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Decaying Λ Cosmologies and Power Spectrum

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Abstract

We investigate the evolution of matter density perturbations in a Universe with a cosmological term that decreases with time as $\Lambda \propto a^{-m}$. For fixed values of Ω_{m0} the power spectrum is constructed and we show that it is only slightly modified when the parameter m is changed from $m = 0$ to $m = 2$. Some properties concerning the peculiar velocity field are also discussed.

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1 Introduction

In spite of its theoretical appeal and remarkable agreement with several sets of observational data, it is nowadays quite generally accepted that the standard cold dark matter (CDM) model[1] must be modified in at least one of its basic assumptions. The main reason is that this model predicts less power in the perturbation spectrum at large scales ($l > 10h^{-1}Mpc$) than inferred from observations.[2] We can find in the literature three important attempts to correct the standard CDM model deficiencies while preserving its qualities. In the first one, generally called "tilted models"[3],[4] , a non flat primordial power spectrum (with more power in large scales) is assumed. A modification in the power spectrum index naturally appears in some inflationary models[4]. A mixture of hot and cold dark matter in a flat space has also been proposed[5] , but it is still not clear if these models are completely consistent with observations.[6] Finally, flat, low energy density models with a cosmological constant are also an alternative to the standard CDM model.[2],[7]

The present interest in the flat cosmological constant model has also appeared motivated by two other reasons. First, a Λ term helps to reconcile inflation with observations. This term could be responsible for the missing mass necessary to "close" the universe. Second, with Λ, it is possible to obtain, for flat universes, a theoretical age in the observed range, even for a high value of the Hubble parameter.[8]

In this paper we shall examine a variant of the cosmological constant model. By introducing a new parameter m, we will explore the possibility that the cosmological term decreases with time as $\Lambda \propto a^{-m}$. Here a is the scale factor of the FRW models. Cosmological models with different expressions for the Λ term have recently been proposed.[9] The dependence on a we use here for the cosmological term, was first introduced by Gasperini[10] and generalizes the $m = 2$ case, suggested by Chen and

Wu[11].

In this work, we study how the evolution of density perturbations is affected by m. We solve the equations for the CDM density contrast in the presence of the decaying Λ for arbitrary values of m. With this solution, we construct the power spectrum and show that it is only slightly modified when m is changed from $m = 0$ to $m = 2$.

The paper is organized as follows: In section 2, the assumptions and basic equations of our models are presented and we discuss how Λ decay models can be viewed as an alternative to implement "exotic" or "loitering" cosmologies. In section 3, the evolution of density perturbations is obtained and previous results concerning the peculiar velocity field are generalized. In section 4 we construct the power spectrum for different values of m and Ω_{m0}, and analyze the results.

2 Decaying Vacuum Cosmological Models

We investigate spatially flat, homogeneous and isotropic cosmologies with a variable cosmological term. The cosmic fluid is assumed to be a non interacting mixture of some kind of cold dark matter and radiation ($p_r = \frac{1}{3}\rho_r$). As a first approximation baryons are not taken into account.

The total energy momentum tensor of the cosmic fluid has the perfect fluid form,

$$T^{\mu}{}_{\nu} = T_r{}^{\mu}{}_{\nu} + T_m{}^{\mu}{}_{\nu} = \text{ diag } (\rho, -p, -p, -p), \tag{1}$$

where $\rho = \rho_r + \rho_m$ is the total energy density (radiation plus CDM) and $p = p_r$ is the thermodynamic pressure.

The behavior of the fluid is governed by Einstein equations with the cosmological term :

$$G^{\mu\nu} = 8\pi G T^{\mu\nu} + \Lambda g^{\mu\nu}. \tag{2}$$

We shall assume Λ to be a time dependent quantity having the following dependence on the scale factor a ,

$$\Lambda = 8\pi G\rho_v = 3\alpha a^{-m}. \tag{3}$$

For the sake of simplicity we shall restrict the parameters α and m to the range $0 \leq m \leq 2$ and $\alpha \geq 0$. The factor 3 was introduced in equation (3) for mathematical convenience.

We also assume that vacuum decays only into relativistic particles (non necessarily photons), such that the matter energy momentum tensor is conserved ($\rho_m \propto a^{-3}$). Hence, due to the Λ decay, radiation is no longer conserved and it is straightforward to show that Bianchi identities imply,

$$\rho_r = \Omega_{r0}H_0^2(\frac{a_0}{a})^4 + \frac{3m\alpha}{8\pi G(4-m)}a^{-m}, \tag{4}$$

where a_0 is the present value of the scale factor and $H_0 = 100h^{-1}Kms^{-1}Mpc^{-1}$ is the present value of the Hubble parameter. In the following, subscripts 0 will always indicate present values.

The first term on the right hand of (4) is the usual conserved radiation term with $\Omega_{r0} = 4.3 \times 10^{-5}h^{-2}$ standing for the present value of its density parameter (we are considering photons and three neutrino species). The second term is related to particle creation and arises due the vacuum decay.

The Einstein equations reduce to two equations, namely,

$$\left(\frac{\dot{a}}{a}\right)^2 = \Omega_{m0}H_0^2\left(\frac{a_0}{a}\right)^3 + \Omega_{r0}H_0^2\left(\frac{a_0}{a}\right)^4 + \Omega_{x0}H_0^2\left(\frac{a_0}{a}\right)^m, \tag{5}$$

and

$$\frac{\ddot{a}}{a} = -\frac{1}{2}\Omega_{m0}H_0^2\left(\frac{a_0}{a}\right)^3 - \Omega_{r0}H_0^2\left(\frac{a_0}{a}\right)^4 + \frac{\alpha(2-m)}{2}\Omega_{x0}H_0^2\left(\frac{a_0}{a}\right)^m, \tag{6}$$

where Ω_{m0} is the matter density parameter and $\Omega_{x0} = \frac{4\alpha H_0^{-2} a_0^{-m}}{(4-m)}$. The above equations were written in a suitable form to be compared with those derived in other models. For instance, if $m = 0$, we recover the usual flat FRW models with a cosmological constant. Further, if we take $m = 2$, we may identify (5) and (6) with Einstein equations of open FRW models. In fact what we have is not a open model. Remember that although the equations are formally the same, here the space is flat. The same kind of equations also appear in some cosmic string models.[12]

As a matter of fact, we would obtain the above equations if we had considered conserved matter, radiation and a fluid with the equation of state, $p_x = \left(\frac{m}{3} - 1\right) \rho_x$, $(0 \leq m \leq 2)$. Some aspects of these models were analyzed in references [13] and [14]. When the space curvature is positive they are usually called *loitering universes.* In this case, the models have a loitering phase in which $\dot{a}=\ddot{a}= 0$. For flat models, however, only $\ddot{a}= 0$ is possible (*coasting models*). Anyway, we remark that vacuum decay models can be thought as another form to implement loitering universes.

3 Perturbation Growth and Peculiar Velocity

Our next step is to describe the evolution of perturbations in models with decaying Λ. These models show three distinct phases according to the dominant component of the energy density in that phase: conserved radiation, matter, or vacuum with its light decay products - thereafter called x-component.

For perturbations well inside the horizon, Newtonian gravity can be applied and, the equation describing the evolution of the CDM component density contrast , $\delta = \frac{\delta \rho_m}{\rho_m}$, is [15]

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} - 4\pi G \rho_m \delta = 0. \tag{7}$$

The solution of (7) depends on the expansion rate $\frac{\dot{a}}{a}$, which, in turn, will depend on the relative contributions of radiation, matter and vacuum to the energy density. It is useful to consider two different regimes. In the first one, conserved radiation or matter is dominant, the x-component contribution remains always the smallest among the three and may be neglected. In the second one, we consider the dominant contributions of matter and x-component, the conserved radiation term is now much smaller than the dominant contributions and may be ignored.

During the first epoch, in which matter and conserved radiation are dominant, we change the variable to $y = \frac{a}{a_{eq}}$, where $a_{eq} = \frac{\Omega_{r0}}{\Omega_{m0}} a_0$ is the scale factor value of the conserved radiation and matter equality , and using the field equations, we get

$$\delta'' + \frac{2+3y}{2y(1+y)}\delta' - \frac{3\delta}{2y(1+y)} = 0. \tag{8}$$

The derivatives are now taken with respect to y . Equation (8) has two linearly independent solutions, the growing (δ_+) and the decreasing (δ_-) modes,

$$\delta_+ \propto 1 + \frac{3}{2}y$$

and

$$\delta_- \propto (1 + \frac{3y}{2}) \ln[\frac{(1+y)^{1/2}+1}{(1+y)^{1/2}-1}] - 3(1+y)^{1/2}.$$

These solutions were originally obtained by Meszaros[16] and by Groth and Peebles[17]. Some of their properties are discussed in references [15] and [18].

So far our cosmological model behaves as the standard one. However, as the universe expands, the a^{-m} term in the field equations becomes more and more noticeable. The model begins to deviate from the standard CDM model when the scale factor reaches the value $a_M = a_0(\frac{\Omega_{r0}}{\Omega_{x0}})^{\frac{1}{4-m}}$,the value of a when conserved radiation and x-component contribute equally to the energy density. In fact, around this epoch CDM (the a^{-3} term in (5) and (6)) effectively dominates the dynamics. However, neglecting now the x-component is no longer a good approximation. Instead, for $a > a_M$, a more accurate procedure would be to neglect the a^{-4} term in (5) and (6).

We are now in the second regime, dominated by matter and vacuum and its decay products. Eq. (7), for the evolution of the density contrast, will now be written as,

$$(3-m)^2(1+w)w^2\delta'' + (3-m)\left[(\frac{w}{2}-m) + w(5-\frac{3m}{2})\right] w\delta' - \frac{3}{2}\delta = 0, \qquad (9)$$

where we introduced the new variable $w = (\frac{a}{a_d})^{3-m}$, with $a_d = (\frac{\Omega_{m0}}{\Omega_{x0}})^{\frac{1}{3-m}}$ denoting the value of the scale factor for the CDM and x-component energy density equality. In (9) derivatives are taken with respect to w .

We stress that, as in the earlier radiation-matter era, perturbations in the radiation component oscillate and in average can be taken equal to zero. Further, as remarked before, the present interest in a Λ term arises mainly as a way to conciliate the theoretical appeal of the inflationary models with observations, which suggest $\Omega_{m0} \simeq 0.2$. The vacuum component is usually assumed to be smooth, otherwise it would be detected by the dynamical methods contributing for the effective value of the density parameter. Hence, in (9), possible perturbations in the cosmological term were also neglected.

The solutions of (9) can be expressed in terms of the hypergeometric function $F(a,b,c,w)$ as,

$$\Delta_+ \propto w^{\frac{1}{3-m}} F\left(\frac{1}{3-m}, \frac{6-m}{6-2m}, \frac{11-2m}{6-2m}, -w\right) \qquad (10)$$

and

$$\Delta_- \propto w^{-\frac{3}{6-2m}} F\left(-\frac{3}{6-2m}, \frac{1-m}{6-2m}, \frac{1-2m}{6-2m}, -w\right). \tag{11}$$

Note that when $m = 0$ we recover the subhorizon solutions for CDM and a cosmological constant[15],[18][19], now expressed in terms of hypergeometric functions. Again, if $m = 2$, the CDM open model[15],[18] solutions are reobtained. It is easy to show that for $w << 1$ ($a << a_d$) we obtain the standard result,

$$\Delta_+ \propto w^{\frac{1}{3-m}} \propto a \tag{12}$$

and

$$\Delta_- \propto w^{-\frac{3}{6-2m}} \propto a^{-\frac{3}{2}}. \tag{13}$$

With the solutions (10) and (11) we can construct the peculiar velocity field. In the linear regime, by taking only the growing mode during the matter and x-component regime, we have[15]

$$\vec{v} = \frac{2\vec{g}}{3H\Omega_m} f,$$

where $\vec{g}$ is the peculiar acceleration and

$$f = \frac{a}{\Delta_+}\frac{\partial \Delta_+}{\partial a} \tag{14}$$

is the Peebles dimensionless function. Neglecting the conserved radiation component we can write the variable w as $w = \frac{1-\Omega_m}{\Omega_m}$ and rewrite (14) as,

$$f = -(3-m)(1-\Omega_m)\frac{\Omega_m}{\Delta_+}\frac{\partial \Delta_+}{\partial \Omega_m}. \tag{15}$$

From the behavior of the logarithmic plot of f as a function of Ω_m (figure 1) we can see that it is a good approximation to take $f \simeq (\Omega_m)^n$. To better illustrate this fact we also show in figure (2) the parameter $n = \frac{\log f}{\log \Omega_m}$ as a function of $\Omega_m$for $m = 0, 1$ and 2. In the limit $\Omega_m \simeq 1$ two previous results present in the literature can be recovered. If $m = 2$ [13], we obtain $f \simeq \Omega_m^{4/7}$ and if $m = 0$[20], we have $f \simeq \Omega_m^{6/11}$. If $m = 1$, we get $f \simeq \Omega_m^{5/9}$. These are special cases of the general expression

$$f \simeq (\Omega_m)^{\frac{6-m}{11-2m}}, \tag{16}$$

obtained by expanding eq.(15) around $\Omega_m = 1$.

By using $\Omega_m = \frac{1}{1+\frac{1-\Omega_{mo}}{\Omega_{mo}}(1+z)^{m-3}}$, the variation of f as function of the redshift z can also be constructed[19,20]. As shown in figure (3), if $z << 1$, there is no appreciable difference between models with the same Ω_{m0}. So, although the dynamical methods allow the determination of Ω_{m0} by measuring the peculiar velocity of objects at low redshift, it will not be possible to distinguish between models with different value of m. In the future, however, the situation may change if a more precise value of Ω_{m0} is achieved and if it becomes possible to perform similar tests at higher redshifts.[20]

4 Power Spectrum

Finally, we want to obtain the power spectrum and to compare the result for different vacuum decay rates (different values of m). To do this, we need to know how the perturbations with different wavelengths evolve from some given initial spectrum to the final spectrum (at some later time t_f). Basically, this evolution will include a stage in which the perturbation is still outside the horizon and the Newtonian equation we have solved must be replaced by its relativistic counterpart. Since we want to focus on the relative behavior of different values of m, we consider a simplified assumption. The spectrum is considered to be a power law at the horizon entrance (t_{enter}), that is:

$$\delta_\lambda(t_{enter}) = A\lambda^\alpha \tag{17}$$

For $\alpha = 3/2$, this corresponds to the scale invariant spectrum, which is predicted by some inflationary models. In fact, the evolution of the perturbation is affected by the cosmological term mainly after its domination over matter. Since this happens very late in the evolution the Universe, for almost all scales of interest, the perturbation is inside the horizon when the x-component dominates and the Newtonian equation is enough to study its growth. Besides, when the x-component is dominating, we generically expect that the largest wavelengths will gradually leave the horizon, as it happens during the inflationary period [1]. Wavelengths that remain inside the horizon up to t_f were already inside when the matter domination finished, and the effect of the x-component on their growth is computed by (7). For these reasons, the simplification made by the choice (17) does not interfere substantially with the relative evolution for different values of m.

Taking (17) with $\alpha = 3/2$ as the initial spectrum, we consider t_{enter} as a function of λ given by the condition:

$$\lambda a(t_{enter}) = \frac{1}{H(t_{enter})} \tag{18}$$

For $a = a_M$, we denote the solution of (18) by λ_M. Thus, λ_M is the wavelength that cross the horizon when conserved radiation and x-component balance their contributions to the total energy density. For those wavelengths that cross the horizon after a_M, that is $\lambda > \lambda_M$, the perturbation grows with Δ_+ after the horizon entrance. For those with $\lambda < \lambda_M$, perturbations grow with δ_+ after the horizon crossing until a_M,

[1]This is strictly true for $m = 0$, $m = 1.0$ but for $m = 2.0$ a few wavelengths do enter the horizon after vacuum domination

when this solution must be matched to the solutions of (9) to give the final growth factor.

By using the top hat window function, the normalization is chosen (for all values of m) by setting the root-mean-square mass fluctuation within spheres of radius $r_o = 8h^{-1}Mpc$ to be equal unity,

$$< (\frac{\delta M}{M})^2 >^{1/2}_{r_o=8h^{-1}Mpc} = 1.$$

The results are shown in figure 4 for both $\Omega_{m0} = 0.2$ and $\Omega_{m0} = 0.4$ together with the minimal cold dark matter model, $\Omega_{m0} = 1.0$ (no baryons). As compared with this minimal cold dark matter model, we observe the expected decreasing power on the small scales and the increasing power on large scales for all values of m. Since $h^2\Omega_{m0}$ is the same for all m's, we have the same λ_{eq} in all cases and the bend in the power spectrum occurs at the same point. For scales larger than λ_{eq}, the three curves become distinct, showing a faster decrease for larger values of m, but, roughly, they all show a more favorable behavior with more power on large scales than the minimal cold dark model.

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Figure Captions

Figure 1: The loglog plot of f as a function of Ω_m is shown for $m = 0$, 1 and 2.

Figure 2: The parameter $n = \frac{log f}{log \Omega_m}$ is displayed as a function of Ω_m for $m=$ 0, 1 and 2. Note that, for values of m larger than 0.05 , n is roughly constant, indicating that, in this limit, $f = (\Omega_m)^n$ is a good approximation.

Figure 3: The variation of f as a function of the redshift z is displayed for two values of the present density parameter ($\Omega_{m0} = 0.2$ and $\Omega_{m0} = 0.4$) and the assigned values of m. For $z << 1$ the difference among models with the same Ω_{m0} is very small: these models can not be distinguished by the present measurement accuracy.

Figure 4: Power spectra for minimal CDM ($\Omega_{m0} = 1.0$) and CDM with decaying cosmological term in two cases: $\Omega_{m0} = 0.2$ and $\Omega_{m0} = 0.4$. The decay rate changes for different values of m (0, 1 and 2). For all models the spectra are normalized using the top hat function with $r_0 = 8h^{-1}Mpc$.

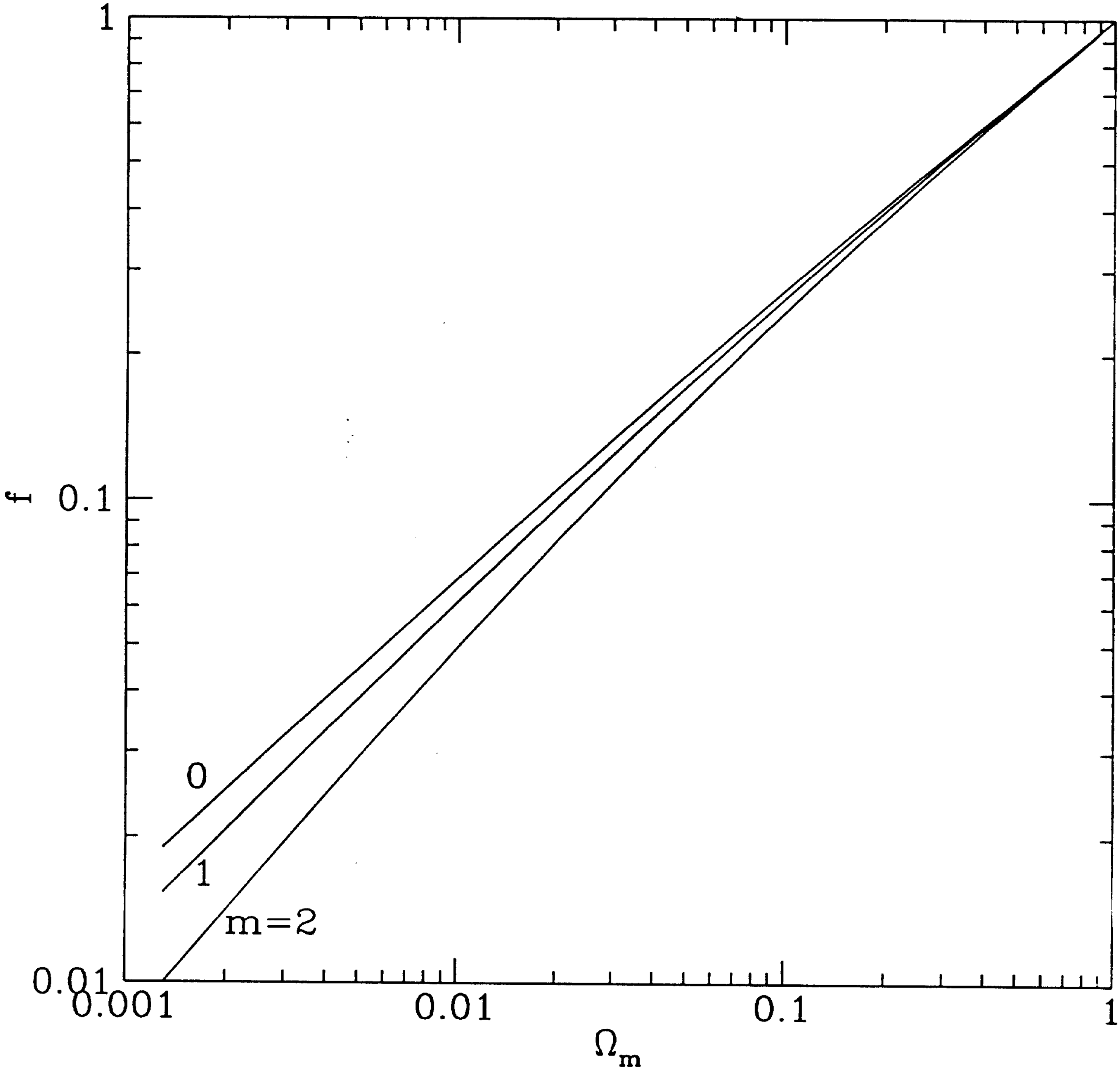

Fig. 1

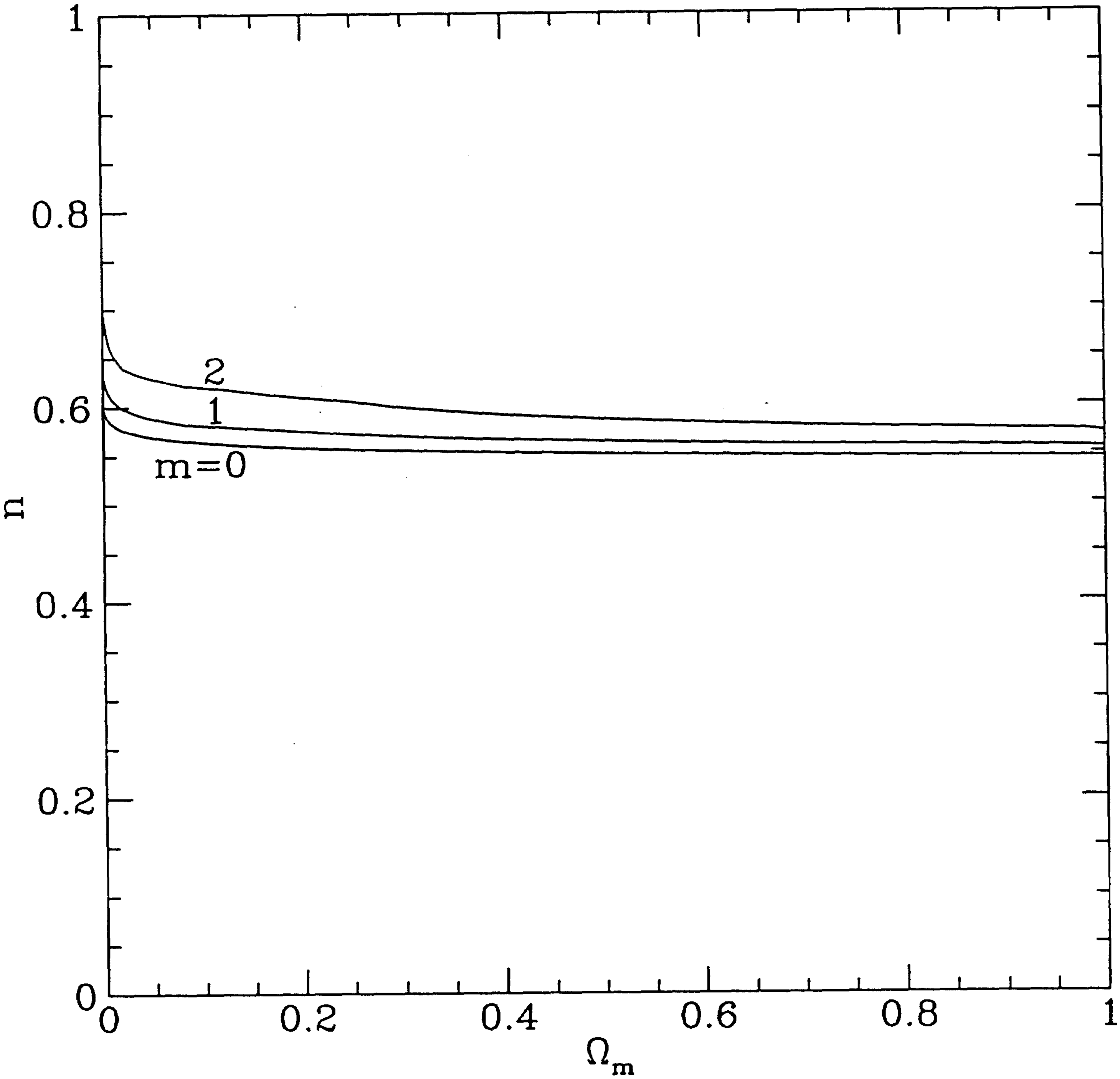

Fig. 2

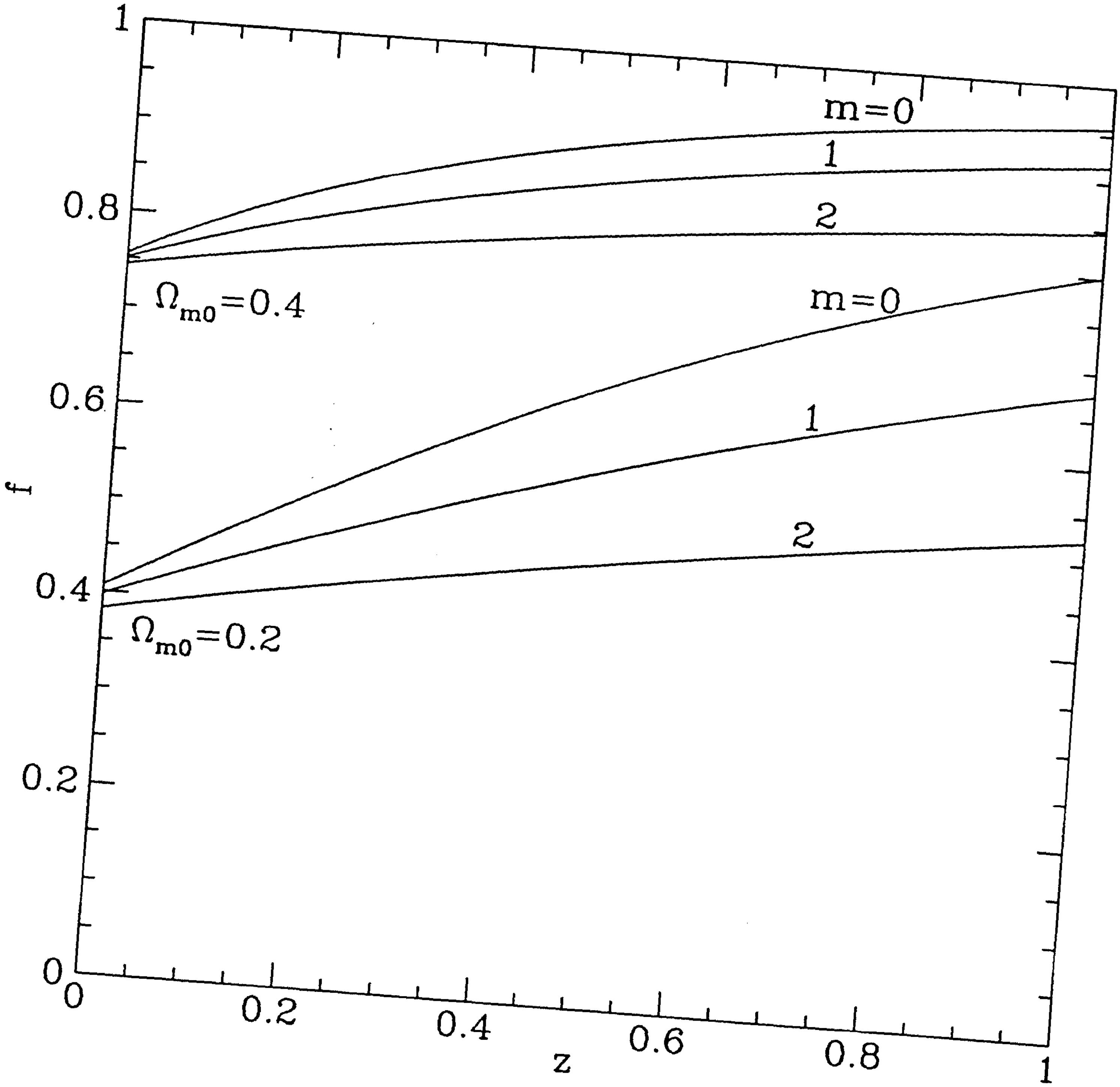

Fig. 3

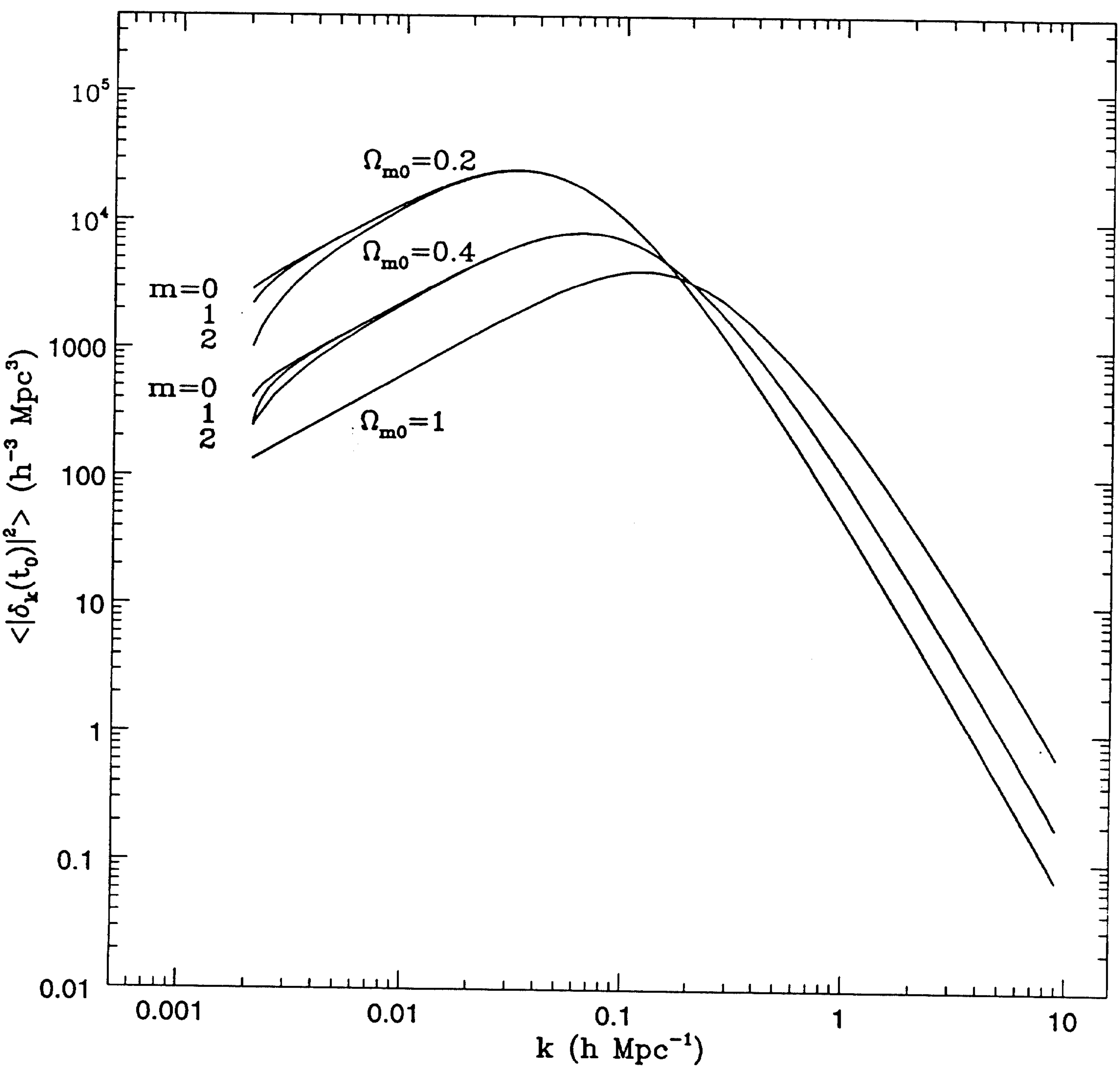
Ω_{m0}=0.2
Ω_{m0}=0.4
Ω_{m0}=1
m=0
1
2
m=0
1
2
10^5
10^4
1000
100
10
1
0.1
0.01
0.001
0.01
0.1
1
10
$<|\delta_k(t_0)|^2>$ (h^{-3} Mpc^3)
k (h Mpc^{-1})

Fig 4