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Universality of Particle Multiplicities

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UNIVERSALITY OF PARTICLE MULTIPLICITIES

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Abstract

We discuss the scaling properties and universality aspects of the rapidity and multiplicity distributions of particles produced in high energy hadronic and e^+e^- interactions. This paper is based on material presented [1] in three lectures on pomeron phenomenology, which included a review of *traditional* soft pomeron physics and selected topics on *hard diffraction* processes probing the structure function of the pomeron.

1 Introduction

In this paper we discuss the inclusive particle distributions and scaling properties in the variables Feynman–x, rapidity–y, and scaled multiplicity $z = n/\bar{n}$. These **(x,y,z)-distributions** are related: Feynman-x scaling leads to a constant rapidity plateau, which in turn results in KNO scaling, i.e. the scaling of the multiplicity by its mean (distribution of $z = n/\bar{n}$ independent of $\bar{n}$). It is shown that the scaling violations observed in all three distributions are also related. The (x, y, z)-relationship leads naturally to a universal description of hadronic and e^+e^- multiplicity distributions, which is traced back to the partonic strucure of the hadrons and the random nature of particle production in high energy collisions.

Below, we define some variables and relationships that we will be using throughout this presentation:

$x = \frac{p_L}{\sqrt{s}/2} = \frac{2p_L}{\sqrt{s}}$	Feynman-x
$y = \ln\frac{E+p_L}{m_T} = \frac{1}{2}\ln\frac{E+p_L}{E-p_L}$	Rapidity
$m_T^2 = p_T^2 + m^2$	Transverse mass squared
$d^3p/E = d^2p_T\,\frac{dp_L}{E} = d^2p_T\,dy$	Phase space element
$\eta \equiv y_{m=0} = -\ln\,\tan(\theta/2)$	Pseudorapidity

$$p_L = m_T \sinh y$$
$$E = m_T \cosh y$$

$$x = (2m_T/\sqrt{s})\sinh y = (2p_T/\sqrt{s})\sinh\eta$$

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2 Feynman scaling

At high energies, the fields to be radiated in an inclusive process, $A + B \to C + X$, are Lorentz contracted in the longitudinal z-direction, so that the field energy eventually becomes a δ-function in z. By Fourier analysis, the field energy is then distributed uniformly in longitudinal momentum space, so that there is equal average amount of energy in any element dp_L, independent of p_L. If in addition the field energy is distributed among the different *kinds* of particles in constant fractions at any given x-value independent of $\sqrt{s}$, then the number of particles C of mass m and energy $E = (m^2 + p_T^2 + p_L^2)^{1/2}$ within dp_L is

$$\frac{d\sigma_{AB}^C}{\sigma_{tot}} = \rho_{AB}^C(p_T, x)\frac{d^2p_T\, dp_L}{E} = \rho_{AB}^C(p_T, x)d^2p_T\, dy$$

The total number of particles, which is obtained by integration of this expression, is the average multiplicity of C. Assuming that the function $\rho_{AB}^C(p_T, x)$ has a limit as $s \to \infty$ and $x \to 0$ (no s-dependence), we obtain for large s

$$\bar{n}_{AB}^C = 2\int_{x=0}^{1}\int_{p_T=0}^{\infty} \rho_{AB}^C(p_T^2, x)\frac{d^2p_T\, dx}{\sqrt{x^2 + \frac{4m_T2}{s}}} = \tilde{\rho}_{AB}^C(x=0)\ \ln\frac{s}{m^2} + constant$$

The factor of two accounts for the fact that the physical region extends to $x = -1$. The tilde in $\tilde{\rho}$ denotes integration over p_T^2, where we have assumed that the p_T is limited and that $\tilde{\rho}(x)$ reaches a constant limit for small x. [2] Thus, under these assumptions, Feynman scaling leads to multiplicities that rise logarithmically with s. For a process to be exclusive, this multiplicity must be suppressed. Assuming Poisson statistics (although not quite right, as will be discussed in Section 5), the probability for zero extra multiplicity goes as $e^{-\bar{n}} \sim 1/s$, which explains why exclusive cross sections fall with increasing energy [1].

3 Rapidity distributions

It was shown above that Feynman scaling leads to multiplicities that rise logarithmically with s. Since $dp_L/E = dy$, and $\ln(s/m^2) = 2\,|y_{max}|$ is the total rapidity space available to particles of mass m, the value of $\tilde{\rho}(x=0)$ represents the height of the plateau of the rapidity distribution (Fig. 1a), and the multiplicity grows with energy in proportion to the phase space. The multiplicity is governed by the value of $\tilde{\rho}(x)$ at $x = 0$, because as $s \to \infty$, $x \to 0$ for all finite longitudinal momenta p_L. The *new* multiplicity comes from the lower and lower x-values contributing around $y = 0$, as can be seen from the expression $(dx/dy)_{y=0} = 2m_T/\sqrt{s}$. This is shown graphically in Fig. 1a, where the expected rapidity distributions for two energies are shown assuming Feynman scaling.

[2] Integrate by parts, noting that $dx/\sqrt{x^2 + 4m_T^2/s} = d\ln(x + \sqrt{x^2 + 4m_T^2/s})$, and that the integral $\int_0^1 \tilde{\rho}(x)\ \ln\, x\ dx$ converges for finite $\tilde{\rho}(0)$.

In the real world, it is found that the rapidity plateau rises logarithmically with s, so that the average multiplicity rises as $\sim \ln^2 s$ (dashed line in Fig. 1a). This behavior violates Feynman scaling and consequently results in KNO scaling violation in multiplicity distributions, as discussed below.

The shape of the pseudorapidity distribution is **not** expected to be flat for particles with mass and a flat y-distribution. The η-distribution is depleted around $\eta = 0$ by an amount depending on the ratio p_T/m and the η-value. The exact shape of the η distribution is given by

$$\frac{dn}{d\eta} = \left(\frac{dn}{dy}\right)_{y=\sinh^{-1}(\frac{p_T}{m_T}\sinh\eta)} \left[\frac{1+\sinh^2\eta}{1+\sinh^2\eta+(m/p_T)^2}\right]^{1/2}$$

where $\eta_{max} = \ln(\sqrt{s}/p_T)$, corresponding to $y_{max} = \ln(\sqrt{s}/m_T)$. Fig. 1b shows the η-distribution, resulting from a flat rapidity distribution, for p_T/m=1.

Experimentally, it is found that the average p_T increases with multiplicity. Thus, the characteristic depletion at $\eta \approx 0$ of the η-distribution of particles above a given p_T threshold, which is usually the experimentally measured distribution, is expected to be more pronounced at low multiplicities.

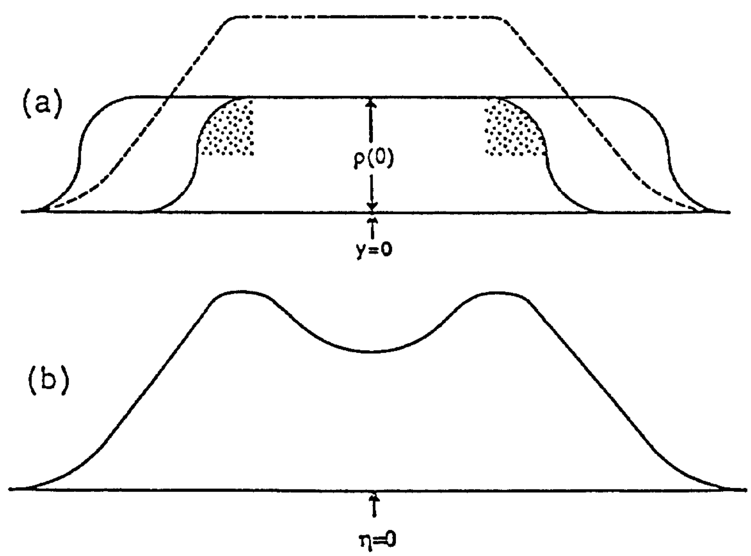


Fig. 1

(a) Rapidity distributions for two cms energies: (solid lines) assuming Feyman Scaling; (dashed line) a more realistic situation at the higher energy.

(b) Pseudorapidity distribution for P_t=m (approximate shape).

4 KNO scaling

Koba, Nielsen and Olesen (KNO) showed [2] that Feynman scaling leads to scaling of multiplicity distributions. Specifically, they showed that the distribution of the number of particles n in the final state should be a function only of the variable $z \equiv n/\bar{n}$, where $\bar{n}$ is the average multiplicity at $\sqrt{s}$. Their result can be expressed as

$$\bar{n}P_n(s) \equiv \bar{n}\frac{\sigma_n(s)}{\sigma_{tot}(s)} = \psi\left(\frac{n}{\bar{n}}\right)$$

where $\sigma_n(s)$ is the cross section for the multiplicity being n.

Formally, this result was derived by proving that all scaled factorial moments, defined as $C^q = \bar{n^q}/\bar{n}^q$, are constant, i.e. independent of s (or $\bar{n}$). In the special case of the second moment, or scaled variance, which we denote by $(D/\bar{n})^2$, KNO scaling predicts that the width D of the multiplicity distribution increases in proportion to the average, $\bar{n}$, as $\bar{n}$ increases.

KNO scaling is a direct consequence of the rapidity plateau remaining constant as the energy increases. The increase of multiplicity comes from the stretching of the available rapidity space. The field "emits" the particles uniformely in y, so that the entire distribution fluctuates up and down with the *total* field energy, in a manner independent of $\sqrt{s}$. Therefore, the multiplicity distribution in general, and the ratio of the width of the distribution to the mean in particular, stay the same.

Feynman scaling violations, which distort the rapidity distribution in the manner illustrated in Fig. 1a (dashed curve), are expected to induce KNO scaling violations. Experimentally, KNO scaling was found to hold at energies up to $\sqrt{s} \sim 60$ GeV (ISR), which was a surprising result given the relatively low energies and the fact that Feynman scaling was observed to be violated, as evidenced by the rise of the rapidity plateau with increasing energy. Later, when data bacame available at energies up to $\sqrt{s} \sim 900$ GeV (SPS), KNO scaling was found to be violated logarithmically with increasing $\sqrt{s}$, and the apparent scaling observed at the lower energies was interpreted as being accidental. Below, we present a model that fits well the observed multiplicity distributions, and provides some insight into KNO scaling and violations.

5 Multiplicity distributions

There has been an enormous amount of data on particle multiplicities, and a large number of papers that offer interpretations. In this note, it is not our purpose to review all the data and theoretical papers. Rather, we present the salient features of the data in a pedagogical fashion, using approximations to make them more comprehensible and accessible for further use, and easier to relate them to the underlying physics principles. For more details, the reader may consult the references provided [3, 4]. This presentation is based mainly on reference [4].

We begin with a discussion of hadronic multiplicities, followed by some comments on e^+e^- annihilation.

5.1 Hadronic multiplicities

Hadronic multiplicities have the following general features:

- **Universality:** Both $\bar{n}$ and the shape of the multiplicity distribution depend only on the hadronic energy M and not on the nature of the hadrons involved in the process. The energy M is $\sqrt{s}$ for colliding hadrons and M_X for diffractive or other subprocesses leading to a distinct hadronic mass (e.g. deep inelastic scattering).

- **Charged/neutral ratio,** $R = \bar{n}_{ch}/\bar{n}_0$: The ratio of charged to neutral multiplicities (π^0's, not γ's from π^0's) is approximately two, as expected from isospin conservation.

- **Average multiplicity:** The average total multiplicity (charged+neutral) increases with M as
  $\bar{n}_{tot} \approx 4 + \ln^2 M$ at high M (GeV)
  $\bar{n}_{tot} \approx 3\sqrt{M}$ for $M <\sim 60$ GeV

- **Charged multiplicity distribution:** Negative binomial.

- **Charged-neutral correlations:** Binomial.

- **Total multiplicity distribution:** Gamma.

- **Scaled width:** The scaled width of hadronic multiplicities is $D/\bar{n} \approx 1/2$ independent of the energy, except for a logarithmic broadening observed at high energies due to KNO scaling violations.

- **KNO scaling:** Approximately observed, violated logarithmically as M increases.

The $\bar{n} \sim \ln^2 M$ dependence shows that the average multiplicity grows by a factor $\ln M$ beyond the one power of $\ln M$ expected from the increse of the rapidity phase space. This extra factor is due to the rising rapidity plateau, which is in turn connected to the rising cross sections with energy, and is also responsible for the observed KNO scaling violations. As we have seen, in Regge theory the total cross section grows as s^ϵ. For $\epsilon \approx 0.1$, $s^\epsilon \approx \ln s$ in the ISR to Tevatron collider energy range, i.e $60 < \sqrt{s} < 1800$ GeV. Thus, it appears that the deviation of the pomeron intecept at $t = 0$ from unity governs not only the rising total cross section, but also the rise of the rapidity plateau, the corresponding rise of the average multiplicity, and KNO scaling violations.

In the parton model, the function $\tilde{\rho}(x)$ we introduced in Section 2.1 corresponds to the $F_2(x)$ structure function measured in deep inelastic scattering. A constant $F_2(x)$ as $x \to 0$ leads to constant cross sections (see "Quarks & Leptons" by Halzen & Martin, p.199). For the cross section to rise as s^ϵ, it is required that $F_2(x)$ be $\sim 1/x^\epsilon$. This results in more partons being concentrated at lower x-values, producing a higher cross section as they become energetically available to create particles ($x_{min} = m^2/s$ for creating a particle of mass m).

KNO scaling restricts the statistical functions that can be used to describe multiplicity distributions. For example, the Poisson distribution, for which the scaled variance is given by $(D/\bar{n})^2 = \bar{n}^{-1}$ is excluded, since $\bar{n}$ is a function of $\sqrt{s}$. The well known Gaussian or Normal distribution can be written in terms of the KNO scaling variable $z = n/\bar{n}$, and $(D/\bar{n})^2$ could be set to a constant, but a Gaussian is also not appropriate for describing multiplicities because it is symmetric in z while multiplicities have a lower bound of zero. A function closely representing a bounded Gaussian is the **Gamma** distribution, which can be written as

$$\bar{n}P_n^G = \frac{k^k}{\Gamma(k)} z^{k-1} e^{-kz}$$

$$z = n/\bar{n}, \quad k^{-1} = (D/\bar{n})^2$$

An example of a Gamma distribution is shown in Fig. 2. For constant k, the Gamma obeys KNO scaling. As we will see below, it describes well the *total* multiplicity. However, it is *not* appropriate for describing *charged* multiplicity distributions, since it is identically zero at $n = 0$ and there is a finite probability for observing no charged particles in the final state.

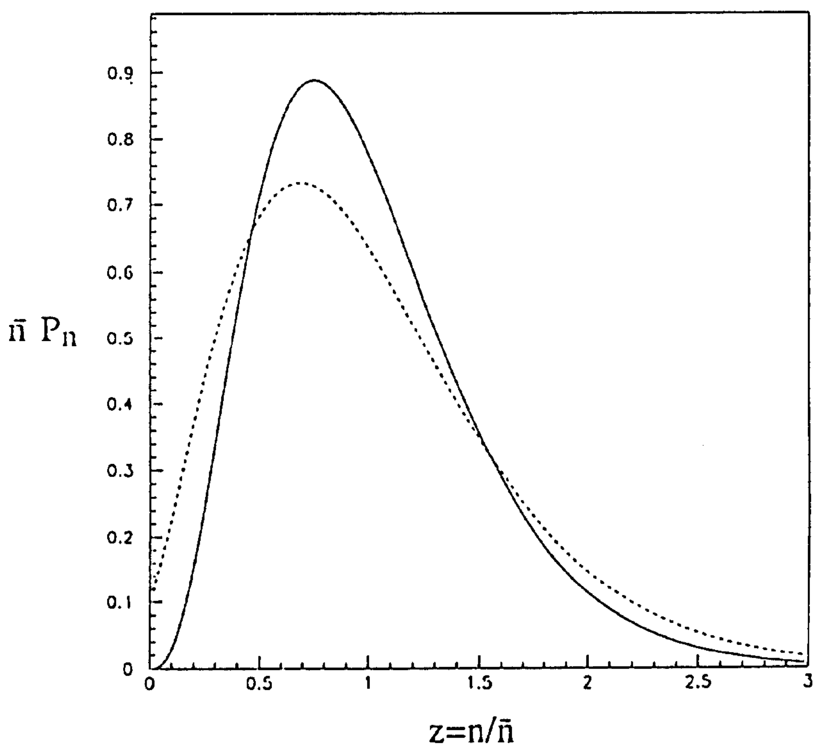


Fig. 2

Gamma (solid line) and Negative Binomial (dashed line) distribution for $\bar{n}$=8 and k=4.

A function that was used successfully to describe *charged* multiplicity distributions is the **Negative Binomial** (NB):

$$P_n = \binom{n+k-1}{n} p^k q^n$$

where P_n is the probability of observing n failures while waiting for k successes, when the probability for success is p and $q \equiv 1-p$. It can also be interpreted as the distribution of successes in a unit of time, when the rate of success is a random variable with a Gamma distribution (rather than constant as for Poisson). The NB can be expressed in terms of k and $\bar{n}$ as follows:

$$\bar{n}P_n = \bar{n}\binom{n+k-1}{n}\frac{1}{(1+\bar{n}/k)^k}\left(\frac{\bar{n}/k}{1+\bar{n}/k}\right)^n$$

$$\left(\frac{D}{\bar{n}}\right)^2 = \frac{1}{k}+\frac{1}{\bar{n}}$$

It is interesting to note that for $\bar{n} \gg k > 1$, $NB \to Gamma$, while for $k \to \infty$ $NB \to Poisson$. The NB is compared to a Gamma in Fig. 2.

The NB also describes well charged multiplicities in restricted pseudorapidity ranges. Fits to data in different pseudorapidity ranges with the NB are shown in Fig. 3 [3].

At high enough $\bar{n}$ the NB is a KNO scaling function if k is constant with energy. By fitting distributions at various energies up to $\sqrt{s} = 900$ GeV, the UA5 collaboration [3] found that the parameter k decreases with energy, in violation of KNO scaling. The UA5 result was expressed as $k^{-1} = a + b\ln\sqrt{s}$, where $a = -0.104 \pm 0.004$ and $b = 0.058 \pm 0.001$.

While adequate for describing *charged* multiplicity distributions, the NB lacks simple physical interpretation. Also, it does not address the question of *correlations* between charged and neutral multiplicities, which must be fitted independently. A distribution that fits simultaneously the entire multiplicity distribution (charged+neutral) *including* the charged to neutral correlations was proposed in 1987 by this author [4]. The main idea was simple: in a hadronic reaction, ficticious particles n^* are produced with a Gamma distribution, and upon production they "decay" to a charged doublet (charge conservation!) or a single neutral particle with a binomial distribution. Setting the "decay" probability to 1/2, the *total* multiplicity distribution remains Gamma, but the distribution obtained for the *charged* multiplicity is found to be practically identical to a NB with the same $\bar{n}_{ch}$. This is not surprising, since the scaled width of the folded Gamma$\oplus$Binomial or *Modified Gamma* (MG), which can be obtained by adding in quadrature the scaled widths of the Gamma and of the Binomial, turns out to be $(D/\bar{n})^2 = k^{-1} + \bar{n}^{-1}$, the same as for the NB! Thus, *the negative binomial behavior of the charged multiplicity arises from a parent Gamma distribution by the binomial interplay between the charged and neutral particles.* Fig. 4 shows fits to data with the MG.

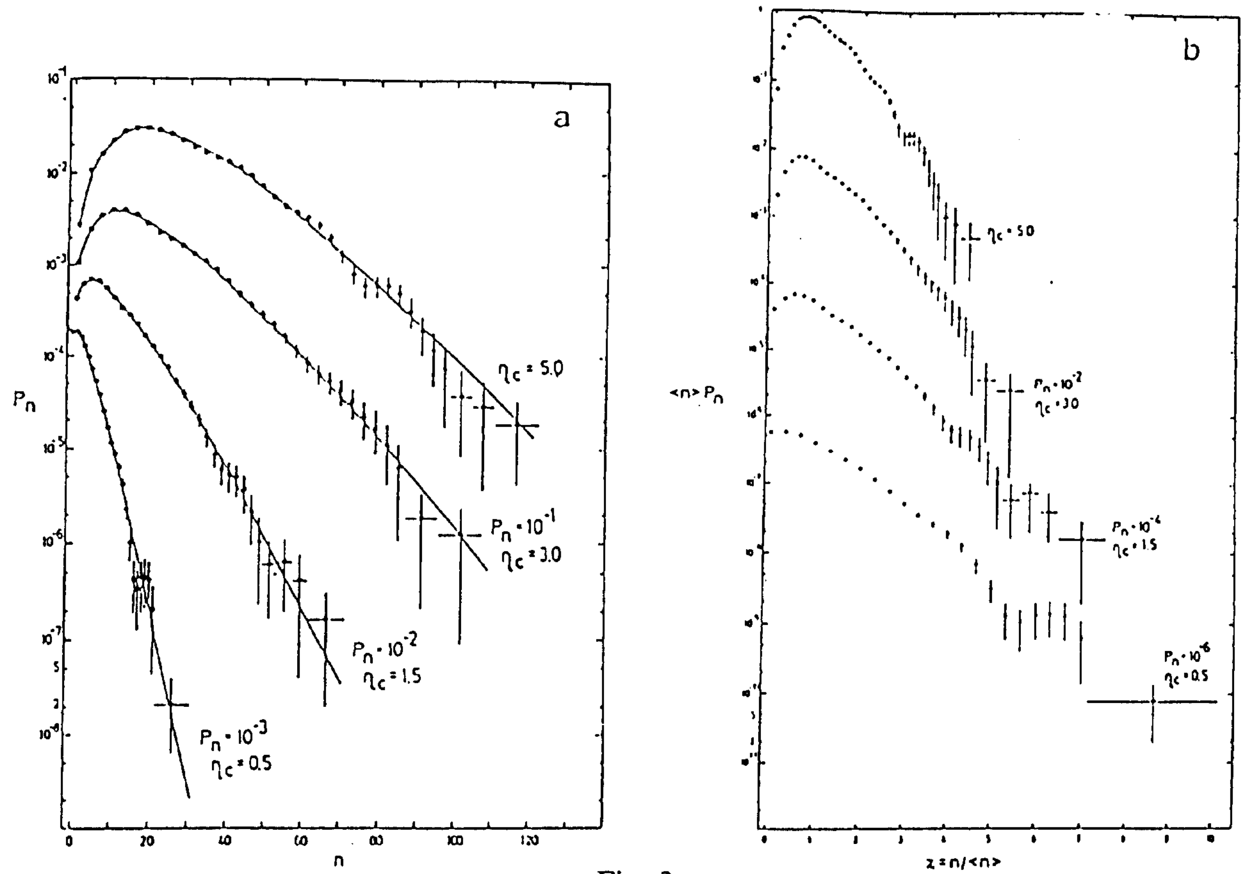


Fig. 3

Charged multiplicity distributions for limited pseudorapidity intervals for proton-antiproton collisionsd at cms energy 540 GeV[3].

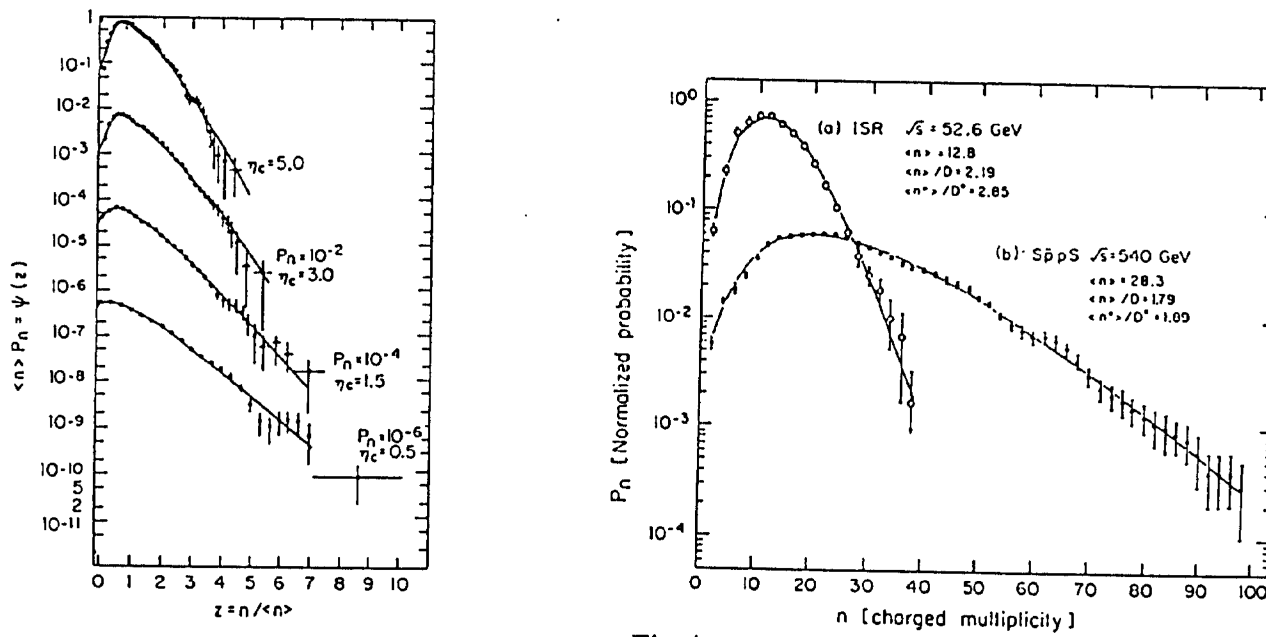


Fig.4

Charged multiplicity distributions: Data fitted with the MG.

By fitting data at energies all the way down to include diffractive "masses" of a few GeV, it was found that the k parameter used in the Gamma function of the MG distribution varies as

$$k^{-1} = a + b\ln(M + c)$$

where a and b are the same as those obtained by UA5 and c = 6 GeV (M here is the *available* mass in GeV, which for a pp collision is $\sqrt{s} - 1.88$ GeV). The increase of k^{-1} with energy represents an explicit KNO scaling violation in the *total* multiplicity. The fact that KNO scaling was found to hold *for charged multiplicities* in the ISR and lower enery range (below 60 GeV) is explained by an accidental "conspiracy" of the k^{-1} and $\bar{n}^{-1}$ terms to give a constant $(D/\bar{n})^2 = k^{-1} + \bar{n}^{-1}$ over this energy range. At higher energies (larger $\bar{n}$), where the contribution of the $\bar{n}^{-1}$ term decreases, the charged multiplicities were found to violate KNO scaling [3].

The MG was also used successfully in fitting data in limited pseudorapidity ranges [4]. In fact, by incorporating it in a minimum bias Monte Carlo simulation, it was found that only two parameters were needed to describe the data in *all pseudorapidity ranges*, namely the $\bar{n}$ and k of the total event multiplicity. Thus, the broadening of the distributions with decreasing pseudorapidity width (Fig. 3) appears to be the result of increased charged to neutral fluctuations, due to the lower number of particles, combined with Poisson-type fluctuations along the rapidity coordinate. The latter are expected in the Feynman model discussed in Section 2.1, in which the field energy is distributed along y with equal a priori probability and therefore the number of particles per unit rapidity obeys Poisson statistics.

When the MG proposal was made, there were no data on charged to neutral multiplicity correlations at high energies. Recently, such data have become available [5], confirming the binomial hypothesis used in the MG (Fig. 5).

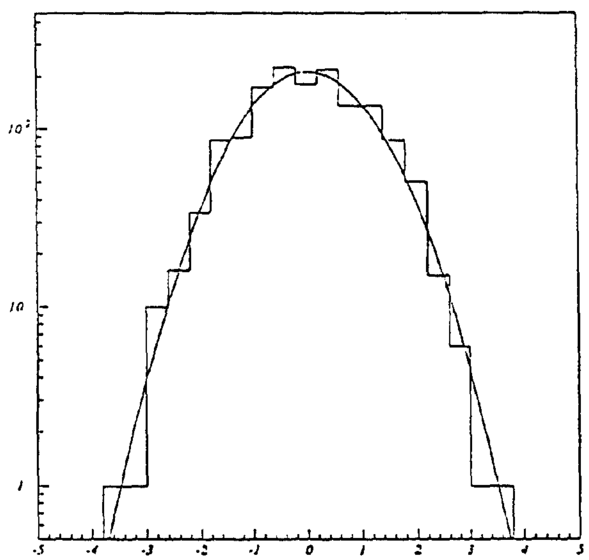
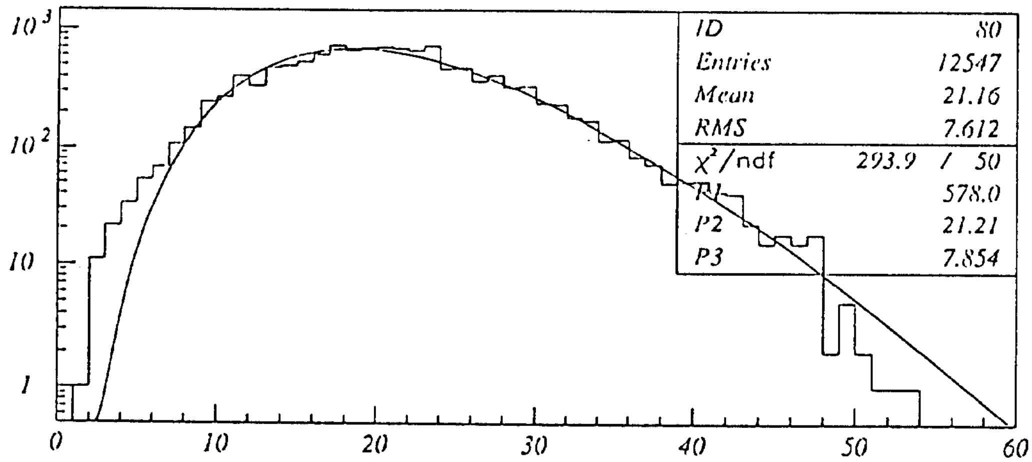


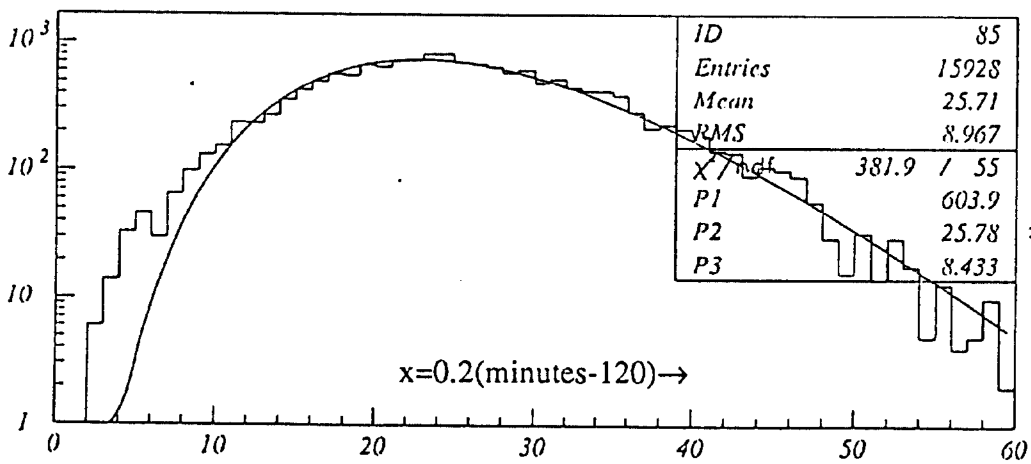
Fig. 5

Charged to neutral multiplicity correlations for $\bar{p}p$ collisions at cms energy of 1.8 TeV [5]: Number of events vs the deviation from the mean measured in standard deviations assuming a binomial distribution.

The MG result has a remarkably simple interpretation: the Gamma distribution arises from a truely random source. In fact, the idea of using the Gamma probability function to describe the total multiplicity came from a study of the distribution of finishing times of the runners of the New York City Marathon. Fig. 6 shows two such distributions, fitted by Gamma functions. The fits are excellent, except for the region of short finishing times, populated by the elite runners. This is a good example of a distribution arising from a random source, represented by the different age groups, training amounts etc. The elite runner pack breaks away visibly from the Gamma distribution. Another feature of these distributions is scaling: the ratio $(D/\bar{n}) = 1/\sqrt{k}$ (k=P3 in the figure) is the same for the two distributions to within 3.5%, although the mean values (parameter P2) differ by 20% (the different average finishing times are due to the temperature difference in the days of the two runs). Thus, the finishing time distributions obey KNO scaling!



Results of 1980 New York Marathon



Results of 1985 New York Marathon

Fig. 6

Finishing time distributions of runners in two New York City Marathons fitted with a Gamma probability function.

5.2 e^+e^- multiplicities

The e^+e^- charged multiplicities differ from the hadronic ones in two ways: they have a more symmetrical shape and they are narrower. This is illustrated in Fig. 7a, in which e^+e^- data are compared with a curve representing the hadronic distribution expected for the same $\bar{n}$. The e^+e^- width is narrower than the hadronic by a factor of $\sim \sqrt{2}$. Realizing that in e^+e^- annihilation the final state consists mostly of two jets, single-jet distributions were also measured. It was found that the distributions of the two jets are almost entirely uncorrelated, and that the width of the single-jet distribution is approximately the same as that of a hadronic distribution at the same $\bar{n}$ as that of the jet, which is one half of $\bar{n}$ of the total event sample (Fig. 7b). The folding of the two single-jet distributions results in a distribution with a $D/\bar{n}$ ratio smaller by a factor of $\sqrt{2}$. Thus, e^+e^- multiplicity distributions are explained in terms of hadronic, extending the universality concept first introduced in reference [6].

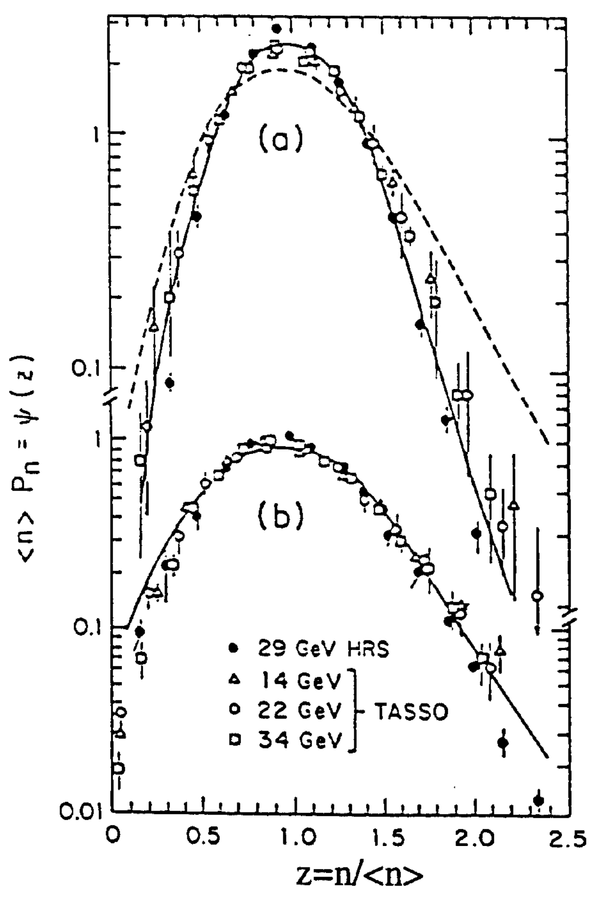


Fig. 7

Comparison of e^+e^- charged multiplicity distributions with curves representing hadronic distributions at the same value of $\bar{n}$ (a) Inclusive sample (b) Single jet distribution.

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