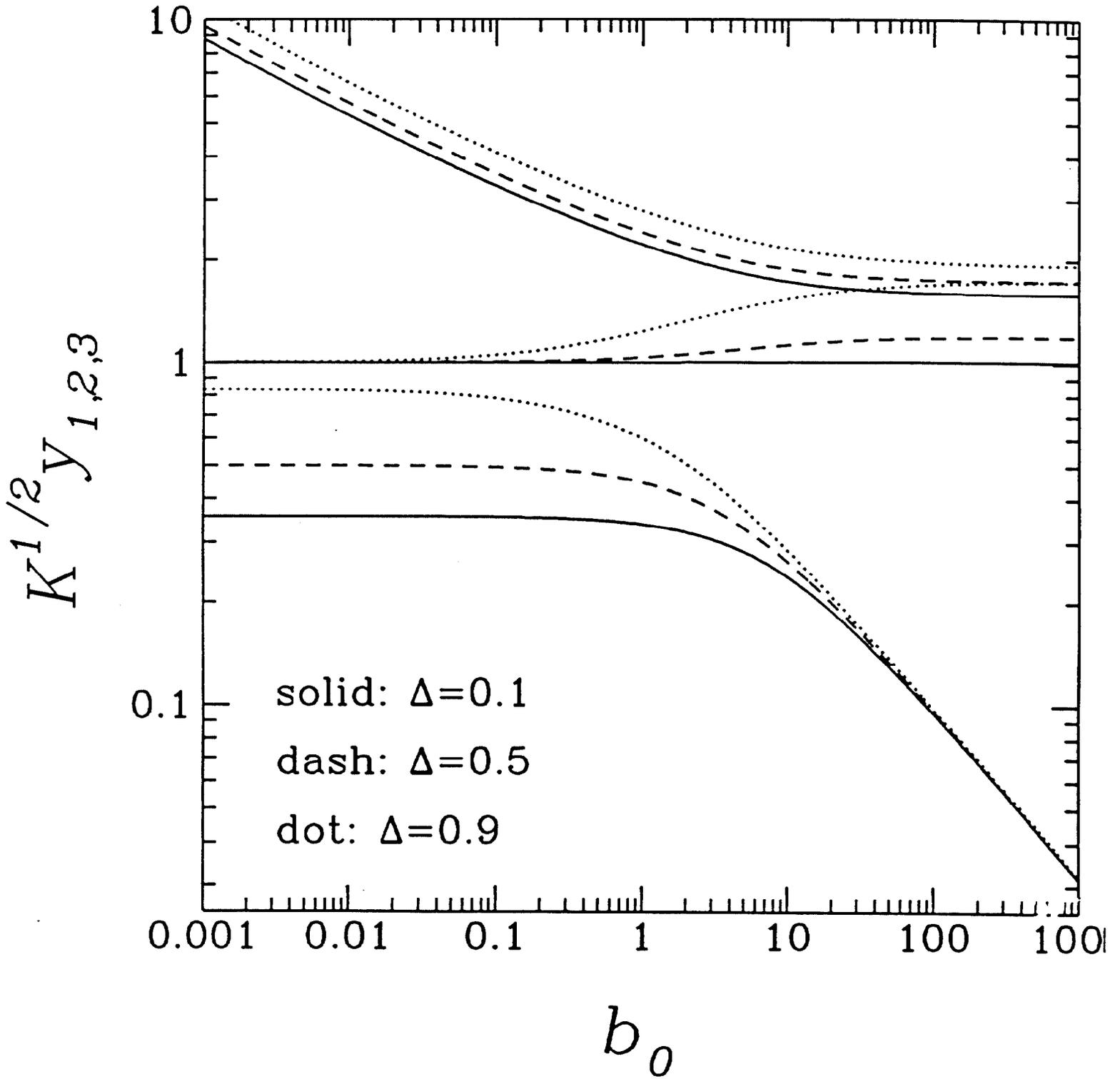


Fig. 4

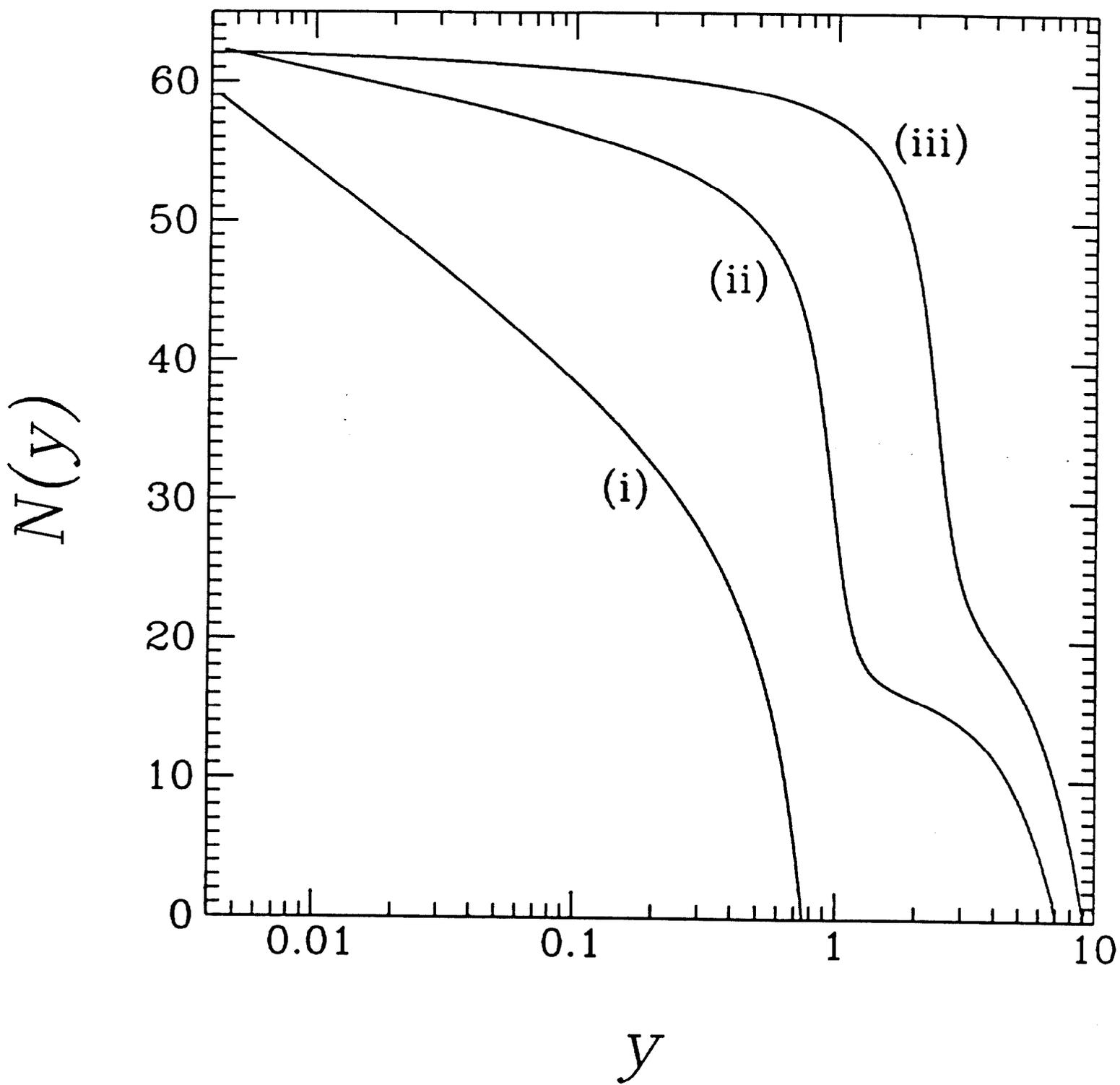


Fig. 5

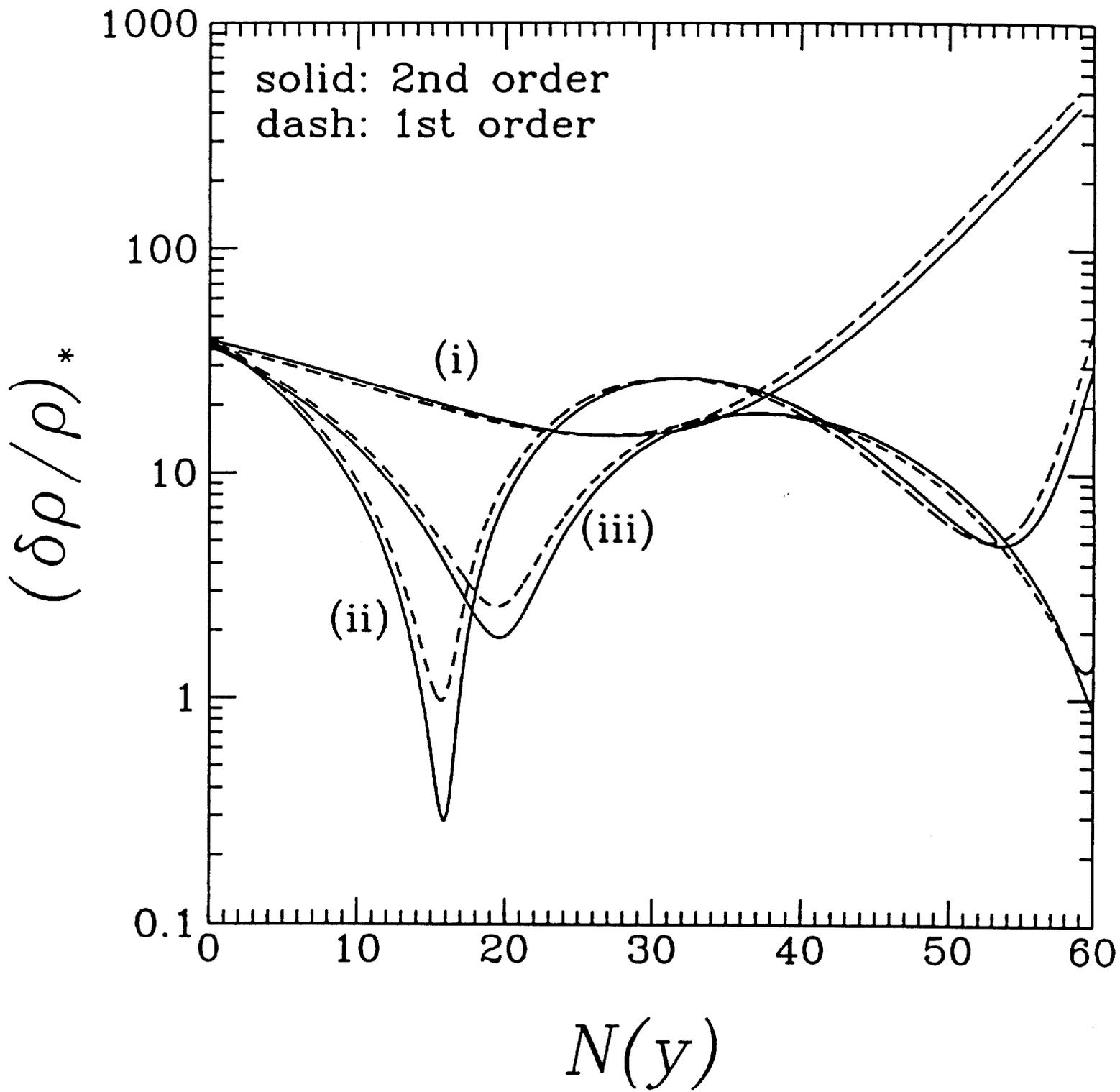


Fig. 6

