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SOLAR NEUTRINOS: SOLAR PHYSICS AND NEUTRINO PHYSICS *

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Abstract

It is shown that the current solar neutrino situation, with results from the Homestake experiment, the Kamiokande experiment, GALLEX and SAGE is unfortunately still ambiguous. The differences between observations and the standard solar theory may still be due to either astrophysical inputs or new neutrino physics. The need for new neutrino physics, MSW or vacuum neutrino mixing is sensitive to the results of the Homestake experiment. If the Homestake experiment is correct, then new neutrino physics is required. A problem with the uncalibrated Homestake experiment would allow an astrophysical solution, which merely consists of a slightly cooler sun than the standard solar model of Bahcall et al. The use of future experiments, SNO, Super Kamiokande and Borexino to resolve this ambiguity are explicity discussed. The measurement of deeper helioseismology modes by GONG will also further constrain solar model solutions.

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1. Introduction

The observed deficit of the solar neutrino flux with respect to the prediction of the standard solar models is one of the major issues of modern physics.[1] Currently, the Homestake experiment,[2] the Kamiokande II and III experiments,[3] the GALLEX experiment,[4] and the SAGE experiment[5] all observe neutrino fluxes lower than the predictions of the standard solar models of Bahcall and Ulrich (hereafter BUSSM),[6] Bahcall and Pinsonèault (BPSSM, which is an updated version of BUSSM with helium diffusion considered),[7] or the standard solar model of Turck-Chièze *et al.* (TSSM).[8] The current solar neutrino experimental status and the theoretical predictions are shown in Table 1. The spectrum of solar neutrinos from different reactions calculated by BUSSM is shown in Figure 1.[9]

There are two basic approaches to explain the deficit. One is new neutrino physics, and the other is to modify the solar model. In the first approach, the MSW (Mikheyev-Smirnov-Wolfenstein) matter mixing is the most natural and robust scenario of new physics,[10] although vacuum neutrino oscillations are also allowed with a narrower range in the parameter space.[11] Under the MSW matter mixing scheme, solar neutrinos (ν_e) are converted into other neutrino species (ν_μ or ν_τ, or new hypothetical sterile neutrinos) through resonant conversions in the sun. To reproduce the fluxes observed by the current experiments, the neutrino mixing parameters are restricted to the diagonal and vertical regions of a triangle in the parameter space ($\Delta m^2 = m_2^2 - m_1^2$ vs. $sin^2 2\theta_{ex}/cos 2\theta_{ex}$, where m_1 and m_2 are the neutrino mass eigenvalues and θ_{ex} is the mixing angle between ν_e and ν_x), assuming standard solar models.[12–16] There are also other proposed new physics solutions, such as the OVV (Okun-Voloshin-Vysotsky) neutrino magnetic moment solution[17] and its extension, spin-flavor precession,[18,19] and the neutrino decay solution.[20]

For the second approach, the simplest technique is to modify the astrophysical input parameters sufficiently to fit the experimental results.[9] The current standard solar models have large uncertainties in certain input parameters. Different choices of input parameters can yield different neutrino fluxes, especially for ^{8}B neutrinos. This is manifested in the comparison of the predictions of the BUSSM, TSSM and BPSSM (shown in Table 2). The differences in the neutrino fluxes in BUSSM (or BPSSM) and TSSM are mainly caused by the different rates adopted for the reaction ^{7}Be$(p,\gamma)^8$B and different opacity coefficients.[7,8] When the same input values are chosen, these models (and the standard solar models of Sieńkiewicz *et al.*,[21] and of Filippone and Schramm[22]) all agree with each other within 0.1 SNU for the ^{37}Cl experiment.[7] Similar agreements are also found for other experiments. Therefore, simply because of the uncertainties in the input parameters instead of the underlying physics of the solar models or the calculation methods exploited, different predictions for the solar neutrino experiments can be drawn. However, such variations in input parameters cannot cause ^{7}Be and especially *pp* neutrino fluxes to be significantly decreased relative to the ^{8}B neutrino flux. Thus, for example, if it is eventually shown that the ^{7}Be neutrino flux is depleted more than the ^{8}B neutrino flux, new neutrino physics is required. At present,

Table 1. Current solar neutrino experimental status.

	Homestake	Kamiokande	Galium
Reactions	$^{37}Cl(\nu_e, e^-)^{37}Ar$	ν-e scattering	$^{71}Ga(\nu_e, e^-)^{71}Ge$
Threshold	0.814 MeV	7.5 MeV[a]	0.233 MeV
Type	Radiochemical Time Integral Energy Integral	Direct Counting Real Time Spectroscopic	Radiochemical Time Integral Energy Integral
BUSSM prediction (theoretical range)	7.9±2.6 SNU[b]	1.0±0.37[c]	132^{+20}_{-17} SNU
TSSM predition(2σ)	6.4±2.6 SNU	0.76±0.40[c]	123±14 SNU
Observed flux (1σ)	2.28±0.23 SNU	0.49±0.04 ± 0.06[c] (K II+K III)	87±14 ± 7 SNU(GALLEX) 70±19 ± 12 SNU (SAGE)

[a]The threshold energy of the recoil electrons. [b]1 SNU= 10^{-36} event/target atom/second. [c]Normalized by the prediction of the BUSSM.

Table 2. Solar neutrino fluxes predicted by BUSSM, TSSM and BPSSM.

Neutrino	BUSSM [TSSM] (BPSSM) Prediction		
Sources	^{37}Cl exp. (SNU)	^{71}Ga exp. (SNU)	Kamiokande(× BUSSM)
pp	0 [0] (0)	70.8 [71.1] (71)	0 [0] (0)
pep	0.2 [0.2] (0.2)	3.0 [3.0] (3)	0 [0] (0)
^{7}Be	1.1 [1.0] (1.2)	34.3 [31.7] (36)	0 [0] (0)
^{8}B	6.1 [4.6] (6.2)	14.0 [10.8] (14)	1.0 [0.76] (1.0)
^{13}N	0.1 [0.1] (0.1)	3.8 [2.4] (3)	0 [0] (0)
^{15}O	0.3 [0.2] (0.3)	6.0 [3.7] (5)	0 [0] (0)
Total	7.9 [6.4] (8.0)	132 [123] (132)	1.0 [0.76] (1.0)

the comparison of the ^{37}Cl experiment (see ^{8}B neutrinos and ^{7}Be neutrinos) to the Kamiokande experiments (see ^{8}B neutrinos only) at face value, implies a greater ^{7}Be neutrino reduction, and thus new neutrino physics. But one should be cautious as we shall see.

Besides the theoretical uncertainties in the solar models, questions have occassionally been cast on the solar neutrino experiments themselves.[23] Because of the weakness of the interaction between neutrinos and matters, it is very difficult to measure the solar neutrino fluxes. Current experiments utilize two general methods to measure neutrino flux[9]: the radiochemical method and the neutrino-electron scattering method. So far, even with detector fiducial masses of 30–600 tons, the average counting rates of the current solar neutrino experiments are of the order 0.1/day. To extract the real events from such small signals and such large volumes of material is an extremely challenging task. Careful calibrations of the experiments are essential. However, among the four current solar neutrino experiments, only the Kamiokande ν-e scattering experiments have been fully calibrated.[24] Therefore, the low rates with respect to the standard solar models of the current radiochemical experiments have to be taken with precaution. Such a statement should in no way detract from the impressive experimental techniques used in each of the experiments and the open and informed manner in which the experiments have attempted to check each potential source of error. In this latter regard, the 20-year pioneering experiment of Davis is particularly notable.

Given all these factors that could lead to ambiguity in the solar neutrino problem, the purpose of this review is to attempt to explore the conditions under which we can attribute the solution to the solar neutrino problem to either solar physics or new neutrino physics, and how we can identify these conditions in experiments. In section 2 we will discuss briefly some occassionally raised uncertainties of the current solar neutrino experiments. In Section 3, we will review major uncertainties of the current solar models. We will discuss how, by modifying the input parameters of the standard solar model, there could be minimal conflict between the Kamiokande experiment as well as the two gallium experiments and the theory, but not between the Homestake experiment and the theory. In section 4, we will do a statistical test to compare the MSW matter mixing solution (representing the new neutrino physics) with modified standard solar models in fitting the experimental results. In section 5, we will discuss future solar neutrino experiments, and show their roles in determining the solution to the solar neutrino problem.

2. The Solar Neutrino Experiments

There are four solar neutrino experiments that are observing or have observed the solar neutrino flux: the Kamiokande neutrino-electron scattering experiments (including the Kamiokande II and the Kamiokande III), the Homestake chlorine capture experiment, the SAGE gallium capture experiment and the GALLEX gallium capture experiment.[25] As mentioned above, only the Kamiokande experiments have been fully calibrated. In order to calibrate the gallium experiments, an MeV β-decay source (^{51}Cr) with a strength of ~ 1 megacurie is needed. GALLEX plans such a calibration in the summer of 1994, and SAGE is also planning a calibration in the not-too-distant future. There is no sufficiently energetic and luminous β-source for calibrating the ^{37}Cl experiment. (Although ^{65}Zn has been suggested, such a source would violate safety regulations.) But calibration has been proposed using LAMPF to expose a similar chlorine tank with energetic neutrinos. Some have even discussed building an accelerator in the Homestake mine to do a calibration.

One point of the skepticism raised regarding the Homestake experiment is the consistency of its results. It was argued that the fact that the observed solar neutrino flux right after two pump failures during 1984–1985 (3.6±0.7 SNU averaging over 1986–88[25]) was unusually higher than the averaged flux before the failures (2.1±0.3 SNU for 1970–1984[25]) might hint for unanticipated systematic uncertainties in the Homestake experiment.[23] However, various tests have been done on the Homestake experiment and no source of unanticipated systematic errors has ever been found.[23,25] Furthermore, the observed flux averaged over 1986 to 1992 has decreased to 2.85±0.3 SNU. Therefore, the unusually high capture rate right after the pump failures is probably just only due to statistical fluctuations, but shifts in sets of runs are curious.

Another debated issue about the Homestake experiment is the anti-correlation between the number of sunspots and the counting rates claimed to be seen by the Homestake experiment,[26] which was not seen by the Kamiokande II during the period 1987–90, when there was a major change in the number of sunspots.[3] The existance of the anti-correlation has been investigated by many groups. Bahcall *et al.* found that the anti-correlation was not significant and was very sensitive to the errors of the data as well as the confidence of the very low counting runs.[27] Filippone and Vogel used a different test and found a similar conclusion.[28] Therefore, the anti-correlation cannot be either comfirmed or ruled out conclusively. This remains true even after the uncertainty of sunspot numbers is included and one assumes that the solar neutrino flux actually directly correlates with the solar magnetic field.[19] Shi, Schramm, Rosner and Dearborn[19] have shown that, given the current conservative limits on the neutrino magnetic moment and the solar magnetic field in the convective zone, the neutrino magnetic moment solution and the resonant spin-flavor precession solution to the solar neutrino problem fail to yield variations over the solar cycle that are large enough to be observable in the chlorine experiment. Therefore any significant correlation or anti-correlation in the chlorine experiment with solar activities may actually suggest unknown systematic effects in the experiment itself. Also, if the anti-correlation does exist, it was argued that its absence in the Kamiokande II experiment might be indicative of intermittent experimental problems in the chlorine experiment.[23] While the significance of a correlated time variation is debatable, most statistical analyses done to date do indicate a non-negligible but unfortunately low probability that the data is from a constant counting rate.[19,27–29] The possible time variation shown in the data of the Homestake experiment with respect to the more or less constant fluxes observed by Kamiokande II might suggest some unexpected systematic errors in the Homestake experiment. However, as mentioned above, no one has been able to indentify the sources of such a variation despite repeated attempts and a variety of tests.

The gallium experiments have the advantage that they can observe the most abundant and the least uncertain solar *pp* neutrinos. Beside the fact that neither of them have been fully calibrated, questions were also raised about the SAGE experiment when results from their first five runs came out.[23] Three of the five runs have best fits of 0 SNU, and none of these five runs has observed the 11-day half-life in the ^{71}Ge decay spectrum.[30] Also, the statistics are too low to draw significant conclusions from them. Even so, the early SAGE result ($20^{+15}_{-20} \pm 32$ SNU) is still in statistical agreement with the GALLEX result (87±14 ± 7 SNU) at the 2σ level. The latest SAGE result averaging over all runs yields 70±19 ± 12 SNU, which agrees well with the GALLEX result.[5]

3. Solar Models

The algorithm of solar models is to evolve a 1 solar mass protostar (a homogeneous cloud of hydrogen and hellium with a small admixture of heavier elements) from about 4.6 billion years ago until now to match the current solar luminosity and radius.[9,31] The structure of the sun is divided into two physical zones: the inner radiative zone, where the energy generated at the solar core is transferred by radiation, and an outer convective zone, where the energy is transferred by turbulant convection. The convective zone is described by a single parameter, the mixing length, which is roughly the size of the density scale height. The mixing length, together with the helium abundance, are two free input parameters to adjust in the standard solar models to match the current sun. Other major input parameters are, the heavy element abundance, the radiative opacity, the nuclear reaction parameters, the solar age, and the equation of state. The details are thoroughly reviewed in Bahcall's *Neutrino Astrophysics*,[9] and sensitivity to assumptions has been systematically explored by Bahcall and his collaborators in a series of papers over the last three decades. Table 3 lists the major different input

Table 3. Major different inputs and the resultant T_c and ^{37}Cl rate for standard solar models.

Models	$S_0(^7\text{Be}+p)$	$S_0(^3\text{He}+^3\text{He})$	Z	T_c	^{37}Cl rate
BUSSM	24.3 eV·b	5.15 MeV·b	0.0196	1.564×10^7 K	7.9 SNU
TSSM	21 eV·b	5.57 MeV·b	0.0197	1.551×10^7 K	6.4 SNU
BPSSM	22.4 eV·b	5.0 MeV·b	0.0196	1.569×10^7 K	8.0 SNU
This work	20.2 eV·b	5.6 MeV·b	0.0150	1.545×10^7 K	4.5 SNU

parameters and results of the BUSSM, TSSM, BPSSM and a modified solar model in this work which will be explained at the end of the section. S_0 in Table 3 represents the cross section of each nuclear reaction. It is related to the cross section $\sigma(E)$ by[9]

$$S(E) = \sigma(E)E\exp(2\pi\eta) \text{ and } S_0 = S(0). \tag{1}$$

The quantity $\eta = Z_1Z_2(e^2/\hbar v)$, where Z_1 and Z_2 are the charges of colliding particles and v is their relative speed.

The standard solar models have been very successfully in agreement with astronomical observations, for example, the spectrum for p-mode oscillations of the sun.[6] (The only exception is the prediction of the amount of the solar surface ^{7}Li depletion relative to observations; this discrepancy is presumably due to the treatment of convective overshoot at the convective/radiative boundary and should have no effect on solar neutrinos.[32]) But it has relatively large uncertainties in predicting the ^{8}B solar neutrino flux, due to their sensitivity to the uncertainties of the input parameters. Table 4 shows the uncertainties in the solar neutrino flux prediction of BUSSM, TSSM and BPSSM.[6–8] Since two of the current solar neutrino experiments, the chlorine experiment and the Kamiokande experiment, are mostly or solely sensitive to the ^{8}B neutrino flux, their theoretical implication is greatly obscured by the uncertainties in

Table 4. Preditions of solar neutrino fluxes at the earth (in $10^8 cm^{-2}s^{-1}$) and their theoretical uncertainties.

Neutrino Flux	BUSSM	TSSM	BPSSM
pp	600(1±0.02)	602	600(1±0.02)
pep	1.4(1±0.05)	1.3	1.43(1±0.04)
^{7}Be	47(1±0.15)	43.3	48.9(1±0.18)
^{8}B	5.8(1±0.37)	4.43	5.69(1±0.43)
^{13}N	6.1(1±0.50)	3.83	4.92(1±0.51)
^{15}O	5.2(1±0.58)	3.15	4.26(1±0.58)

the neutrino flux prediction. Two currently uncertain input parameters critically affect the ^{8}B neutrino flux[6,9]: (1) the uncertainty in the extrapolated cross section for the ^{7}Be(p,γ)^{8}B reaction. The ^{7}Be(p,γ)^{8}B reaction produces ^{8}B which emits a neutrino as a decay product. Therefore, its rate linearly affects the calculated ^{8}B neutrino flux. (2) the primordial heavy element abundance Z and its effect on calculated radiative opacity. A higher Z and a higher opacity increase the temperature gradient and therefore lead to a higher core temperature T_c inside the sun. It was shown by Bahcall and Ulrich that the ^{8}B neutrino flux $\phi(^8B) \propto T_c^{18}$.[6,9] Therefore a tiny difference in T_c will lead to a significant difference in ^{8}B neutrino flux.

The uncertainty in the ^{7}Be neutrino flux comes primarily from its temperature dependence. Its flux $\phi(^7Be) \propto T_c^8$.[6,9] Since the ^{7}Be neutrino is predicted to contribute considerably in the chlorine and gallium experiments (see Table 2), its uncertainty also needs to be considered carefully.

The CNO neutrinos have large uncertainties also due to their sensitive T_c dependence.[6,9] But since they are minor contributions in the experiments, their uncertainties don't play a major role in the solar neutrino problem.

The *pp* neutrinos are the most abundant and the least uncertain neutrino source among the solar neutrinos. Due to the constraints of the solar model observables, its flux $\phi(pp) \propto T_c^{-1.2}$.[6,9] It consists of more than half of the neutrino events that the gallium experiments can see. An observed rate lower than the predicted *pp* neutrino contribution in the gallium experiments will definitely indicate that the *pp* neutrinos are depleted, and new neutrino physics would need to be introduced. From energetic considerations, regardless of the detail of solar models, the requirement of reproducing the observed solar luminosity through nuclear reactions will yield a minimal neutrino flux of about 80 SNU in the gallium experiments (in which case only *pp* and *pep* neutrinos are produced).[33] Therefore, an observed solar neutrino flux significantly lower than 80 SNU in the gallium experiments would be a smoking gun of new neutrino physics. So far, neither of the two gallium experiments yields a neutrino flux significantly lower than 80 SNU with confidence. Therefore the fact that gallium experiments observed a solar neutrino flux which is lower than the standard model predicion is not a compelling evidence of new neutrino physics. For the Kamiokande experiment, because of the large uncertainties in the ^{8}B neutrinos, the observed deficit can be easily avoided by modifying the standard solar model within a reasonable uncertainty range. However, for the ^{37}Cl experiment, a predicted rate below 4 SNU is definitely problematic for any pure solar model input parameter variation. Let us now look in more detail at the crucial solar model input parameters.

The reaction rate of ^{7}Be(p,γ)^{8}B that affects the ^{8}B neutrino flux is obtained by extrapolating the measured cross section at high energy (110 keV to 4000 keV) to the astophysical interesting energy of $\sim$ 20 keV. Its true uncertainty is difficult to extract. Experiments at relatively high energy were done and were improved gradually over time. The extrapolation calculation has also been improved over the course of more than two decades. BUSSM used $S(0)$ = 24.3 eV$\cdot$ b, based upon the extrapolation method of Tombrello (1965),[34] which only included the s-wave extrapolation. TSSM adopted $S(0) = 21 \pm 3$eV$\cdot$b from Barker's extrapolation including both the s- and d-waves.[35] The most recent calculation including both the s- and d-waves done by Johnson *et al.* gives an average $S(0)$ for six experiments $S(0) = 22.4 \pm 2.1$ eV$\cdot$b,[36] which is adopted by BPSSM. Among the six experiments, the two experiments done by Kavanagh[37] in 1969 and Filippone[38] in 1983 did measurements at low energy below 430 keV, which is crucial to the extrapolation.[36] And these two experiments disagree with each other at low energy, yielding a $S(0)$ of 25.2 $\pm$2.4 eV$\cdot$b and 20.2$\pm$2.3 eV$\cdot$b respectively.[36] Therefore it is questionable whether we should use the average value for these experiments, since they seem to be systematically different. If the value of the Filippone experiment, which is more recent and more carefully described (and with which the preliminary results of a new experiment of Gai *et al.* carried out in Germany seems to agree[39]), is taken, the ^{8}B neutrino flux in BUSSM will decrease by 17%, from 6.1 SNU to 5.1 SNU for the chlorine experiment.[23] The total flux predicted by BUSSM is then 6.9 SNU. The ^{8}B neutrino flux in BPSSM will decrease by 10%, giving a total flux of 7.4 SNU. The TSSM prediction will also decrease from a total of 6.4 SNU to 6.2 SNU.

Further reductions in neutrino fluxes can be achieved by lowering the T_c. Various non-standard solar models[9] have been proposed to yield a lower T_c and hence lower neutrino flux relative to the standard solar models. For example, in the WIMPs (Weakly Interacting Massive Particles) model, the WIMPs captured by the sun can transport the nuclear energy produced at the core and lead to a smaller temperature gradient and a lower T_c; in the strong magnetic field model, a magnetic field of 10^9 Gauss at the center of the sun will provide part of the pressure to support the sun, and therefore lower the T_c. These models are not as successful as the standard solar models in fitting standard solar observational properties and are rather *ad hoc*.[9] In the context of standard solar models, a lower T_c can be achieved by lowering the heavy element abundance Z and the opacity. The solar element abundances are obtained from the spectrometry of the solar surface and the measurements from the meteorites, assuming the abundances there are primordial.[40] For most of the elements, the two types of measurements agree within 9%. But for iron, which contributes $\sim$ 20% of the opacity, the two measurements are about 4σ away from each other. The iron to hydrogen abundance ratio from the photospheric measurement is $4.68(\pm 0.33) \times 10^{-5}$, while the meteorite measurement gives $3.24(\pm 0.075) \times 10^{-5}$. If the lower value of the meteorite measurement is used, the contribution to the opacity from the iron will be substantially reduced, which in turn yields a lower core temperature and a lower neutrino flux. For example, Bahcall and Pinsoneault found that their solar model without helium diffusion yielded 8.5 SNU for the chlorine experiment when the photospheric value was used, and 7.2 SNU when the meteoritic value was used.[7] Furthermore, Marx and Dearborn have discussed possible depletion in the iron contribution to the opacity due to proposed iron-clustering in the solar interior.[41] Such clustering could further reduce the solar opacity and thus T_c (but would still not yield a ^{37}Cl result below 3 SNU).

In an attempt to construct a low flux solar model that is consistent with the experiments, we have evolved the solar code of Dearborn. We adopt $S(0) = 20.2$eV·b for the reaction ^{7}Be(p,γ)^{8}B, and values of Caughlan and Fowler[42] for other rates. We also adopt the OPAL opacity table with an Anders and Grevesse meteoritic mixture.[40] By chosing Z=0.015, X=0.723, our model yields a neutrino flux of 4.5 SNU for the chlorine experiment. The ^{8}B contribution is 3.3 SNU, a nearly 50% reduction with respect to those of BUSSM and BPSSM. The ^{7}Be neutrinos contribute 0.9 SNU. The remaining 0.3 SNU is almost entirely from CNO neutrinos. The predicted event rate for the Kamiokande experiments is 0.54 of the BUSSM prediction. The prediction for the gallium experiments is about 113 SNU. These predictions agree with the Kamiokande results very well and agree with GALLEX and SAGE within 2σ. However, compared with the chlorine result, our prediction is still more than 3σ away. It is very difficult to construct a solar model with less than 4 SNU for the chlorine experiment by modifying solar model inputs. Therefore, given both the theoretical uncertainties and the experimental uncertainties, it is conceivable that there could be no significant conflict between the experiments and such a minimized solar model, except for the chlorine experiment. The core temperature T_c of this model is 1.545×10^7K, compared to 1.569×10^7K of BPSSM and 1.551×10^7K of TSSM (Table 3). Obviously, the heavy element abundance adopted is lower than those of other standard solar models. But it is only 2.4σ below the heavy element abundance obtained by Anders and Grevesse based on the meteoritic measurement ($Z = 0.01886 \pm 0.0016$) or 2.7σ below the abundance based on the solar photospheric measurement ($Z = 0.01941 \pm 0.0016$).[40]

We also calculated the p-mode frequencies of our model. Following the same fitting procedure of Bahcall and Ulrich,[6] we get for our solar model ν_{00}=2622.9μHz, $\Delta\nu_0 = 142.6\mu$Hz, and $\delta_{02} = 8.5\mu$Hz, compared with those obtained from observations, ν_{00}=2630.1$\pm$0.4μHz, $\Delta\nu_0 = 134.5 \pm 0.2\mu$Hz, and $\delta_{02} = 9.6 \pm 0.4\mu$Hz.[6] While the discrepancies in ν_{00} and $\Delta\nu_0$ don't mean much due to the high sensitivity of predictions to the solar atmosphere modeling (which is not fully understood), the discrepancy in δ_{02} which mainly depends on the solar interior is certainly problematic and needs further investigation.

4. The Current Status of the Solar Neutrino Problem

It has been reasonably argued by Bahcall and Bethe[43] that the disagreement between the results of the ^{37}Cl experiment (29% $\pm$ 3% the BUSSM prediction) and the Kamiokande (49% $\pm$ 8% the BUSSM prediction) implies neutrino oscillations. Obviously, assuming the full validity of both experiments, if nothing happens after the solar neutrinos are produced, the ^{8}B neutrino contribution to the ^{37}Cl experiment is at most 2.3$\pm$0.2 SNU, corresponding to 38% $\pm$ 4% the BUSSM prediction. It marginally agrees with the Kamiokande result within 2σ level. But if ν_e oscillates into ν_μ or ν_τ, the ν_μ or ν_τ will account for 62% of the total neutrino flux and will interact with the electrons via neutral currents, with $\sigma(\nu_{\mu\ or\ \tau}\text{-e})/\sigma(\nu_e\text{-e}) = 1/6 \sim 1/7$.[9] So the total rate inferred from the Homestake experiment for the Kamiokande experiment would be $38\% + 62\% \times 1/7 \approx 47\%$ the BUSSM prediction, which agrees with the Kamiokande measurement extremely well. Similar arguments apply to BPSSM and marginally to TSSM, but not to our minimum flux solar model since our model has already agreed very well with the Kamiokande experiment and left no room for extra neutrino species in it.

Because the ^{37}Cl experiment observes both the ^{8}B neutrinos and the ^{7}Be neutrinos, and the Kamiokande experiment detects only the ^{8}B neutrinos, a stronger argument for new neutrino physics applying to all known solar models comes from the ^{7}Be neutrino flux in the comparison of the two experiments. Without resorting to any specific new physics, if the chlorine experiment yield is smaller or equal to the yield of the Kamiokande experiments with respect to the solar model prediction, the ^{7}Be neutrinos are entirely missing. No known astrophysical solution can explain a higher ^{7}Be neutrino reduction than ^{8}B neutrino reduction relative to the standard solar model prediction. A lower central temperature in the sun (which is the spirit of the non-standard solar models metioned in the previous section) doesn't help because the ^{8}B neutrinos are more temperature sensitive than the ^{7}Be neutrinos. In terms of nuclear physics, the ^{7}Be neutrinos and the ^{8}B neutrinos are produced by two different channels of the ^{7}Be nuclear reaction (^{7}Be(e,ν_e)^{7}Li for ^{7}Be neutrinos; ^{7}Be(p,γ)^{8}B and subsequent ^{8}B β-decay for ^{8}B neutrinos).[9] If one supresses the ^{7}Be neutrinos by supressing the overall ^{7}Be production (e.g., by increasing the ^{3}He(^{3}He,2p)^{4}He rate), the ^{8}B neutrinos will be reduced by the same factor. If one supresses the branching ratio that produces ^{7}Be neutrinos, ^{8}B neutrino flux will increase, opposite to the experimental implication. Therefore, the current experimental results, at face value, suggest new physics, if not in particle physics, then in fundamental nuclear physics.

Since the most controversial experiment, the Homestake experiment, combined with the Kamiokande experiment, may provide us the only strong evidence so far for new neutrino physics, it is interesting to see how the situation of an astrophysical solution vs. new neutrino physics could change if, say, unanticipated systematic errors did exist in the Homestake experiment. For example, in the chlorine experiment, if only runs after the pump failures are considered, which yield an average of 2.85$\pm$0.3 SNU, the above ^{7}Be neutrino vs. ^{8}B neutrino argument for new physics may not be as strong when the experimental and theoretical uncertainties are fully taken into account.

In order to show the sensitivity of the current solar neutrino situation to the experimental results, we represent the astrophysical solution by solar models with modified core temperatures T_c, assuming the neutrino fluxes follow the core temperature dependence shown by Bahcall and Ulrich,[6] and compare them with the solution of MSW mixing between ν_e and non-sterile ν_x, which is represented by $\Delta m^2 = m_2^2 - m_1^2$, where m_1 and m_2 are the two mass eigenvalues, and $sin^2 2\theta$, where θ is the mixing angle.

Generally, the core temperature can be as low as $\sim 1.54\times10^7$ K ($\sim 0.98T_c$(BUSSM)) in the uncertainty range of standard solar models.[9] Additional T_c reductions may be achieved by non-standard solar models. But below $0.98T_c$(BUSSM), the temperature dependence of different neutrino sources may vary. Therefore, our calculation is for illustrative purposes when T_c goes beyond the standard solar model range. However, the qualitative conclusion of our calculation remains as long as the ^{8}B neutrino flux is much more sensitively dependent on the core temperature than any other neutrino souces. We have used the rate calculated by Johnson from Filippone's experiment as the ^{7}Be$(p,\gamma)^8$B rate for all solar models in the following discussion. Therefore when T_c=T_c(BUSSM), BUSSM yields 6.9 SNU instead of 7.9 SNU.

The MSW solution with astrophysical uncertainties (uncertainties in standard solar models), for both the two-family and three-family mixing, has been discussed by Shi, Schramm and Bahcall.[12] It is found that the three-family MSW solution doesn't alter the basic picture of the two-family MSW solution, and astrophysical uncertainties significantly broaden the allowed parameter space of the solutions. Figure 2(a) and Figure 2(b) show our updated calculation of the two-family MSW solution and vaccum solution for 1000 solar models in a similar way to ref..[12] In the following comparison between the modified solar models and MSW solutions, however, we only consider two-family MSW mixings with BUSSM for the MSW solutions.

We have performed a Bayesian analysis[44] to the two solutions (the MSW solution and the solar model solution) for the following two experimental situations: (1) the experimental result as shown in Table 1; (2) same as case (1) except a higher Homestake rate 2.85±0.3 SNU (which is the average rate over the period after the pump failures). We found that the MSW solution is favored by $\mathcal{O}(100)$ to 1 over low T_c solar model solutions with T_c varying between $0.95T_c$(BUSSM) and T_c(BUSSM) in case (1). But in case (2), the MSW solutions and the low T_c solar models are comparable in fitting the data.

Therefore, if the current solar neutrino experimental results are taken at face value as in Table 1, we find that the astrophysical solutions may be disfavored. But if the chlorine result is higher than listed in Table 1, the cooler sun solutions may fit the data fairly well. In other words, the astrophysical solution to the solar neutrino problem may, given the current experimental situation, remain plausible and viable. Obviously, more experiments need to be done to rule out decisively any of the alternative solutions.

5. Future Prospects

In order to clear the ambiguity around the new neutrino physics solutions vs. solar model solutions and to resolve the different solutions, in addition to the calibrations needed to be done for current experiments, information from next generation experiments is necessary. The additional information should include: (1) the spectral shape of the incoming solar neutrinos; (2) statistically significant data on the neutral current component in the scattering events; (3) the diurnal and seasonal variations of signals; (4) limits on possible antineutrino components in the solar neutrino flux; and (5) new helioseismic data probing deeper structure of the sun.

The spectral shape of the solar neutrino flux provides one possible way to distinguish the new neutrino physics solution from the astrophysical solution. Different solutions generate different neutrino spectra at the detectors. The astrophysical solution will reduce the ^{8}B neutrino spectrum uniformly with respect to the BUSSM. Other neutrino fluxes remain more or less the same. In the MSW solution, the diagonal region ($3 \times 10^{-6}\mathrm{eV}^2 < \Delta m^2 < 2 \times 10^{-5}\mathrm{eV}^2$, $\Delta m^2 sin^2 2\theta \sim 10^{-7}\mathrm{eV}^2$) has a larger depletion rate at low energies than at high energies, while the vertical solution ($2 \times 10^{-6}\mathrm{eV}^2 < \Delta m^2 < 10^{-4}\mathrm{eV}^2$, $sin^2 2\theta > 0.5$) yields a uniform depletion over the whole energy range. The vacuum mixing solution ($\Delta m^2 \sim 10^{-10}\mathrm{eV}^2$, $sin^2 2\theta > 0.7$) also shows a distortion in spectrum with respect to the β-spectrum predicted by standard solar models.[15] The spectrum of the neutrino magnetic moment solution is sensitive to the magnetic field configuration in the solar interior, which is largely unknown. But a neutrino magnetic moment has been ruled out as a viable solution if the current constraints on solar magnetic field and neutrino magnetic moments hold.[19] The neutrino decay solution, in which the solar neutrinos decay on their way to the earth, will have a lower depletion rate at higher energies because the life-time of neutrinos is proportional to their energies [15]. This solution seems also to have been ruled out by a combination of current experimental results.[20]

So far, only the Kamiokande experiments have spectral resolutions of ~22% at 10 MeV.[24] But because of its low statistics, the spectrum obtained by the Kamiokande experiment shows no appreciable distortion with respect to the prediction of standard solar models and is consistent with all currently allowed solutions, which do not show large distortion over the ^{8}B neutrino spectrum above 8 MeV.

The neutral current interaction is indiscriminate to different flavors. Therefore, the neutral current events include contributions from ν_e, ν_μ and ν_τ. The ν_e contribution, however, can be calculated from charged current events, which involve only ν_e. Any neutral current events in excess of the ν_e contribution clearly imply the presence of neutrinos of other flavors into which the solar neutrinos oscillate. So far, only the Kamiokande experiments can see neutral current events, but they cannot distinguish them from the charged current events. Though the charged current events can be roughly inferred from the chlorine experiment, there is no independent method to obtain evidence of oscillation.

The diurnal variation is a unique signature of MSW matter mixing. For the region with $10^{-6}eV^2 < \Delta m^2 < 10^{-5}eV^2$ on the MSW parameter space, resonances could occur when neutrinos propogate through the earth. Such resonances convert ν_e to other flavors and vice versa, which is the so-called earth regeneration effect.[45] If the ν_e component in the solar neutrinos is not 1/2 when they reach the earth, such an effect will make a difference between daytime event rates and night time event rates in direct counting experiments. It will also cause an extra seasonal variation besides that from the usual $1/R^2$ effect (where R is the distance between the earth and the sun). The vacuum oscillations also show the extra seasonal variation, especially in the monochromatic low-energy ^{7}Be neutrinos, due to the sensitivity of the depletion rate to the distance between the sun and the earth.[15] But there is no day-night variation in this case. For astrophysical solutions, however, there is no time variation in the neutrino flux except that from the $1/R^2$ effect. The diurnal and seasonal variations for ^{8}B neutrinos have been searched for in the Kamiokande II experiment. The result yields no significant variations and rules out the area $2 \times 10^{-6}eV^2 < \Delta m^2 < 10^{-5}eV^2$ on the vertical region of the MSW solution.[46] Obviously, more statistics are needed to draw a more decisive conclusion on time variation.

The existence of an antineutrino component in the solar neutrino flux is a unique signature of majorana neutrinos with magnetic moments. If electron neutrinos are majorana neutrinos and possess magnetic moments, left-handed solar neutrinos can be converted to right-handed antineutrinos in the solar magnetic field.[18] Current solar neutrino experiments are not sensitive to the signature of solar antineutrinos.

If solar neutrinos oscillate into sterile neutrinos, the neutral current test cannot resolve neutrino oscillations from astrophysical solutions. Spectral distortions and time variations are then tests for this resolution. But since a large angle MSW mixing between electron neutrinos and sterile neutrinos yields no appreciable spectral distortion and time variation, the currently available tests cannot distinguish it from astrophysical solutions. Fortunately, constraints from Big Bang Nucleosynthesis (shown in Figure 2(b)) excluded the large angle MSW mixing between electron neutrinos and sterile neutrinos since such mixing yields a higher primordial helium abundance than observed, this ambiguity no longer exits.[47]

The next generation of solar neutrino experiments could provide the additional information to resolve the solar neutrino problem. Among them, four experiments have been approved or proposed for construction and are widely discussed in the literature: the Super Kamiokande, SNO (Sudbury Neutrino Observatory), Borexino, and the Iodine experiment. The first three are direct counting, real-time experiments, while the Iondine experiment is radiochemical.

The Super Kamiokande is an upgraded version of the Kamiokande II. With a fiducial mass about 30 times larger than the Kamiokande II, and a lower threshold, the Super Kamiokande is estimated to have an event rate $\sim$ 50 times higher than the Kamiokande II. It will provide significantly better statistics in terms of determining the neutrino flux, the spectrum shape and any diurnal or seasonal variation.

SNO ultilizes 1 kton of heavy water (D_2O) to interact with solar neutrinos. It has the following reactions: (1) $\nu_e + \mathrm{d} \rightarrow \mathrm{p} + \mathrm{p} + e^-$, via the charged current; (2) ν-e elastic scattering via both the charged and the neutral currents; and (3) $\nu + \mathrm{d} \rightarrow \nu + \mathrm{p} + \mathrm{n}$ via the neutral current only. Reactions (2) and (3) can see not only ν_e, but also other flavors of neutrinos. By comparing the neutral current events and the charged current events measured from reactions (1), (2) and (3), neutrino oscillations can then be tested. SNO can also detect the existence of solar antineutrinos via the reaction $\bar{\nu}_e + \mathrm{d} \rightarrow e^+ + n + n$, which provides unique evidence for the majorana neutrino magnetic moment solution. SNO also has better spectral resolution and time variation resolution than current experiments because of the higher energy resolution and event rate.

Borexino[48] sees mainly the low-energy high-flux ^{7}Be neutrinos with a rate of up to 50 events/day, which is unique among the solar neutrino experiments. It uses the same ν-e scattering reaction as in the Kamiokande experiment, but exploits liquid scintillator instead of water as its target. Comparing the low energy ^{7}Be neutrino flux observed by Borexino with the high energy ^{8}B neutrino flux observed by other experiments, one may have a good picture of the depletion rates at different parts of the spectrum, and a possible resolution of different solutions. Borexino will also be capable of detecting the strong seasonal variation of ^{7}Be neutrinos exhibited by the vaccum neutrino mixing solutions. Finally, Borexino has sensitivity to solar antineutrinos by detecting e^+ produced via the reaction $\bar{\nu}_e + p \rightarrow e^+ + n$.

The Iodine experiment[25,49] is very similar to the chlorine experiment. Via the reaction $^{127}\mathrm{I}(\nu_e,e^-)^{127}\mathrm{Xe}$ with a threshold E_{th}=0.789 MeV, solar neutrinos are absorbed by the NaI desolved in water. ^{127}Xe is extracted in the same way as ^{37}Ar in the chlorine experiment. With a threshold similar to the chlorine experiment, the iodine experiment is also mostly sensitive to ^{8}B neutrinos, plus some low energy ^{7}Be neutrinos and CNO neutrinos. Because of its similarity with the chlorine experiment, the iodine experiment can be used to check the chlorine capture rate and the time variation of the rate, which have drawn controversies so far. Its advantages are that first, the neutrino capture cross section of the iodine is several times larger than that of the chlorine; and second, with the availability of the technology used in the chlorine experiment, the iodine experiment can be put into operation in a matter of months. It is also argued that backgrounds should be relatively low.

Table 5 summarizes the signatures of different solutions including the solar model solutions in the future experiments. It can be seen that with the next generation of solar neutrino experiments, different viable solutions to the solar neutrino experiments can be clearly distinguished.

Also of importance is the next generation helioseismic data from GONG which should probe deeper solar structure than the current experiments.[50] Presently measured modes penetrate only the outer part of the sun and are not sensitive to the inner neutrino producing region. Deeper modes measured by GONG can probe the inner region of the sun and severely constrain solar model solutions.

In summary, while the present solar neutrino situation is still ambiguous, the next round of experiments should hopefully resolve the situation.

Table 5. Signatures of solar neutrino solutions in future experiments.

Solutions	Spectral distortion (Super-K, SNO)	Neutral Current Event (SNO)	Seasonal variation[a] of ^{7}Be ν (Borexino)
Small angle MSW $\nu_e \leftrightarrow \nu_\mu$	Yes	Yes	No
Large angle MSW $\nu_e \leftrightarrow \nu_\mu$	No	Yes	Possible
Vaccum mixing $\nu_e \leftrightarrow \nu_\mu$	Yes	Yes	Yes
Small angle MSW $\nu_e \leftrightarrow \nu_s$	Yes	No	No
Vaccum mixing $\nu_e \leftrightarrow \nu_s$	Yes	No	Yes
Solar model solutions	No	No	No

[a]Other than the usual $1/R^2$ variation.

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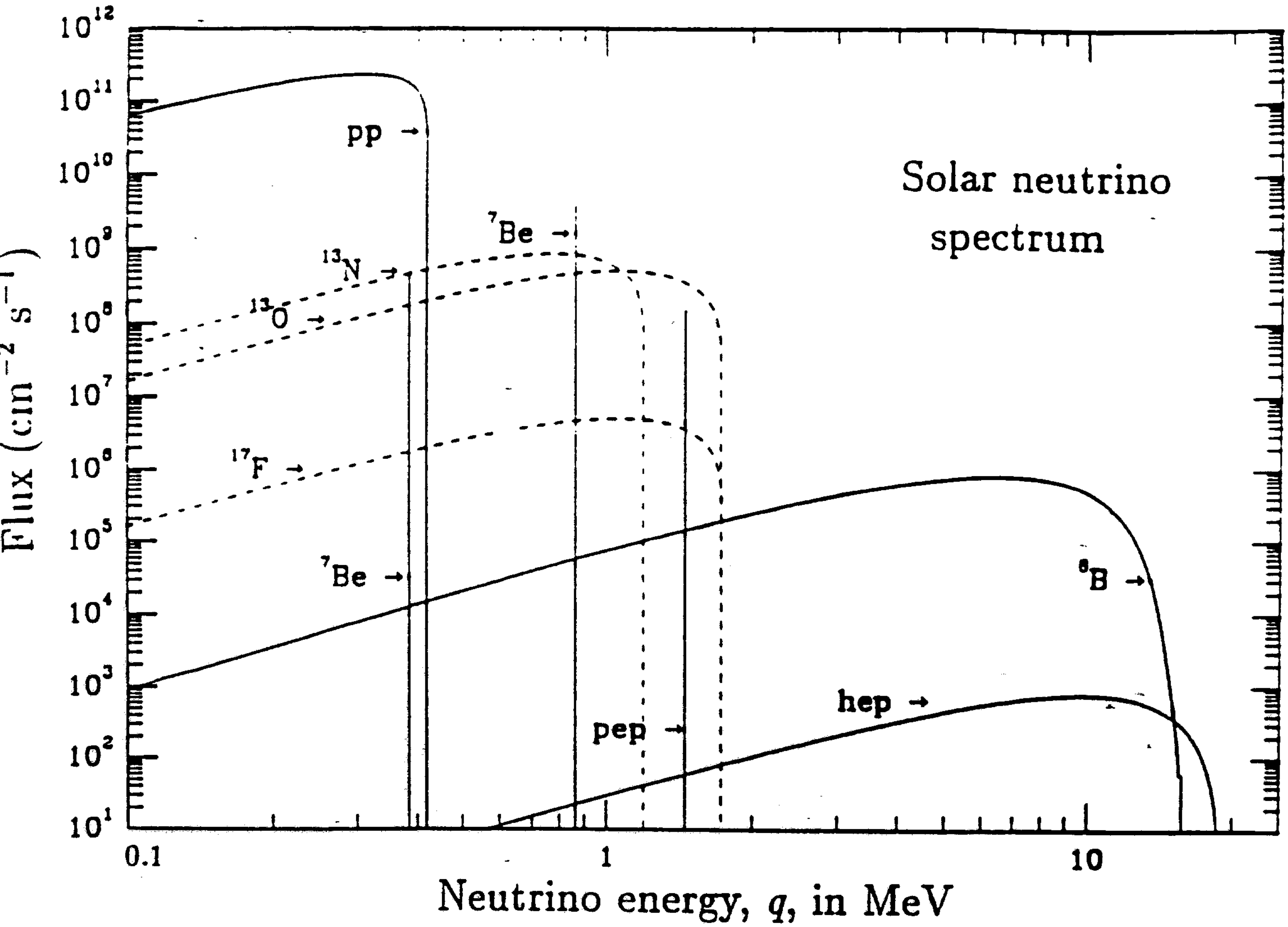
Figure Captions:

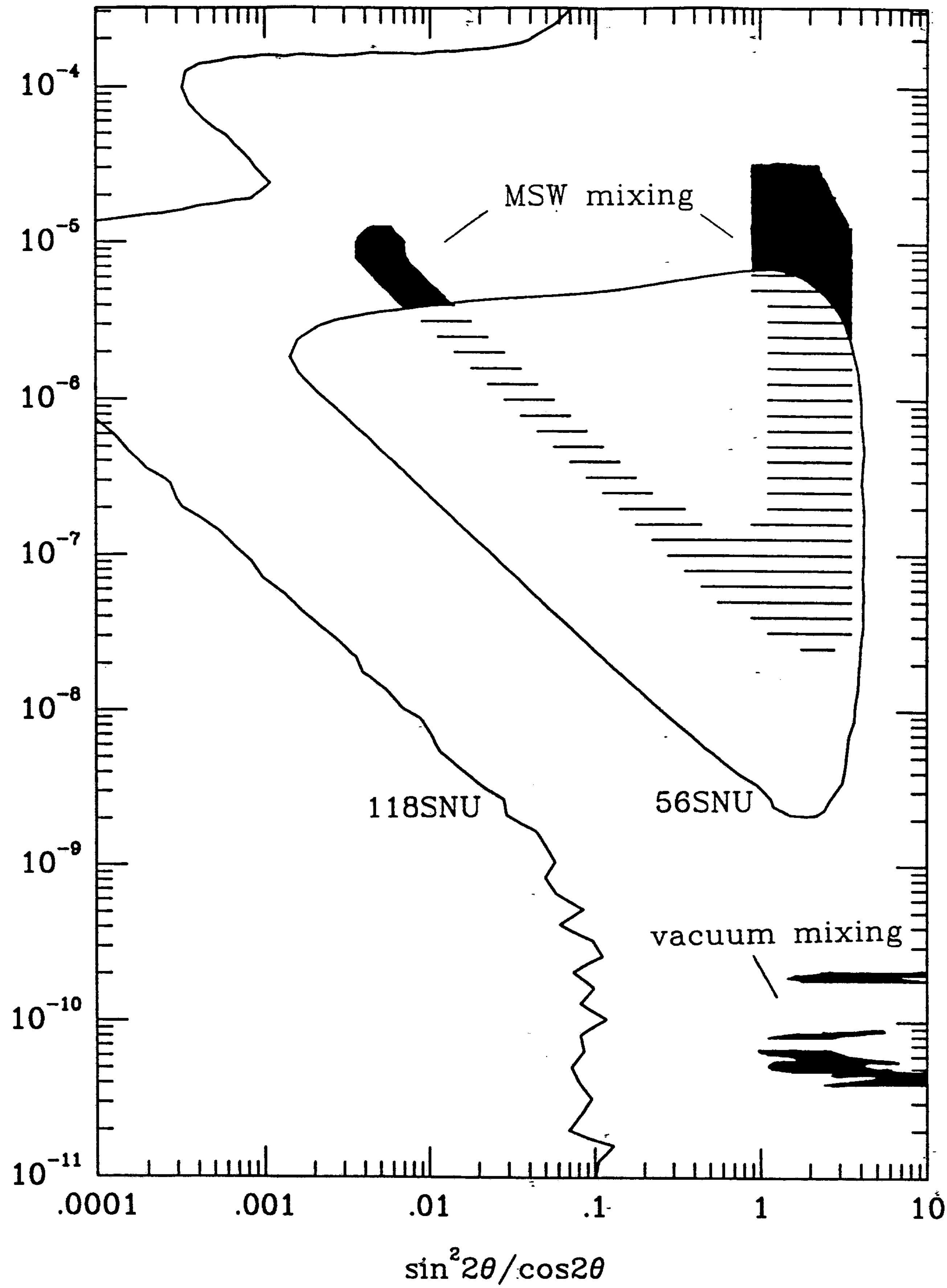
Figure 1. The solar neutrino spectrum as calculated by BUSSM.

Figure 2. (a) For ν_e-ν_μ (or ν_τ) mixings, the shaded regions (including the blacken regions) are the overlaps between parameter spaces allowed by the Homestake experiment and the Kamiokande experiment, allowing for solar model uncertainties, at 95% C.L. The solid contours are iso-SNU contours for gallium experiments, allowing for solar model uncertainties, at 95% C.L. The blacked regions are the overlaps of regions allowed by the Homestake experiment, Kamiokande experiment and GALLEX. (b) The same with (a) but for ν_e-ν_s mixings. The dashed line shows the constraint from Big Bang Nucleosynthesis.



Solar neutrino spectrum
Flux (cm^{-2} s^{-1})
Neutrino energy, q, in MeV
pp →
^{7}Be →
^{13}N →
^{15}O →
^{17}F →
^{7}Be →
^{8}B →
hep →
pep →
0.1
1
10
10^{12}
10^{11}
10^{10}
10^{9}
10^{8}
10^{7}
10^{6}
10^{5}
10^{4}
10^{3}
10^{2}
10^{1}

Fig. 1



MSW mixing
118SNU
56SNU
vacuum mixing
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
$\Delta m^2 (eV^2)$
.0001
.001
.01
.1
1
10
$\sin^2 2\theta / \cos 2\theta$

Fig. 2(a)

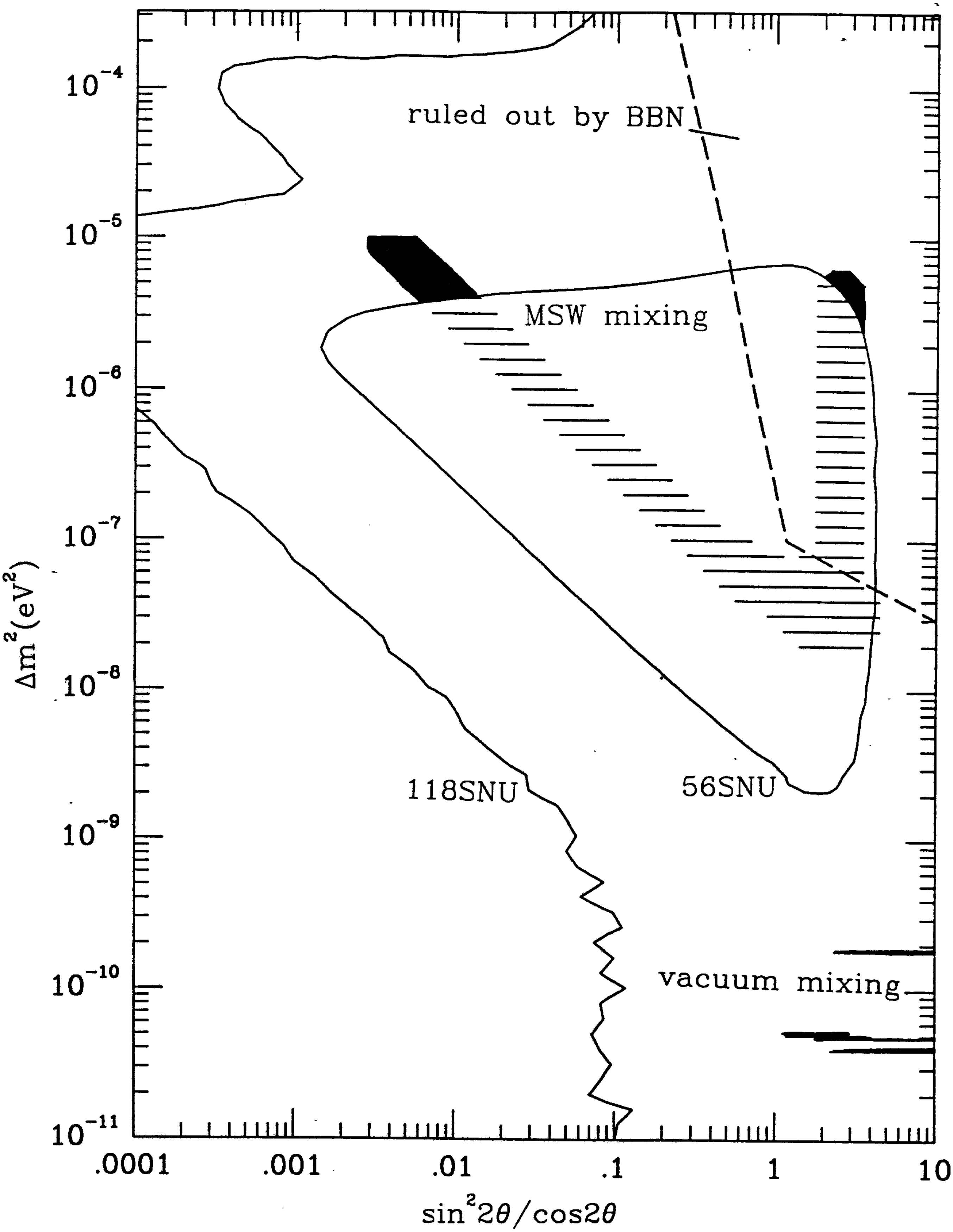

ruled out by BBN
MSW mixing
118SNU
56SNU
vacuum mixing
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
$\Delta m^2(eV^2)$
.0001
.001
.01
.1
1
10
$\sin^2 2\theta/\cos 2\theta$

Fig. 2(b)