



Can MACHOs Probe the Shape of the Galaxy Halo?

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ABSTRACT: We study the prospects for the current microlensing searches, which have recently discovered several candidates, to yield useful information about the flattening of the Galaxy dark matter halo. Models of HI warps and N-body simulations of galaxy formation suggest that disks commonly form in oblate halos with a tilt between the disk and halo symmetry axes. The microlensing optical depth for the Large Magellanic Cloud depends sensitively on the disk-halo tilt angle in the Milky Way. We conclude that a much larger spread in values for $\tau(LMC)$ is consistent with rotation curve constraints than previously thought, and that the ratio $\tau(SMC)/\tau(LMC)$ of the optical depth to the Small and Large Magellanic clouds is not a clean test of halo flattening.

Subject Headings: dark matter - gravitational lensing - Magellanic Clouds

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1. Introduction

The nature of the dark matter in galaxy halos is still unknown, the current choice being between WIMPs (non-baryonic particles such as axions or supersymmetric neutralinos) or MACHOs (massive astrophysical compact halo objects such as brown dwarfs or more massive stellar remnants), or some combination of the two. Recently three candidate microlensing events, the presumed signature of MACHOs in our galaxy, have been observed in the direction of the Large Magellanic Cloud (LMC) by the MACHO (Alcock et al. 1993) and EROS (Aubourg et al. 1993) collaborations, and a fourth event toward the Galactic bulge has been found by the OGLE collaboration (Udalski et al. 1993). The idea of detecting compact halo objects by observing the amplification of stellar magnitude, i.e., gravitational microlensing, when a MACHO passes near the line of sight to a monitored star was first suggested by Paczyński (1986).

If MACHOs constitute the bulk of the halo (see Gates & Turner 1993), it is clearly of interest to ask what can be learned about the Milky Way halo from the microlensing experiments over the next several years. This involves study of how such quantities as the microlensing optical depth τ , event rate Γ , average event time duration $\langle t_e \rangle$, and the distributions in event time and lens mass depend on halo parameters such as core radius and velocity dispersion. Exploratory studies along these lines were made by Griest (1991) for spherical halo models and by Sackett & Gould (1994, hereafter SG) for flattened halo models. Imposing constraints from the Galactic rotation curve and varying assumptions about the Galactic disk and bulge within reasonable limits, SG found that the microlensing optical depth for the LMC is relatively insensitive to the halo model, only varying over the range $\tau(LMC) \simeq 2 - 5 \times 10^{-7}$. Moreover, they found that $\tau(LMC)$ is essentially independent of halo flattening while $\tau(SMC)$ is not, and proposed the ratio of optical depths to the small and large clouds, $\tau(SMC)/\tau(LMC)$, as a robust measure of halo ellipticity, independent of other unknown halo parameters.

In this Letter, we consider microlensing in a flattened halo from a new angle, so to speak. Although the shape of galaxy halos is still an open question (Ashman 1992), several arguments suggest that non-spherical halos may be the norm for spiral galaxies. Flattened halos have been invoked in studies of polar-ring galaxies (Sackett and Sparke 1990, Sackett 1991) and of HI warps in spiral disks (e.g., Hofner & Sparke 1991, Casertano 1991). In particular, it has been suggested that warps are bending modes (Sparke & Casertano 1988), in which a disk precesses into a warped configuration because it is initially tilted with respect to the symmetry plane of an oblate halo (Toomre 1983, Dekel & Shlosman 1983; for recent discussions see Casertano 1991, Hofner & Sparke 1991, Binney 1992). So far, this idea is in reasonable agreement with warp observations (Briggs 1990, Bosma 1991), predicting both a straight line of nodes in the inner parts of disks and that concentrated halos inhibit warping. Although complex issues, such as the halo response to the changing potential of the disk, require further study (Casertano 1991, Binney 1992), misalignment between the symmetry axes of flattened halos and disks remains a promising scenario for explaining warps. We also note that oblate halos with misaligned, warped disks appear to be a common outcome of N-body simulations of galaxy formation in cold dark matter

models (Dubinski & Carlberg 1991, Katz & Gunn 1991) and of galaxy mergers (Hernquist 1989), which might have relevance here if MACHOs form sufficiently early in the history of the galaxy.

Here we study microlensing for an axisymmetric flattened halo model (Binney & Tremaine 1987), taking into account the anisotropy of the velocity dispersion consistent with the halo density distribution (Evans 1993). Since the Milky Way is known to be warped beyond the solar circle (Henderson, Jackson, & Kerr 1982), we consider an oblate halo whose plane of symmetry does not necessarily coincide with the galaxy disk plane. Bearing in mind the complexity of the Milky Way warp (e.g., in the south, the HI disk curves back on itself), we consider a moderate range of misalignment angles between disk and halo. In contrast with the case of a flattened halo coplanar with the disk (Sackett & Gould 1994), allowance for disk-halo misalignment significantly widens the spread in the LMC microlensing optical depth $\tau(LMC)$ and makes the τ_{SMC}/τ_{LMC} ratio test for halo flattening, while still potentially useful, significantly less robust (more ambiguous). These effects are easily understood on geometric grounds.

2. The Halo Model and Rotation Curve Constraints

We consider an axisymmetric flattened halo derived from the logarithmic potential (Binney 1981, Binney & Tremaine 1987)

$$\Phi = -\frac{v_0^2}{2} \ln \left(R_c^2 + R^2 + z^2 q^{-2} \right), \quad (2.1)$$

where (R, z, ϕ) are cylindrical coordinates, R_c is the core radius, q is the axis ratio of the spheroidal equipotentials, and v_0 is the rotation velocity at infinite radius in the equatorial plane ($z = 0$) of the halo. The corresponding density profile is given by

$$\rho(R, z) = \frac{v_0^2}{4\pi G q^2} \frac{(2q^2 + 1)R_c^2 + R^2 + (2 - q^{-2})z^2}{(R_c^2 + R^2 + z^2 q^{-2})^2}. \quad (2.2)$$

The asymptotic axis ratio of the equidensity contours is $1 : 1 : q^{-3}$. Note that in the limit $R_c \rightarrow 0$ and $q \rightarrow 1$, (2.2) reduces to a singular isothermal sphere. The unique even part of the distribution function $f(E, L_z^2)$ can be recovered from $\rho(R, \Phi)$ by a double Laplace inversion (Lynden-Bell 1962) or by contour integration techniques (Hunter & Quian 1993) to give (Evans 1993):

$$f(E, L_z^2) = (AL_z^2 + B) \exp(4E/v_0^2) + C \exp(2E/v_0^2), \quad (2.3)$$

where $E = \Phi - v^2/2$, $L_z = Rv_\phi$, and the parameters $A \equiv (2/\pi)^{5/2}(1 - q^2)/(Gq^2v_0^3)$, $B \equiv (2/\pi^5)^{1/2}(R_c^2/Gq^2v_0)$ and $C \equiv (2q^2 - 1)/(4\pi^{5/2}Gq^2v_0)$. Adding arbitrary terms odd in the z -component of angular momentum L_z to this distribution function does not change the density, but determines the total angular momentum of the halo. The odd part of

the distribution function can be determined uniquely given a rotation law $\langle v_\phi \rangle (R)$ (Lynden-Bell 1962, Evans 1993). In this letter we consider the case of no net streaming, so the distribution function is given by its even part (2.3); this is positive definite as long as $0.707 \leq q \leq 1.08$. We focus on the case of oblate halos, with $q \leq 1$.

To apply this halo model to the Milky Way, we must consider the restrictions on the parameters R_c and v_0 imposed by the observed rotation curve of the Galaxy. The contribution of the halo to the rotation velocity in the disk plane is found from (2.1),

$$v = v_0 \left(\frac{r^2}{a^2 + r^2} \right)^{1/2}, \quad (2.4)$$

where r is the radial distance in the plane of the disk, $a^2 = R_c^2 / (q^{-2} \sin^2 \theta + \cos^2 \theta)$, and θ is the tilt angle between the symmetry plane of the halo and the disk. To find the rotation curve, we add the halo contribution (2.4) in quadrature with those from the visible components of the Galaxy, i.e., the disk, the spheroid, and a central component. For the visible components, we use the model of Bahcall, Schmidt & Soneira (Bahcall, Schmidt, & Soneira 1982, 1983, Bahcall and Soneira 1980). This corresponds approximately to the ‘light disk’ model used in SG; our conclusions would not change qualitatively if we considered a range of models for the visible components. The rotation curve of the Milky Way is known to be approximately flat between 3 and 16 kpc (for a review, see Fich and Tremaine 1991); we impose the constraint that the circular velocity $v(r)$ in the disk lie between 200 and 240 km/sec over the interval $r = 3 - 16$ kpc (this is similar but not identical to the constraint imposed by SG). This leads to an allowed region in the (a, v_0) plane contained within the bounds $0 < a < 7.7$ kpc and $140 \text{ km/sec} < v_0 < 214 \text{ km/sec}$. The shape of the constraint region is such that lower values of a are paired with lower values of v_0 , and large a with higher v_0 .

If the halo is tilted with respect to the disk, the second relevant parameter describing the halo orientation, in addition to the tilt angle θ , is the angle ψ between the “unobservable” line of nodes (the intersection of the halo symmetry plane with the disk) and the sun-galactic center line (SGCL). We set this angle to be the observed angle between the SGCL and the “observable” line of nodes (the intersection of the plane of the outer disk with that of the inner disk), which is about 10 degrees due north (Henderson, Jackson, & Kerr 1982). We have also investigated other choices of ψ ; the choice above happens to have roughly the largest impact on the microlensing rates, so results for other ψ will be bracketed by those shown below.

We must choose a reasonable range of values for the tilt angle θ . The warp angle, defined as the angle between the plane of the outer disk and that of the inner disk, is about 18 degrees for the Milky Way (Henderson, Jackson, & Kerr 1982). In the “modified tilt mode” model (Sparke and Casertano 1988), the relation between the warp angle and the unobservable tilt angle θ depends on the parameters of the halo and the disk. Roughly speaking, for small R_c (in units of the disk scale length), the warp angle increases with radius (“Type I” mode), whereas in the opposite case the warp angle decreases with radius (“Type II” mode). The parameters of the Milky Way naively appear to favor the

Type I mode, but the complexity of the Galactic warp and the simplicity of the model mitigate against giving this much weight. (The constraint that the mode be discrete in principle constrains the core radius and halo ellipticity, but this does not constrain the halo parameter space more strongly than the rotation curve above.) For a Type I mode, θ can be smaller or larger than the warp angle (depending on the halo mass compared to the disk mass, and on the flattening). For a Type II mode, θ is always greater than the warp angle. Therefore, we consider values of θ between -30 and $+30$ degrees to cover a plausible range. Positive values of θ denote tilting the halo symmetry plane “towards the LMC”, whereas negative values correspond to tilting “away from the LMC”. This geometric picture accounts qualitatively for the results below.

3. Microlensing Results

We now use the constrained halo models above to study microlensing. The optical depth is given by the number of MACHOs inside the “microlensing tube” (Griest 1991):

$$\tau = \int_0^{x'_h L} \frac{\rho(R(x), z(x))}{m} \pi u_T^2 R_E^2(x) dx, \quad (3.1)$$

where m is the MACHO mass, $R_E = 2(Gmx(L-x)/c^2L)^{1/2}$ is the Einstein ring radius, L is the distance to the lensed star, u_T is the threshold impact parameter in units of the Einstein radius, x denotes the distance along the line of sight, and $x'_h L$ is the lesser of the extent of the halo and L . Studies using high velocity stars to infer the local escape speed (Fich & Tremaine 1991) suggest that the truncation radius of the halo is in excess of 35 kpc; therefore, since 90% of the microlensing occurs between 5 and 30 kpc, extending the halo out to the LMC or SMC will introduce negligible error. We thus take x'_h to be 1. (It would be useful to check this by using the physically truncated lowered Evans models of Kuijken and Dubinski 1993).

In Figure 1, we show the LMC optical depth τ_{LMC} (assuming $l = 280.5^\circ$, $b = -32.9^\circ$, and $L = 50$ kpc) as a function of core radius, for different values of the halo-disk tilt angle θ for an E6 model (with asymptotic axis ratio equal to 0.4, and $q = 0.737$). For comparison, we show the spherical halo (E0) values (solid curves). For each θ , we show the maximum and minimum values of τ_{LMC} allowed by the rotation curve constraints. Note that the LMC optical depth is insensitive to halo flattening without tilt (the E0 and E6 $\theta = 0$ curves are nearly identical), and for $\theta = 0$ the allowed range in optical depth is $\tau(LMC) \simeq 3 - 6 \times 10^{-7}$, both in good agreement with SG. On the other hand, the LMC optical depth is quite sensitive to the tilt angle: $\tau(LMC)$ increases with θ , reflecting the increasing mass between us and the LMC. As a result, for $-30^\circ < \theta < 30^\circ$, the spread in τ_{LMC} is increased to an order of magnitude, $\tau(LMC) \simeq 1.2 - 10 \times 10^{-7}$.

As pointed out by SG, the ratio τ_{SMC}/τ_{LMC} is independent of the rotation curve constraints (because $\rho \propto v_0^2$). In Figure 2, we show the ratio τ_{SMC}/τ_{LMC} as a function of

R_c for each θ (we assume $l = 302.8^\circ$, $b = -44.3^\circ$, and $L = 63kpc$ for the SMC). For the spherical E0 model, we find the ratio is larger than 1.45, while for the untilted E6 model it is below 1.0, in agreement with SG. This large difference was the basis of SG's proposal to use this ratio as a test of halo ellipticity. However, for $\theta = 30^\circ$, the E6 model ratio is about 1.22-1.24, significantly closer to the E0 value of 1.46. SG estimate that the fractional precision in this ratio measurement would be at best 10 % due to statistical fluctuations, and perhaps a factor of two larger, depending on the extent to which MACHOs in the LMC contaminate and can be removed from the sample. Consequently, there appears to be some degeneracy in the ratio test: a measurement of τ_{SMC}/τ_{LMC} in the range 1.2 – 1.4 would not reliably determine the shape of the Galaxy halo. Nevertheless, it would still help narrow down the range of possibilities in the ellipticity-orientation parameter space. On the other hand, a measurement of the ratio which came out below unity would definitely point to halo flattening, although it would provide little information on the tilt, which is unfortunate given the insights this might provide on the dynamics of warps.

Recently, Gould, Miralda-Escudé, and Bahcall (1993) have discussed tests to distinguish microlensing by disk and halo populations. For MACHOs in a disk, they find $\tau_{SMC}/\tau_{LMC} = 0.6$, substantially below their values of 1.47 for a spherical halo and 0.96 for a flattened coplanar E6 halo. We find that this ratio can be as low as 0.84 for an E6 halo with $\theta = -15^\circ$, so this particular test can still potentially discriminate between halo and disk populations, although with less confidence.

Another quantity of interest is the microlensing event rate Γ , the rate at which MACHOs enter the microlensing tube (Griest 1991):

$$\Gamma = \int \frac{f(\mathbf{v}, R(x), z(x))}{m} v_r^2 \cos \phi u_T R_E(x) dv_r dv_x d\phi dx d\alpha . \quad (3.2)$$

Here v_r is the transverse velocity of the MACHOs in the frame of the microlensing tube, ϕ is the angle between v_r and the normal to the surface of the tube, and α is the polar angle in the plane normal to the line of sight (see Griest 1991; we do not include the motion of the Earth or of the LMC). The microlensing rate for the LMC looks qualitatively similar to the optical depth results of Fig. 1: there is an order of magnitude spread in the values of Γ_{LMC} , over the range $0.25 - 2.9 \times 10^{-6} (m/M_\odot)^{-1/2} u_T$ events/yr. In Figure 3 we show the ratio $\Gamma_{SMC}/\Gamma_{LMC}$ for different values of θ as a function of R_c for E6 and E0 halos, taking into account the rotation curve constraints. The dependence of the rate ratio on R_c differs from that for the optical depth, because the velocity dispersion is tied to the other halo parameters through (2.3) and the rotation curve; the velocity anisotropy plays a relatively minor role. But the dependence of the rate ratio on halo-disk tilt angle is qualitatively similar to that for the optical depth. Note that, in the absence of independent information on the halo core radius, the degeneracy between the spherical and flattened tilted models is more severe for the rate ratio than for the optical depth ratio.

Finally, we calculate the relative probability $P(m, t_e)$ of models characterized by mass m giving rise to an event duration of t_e . Assuming that all the MACHOs have the same mass, this is given by (Griest 1991):

$$P(m, t_e) = \frac{\Delta t_e}{\Gamma} \int \frac{f}{m} (v_0^2 \beta)^2 y^{3/2} (1 - y)^{-1/2} dv_x dy dx d\alpha, \quad (3.3)$$

where $v_0^2 \beta \equiv (2R_E u_T / t_e)^2$, and $\beta y \equiv v_r^2 / v_0^2$. The constant Δt_e is chosen so that the maximum of $P(m, t_e)$ is 1. For comparison with the results of Griest (1991), we show this quantity in Fig. 4 for $t_e = 0.3$ yr, assuming $u_T = 1$ and $R_c = 4$ kpc. The mass scales as $(t_e / 0.3)^2 / u_T^2$. The results show that the inferred mass is relatively insensitive to halo flattening or tilt, but that the allowed spread in halo velocity dispersion at fixed R_c gives rise to an additional factor of ~ 1.6 uncertainty in the mass.

4. Conclusion

Motivated in part by models of HI disk warps and by N-body simulations of galaxy formation, we have studied microlensing in an oblate halo that is not coplanar with the disk. Imposing constraints from the observed Galaxy rotation curve, we find that the lensing optical depth for the LMC is sensitive to the tilt angle between disk and halo, and can be twice as large for moderate tilt angle as for coplanarity. The resulting spread in LMC optical depth is roughly an order of magnitude, $\tau(LMC) \simeq 1 - 10 \times 10^{-7}$ for models consistent with the flatness of the rotation curve. The tilt angle also affects the ratio of the optical depth to the SMC and LMC, rendering it a less robust test of halo flattening. While a small ratio, $0.8 < \tau_{SMC} / \tau_{LMC} < 1$ would unambiguously indicate flattening, a larger ratio does not cleanly indicate sphericity: there is a near-degeneracy for spherical and flattened tilted halos. On the other hand, a very large optical depth for the LMC, $\tau > 7 \times 10^{-7}$, would help break this degeneracy and would point to a flattened halo with tilt.

We can envision a number of directions which could be taken to improve upon or extend the results discussed here. These include the study of triaxial halos, self-consistent modelling of the disk-halo system (the halo model discussed here is an equilibrium solution if the gravitational field of the disk is neglected), and the exploration of other observational tests of disk-halo tilt in the Milky Way.

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Figure Captions

Figure 1. LMC optical depth as a function of halo core radius R_c for spherical E0 and flattened E6 halos, assuming $u_T = 1$ (for other values, $\tau \propto u_T^2$). For each tilt angle θ , we show the maximum and minimum values of the optical depth consistent with the rotation curve constraints.

Figure 2. Ratio of optical depth toward the Small and Large Magellanic clouds, τ_{SMC}/τ_{LMC} , as a function of core radius for E6 halos (with different tilt angles) and the spherical E0 halo.

Figure 3. Ratio of microlensing rates for the Small and Large clouds, $\Gamma_{SMC}/\Gamma_{LMC}$, as a function of core radius, for E6 halos (with different tilt angles) and the E0 halo.

Figure 4. Relative probability of models characterized by MACHO mass m giving rise to an event duration $t_e = 0.3$ yr, assuming $u_T = 1$ and halo core radius $R_c = 4$ kpc. The curves denote the same models as shown in Fig. 1, and the two curves for each model correspond to the maximum and minimum halo velocities for this core radius.

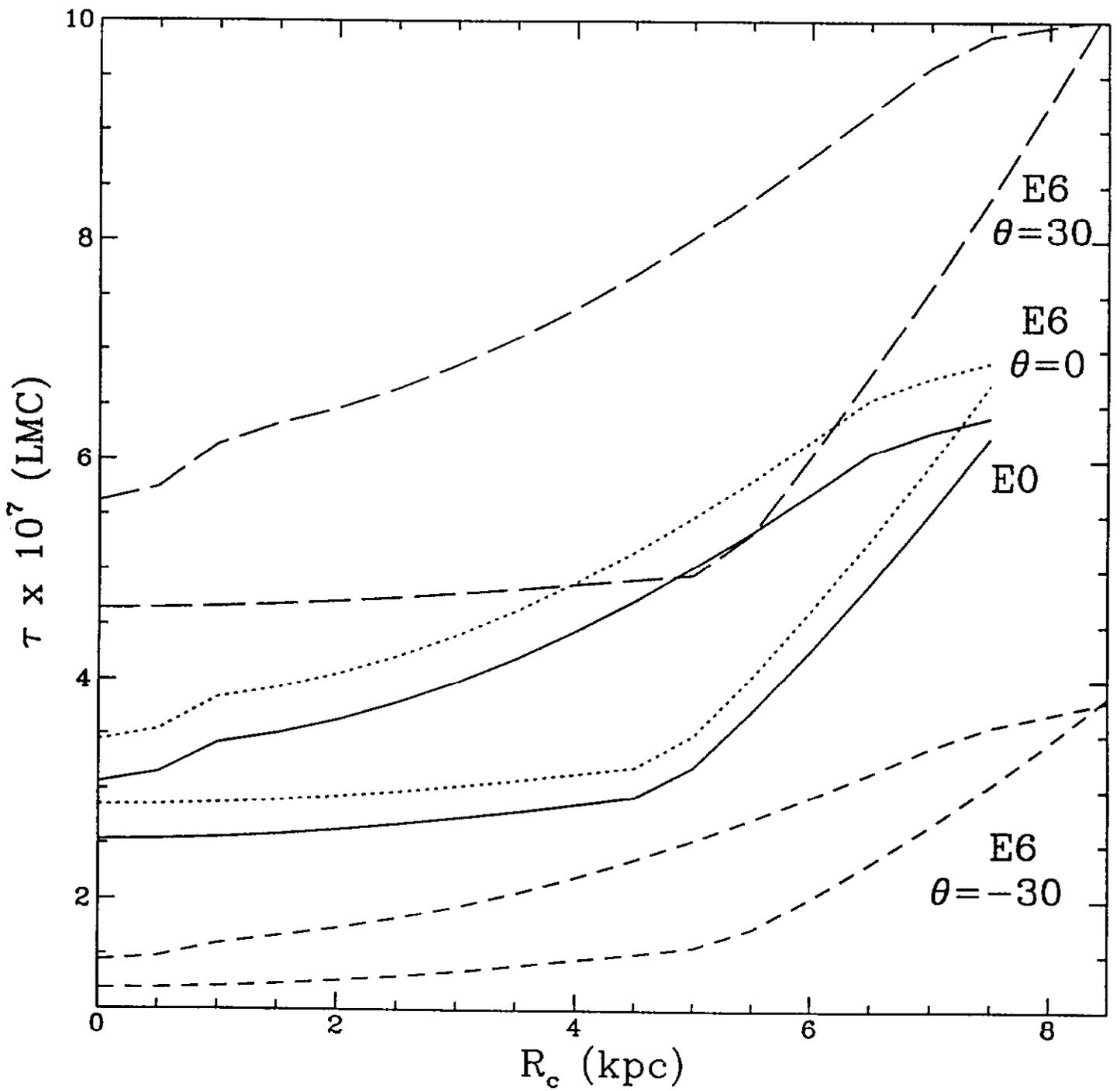


Figure 1

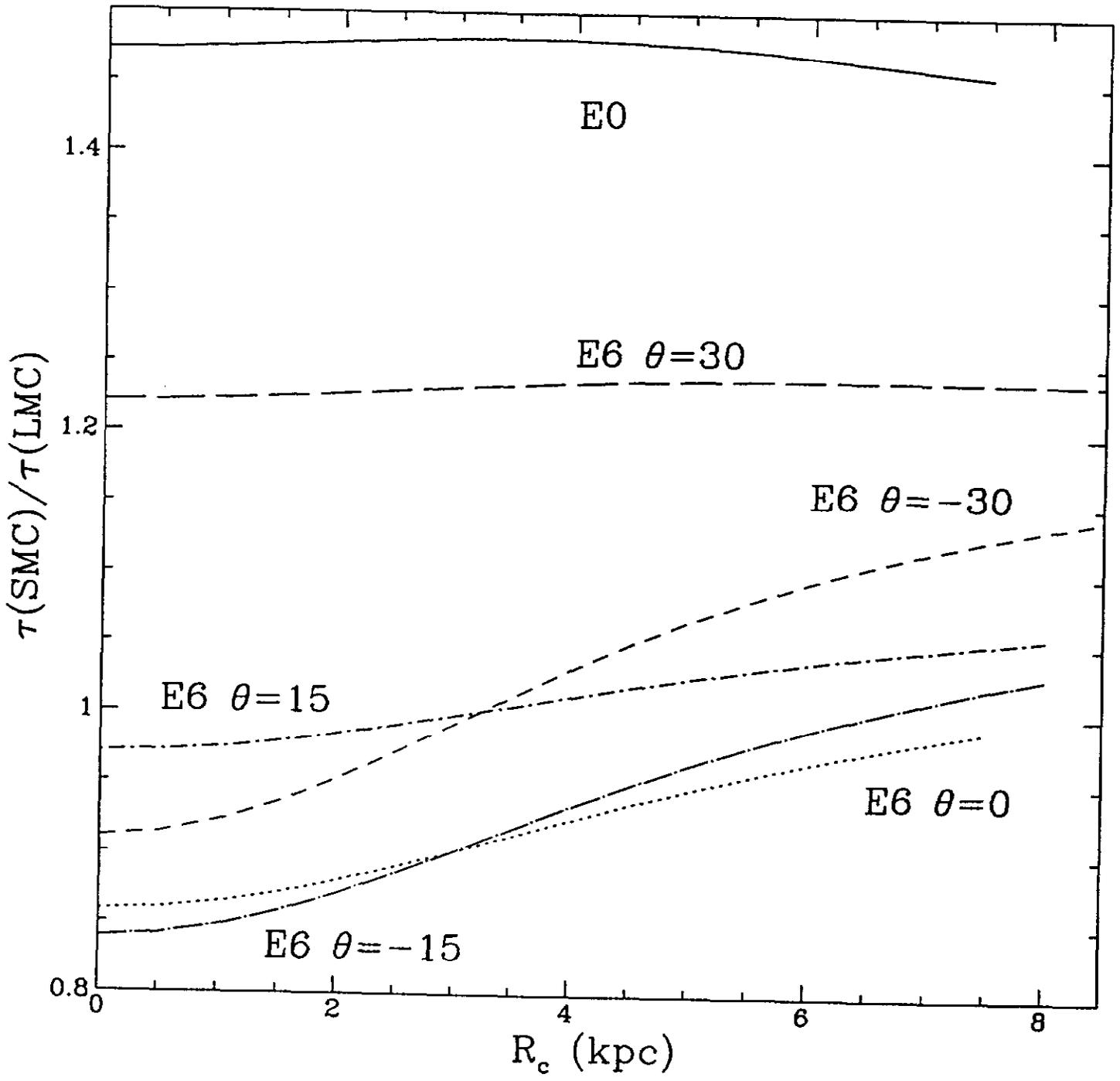


Figure 2

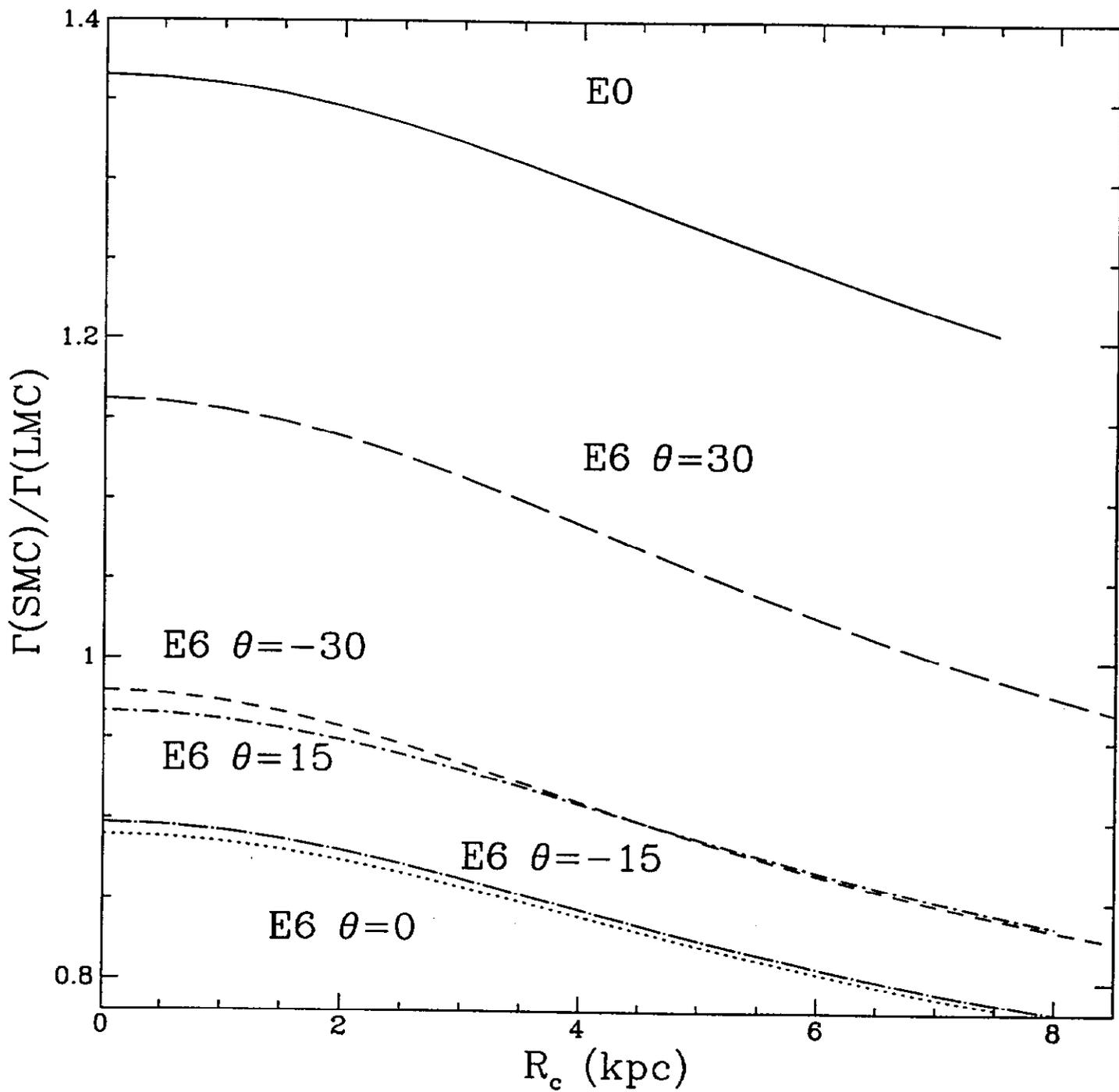


Figure 3

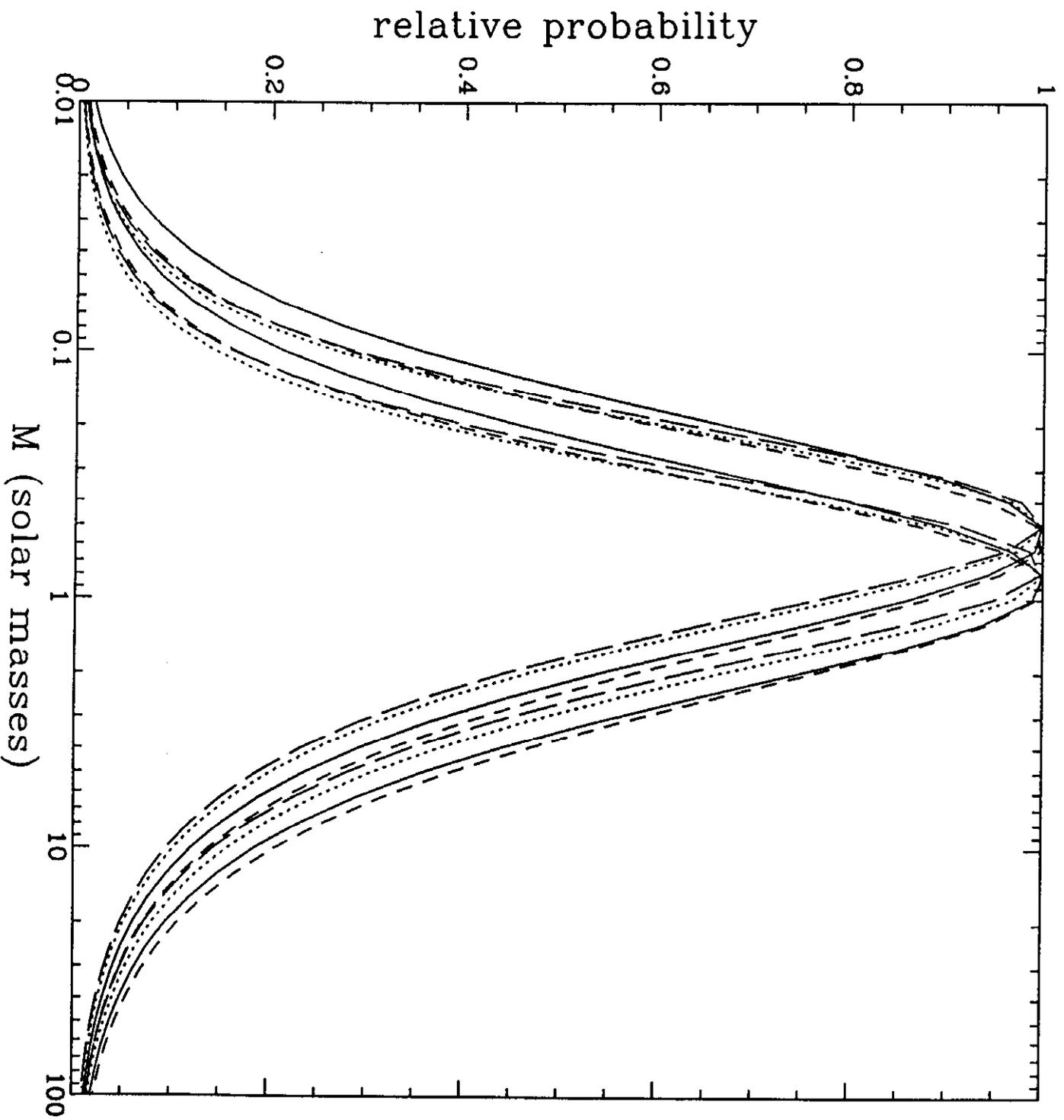


Figure 4