

hep-ph/9403401

FERMILAB-PUB-94/040-T

NUHEP-TH-94-4

UCD-94-3

March 1994

Gluon Fragmentation into P-wave Heavy Quarkonium

Eric Braaten¹

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, IL 60510

Tzu Chiang Yuan

Davis Institute for High Energy Physics

Department of Physics, University of California, Davis, CA 95616

Abstract

The fragmentation functions for gluons to split into P-wave heavy quarkonium states are calculated to leading order in the QCD coupling constant. Long-distance effects are factored into two nonperturbative parameters: the derivative of the radial wavefunction at the origin and a second parameter related to the probability for a heavy-quark-antiquark pair that is produced in a color-octet S-wave state to form a color-singlet P-wave bound state. The fragmentation probabilities for a high transverse momentum gluon to split into the P-wave charmonium states χ_{c0} , χ_{c1} , and χ_{c2} are estimated to be 0.4×10^{-4} , 1.8×10^{-4} , and 2.4×10^{-4} , respectively. This fragmentation process may account for a significant fraction of the rate for the inclusive production of χ_{cJ} at large transverse momentum in $p\bar{p}$ colliders.

¹on leave from Dept. of Physics and Astronomy, Northwestern University, Evanston, IL 60208



gluon can split into χ_{cJ} at order α_s^2 through the Feynman diagram in Figure 1, gluon splitting into the h_c occurs first at one order higher in α_s . Since 1^{+-} states of heavy quarkonium like the h_c are difficult to observe experimentally, we concentrate in this paper on the J^{++} states.

In charmonium, the charmed quark and antiquark are nonrelativistic with typical velocity $v \ll 1$ and typical separation $1/(m_c v)$. Our calculation of the fragmentation function is based on separating short-distance effects involving the scale $1/m_c$ from long-distance effects involving scales of order $1/(m_c v)$ or larger. There are two distinct mechanisms that contribute to the fragmentation function at leading order in v [8], and we will refer to them as the *color-singlet mechanism* and the *color-octet mechanism*. The *color-singlet mechanism* is the production of a $c\bar{c}$ pair in a color-singlet 3P_J state with separation of order $1/m_c$ in the quarkonium rest frame. The subsequent formation of the χ_{cJ} is a long-distance process with probability of order v^5 . In addition to the volume factor $(m_c v/m_c)^3$, there is an extra suppression factor of v^2 from the wavefunction of the P-state near the origin. The *color-octet mechanism* is the production of a $c\bar{c}$ pair in a color-octet 3S_1 state with separation of order $1/m_c$. The subsequent formation of the χ_{cJ} can proceed either through the dominant $|c\bar{c}\rangle$ component of the χ_{cJ} wavefunction or through the small $|c\bar{c}g\rangle$ component, which has a probability of order v^2 . In the first case, the $c\bar{c}$ pair must radiate a soft gluon to make a transition to the color-singlet ${}^3P_J |c\bar{c}\rangle$ state. In the second case, a soft gluon must combine with the $c\bar{c}$ pair to form a color-singlet $|c\bar{c}g\rangle$ state. In either case, the probability is of order v^5 , with a volume factor of v^3 and an additional suppression of v^2 coming either from the probability of radiating a soft gluon or from the small probability of the $|c\bar{c}g\rangle$ component of the wavefunction. Since the color-singlet mechanism and the color-octet mechanism contribute to the fragmentation function at the same order in v , they must both be included for a consistent calculation.

Separating effects due to short distances of order $1/m_c$ from those of longer distance scales requires the introduction of an arbitrary factorization scale Λ in the range $m_c v \ll \Lambda \ll m_c$. The fragmentation functions for heavy quarkonium satisfy factorization formulas that involve this arbitrary scale. At leading order in v^2 , the factorization formula for the

fragmentation function for $g \rightarrow \chi_{cJ}$ has two terms:

$$D_{i \rightarrow \chi_{cJ}}(z, m_c) = \frac{H_1}{m_c} d_1^{(J)}(z, \Lambda) + (2J + 1) \frac{H'_8(\Lambda)}{m_c} d_8(z), \quad (1)$$

where H_1 and $H'_8(\Lambda)$ are nonperturbative long-distance factors associated with the color-singlet and color-octet mechanisms, respectively. The short-distance factors $d_1^{(J)}(z, \Lambda)$ and $d_8(z)$ can be calculated using perturbation expansions in $\alpha_s(m_c)$. They are proportional to the fragmentation functions for a gluon to split into a $c\bar{c}$ pair with vanishing relative momentum and in the appropriate color-spin-orbital state: color-singlet 3P_J state for $d_1^{(J)}$ and color-octet 3S_1 state for d_8 . Note that in the factorization formula (1), the only dependence on Λ is in $d_1^{(J)}$ and H'_8 . This simple form holds if the coefficients are calculated at most to next-to-leading order in α_s . Beyond that order, d_8 also acquires a weak dependence on Λ and J .

The nonperturbative parameters H_1 and H'_8 can be rigorously defined as matrix elements of 4-quark operators in nonrelativistic QCD [9]. Their dependence on Λ is given by renormalization group equations whose coefficients can be calculated as perturbation series in $\alpha_s(\Lambda)$ [8]. To order α_s , H_1 is scale invariant and H'_8 satisfies

$$\Lambda \frac{d}{d\Lambda} H'_8(\Lambda) = \frac{16}{27\pi} \alpha_s(\Lambda) H_1. \quad (2)$$

If the factorization scale Λ is chosen to be much less than m_c , this equation can be used to sum up large logarithms of m_c/Λ :

$$H'_8(m_c) = H'_8(\Lambda) + \frac{16}{27\beta_0} \log\left(\frac{\alpha_s(\Lambda)}{\alpha_s(m_c)}\right) H_1, \quad (3)$$

where $\beta_0 = (33 - 2n_f)/6$ is the first coefficient in the beta function for QCD with n_f flavors of light quarks. The parameter H_1 can be related to the derivative of the nonrelativistic radial wavefunction at the origin for the P-wave states:

$$H_1 \approx \frac{9}{2\pi} \frac{|R'_P(0)|^2}{m_c^4} (1 + \mathcal{O}(v^2)). \quad (4)$$

This parameter can be determined phenomenologically from the annihilation rates of the χ_{cJ} states. Using recent high precision measurements of the light hadronic decay rates of χ_{c1} and χ_{c2} , H_1 has been determined to be approximately 15 MeV [10]. The parameter H'_8 was introduced in Ref. [11] in a calculation of the rate for the decay $b \rightarrow \chi_{cJ} + X$, which also receives contributions from both the color-singlet and color-octet mechanisms for χ_{cJ} production. The prime on H'_8 is a reminder that this parameter is not related in any rigorous way to the corresponding parameter H_8 that appears in decays of the χ_{cJ} states into light hadrons. Using data on the inclusive decays of B -mesons into charmonium, its value for $\Lambda = m_c$ has been estimated to be $H'_8(m_c) \approx 3$ MeV [11]. This parameter also enters into the inclusive decay rate of the Υ into P-wave charmonium states [12].

We now turn to the calculation of the coefficient $d_1^{(J)}(z, \Lambda)$ in the color-singlet contribution to the fragmentation function. We follow the method and notation of Ref. [2]. Let \mathcal{A}_α denote the amplitude for $g^* \rightarrow c\bar{c}({}^3P_{J,\perp}) + g$ corresponding to the Feynman diagram in Figure 1. The $c\bar{c}$ pair have almost equal momenta, and are in a color-singlet 3P_J state. The amplitude \mathcal{A}_α can be written down in terms of $R'_p(0)$ using standard Feynman rules for quarkonium processes [13]. Multiplying \mathcal{A}_α by its complex conjugate and summing over final colors and spins, we obtain the following generic form

$$\sum \mathcal{A}_\alpha \mathcal{A}_\beta^* = \frac{H_1}{m_c} \left(A_J(s)(-g_{\alpha\beta}) + B_J(s)p_\alpha p_\beta + C_J(s)(p_\alpha q_\beta + q_\alpha p_\beta) + D_J(s)q_\alpha q_\beta \right), \quad (5)$$

where p and q are the 4-momenta of the $c\bar{c}$ pair and the fragmenting gluon g^* , respectively, and $s = q^2$. The strategy is to reduce this expression in the limit $q_0 \gg m_c$ to the polarization sum $(-g_{\alpha\beta} + \dots)$ for an on-shell gluon multiplied by a function of s and z , where z is the longitudinal momentum fraction of the $c\bar{c}$ pair relative to the fragmenting gluon. Terms in (5) that are proportional to q_α or q_β can be dropped, because in axial gauge, q_α and q_β are of order m_c^2/q_0 when contracted with the numerator of the propagator of the virtual gluon. In the $p_\alpha p_\beta$ term, we can set $p = zq + p_\perp$, where p_\perp is the transverse part of the 4-vector p . After averaging over the directions of p_\perp , $p_\alpha p_\beta$ can be replaced by $-g_{\alpha\beta} \vec{p}_\perp^2/2$, up to terms that are suppressed in axial gauge. The terms in (5) that contribute to fragmentation then

reduce to

$$\sum \mathcal{A}_\alpha \mathcal{A}_\beta^* \approx \frac{H_1}{m_c} \left(A_J(s) + \frac{\vec{p}_\perp^2}{2} B_J(s) \right) (-g_{\alpha\beta}). \quad (6)$$

Energy-momentum conservation in the form $s = (\vec{p}_\perp^2 + 4m_c^2)/z + \vec{p}_\perp^2/(1-z)$ can be used to eliminate \vec{p}_\perp^2 in (6) in favor of s and z . The fragmentation probability is obtained by dividing the coefficient of $(-g_{\alpha\beta})$ by s^2 for the propagator of the virtual gluon, and then integrating over the phase space of the $c\bar{c}$ pair and the gluon in the final state. The phase space integral can be expressed compactly in terms of integrals over s and z [2]. The resulting expression for the integral over z of $d_1^{(J)}(z, \Lambda)$ is

$$\int_0^1 dz d_1^{(J)}(z, \Lambda) = \frac{1}{16\pi^2} \int_{s_{\min}(\Lambda)}^\infty ds \int_{4m_c^2/s}^1 dz \frac{1}{s^2} \left(A_J(s) + \frac{(1-z)(zs - 4m_c^2)}{2} B_J(s) \right). \quad (7)$$

We have anticipated the presence of an infrared divergence associated with a soft gluon in the final state by imposing a lower cutoff Λ on the energy of the gluon in the quarkonium rest frame. This translates into a lower limit on s : $s_{\min}(\Lambda) = 4m_c^2(1 + \Lambda/m_c)$. The calculations of the functions $A_J(s)$ and $B_J(s)$ in (7) involves some rather complicated algebra, but the final results are relatively simple. Interchanging orders of integration in (7), we can read off the functions $d_1^{(J)}(z, \Lambda)$:

$$d_1^{(J)}(z, \Lambda) = \frac{\alpha_s^2}{27} \int_{4m_c^2/z}^\infty ds \frac{m_c^2}{s^2(s - 4m_c^2)^4} f_J(s, z), \quad z < \left(1 + \frac{\Lambda}{m_c}\right)^{-1}, \quad (8)$$

$$= \frac{\alpha_s^2}{27} \int_{s_{\min}(\Lambda)}^\infty ds \frac{m_c^2}{s^2(s - 4m_c^2)^4} f_J(s, z), \quad z > \left(1 + \frac{\Lambda}{m_c}\right)^{-1}, \quad (9)$$

where

$$f_0(s, z) = (s - 12m_c^2)^2 \left[(s - 4m_c^2)^2 - 2(1-z)(zs - 4m_c^2)s \right], \quad (10)$$

$$f_1(s, z) = 6s^2 \left[(s - 4m_c^2)^2 - 2(1-z)(zs - 4m_c^2)(s - 8m_c^2) \right], \quad (11)$$

$$f_2(s, z) = 2 \left[(s - 4m_c^2)^2 (s^2 + 96m_c^4) - 2(1-z)(zs - 4m_c^2)s(s^2 - 24sm_c^2 + 96m_c^4) \right] \quad (12)$$

For $z < (1 + \Lambda/m_c)^{-1}$, the integral over s in (8) can be calculated straightforwardly. The cutoff Λ can be set to zero everywhere except in terms proportional to $1/(1-z)$, which

diverge upon integrating over z . In the $1/(1-z)$ terms, the limit $\Lambda \ll \bar{m}_c$ must be taken more carefully, and it gives rise to a plus-distribution:

$$\frac{1}{1-z} \theta\left(1-z-\frac{\Lambda}{m_c}\right) \rightarrow \frac{1}{(1-z)_+} - \log\frac{\Lambda}{m_c} \delta(1-z). \quad (13)$$

For $z > (1+\Lambda/m_c)^{-1}$, the limit $\Lambda \ll m_c$ can be taken only after evaluating the integral over s in (9). This gives rise to additional endpoint contributions proportional to $\delta(1-z)$. Our final result for the short distance factor multiplying H_1/m_c in the fragmentation function is

$$d_1^{(J)}(z, \Lambda) = \frac{2\alpha_s^2}{81} \left[(2J+1) \frac{z}{(1-z)_+} + \left(Q_J - (2J+1) \log\frac{\Lambda}{m_c} \right) \delta(1-z) + P_J(z) \right], \quad (14)$$

where the coefficients Q_J are

$$Q_0 = \frac{13}{12}, \quad Q_1 = \frac{23}{8}, \quad Q_2 = \frac{121}{24}, \quad (15)$$

and the functions $P_J(z)$ are

$$P_0(z) = \frac{z(85-26z)}{8} + \frac{9(5-3z)}{4} \log(1-z), \quad (16)$$

$$P_1(z) = -\frac{3z(1+4z)}{4}, \quad (17)$$

$$P_2(z) = \frac{5z(11-4z)}{4} + 9(2-z) \log(1-z). \quad (18)$$

We next consider the color-octet coefficient $d_8(z)$ in the fragmentation formula (1). At leading order in α_s , this contribution to the fragmentation function comes from the subprocess $g^* \rightarrow c\bar{c}({}^3S_1, \underline{8})$ given by the Feynman diagram in Figure 2. The c and \bar{c} have equal momenta $q/2$, and are in a color-octet 3S_1 state. The projection onto this state can be reduced to a simple Feynman rule:

$$v(q/2)\bar{u}(q/2) \rightarrow \frac{R_8(0)}{\sqrt{16\pi m_c}} T_{ij}^a \not{\epsilon}(q)(\not{q} + 2m_c), \quad (19)$$

where $\epsilon^\mu(q)$ is the polarization 4-vector of the 3S_1 state and i, j , and a are the color indices of the quark, antiquark, and color-octet state, respectively. The parameter $R_8(0)$ is a fictitious

“color-octet radial wavefunction at the origin” related to the nonperturbative matrix element $H'_8(\Lambda)$ by $H'_8 = (2/3\pi)|R_8(0)|^2/m_c^2$. The square of the amplitude \mathcal{A}_α for the subprocess $g^* \rightarrow c\bar{c}$, summed over final state colors and spins, is

$$\sum \mathcal{A}_\alpha \mathcal{A}_\beta^* = 6\pi\alpha_s m_c^3 H'_8(\Lambda) \left(-g_{\alpha\beta} + \frac{q_\alpha q_\beta}{4m_c^2} \right). \quad (20)$$

The $q_\alpha q_\beta$ term can be dropped because q_α is of order m_c^2/q_0 when contracted with the numerator of the virtual gluon propagator in axial gauge. The expression therefore reduces to the polarization sum $(-g_{\alpha\beta} + \dots)$ for an on-shell gluon multiplied by $6\pi\alpha_s m_c^3 H'_8$. Dividing by $(4m_c^2)^2$ for the virtual gluon propagator, we obtain the fragmentation probability $(3\pi\alpha_s/8)H'_8/m_c$. This probability can be identified with the second term in (1), integrated over z and summed over $J = 0, 1, 2$. This term in the fragmentation function contributes only at the endpoint $z = 1$. We can therefore identify the function $d_8(z)$ in (1) to be

$$d_8(z) = \frac{\pi\alpha_s}{24} \delta(1-z). \quad (21)$$

The total fragmentation function at leading order in α_s is given by the factorization formula (1), with the color-singlet coefficient given in (14) and the color-octet coefficient given in (21). To avoid large logarithms of m_c/Λ in the color-singlet coefficient, we can choose $\Lambda = m_c$. We thus arrive at the final expressions for the fragmentation functions of gluon splitting into χ_{cJ} to leading order in α_s :

$$\begin{aligned} D_{g \rightarrow \chi_{cJ}}(z, 2m_c) &\approx \frac{2\alpha_s^2(2m_c) H_1}{81 m_c} \left[(2J+1) \frac{z}{(1-z)_+} + Q_J \delta(1-z) + P_J(z) \right] \\ &+ (2J+1) \frac{\pi\alpha_s(2m_c) H'_8(m_c)}{24 m_c} \delta(1-z), \end{aligned} \quad (22)$$

where Q_J and $P_J(z)$ given by (15)-(18). The choice of the scale μ in the running coupling constant is independent of the choice of factorization scale Λ . We have followed Ref. [2] in choosing $\mu = 2m_c$, which is the minimum value of the invariant mass of the virtual gluon. If we wish to use a value for the factorization scale Λ in (22) which is significantly smaller than m_c , we should use the solution (3) to the renormalization group equation for $H'_8(\Lambda)$ to sum up the leading logarithms of m_c/Λ .

Rough estimates of the gluon fragmentation contribution to the production of the χ_{cJ} states at large transverse momentum in any high energy process can be obtained by multiplying the cross sections for producing gluons with transverse momentum larger than $2m_c$ by appropriate fragmentation probabilities. Integrating the initial fragmentation functions (22) over z , we obtain the probabilities

$$P_{g \rightarrow \chi_{cJ}} \approx -R_J \frac{\alpha_s^2(2m_c)H_1}{108m_c} + (2J+1) \frac{\pi\alpha_s(2m_c)H'_g(m_c)}{24m_c}, \quad (23)$$

where $R_0 = 5$, $R_1 = 4$, and $R_2 = 16$. Notice that with the choice $\Lambda = m_c$ for the factorization scale, the color-singlet pieces give rise to negative contributions to the initial fragmentation probabilities. Requiring that all the probabilities (23) be positive, we obtain an interesting lower bound on $H'_g(m_c)$:

$$H'_g(m_c) > \frac{10\alpha_s(2m_c)}{9\pi} H_1. \quad (24)$$

Using $H_1 \approx 15$ MeV, $m_c = 1.5$ GeV, and $\alpha_s(2m_c) = 0.26$, we find $H'_g(m_c) > 1.4$ MeV. The estimate $H'_g(m_c) \approx 3$ MeV obtained in Ref. [11] is consistent with this lower bound. Using the value $H'_g(m_c) \approx 3$ MeV, our estimates for the the initial fragmentation probabilities in (23) are $0.4 \cdot 10^{-4}$, $1.8 \cdot 10^{-4}$, and $2.4 \cdot 10^{-4}$ for χ_{c0} , χ_{c1} , and χ_{c2} , respectively. The production of χ_{cJ} states contributes to the inclusive rate for production of the 1^{--} charmonium state J/ψ through the radiative decay $\chi_{cJ} \rightarrow J/\psi + \gamma$. Multiplying the fragmentation probabilities given above by the appropriate radiative branching fractions of 0.7%, 27%, and 14%, we find that the probability of a J/ψ in a gluon jet is approximately 8×10^{-5} . This is more than an order of magnitude larger than the probability 3×10^{-6} for the direct fragmentation of a gluon into J/ψ that was obtained in Ref. [2].

The methods used above to calculate the fragmentation functions $D_{g \rightarrow \chi_{cJ}}(z)$ can also be used to calculate the distribution of the transverse momentum p_\perp of the χ_{cJ} relative to the gluon jet. This transverse momentum is related to the the invariant mass s of the gluon jet by $s = (\vec{p}_\perp^2 + 4m_c^2)/z + \vec{p}_\perp^2/(1-z)$. For the color-singlet contribution, the s -distribution is obtained by integrating over z in (7). For the color-octet contribution, the s -distribution

is a delta-function at $s = 4m_c^2$. Adding these two contributions we obtain

$$\frac{dP_{g \rightarrow \chi_{c0}}}{ds} = \frac{2\alpha_s^2 m_c H_1}{81} \frac{(s - 12m_c^2)^2}{s^3(s - 4m_c^2)} \theta(s - s_{\min}(\Lambda)) + \frac{\pi\alpha_s H'_8(\Lambda)}{24m_c} \delta(s - 4m_c^2), \quad (25)$$

$$\frac{dP_{g \rightarrow \chi_{c1}}}{ds} = \frac{4\alpha_s^2 m_c H_1}{27} \frac{s + 4m_c^2}{s^2(s - 4m_c^2)} \theta(s - s_{\min}(\Lambda)) + 3 \frac{\pi\alpha_s H'_8(\Lambda)}{24m_c} \delta(s - 4m_c^2), \quad (26)$$

$$\frac{dP_{g \rightarrow \chi_{c2}}}{ds} = \frac{4\alpha_s^2 m_c H_1}{81} \frac{s^2 + 12sm_c^2 + 96m_c^4}{s^3(s - 4m_c^2)} \theta(s - s_{\min}(\Lambda)) + 5 \frac{\pi\alpha_s H'_8(\Lambda)}{24m_c} \delta(s - 4m_c^2), \quad (27)$$

where $s_{\min}(\Lambda) = 4m_c^2(1 + \Lambda/m_c)$. Integrating over s , we recover the fragmentation probabilities given in (23). The cutoff-dependence of the color-singlet terms in (25)-(27) is cancelled by the Λ -dependence of the parameter $H'_8(\Lambda)$ in the color-octet terms. The color-singlet terms in these invariant mass distributions were obtained previously by Hagiwara, Martin and Stirling [14], up to an error of 4π in the overall coefficient. They did not include the color-octet contributions, so their answers were sensitive to the value of the infrared cutoff Λ . In the region near the lower endpoint $s = 4m_c^2$, the distributions (25)-(27) must of course be smeared over an appropriate range in p_\perp before they can be compared with experimental data.

We have calculated the fragmentation functions for gluons to split into P-wave quarkonium states to leading order in α_s . The fragmentation functions satisfy a factorization formula with two nonperturbative parameters H_1 and H'_8 which can be determined from other processes involving the annihilation and production of P-wave states. These fragmentation functions are universal and can be used to calculate the rates for the direct production of P-wave states at large transverse momentum in any high energy process. They are also needed to calculate the total production rate of the 1^{--} states from the fragmentation mechanism, since the P-wave states have significant rates for transitions to the 1^{--} states. The fragmentation probabilities for $g \rightarrow \chi_{c1}$ and $g \rightarrow \chi_{c2}$ were estimated to be on the order of 10^{-4} . This is large enough that gluon fragmentation into χ_{cJ} should account for a significant fraction of the χ_{cJ} 's that are observed at large transverse momentum in hadron colliders. Fragmentation into χ_{cJ} followed by its radiative decay may also account for a significant fraction of the J/ψ 's that are produced at large p_T .

While this paper was being written, we received a paper by Ma [15], in which the color-singlet term in the fragmentation function for $g \rightarrow \chi_{c1}$ is calculated for longitudinally and transversely polarized χ_{c1} separately. After summing over polarizations, his result agrees with ours except near the endpoint $z = 1$. In the endpoint region, Ma's fragmentation function is sensitive to an infrared cutoff.

This work was supported in part by the U.S. Department of Energy, Division of High Energy Physics, under Grants DE-FG02-91-ER40684 and DE-FG03-91ER40674 and also by the Texas National Research Laboratory Commission Grant RGFY93-330.

References

- [1] See, for example, R. Baier and R. Rückl, *Z. Phys.* **C19**, 251 (1983).
- [2] E. Braaten and T.C. Yuan, *Phys. Rev. Lett.* **71**, 1673 (1993).
- [3] G. Curci, W. Furmanski, and R. Petronzio, *Nucl. Phys.* **B175**, 27 (1980); J.C. Collins and G. Sterman, *Nucl. Phys.* **B185**, 172 (1981); J.C. Collins and D.E. Soper, *Nucl. Phys.* **B194**, 445 (1982).
- [4] E. Braaten, K. Cheung, and T.C. Yuan, *Phys. Rev.* **D48**, 4230 (1993); *Phys. Rev.* **D48**, R5049 (1993).
- [5] C.-H. Chang and Y.-Q. Chen, *Phys. Lett.* **B284**, 127 (1992); *Phys. Rev.* **D46**, 3845 (1992).
- [6] A.F. Falk, M. Luke, M.J. Savage, and M.B. Wise, *Phys. Lett.* **B312**, 486 (1993); *Phys. Rev.* **D49**, 555 (1994).
- [7] Y.-Q. Chen, *Phys. Rev.* **D48**, 5158 (1993); T.C. Yuan, U.C.-Davis preprint UCD-94-2 (April, 1994).
- [8] G.T. Bodwin, E. Braaten, and G.P. Lepage, Fermilab preprint FERMILAB-PUB-94-073-T (in preparation).

- [9] W.E. Caswell and G.P. Lepage, Phys. Lett. **167B**, 437 (1986).
- [10] G.T. Bodwin, E. Braaten, and G.P. Lepage, Phys. Rev. **D46**, 1914 (1992).
- [11] G.T. Bodwin, E. Braaten, T.C. Yuan, and G.P. Lepage, Phys. Rev. **D46**, R3703 (1992).
- [12] H. Trottier, Phys. Lett. **B320**, 145 (1994).
- [13] J.H. Kühn, J. Kaplan, and E.G.O. Safiani, Nucl. Phys. **B157**, 125 (1979); B. Guberina, J.H. Kühn, R.D. Peccei, and R. Rückl, Nucl. Phys. **B174**, 317 (1980).
- [14] K. Hagiwara, A.D. Martin, and W.J. Stirling, Phys. Lett. **B267**, 527 (1991) and erratum (to be published).
- [15] J.P. Ma, Melbourne preprint UM-P-94/01.

Figure Captions

1. Feynman diagrams for $g^* \rightarrow c\bar{c} + g$ which contribute to the color-singlet term in the fragmentation function for $g \rightarrow \chi_{cJ}$.
2. Feynman diagram for $g^* \rightarrow c\bar{c}$ which contributes to the color-octet term in the fragmentation function for $g \rightarrow \chi_{cJ}$.

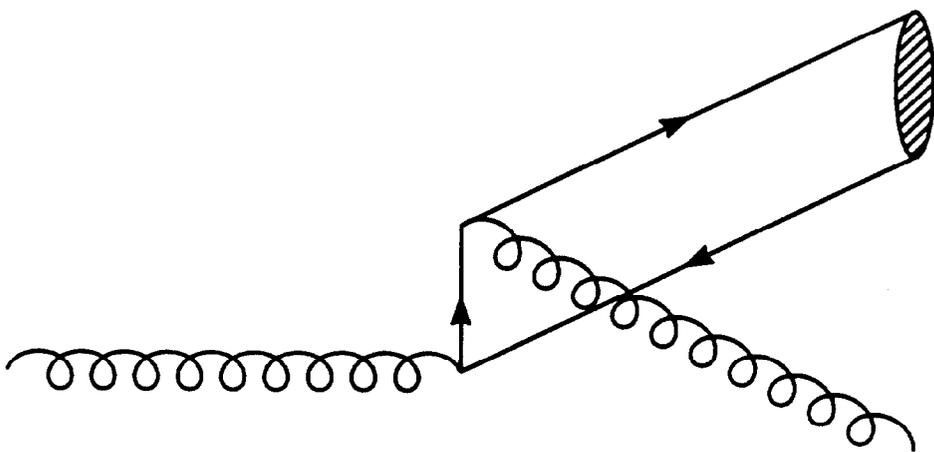
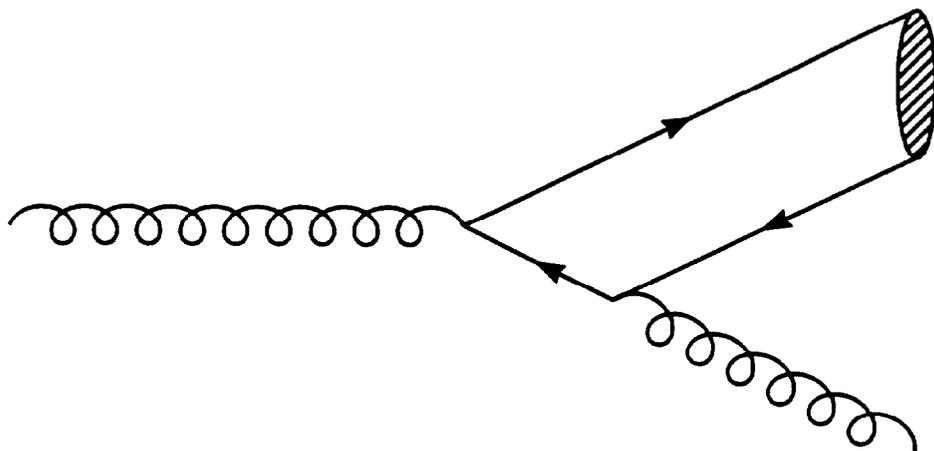


Figure 1

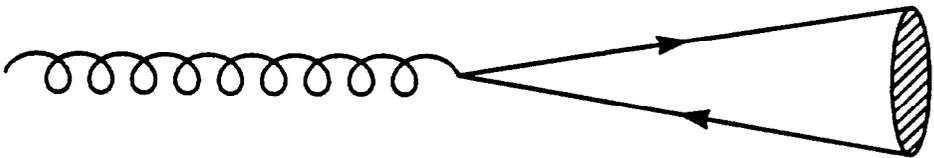


Figure 2