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One-Loop Effective Potential for SO(10) GUT Theories in de Sitter Space

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ABSTRACT

Zeta-function regularization is applied to evaluate the one-loop effective potential for $SO(10)$ grand-unified theories in de Sitter cosmologies. When the Higgs scalar field belongs to the 210-dimensional irreducible representation of $SO(10)$, attention is focused on the mass matrix relevant for the $SU(3) \otimes SU(2) \otimes U(1)$ symmetry-breaking direction, to agree with low-energy phenomenology of the particle-physics standard model. The analysis is restricted to those values of the tree-level-potential parameters for which the absolute minima of the classical potential have been evaluated. As shown in the recent literature, such minima turn out to be $SO(6) \otimes SO(4)$- or $SU(3) \otimes SU(2) \otimes SU(2) \otimes U(1)$-invariant. Electroweak phenomenology is more naturally derived, however, from the former minima. Hence the values of the parameters leading to the alternative set of minima have been discarded. Within this framework, flat-space limit and general form of the one-loop effective potential are studied in detail by using analytic and numerical methods. It turns out that, as far as the absolute-minimum direction is concerned, the flat-space limit of the one-loop calculation about a de Sitter background does not change the results previously obtained in the literature, where the tree-level potential in flat space-time was studied. Moreover, when curvature effects are no longer negligible in the one-loop potential, it is found that the early universe remains bound to reach only the $SO(6) \otimes SO(4)$ absolute minimum.

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1. Introduction

Over the last ten years, the idea by Coleman and Weinberg (1973) on radiative corrections at the origin of spontaneous symmetry breaking has played a very important role also in cosmology. Since the singularity theorems by Penrose, Hawking and Geroch show that Einstein's general relativity leads to the occurrence of singularities in a generic way, modern theoretical cosmology has focused instead on quantum theories of gravitation (Hawking and Ellis 1973, Gibbons and Hawking 1993, Esposito 1994). As a first step of this program, one faces the problem of making sense of quantum field theory in curved space-time (Birrell and Davies 1982), where matter fields are quantized via algebraic or canonical or path-integral methods, whereas gravity provides a curved, fixed background 4-geometry.

In Allen (1983) and Allen (1985) the one-loop approximation of path integrals in curved space was applied to study massless scalar electrodynamics and $SU(5)$ non-Abelian gauge fields in de Sitter space. For this purpose, the author used zeta-function regularization (Hawking 1977, Esposito 1994), and was able to show that the inflationary universe can only slide into either the $SU(3) \otimes SU(2) \otimes U(1)$ or the $SU(4) \otimes U(1)$ extremum, in the case of $SU(5)$ gauge models. In his analysis, Allen (1983, 1985) was dealing with Wick-rotated path integrals, leading to a Riemannian background 4-geometry with S^4 topology and constant scalar curvature, i.e. the Euclidean-time version of de Sitter space-time.

More recently, work by the authors (Buccella *et al* 1992, Esposito *et al* 1993) has tried to improve the analysis appearing in Allen (1985). The main results of our recent papers are as follows.

(i) The technique originally developed in Buccella *et al* (1980) to study spontaneous symmetry breaking of $SU(n)$ for renormalizable polynomial potentials has been generalized, for $SU(5)$, to a de Sitter background.

(ii) Algebraic and numerical techniques, jointly with Dirac's theory of constrained Hamiltonian systems, have been used to prove that, to be consistent with the experimentally established electroweak standard model and with inflationary cosmology, the residual gauge symmetry group of the early universe, during the whole de Sitter era, is $SU(3) \otimes SU(2) \otimes U(1)$.

Thus, a deeper understanding of the results in Allen (1985) was obtained, and the numerical integration of the corresponding field equations confirmed the existence of a sufficiently long de Sitter phase of the early universe. However, since the technique described in Allen (1985) enables one to evaluate the one-loop effective potential for *all* non-Abelian gauge theories in de Sitter space, a naturally occurring question is whether one can repeat this analysis in the case of physically more relevant GUT theories in de Sitter cosmologies. For this purpose, our paper studies the one-loop effective potential of $SO(10)$ GUT theories.

$SO(10)$ gauge theories as unified models for strong, electromagnetic and weak interactions (Fritzsch and Minkowski 1975, Tuan 1992) have been studied over many years for their interesting physical properties. There are several motivations for this choice and we just review some of them. Unlike $SU(5)$ in fact, where a complete fermionic family has to be classified within two different irriducible representations of dimension 10 and 5

respectively, the $SO(10)$ theory is able to accommodate in the 16-dimensional spinor representation one fermionic family plus a right-handed neutrino. Furthermore, the orthogonal groups like $SO(10)$ are anomaly-free with the exception of $SO(6)$.

Finally, the strongest motivation for this choice can be found in the predictions for nucleon lifetimes. In this case in fact, $SO(10)$ models enable one to obtain higher values for the masses of the lepto-quarks which mediate proton decay and which were predicted to be too low, with respect to the experimental lower limit, in the minimal $SU(5)$ model. This property is essentially related to the presence of an intermediate symmetric phase between the $SO(10)$ symmetry at GUT scale and the $SU(3) \otimes SU(2) \otimes U(1)$ symmetry at weak scale, and it provides the high flexibility of these models with respect to the simpler $SU(5)$ theory. In particular, recently, the more precise determination of the gauge coupling constants at the scale M_Z (i.e. the mass of the Z^0 gauge boson) has been used to show that the three running coupling constants $\alpha_3(\mu), \alpha_2(\mu), \alpha_1(\mu)$ of $G \equiv SU(3) \otimes SU(2) \otimes U(1)$, if only standard-model particles contribute to the renormalization-group equations, meet at three different points, and only the matching points of $\alpha_2(\mu)$ and $\alpha_3(\mu)$ correspond to a value of the scale sufficiently high to agree with the experimental lower limit on proton decay: $t_{p \to e^+ \pi^0} > 9 \cdot 10^{32}$ years (Aguilar-Benitez *et al* 1992), which corresponds to a mass for the lepto-quarks which mediate that decay of about $3 \cdot 10^{15}$ GeV (Buccella *et al* 1989). Indeed it has been observed that $SO(10)$ is very promising to modify the $SU(5)$ predictions in such a way to prevent conflict with experiment due to its intermediate symmetry group G' containing $SU(2)_R$ and (or) $SU(4)_{PS}$ (see below).

A complete analysis of the possible symmetry-breaking patterns with Higgs particles in representations with dimension ≤ 210 and with only an intermediate symmetry group G' between G and $SO(10)$ has led to only four different possibilities for a physically relevant $SO(10)$ unified model (Buccella 1988) [table I]. With the notation of table I, $SU(3)_C$ is the colour group, $SU(2)_{L,R}$ are the *left* and *right* $SU(2)$ groups whose representations differ by their behaviour with respect to helicity. Moreover, $B - L$ is the difference between baryonic and leptonic number, D is the discrete left-right interchanging symmetry introduced in Kuzmin and Shaposhnikov (1980), and $SU(4)_{PS}$ denotes the $SU(4)$ Pati-Salam group (Pati and Salam 1973). For these models, using the one-loop approximation for the renormalization-group equations, the upper limit for the values of the symmetry-breaking scales of $SO(10)$ (M_X), and of G' (M_R), is reported for the different models in their minimal formulation [table II].

As one can see, both models without D symmetry yield sufficiently high values for the scale M_X, and the model with $G' \supset SU(4)_{PS}$ predicts $M_R = 10^{11}$ GeV, while the one with $G' \supset SU(3)_C \otimes U(1)_{B-L}$ gives rise to a value about two orders of magnitude smaller. Using these results and their implications for proton decay we can safely restrict our analysis of the cosmological implications of $SO(10)$ GUT model, to the ones which appear physically more relevant and which contain the Higgs field in the 210-dimensional irreducible representation.

Our paper is organized as follows. Section 2, aimed at cosmologists who are not familiar with grand-unified theories, describes the basic elements of $SO(10)$ GUT models in particle physics, the tree-level potential for the 210-dimensional irreducible representation,

and the mass matrix relevant for the $SU(3) \otimes SU(2) \otimes U(1)$ symmetry-breaking direction. Section 3, after a review of the Faddeev-Popov form of the amplitudes in quantum field theory, derives the one-loop form of the effective potential for $SO(10)$ GUT theories studied about a de Sitter background. The numerical analysis of the corresponding flat-space limit is then carried out in section 4. Section 5 studies by numerical methods the one-loop effective potential in the region where no asymptotic expansion for infinite or vanishing curvature of the special function occurring in such a potential can be made. Results and concluding remarks are presented in section 6.

2. SO(10) GUT theory in flat space-time

The group $SO(10)$ is defined as the set of 10×10 orthogonal matrices with unit determinant, and with the usual product rules. It has 45 generators here denoted by T_{ij} $(i, j = 0, 1, \ldots, 9)$ obeying the following commutation relations:

$$\Big[T_{jk}, T_{lm}\Big] = i\Big(\delta_{jl}\, T_{mk} + \delta_{jm}\, T_{kl} + \delta_{kl}\, T_{jm} + \delta_{km}\, T_{lj}\Big) \quad . \tag{2.1}$$

Considering the vector irreducible representation ($\underline{10}$) of the group, which we indicate by φ_l, the action of the generators T_{jk} on it is given by

$$T_{jk}\, \varphi_l \equiv i\Big(\delta_{kl}\, \varphi_j - \delta_{jl}\, \varphi_k\Big) \quad . \tag{2.2}$$

Analogously, the action of T_{kl} on the Kronecker product of two representations φ_i and ψ_j is given by

$$T_{kl}\Big[\varphi_i \otimes \psi_j\Big] \equiv \Big(T_{kl}\, \varphi_i\Big) \otimes \psi_j + \varphi_i \otimes \Big(T_{kl}\, \psi_j\Big) \quad . \tag{2.3}$$

To construct a satisfactory gauge theory based on the $SO(10)$ local symmetry, which has the proper residual symmetry in the low-energy limit, we need a Higgs mechanism to break the symmetry spontaneously (O'Raifeartaigh 1986). This is based on the presence of a fundamental scalar particle (Higgs field), belonging to one or more irreducible representations (hereafter referred to as IRR's) of the gauge group, whose dynamics is ruled by a Higgs potential. In the present case, we are going to study the most general, renormalizable and conformally invariant Higgs potential constructed by using only the IRR $\underline{210}$, which is obtained by the completely anti-symmetrized product of four different $\underline{10}$'s as

$$\Phi_{abcd} = N\, \mu_{[a} \otimes \nu_b \otimes \rho_c \otimes \sigma_{d]} \tag{2.4}$$

where N is the normalization constant. The $\underline{210}$ IRR has four independent quartic invariants, i.e. $\|\phi\|^4$ and three non-trivial invariants (see (2.10)-(2.12)), hence the Higgs potential we are going to construct will be a function of these. Multiplying two $\underline{210}$ and symmetrizing one gets the Clebsch-Gordan decomposition

$$(\underline{210} \otimes \underline{210})_{sym.} = \underline{1} \oplus \underline{45} \oplus \underline{54} \oplus \underline{210} \oplus \underline{770} \oplus (\underline{1050} \oplus \overline{\underline{1050}}) \oplus \underline{4125} \oplus \underline{8910} \oplus \underline{5940} \tag{2.5}$$

where $\underline{45}, \underline{54}, \underline{770}$ and so on denote the IRR's with dimension 45, 54, 770 respectively.

The IRR's $\underline{45}$, $\underline{210}$ and $(\underline{1050} \oplus \overline{\underline{1050}})$ give no contribution along the $SO(6) \otimes SO(4)$-invariant direction. This can be easily understood noticing that $\underline{45}$ and $(\underline{1050} \oplus \overline{\underline{1050}})$ do not contain singlets along the above direction and that the only singlet contained in the $\underline{210}$ representation is such that

$$C_{(1,1,1)\,(1,1,1)\,(1,1,1)}^{\underline{210}\;\;\underline{210}\;\;\underline{210}} = 0 \quad . \tag{2.6}$$

With our notation, $(1,1,1)$ is the only singlet with respect to the $SU(4) \otimes SU(2) \otimes SU(2)$ group contained in the $\underline{210}$ representation. Moreover, the left-hand side of (2.6) is the Clebsch-Gordan coefficient (Cornwell 1984b) corresponding to the decomposition $(1,1,1)_{\underline{210}} \otimes (1,1,1)_{\underline{210}} \to (1,1,1)_{\underline{210}}$.

Hence the only quartic non-trivial invariants, apart from the fourth power of the $\underline{210}$ norm which is isotropic in the space of IRR's and thus cannot discriminate among the invariant directions, are $\|(\phi\phi)_{\underline{45}}\|^2$, $\|(\phi\phi)_{\underline{210}}\|^2$ and $\|(\phi\phi)_{\underline{1050}}\|^2$ (where for example, the symbol $(\phi\phi)_{\underline{45}}$ stands for the $\underline{45}$ IRR contained in the product of two $\underline{210}$).

By virtue of the above considerations, the most general renormalizable and conformally invariant Higgs potential, made out of the $\underline{210}$ representation only, turns out to be a linear combination of the above invariants, with arbitrary coefficients g_1, g_2, g_3 and λ

$$V(\phi) = g_1 \ \|(\phi\phi)_{\underline{45}}\|^2 + g_2 \ \|(\phi\phi)_{\underline{210}}\|^2 + g_3 \ \|(\phi\phi)_{\underline{1050}}\|^2 + \lambda \ \|\phi\|^4 \quad . \tag{2.7}$$

The IRR $\underline{1050}$ is quite complicated. We thus prefer to express the term $\|(\phi\phi)_{\underline{1050}}\|^2$ as a function of the representations $\underline{45}$, $\underline{54}$ and $\underline{210}$

$$\|(\phi\phi)_{\underline{1050}}\|^2 = -\frac{35}{6}\|(\phi\phi)_{\underline{45}}\|^2 - \frac{7}{3}\|(\phi\phi)_{\underline{54}}\|^2 + \frac{5}{4}\|(\phi\phi)_{\underline{210}}\|^2 + \frac{1}{10}\|\phi\|^4 \quad . \tag{2.8}$$

In other words, since the space of group invariants is a vector space, we can evaluate the components of $\|(\phi\phi)_{\underline{1050}}\|^2$ along the basis vectors. Thus, by inserting (2.8) in (2.7) we get the flat-space potential

$$\begin{aligned} V(\phi) = & \left(g_1 - \frac{35}{6}g_3\right) \ \|(\phi\phi)_{\underline{45}}\|^2 + \left(g_2 + \frac{5}{4}g_3\right) \ \|(\phi\phi)_{\underline{210}}\|^2 \\ & - \frac{7}{3}g_3 \ \|(\phi\phi)_{\underline{54}}\|^2 + \left(\frac{1}{10}g_3 + \lambda\right) \ \|\phi\|^4 \quad . \end{aligned} \tag{2.9}$$

To clarify the definitions of $(\phi\phi)_{\underline{45}}$, $(\phi\phi)_{\underline{210}}$ and $(\phi\phi)_{\underline{54}}$, we point out that from the symmetrized product of two IRR's $\underline{210}$ (ϕ_{abcd}) it is possible, using the Levi-Civita symbol $\epsilon_{i_0 \ldots i_9}$, to construct the IRR (hereafter we sum over repeated indices)

$$(\underline{45})_{ab} = C^{\underline{210}}_{cdef}\,{}^{\underline{210}}_{ghil}\,{}^{\underline{45}}_{ab}\ \phi_{cdef}\ \phi_{ghil} = \frac{1}{\sqrt{70}}\epsilon_{abcdefghil}\ \phi_{cdef}\ \phi_{ghil} \quad . \tag{2.10}$$

Analogously, using the $SO(10)$ invariance of the Levi-Civita symbol, the $\underline{210}$ representation can be denoted by 4, or, equivalently, 6 indices of the completely antisymmetric tensor

$$(\underline{210})_{abcd} = (\underline{210})_{efghil} = C^{\underline{210}}_{abmn}\,{}^{\underline{210}}_{mncd}\,{}^{\underline{210}}_{efghil}\ \phi_{abmn}\ \phi_{cdmn}$$

$$= \frac{1}{\sqrt{90}}\epsilon_{abcdefghil}\ \phi_{abmn}\ \phi_{cdmn} \tag{2.11}$$

and

$$(\underline{54})_{ab} = \frac{1}{\sqrt{112}}(\phi_{amno}\ \phi_{bmno}\ +\ \phi_{bmno}\ \phi_{amno}) \qquad a \neq b \quad . \tag{2.12}$$

If $a = b$ we have 9 more terms orthogonal to the trace, here omitted for the sake of brevity.

Starting from the general potential (2.9), a complete analysis of its absolute minima would require first of all, the computation of the above potential along every direction of possible residual symmetry and secondly the determination of the ranges for the parameters g_i corresponding to the different residual symmetries for the absolute minima. This is exactly what was done in the case of $SU(5)$ (Allen 1985, Buccella *et al* 1992, Esposito *et al* 1993) with the Higgs scalar field in the adjoint representation.

Unfortunately, the technical difficulties due to the complexity of the group $SO(10)$ with respect to the unitary ones and the size of the IRR used, make it impossible to extend the previous analysis to the present case. For this reason, at least at this stage,

we restrict our considerations to the study of the modifications, induced by one-loop and curvature effects (section 3), of the symmetry-breaking pattern, for choices of the parameters g_i corresponding to absolute minima of the potential at tree-level, invariant under the residual-symmetry group $SU(3) \otimes SU(2) \otimes U(1)$. These are the only ones relevant for particle physics in flat space, because they predict the correct low-energy-limit phenomenology.

The most general singlet ϕ_0 with respect to the group $SU(3) \otimes SU(2) \otimes U(1)$ contained in the $\underline{210}$ representation is

$$\phi_0 = \frac{z_1}{\sqrt{3}}\left(\phi_{1234} + \phi_{3456} + \phi_{5612}\right)$$

$$+ \frac{z_2}{\sqrt{6}}\left(\phi_{1278} + \phi_{3478} + \phi_{5678} + \phi_{1290} + \phi_{3490} + \phi_{5690}\right) + z_3\phi_{7890} \qquad (2.13)$$

where $\left(z_1^2 + z_2^2 + z_3^2\right) = 1$. Such a decomposition can be obtained by using the generators described in appendix A. Varying the z_i parameters in their ranges, we get the following residual-symmetry groups (see comments in section 1):

$$z_2 = 0 \rightarrow SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \qquad (2.14a)$$

$$z_2 = z_3 = 0 \rightarrow SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \times D \qquad (2.14b)$$

$$z_1 = z_2 = 0 \rightarrow SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \qquad (2.14c)$$

$$\frac{z_1}{\sqrt{3}} = \frac{z_2}{\sqrt{6}} = z_3 \rightarrow SU(5) \otimes U(1) \qquad (2.14d)$$

$$\text{otherwise} \rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_{T_{3R}} \otimes U(1)_{B-L} \qquad (2.14e)$$

where T_{3R} is the z-component of the $SU(2)_R$ group.

Substituting (2.13) in (2.9) we get in flat space-time

$$\widehat{V} \equiv V(\phi_0) = \left(\frac{\alpha}{8} f_\alpha + \frac{\gamma}{4} f_\gamma + \frac{\delta}{9} f_\delta + (\lambda - \delta)\right) \|\phi_0\|^4 \tag{2.15}$$

where

$$\alpha \equiv \frac{4}{945}\Big(-108 g_1 + 28 g_2 + 140 g_3 \Big) \tag{2.16}$$

$$\gamma \equiv \frac{8}{35} g_1 \tag{2.17}$$

$$\delta \equiv -\frac{1}{10} g_3 \tag{2.18}$$

$$f_\alpha \equiv \left(z_1^2 + z_2^2\right)^2 + z_2^2\left(2z_1 + \sqrt{3} z_3\right)^2 + \frac{3}{4} z_2^4 \tag{2.19}$$

$$f_\gamma \equiv \left(z_1 z_3 + \frac{z_2^2}{\sqrt{3}}\right)^2 + (z_1 z_2)^2 + f_\alpha \tag{2.20}$$

$$f_\delta \equiv 30\left(z_1 z_3 + \frac{z_2^2}{\sqrt{3}}\right)^2 + 30 z_1^2 z_2^2 + \left(2z_1^2 - \frac{z_2^2}{2} - 3z_3^2\right)^2$$

$$+ 5\left(z_1^2 + z_2^2\right)^2 + 5 z_2^2\left(2z_1 + \sqrt{3} z_3\right)^2 + \frac{15}{4} z_2^4 \quad . \tag{2.21}$$

Since in the following analysis δ is always negative and α may take negative values, the tree-level potential (2.15) is unbounded from below, unless we impose the restriction

$$\lambda \geq \frac{|\alpha|}{8}\left(f_\alpha\right)_{\max} + \frac{|\delta|}{9}\left(f_\delta\right)_{\max} \quad . \tag{2.22}$$

Note also that contributions proportional to a cubic term in the potential (denoted by β in Acampora *et al* (1993)) are set to zero, since we are assuming conformal invariance of

our model (Buccella *et al* 1992, Esposito *et al* 1993). In the models proposed in Acampora *et al* (1993) a complete study of the potential at tree-level for the case $z_2 = 0$ has been carried out, including the range of the bare-potential parameters such that the absolute minimum lies in the two-dimensional surface ($z_2 = 0$).

However, since we are interested in the modification of the bare potential produced at one-loop by de Sitter curvature, we can use only part of the inequalities appearing in Acampora *et al* (1993). More precisely, the parameters are bound to lie in regions where the mass spectrum is positive and the first derivatives of the effective potential vanish.

Thus, the allowed ranges for the parameters become

(1) $z_1 = 0 \;\;,\;\; z_3 = 1 \;\Rightarrow\; SO(6) \otimes SO(4) \sim SU(4) \otimes SU(2)_L \otimes SU(2)_R$

$$\gamma > 0 \;\; \delta < 0 \;\; \beta = 0 \;\; \alpha > -2\gamma$$

(2) $z_1^2 + z_3^2 = 1 \;\Rightarrow\; SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$

$$\alpha > 0 \;\; \beta = 0 \;\; -\frac{3}{5}\alpha < \gamma < -\frac{1}{2}\alpha$$

$$\frac{3\Big(9\alpha^2 + 9\alpha\gamma - 18\gamma^2 + 4\Big(3\alpha + 7\gamma\Big)\sqrt{-3\gamma(\alpha+\gamma)}\Big)}{320\Big(\gamma - \sqrt{-3\gamma(\alpha+\gamma)}\Big)} < \delta < \frac{3\gamma^2}{10(3\alpha+\gamma)} \;.$$

In the case $z_1 = 1, z_3 = 0$ one finds that it is impossible to get positive mass for both (6,2,2,-2/3) and (1,2,2,2). In fact this is a saddle point in the space representation. Indeed, since we are interested in the case when the intermediate symmetry group contains $SU(4)_{PS}$ for the reasons described in the introduction, we can restrict our analysis to case (1).

For our purposes we need to compute the mass matrix for the gauge bosons. As it is well known, this comes from the kinetic term for the Higgs field when we expand this scalar field around its vacuum expectation value ϕ_0

$$\left(D_\mu\phi_0, D^\mu\phi_0\right) = \mathcal{G}^2\left(T_{ab}\phi_0, T_{cd}\phi_0\right)A_\mu^{ab}A^{cd\,\mu} \tag{2.23}$$

where square brackets denote the scalar product in the 210-dimensional space and $\mathcal{G}$ is the gauge coupling constant of the $SO(10)$ group. Since the general form of ϕ_0 is given, we can evaluate the action of the 45 generators T_{ab} on it. In particular, by taking the decomposition of the adjoint representation $\underline{45}$ under the group $SU(3)\otimes SU(2)\otimes U(1)$ one finds

$$\begin{aligned}\underline{45} = &(8,1,0) \oplus (1,3,0) \oplus (1,1,1) \oplus (1,1,0) \oplus (1,1,-1)\\ &\oplus (1,1,0) \oplus (3,1,2/3) \oplus (\bar{3},1,-2/3) \oplus (3,2,5/6) \oplus (\bar{3},2,-5/6)\\ &\oplus (3,2,1/6) \oplus (\bar{3},2,-1/6) \quad .\end{aligned} \tag{2.24}$$

The readers not familiar with this group-theoretical calculation are not expected to repeat the lengthy derivation, but they only need to know that, using a standard notation in particle physics, (l,r,x) denotes the tensor which behaves as an l-dimensional representation under $SU(3)$, r-dimensional under $SU(2)$ and takes a value$=x$ when acted upon by the $U(1)$ generator. By virtue of the Wigner-Eckart theorem (Cornwell 1984a) the scalar product on the right-hand side of (2.23) is given by the product of $\|\phi_0\|^2$ with a Clebsch-Gordan coefficient. Thus, defining $m^2 \equiv \mathcal{G}^2\|\phi_0\|^2$, and evaluating the Clebsch-Gordan coefficients, one finds that the non-vanishing eigenvalues $m^2_{(l,r,x)}$ are $m^2_{(1,1,1)} =$

$m^2_{(1,1,-1)} = m^2\left[\frac{z_2^2}{2}\right]$ with degeneracy 1, $m^2_{(3,1,2/3)} = m^2\left[\frac{2}{3}\left(z_1^2+z_2^2\right)\right]$ with degeneracy 3, $m^2_{(3,2,1/6)} = m^2\left[\frac{2}{3}z_1^2+\frac{z_2^2}{2}+z_3^2-\sqrt{\frac{2}{3}}\ z_2 z_3\right]$ with degeneracy 6, and $m^2_{(3,2,-5/6)} = m^2\left[\frac{2}{3}z_1^2+\frac{z_2^2}{2}+z_3^2+\frac{2\sqrt{2}}{3}z_1 z_2+\sqrt{\frac{2}{3}}\ z_2 z_3\right]$ with degeneracy 6 as well. Note that this is the mass matrix relevant for the $SU(3)\otimes SU(2)\otimes U(1)$ symmetry-breaking direction. This choice is motivated by low-energy phenomenology of the particle-physics standard model, and all groups containing $SU(3)\otimes SU(2)\otimes U(1)$ lead to the same kind of mass matrix (of course, the z_i parameters take different values for different groups). Hence we only rely on the ϕ_0 singlet appearing in (2.13).

3. One-loop effective potential in de Sitter space

The (formal) quantization of gauge theories via Feynman path integrals leads to quantum amplitudes involving not only the Wick-rotated action, but also gauge-averaging terms and corresponding ghost fields. For example, in the case of vacuum Maxwell theory in four-dimensions, one deals with amplitudes of the form

$$Z[g] \equiv \int \widetilde{\mu}_1[A]\ \delta\Big[\Phi(A)\Big]\ \det\left[\frac{\delta\Phi(A)}{\delta\epsilon}\right]\ \exp\left[-\int_{M_4}\frac{1}{4}F_{\mu\nu}F^{\mu\nu}\ \sqrt{\det g}\ d^4x\right] \quad . \tag{3.1}$$

With our notation, $A \equiv A_\mu dx^\mu$ is the gauge-connection one-form, $\widetilde{\mu}_1[A]$ a suitable measure on the space of such connections, $\Phi(A)$ an arbitrary functional of the gauge-connection,

$F_{\mu\nu} \equiv \nabla_\mu A_\nu - \nabla_\nu A_\mu$ the electromagnetic-field tensor corresponding to the Maxwell-curvature two-form $F \equiv dA$. Moreover, g is the Riemannian (i.e. positive-definite) background four-metric, and ∇_μ is the covariant derivative obtained from the affine connection ∇ existing on the background four-geometry $\Big(M_4, g\Big)$. The amplitudes (3.1) are more conveniently re-expressed as

$$Z[g] = \int \widetilde{\mu}_1[A]\, \widetilde{\mu}_2[c, c^*] \exp\Big(- \widetilde{I}_E \Big) \tag{3.2}$$

where

$$\widetilde{I}_E = I_{gh} + \int_{M_4} \left[\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\widetilde{\alpha}}{2} \Big[\Phi(A) \Big]^2 \right] \sqrt{\det g}\, d^4 x \quad . \tag{3.3}$$

In (3.2)-(3.3), $\widetilde{\alpha}$ is a positive real number, I_{gh} the ghost action corresponding to the form chosen for $\Phi(A)$ (Itzykson and Zuber 1985), and $\widetilde{\mu}_2[c, c^*]$ is a suitable measure on the space of ghost fields. The term $\frac{\widetilde{\alpha}}{2}\Big[\Phi(A)\Big]^2$ in the total Euclidean action is usually referred to in the literature as gauge-fixing term, since its form is clearly suggested by what would be set to zero at the classical level. However, since in the non-Abelian case no gauge choice exists at the classical level for the S^4 topologies we are interested in (Jungman 1992, Esposito *et al* 1993), we use instead the terminology *gauge-averaging*. By this we mean a contribution to the action which enables one to extract the volume of the gauge group and to obtain one-loop quantum amplitudes manifestly independent of the choice of $\Phi(A)$. The effects of the longitudinal and transverse parts of the gauge-potential are then decoupled, and no cross-terms occur if $\Phi(A)$ is suitably chosen (Allen 1983, Allen 1985).

Within the framework of inflationary cosmology, the quantization of non-Abelian gauge fields has been recently studied in the case of $SU(5)$ GUT theories (Allen 1985, Buccella *et al* 1992, Esposito *et al* 1993). In this case one starts from a bare, Euclidean-time Lagrangian

$$L = \frac{1}{4}\mathrm{Tr}\Big(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}\Big) + \frac{1}{2}\mathrm{Tr}\Big(D_\mu\varphi\Big)\Big(D^\mu\varphi\Big) + V_0(\varphi) \tag{3.4}$$

where both the gauge-potential A^μ and the Higgs scalar field φ are in the adjoint representation of $SU(5)$. Note that boldface characters are used to denote the curvature 2-form $\mathbf{F}$ in the non-Abelian case, to avoid confusion with the curvature 2-form F in the Abelian case. The background 4-geometry is de Sitter space with S^4 topology. The background-field method is then used, jointly with the gauge-averaging term first proposed by 't Hooft

$$L_g = \frac{\widetilde{\alpha}}{2}\mathrm{Tr}\Big(\nabla_\mu A^\mu - i\mathcal{G}\widetilde{\alpha}^{-1}[\varphi_0,\varphi]\Big)^2 \quad . \tag{3.5}$$

This particular choice is necessary to eliminate in the total action cross-terms involving $\mathrm{Tr}\Big(\nabla_\mu A^\mu\Big)$ and the *commutator* $[\varphi_0,\varphi]$, where φ_0 is a constant background Higgs field. After sending $\widetilde{\alpha} \to \infty$ (Landau condition), and denoting by $\Omega = \frac{8}{3}\pi^2 a^4$ the volume of a 4-sphere of radius a, the resulting one-loop effective potential is (Allen 1985)

$$\begin{aligned} V(\varphi_0) = V_0(\varphi_0) &+ \frac{1}{2\Omega}\log\ \det\mu^{-2}\Big[\delta_{ab}\Big(-g_{\mu\nu}\Box + R_{\mu\nu}\Big) + g_{\mu\nu}M^2_{ab}(\varphi_0)\Big] \\ &+ \frac{1}{2\Omega}\log\ \det\mu^{-2}\Big[-\delta_{ab}\Box + \frac{\partial^2 V_0}{\partial\varphi_a\partial\varphi_b}\Big|_{\varphi_0}\Big] \end{aligned} \tag{3.6}$$

since the ghost determinant cancels the longitudinal one.

One-loop effective potential for $SO(10)$ GUT theories in de Sitter space

To understand how to generalize (3.6) to $SO(10)$ GUT theories, we have to bear in mind only the first line of (3.6), since, by virtue of the Coleman-Weinberg mechanism, only gauge-field loop diagrams contribute to the symmetry-breaking pattern in the early universe (Allen 1983, Allen 1985, Buccella *et al* 1992). Denoting by ψ the logarithmic derivative of the Γ function, and defining the functions $\mathcal{A}$ and P by means of

$$\mathcal{A}(z) \equiv \frac{z^2}{4} + \frac{z}{3} - \int_2^{\frac{3}{2}+\sqrt{\frac{1}{4}-z}} y\Big(y - \frac{3}{2}\Big)(y-3)\psi(y)\, dy$$

$$- \int_1^{\frac{3}{2}-\sqrt{\frac{1}{4}-z}} y\Big(y - \frac{3}{2}\Big)(y-3)\psi(y)\, dy \tag{3.7}$$

$$P(z) \equiv \frac{z^2}{4} + z \tag{3.8}$$

one thus finds for the $SU(5)$ model (Allen 1985)

$$V(\varphi_0) = V_0(\varphi_0) - \frac{1}{2\Omega}\sum_{l=1}^{24}\Big[\mathcal{A}(a^2 m_l^2) + P(a^2 m_l^2)\log(\mu^2 a^2)\Big] \tag{3.9}$$

where the m_l^2 are the 24 eigenvalues of the mass matrix M_{ab}^2.

In the case of the $SO(10)$ GUT model, the same method used for $SU(5)$ in Allen (1985) shows that the one-loop effective potential V takes the form (see appendix B)

$$V = \widehat{V}_c - \frac{1}{2\Omega}\sum_{i=1}^{45}\Big[\mathcal{A}(a^2 m_i^2) + P(a^2 m_i^2)\log(\mu^2 a^2)\Big] \tag{3.10}$$

where (cf (2.15))

$$\widehat{V}_c = \Big(\frac{\alpha}{8} f_\alpha + \frac{\gamma}{4} f_\gamma + \frac{\delta}{9} f_\delta + (\lambda - \delta)\Big)\|\phi_0\|^4 + \frac{R}{12}\|\phi_0\|^2 \quad . \tag{3.11}$$

The corresponding one-loop effective potential in (3.10) is obtained by inserting the following formulae:

$$\sum_{i=1}^{45} \mathcal{A}(a^2 m_i^2) = 6\mathcal{A}\left[a^2 m^2 \left(\frac{2}{3} z_1^2 + \frac{z_2^2}{2} + z_3^2 + \frac{2\sqrt{2}}{3} z_1 z_2 + \sqrt{\frac{2}{3}}\, z_2 z_3\right)\right]$$

$$+ 6\mathcal{A}\left[a^2 m^2 \left(\frac{2}{3} z_1^2 + \frac{z_2^2}{2} + z_3^2 - \sqrt{\frac{2}{3}}\, z_2 z_3\right)\right]$$

$$+ \mathcal{A}\left[a^2 m^2 \frac{z_2^2}{2}\right] + 3\mathcal{A}\left[a^2 m^2 \frac{2}{3}\left(z_1^2 + z_2^2\right)\right] \tag{3.12}$$

$$\sum_{i=1}^{45} P(a^2 m_i^2) = a^2 m^2 \left(10 z_1^2 + 4\sqrt{2}\, z_1 z_2 + \frac{17}{2} z_2^2 + 12 z_3^2\right)$$

$$+ a^4 m^4 \left(\frac{5}{3} z_1^4 + \frac{4}{3}\sqrt{2}\, z_1^3 z_2 + 4 z_1^2 z_2^2\right.$$

$$+ \sqrt{2}\, z_1 z_2^3 + \frac{55}{48} z_2^4 + \frac{4}{\sqrt{3}} z_1 z_2^2 z_3 + 4 z_1^2 z_3^2$$

$$\left. + 2\sqrt{2} z_1 z_2 z_3^2 + 5 z_2^2 z_3^2 + 3 z_3^4\right) \quad . \tag{3.13}$$

4. Flat-space limit

The one-loop effective potential (3.10)-(3.13) can hardly be used for an analytic or numerical study of the absolute minima, since it involves a large number of complicated contributions. We therefore begin by studying its flat-space limit, i.e. its asymptotic behaviour when the 4-sphere radius a tends to ∞. The corresponding asymptotic form of

$\mathcal{A}(z)$ is (Allen 1985)

$$\mathcal{A}(z) \sim -\left(\frac{z^2}{4} + z + \frac{19}{30}\right)\log(z) + \frac{3}{8}z^2 + z + \text{const.} + \mathrm{O}(z^{-1}) \quad . \tag{4.1}$$

For the purpose of numerical analysis at $a \to \infty$, the expansion (4.1) can be further approximated as

$$\mathcal{A}(z) \sim \frac{z^2}{8}\Big(3 - \log(z^2)\Big) \quad . \tag{4.2}$$

Thus, defining (cf end of section 2)

$$h_1 \equiv \frac{2}{3}z_1^2 + \frac{z_2^2}{2} + z_3^2 + \frac{2\sqrt{2}}{3}z_1 z_2 + \sqrt{\frac{2}{3}}\, z_2 z_3 \tag{4.3}$$

$$h_2 \equiv \frac{2}{3}z_1^2 + \frac{z_2^2}{2} + z_3^2 - \sqrt{\frac{2}{3}}\, z_2 z_3 \tag{4.4}$$

$$h_3 \equiv \frac{z_2^2}{2} \tag{4.5}$$

$$h_4 \equiv \frac{2}{3}\Big(z_1^2 + z_2^2\Big) \tag{4.6}$$

$$h_5^2 \equiv \frac{3}{2}h_1^2 + \frac{3}{2}h_2^2 + \frac{h_3^2}{4} + \frac{3}{4}h_4^2 \tag{4.7}$$

$$y \equiv \frac{m}{\mu} \tag{4.8}$$

equations (3.10)-(3.13) and (4.2) lead to

$$\frac{V}{\mu^4} \sim \frac{\widehat{V}}{\mu^4} - \frac{3}{8\pi^2}y^4\left[\frac{3}{4}h_1^2\Big(3 - \log(h_1^2)\Big) + \frac{3}{4}h_2^2\Big(3 - \log(h_2^2)\Big)\right.$$
$$\left. + \frac{h_3^2}{8}\Big(3 - \log(h_3^2)\Big) + \frac{3}{8}h_4^2\Big(3 - \log(h_4^2)\Big) - h_5^2\log\Big(y^2\Big)\right] \quad . \tag{4.9}$$

The problem now arises to find the absolute minima of the potential (4.9) by numerical methods. Since z_1, z_2, z_3 lie on a unit 2-sphere, they can be expressed as $z_1 = \sin(\theta)\cos(\varphi), z_2 = \sin(\theta)\sin(\varphi), z_3 = \cos(\theta)$. For given values of the parameters $\alpha, \gamma, \lambda, \delta$ appearing in (3.11), we have thus to minimize with respect to θ, φ, y. For this purpose, we point out that $y_{\min}$ should be ≤ 1, since it is the ratio of the gauge-boson mass to the cut-off value. Hence one gets a further restriction on λ which, combined with the inequality (2.22), yields the sufficient condition

$$\lambda \geq \lambda_0 + \frac{|\alpha|}{8}\Big(f_\alpha\Big)_{\max} + \frac{|\delta|}{9}\Big(f_\delta\Big)_{\max} \quad . \tag{4.10}$$

With our notation, λ_0 is given by

$$\lambda_0 \equiv \frac{3\mathcal{G}^4}{8\pi^2}\Bigg[\frac{3}{4}h_1^2\Big(3-\log(h_1^2)\Big) + \frac{3}{4}h_2^2\Big(3-\log(h_2^2)\Big)$$

$$+ \frac{h_3^2}{8}\Big(3-\log(h_3^2)\Big) + \frac{3}{8}h_4^2\Big(3-\log(h_4^2)\Big) - \frac{1}{2}h_5^2\log\Big(y^2\Big)\Bigg]_{\min(\theta,\varphi)} - \widetilde{f}_{\min} \tag{4.11}$$

where $\widetilde{f}$ is the function such that $\Big(\widetilde{f}+\lambda\Big)y^4 = \widehat{V}$. The corresponding numerical analysis, carried out by using the MINUIT minimization program available in the CERN libraries, shows that the absolute minimum always lies in the $\theta = 0$ direction. This is the $SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$ symmetry-breaking direction (see (2.14c)). Thus, as far as the absolute-minimum direction is concerned, the flat-space limit of the one-loop calculation about a de Sitter background does not change the results found in Buccella *et al* (1986), where the tree-level potential in flat space-time was studied. Remarkably, since the value of y leading to the absolute minimum of V in the presence of symmetry breaking has been found to be

$y_{\min} \in [0.4, 0.8]$ in the regions where the inequality (4.10) is satisfied, one can evaluate μ from (4.8) as

$$\mu = \frac{M_X}{y_{\min}} \quad . \tag{4.12}$$

This formula for μ can be used to derive the values of the 4-sphere radius corresponding to given values of the dimensionless parameter μa (see below).

To complete this section, we find it helpful for the reader to evaluate the behaviour of the flat-space-limit one-loop effective potential as $\theta \to 0$, since $\theta = 0$ yields the absolute-minimum direction as we just said. The analytic calculation shows that the potential V in (4.9) obeys the relation

$$\lim_{\theta \to 0} V(\lambda, y, \theta) \equiv V_{\lim} = \frac{y^4}{\pi^2}\left[\frac{625}{4}\lambda - \frac{27}{16} + \frac{9}{4}\log(y)\right] \quad . \tag{4.13}$$

The corresponding behaviours of $V_{\lim}$ for various values of λ ($\lambda = 0.03, 0.02, 0.015, 0.012$) are plotted in figure 1, when $y \in [0, 1]$.

5. Numerical evaluation of the absolute minima

As one can see from equations (3.10)-(3.13), the one-loop effective potential for our cosmological model takes a complicated form, and it is not clear whether curvature can modify the results of section 4, once the same values for $\alpha, \gamma, \delta, \lambda$ have been chosen. The corresponding absolute minima have been evaluated using again the MINUIT minimization

program and choosing different values for the dimensionless parameter μa, since it is convenient to work with the dimensionless form of the one-loop potential, obtained dividing (3.10)-(3.13) by μ^4. Of course, the parameters in the effective potential are $\alpha, \gamma, \delta, \lambda, \mu a$, whereas the arguments are y, θ, φ.

Interestingly, if μa is ≤ 1, the term $\frac{R}{12}\|\phi\|^2$ in the potential (3.10)-(3.13) dominates over all other contributions, and hence does not lead to any symmetry breaking. Thus, only intermediate values of μa are relevant for the symmetry-breaking pattern. In this case the absolute minima are still found to be $SO(6) \otimes SO(4)$-invariant (for them $\theta = 0$), providing the inequality (4.10) is satisfied. In figures 2-3, obtained setting $\mu a = 30, 300$ respectively, the one-loop effective potential is plotted as a function of y when $\alpha = \gamma = \delta = 0$. Note that these particular values are chosen since the flat-space effective potential (4.13) is independent of α, γ, δ. Hence α, γ, δ can be set to zero for simplicity when curvature vanishes, whereas in the presence of curvature they are set to zero to compare the flat-space analysis with the de Sitter case.

A naturally occurring question is what can be learned from the comparison of figure 1 with figures 2-3. Indeed, the values of the independent variable y for which the absolute minima are attained are modified in the presence of curvature. The smaller μa (stronger curvature), the more substantial the change of the shape of our plots. In particular, figure 2 shows that for $\mu a = 30$ and $\alpha = \gamma = \delta = 0$ no symmetry breaking occurs even though for other choices of values for α, γ, δ, non-trivial absolute minima are present. By contrast, from figure 3, corresponding to $\mu a = 300$, the effects of curvature on the absolute minima of the potential can be easily seen (cf figure 1).

Remarkably, our numerical investigation shows that, providing the mass matrix is positive-definite, the potential is bounded from below, and the gauge-boson mass remains smaller than the cut-off value, the absolute-minimum direction remains $SO(6) \otimes SO(4)$-invariant in flat or de Sitter space, if the tree-level potential has this invariance property. To help the reader, table III shows, for the same values of the parameters used in figure 3, the values taken by $y_{\min}$ and the corresponding values of the dimensionless one-loop effective potential. The θ and φ entries are omitted since $\theta = 0$ and φ is undetermined in the presence of spontaneous symmetry breaking along the $SO(6) \otimes SO(4)$ direction.

6. Results and concluding remarks

The main results of our investigation are as follows.

First, the one-loop effective potential of $SO(10)$ GUT theories in de Sitter space has been obtained for the first time. This analytic result represents the continuation of the program initiated in Allen (1985), where the tools necessary for any non-Abelian gauge theory in de Sitter space were described in detail. Note that, while (3.10) holds for any irreducible representation of $SO(10)$, (3.11) relies on the $\underline{210}$ representation, and (3.12)-(3.13) lead to a *particular* form of such potential, once $SU(3) \otimes SU(2) \otimes U(1)$ invariance for the mass matrix is required to agree with electroweak symmetry.

Second, the flat-space limit of the corresponding Coleman-Weinberg effective potential has been evaluated for the $\underline{210}$ representation.

Third, the numerical analysis of absolute minima has been carried out in the case of the mass matrix relevant for low-energy-limit phenomenology. Interestingly, de Sitter curvature does not affect the flat-space symmetry-breaking pattern, leading only to the $SO(6) \otimes SO(4)$ symmetry-breaking direction.

A naturally occurring question is whether the analytic study of absolute minima can be performed, to check the results of our numerical investigation. In principle, this research appears possible, although it goes beyond the author's computational skills, due to the many parameters appearing in the $SO(10)$ effective potential. For the time being, we should emphasize that our results, although obtained after a time-consuming numerical analysis, remain preliminary.

It has been our task to work under the restrictive conditions summarized at the end of section 5, while other forms of the mass matrix remain unknown in the literature. Thus, a complete mathematical treatment similar to what was done in Allen (1985) for $SU(5)$ theories is lacking, and appears to be a topic for further research. Moreover, since the Higgs field (if it exists) is actually varying in time, it appears necessary to evaluate the one-loop effective potential of non-Abelian gauge theories in closed FRW cosmologies, de Sitter being just a particular case. This more complicated analysis would supersede the approximations made in Allen (1983) and Esposito *et al* (1993) to study the evolution of the early universe.

Appendix A

The readers not familiar with group-theoretical methods in particle physics may find it useful to know that the generators used to obtain the decomposition (2.13) of the ϕ_0 singlet are such that (see (2.1)-(2.3))

$$T_{13} + T_{24} = \lambda_2 \tag{A.1}$$

$$T_{12} - T_{34} = \lambda_3 \tag{A.2}$$

$$T_{12} + T_{34} - 2T_{56} = \sqrt{3}\,\lambda_8 \tag{A.3}$$

$$T_{14} - T_{23} = \lambda_1 \tag{A.4}$$

$$T_{15} + T_{26} = \lambda_5 \tag{A.5}$$

$$T_{16} - T_{25} = \lambda_4 \tag{A.6}$$

$$T_{35} + T_{46} = \lambda_7 \tag{A.7}$$

$$T_{36} - T_{45} = \lambda_6 \tag{A.8}$$

where λ_i are the Gell-Mann matrices for the $SU(3)_C$ group (Cornwell 1984b). Moreover, for the *right* part of the $SU(2)$ group contained in $SO(4)$, one finds

$$\frac{1}{2}\Big(T_{79} - T_{80}\Big) \equiv T_R^1 \tag{A.9}$$

$$-\frac{1}{2}\Big(T_{70} + T_{89}\Big) \equiv T_R^2 \tag{A.10}$$

$$-\frac{1}{2}\Big(T_{78} + T_{90}\Big) \equiv T_R^3 \tag{A.11}$$

whereas for the *left* part of the $SU(2)$ group in $SO(4)$ one obtains

$$\frac{1}{2}\Big(T_{79}+T_{80}\Big) \equiv T_L^1 \tag{A.12}$$

$$-\frac{1}{2}\Big(T_{70}-T_{89}\Big) \equiv T_L^2 \tag{A.13}$$

$$-\frac{1}{2}\Big(T_{78}-T_{90}\Big) \equiv T_L^3 \quad . \tag{A.14}$$

Appendix B

To obtain the one-loop effective potential (3.10)-(3.11) one starts from the bare, Euclidean-time Lagrangian (cf (3.4))

$$L = \frac{1}{4}\mathrm{Tr}\Big(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}\Big) + \frac{1}{2}\mathrm{Tr}\Big(D_\mu\phi\Big)\Big(D^\mu\phi\Big) + V_0(\phi) \tag{B.1}$$

where $D_\mu \equiv \partial_\mu - i\mathcal{G}A^{ab}_\mu\, T^{ab}\ \forall a,b = 0,...,9$. According to the background-field method, one expands the field ϕ as

$$\phi = \phi_0 + \widetilde{\phi} \tag{B.2}$$

where ϕ_0 is the background value, and $\widetilde{\phi}$ is a perturbation. The 4-metric g is also expanded as in Allen (1983, 1985). The resulting one-loop form of L, i.e. the Lagrangian quadratic in the perturbations, is

$$L^{(1)} = \frac{1}{4}\mathrm{Tr}\Big(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}\Big) + \frac{1}{2}\mathrm{Tr}\Big(D_\mu\widetilde{\phi}\Big)\Big(D^\mu\widetilde{\phi}\Big)$$

$$+\frac{i\mathcal{G}}{2}\left[\left(\nabla_\mu A^\mu\right)^{lm}\right]\left[<\tilde{\phi}\mid T^{lm}\mid\phi_0>-<\phi_0\mid T^{lm}\mid\tilde{\phi}>\right]$$

$$+\frac{\mathcal{G}^2}{2}A_\mu^{lm}<\phi_0\mid T^{lm}T^{pq}\mid\phi_0>A^{pq\,\mu}+V_0+\frac{1}{2}\left(\tilde{\phi}\right)^2\frac{\partial^2 V_0}{\partial\phi^2}\bigg|_{\phi=\phi_0}\quad . \qquad (B.3)$$

Moreover, the gauge-averaging term we are looking for is (cf Allen 1985)

$$L_{\text{gauge}}=\frac{\tilde{\alpha}}{2}\mathrm{Tr}\left[\left(\nabla_\mu A^\mu\right)^{lm}+\tilde{\beta}\left(<\tilde{\phi}\mid T^{lm}\mid\phi_0>-<\phi_0\mid T^{lm}\mid\tilde{\phi}>\right)\right]^2\quad . \qquad (B.4)$$

By virtue of equations (B.3)-(B.4), cross-terms disappear in $L^{(1)}+L_{\text{gauge}}$ if and only if $\tilde{\beta}=-\frac{i\mathcal{G}}{2}\tilde{\alpha}^{-1}$. This leads to

$$L^{(1)}+L_{\text{gauge}}=\frac{1}{4}\mathrm{Tr}\left(\mathbf{F}_{\mu\nu}\mathbf{F}^{\mu\nu}\right)+\frac{1}{2}\mathrm{Tr}\left(D_\mu\tilde{\phi}\right)\left(D^\mu\tilde{\phi}\right)$$

$$+\frac{\tilde{\alpha}}{2}\mathrm{Tr}\left[\left(\nabla_\mu A^\mu\right)^{lm}\right]^2-\frac{\mathcal{G}^2}{8\tilde{\alpha}}\mathrm{Tr}\left[<\tilde{\phi}\mid T^{lm}\mid\phi_0>-<\phi_0\mid T^{lm}\mid\tilde{\phi}>\right]^2$$

$$+\frac{\mathcal{G}^2}{2}A_\mu^{lm}<\phi_0\mid T^{lm}T^{pq}\mid\phi_0>A^{pq\,\mu}+V_0+\frac{1}{2}\left(\tilde{\phi}\right)^2\frac{\partial^2 V_0}{\partial\phi^2}\bigg|_{\phi=\phi_0}\quad . \qquad (B.5)$$

By splitting the gauge potential into transverse and longitudinal part on the S^4 background, and following Allen (1985), one obtains an equation similar to (3.6), where the mass matrix has 45 eigenvalues rather than 24. Hence (3.10) is proved.

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Figure captions:

Figure 1. Flat-space limit of the dimensionless one-loop effective potential at $\theta = 0$ (4.13) versus y (4.8) is here shown. The curves correspond to the λ-values 0.03, 0.02, 0.015, 0.012 respectively.

Figure 2. The dimensionless form of the one-loop effective potential in de Sitter space at $\theta = 0$, and $\mu a = 30$ versus y is here shown. The curves correspond to $\alpha, \gamma, \delta = 0$ and the same values of λ of figure 1.

Figure 3. The one-loop potential of figure 2 is evaluated for $\mu a = 300$.

Table captions:

Table I. The intermediate symmetries, the Higgs directions and the IRR's of $SO(10)$ used for the Higgs scalar fields are here reported for the most physically relevant $SO(10)$ GUT models (Acampora *et al* 1993). With our notation, ω_{ab} denotes the 54-dimensional irreducible representation of $SO(10)$.

Table II. For the same models of table I, the masses of gauge bosons are shown (Acampora *et al* 1993).

Table III. For the same values of the parameters used in figure 3, the values taken by $y_{\min}$ and by the dimensionless one-loop effective potential are shown.

TABLE I

G'	Higgs direction	Representation
$SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \times D$	$2\,(\omega_{11} + \ldots + \omega_{66}) - 3\,(\omega_{77} + \ldots + \omega_{00})$	$\underline{54}$
$SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$	$\Phi_T = \Phi_{7890}$	$\underline{210}$
$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \times D$	$\Phi_L = \frac{(\Phi_{1234}+\Phi_{1256}+\Phi_{3456})}{\sqrt{3}}$	$\underline{210}$
$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$	$\Phi(\theta) = \cos(\theta)\ \Phi_L + \sin(\theta)\ \Phi_T$	$\underline{210}$

TABLE II

G'	$M_X/10^{15}\ GeV$	$M_R/10^{11}\ GeV$
$SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R \times D$	$0.55 \cdot 1.64^{0\pm1}$	$343.70 \cdot 1.25^{0\pm1}$
$SU(4)_{PS} \otimes SU(2)_L \otimes SU(2)_R$	$5.30 \cdot 1.87^{0\pm1}$	$1.45 \cdot 2.09^{0\pm1}$
$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \times D$	$1.64 \cdot 2.83^{0\pm1}$	$0.32 \cdot 1.81^{0\pm1}$
$SU(3)_C \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$	$11.26 \cdot 2.06^{0\pm1}$	$0.03 \cdot 3.34^{0\pm1}$

TABLE III

λ	y_{min}	$V(y_{min})/\mu^4$
0.030	0	no symmetry breaking
0.020	0	no symmetry breaking
0.015	0.22	$-0.6 \cdot 10^{-4}$
0.012	0.32	$-0.26 \cdot 10^{-3}$

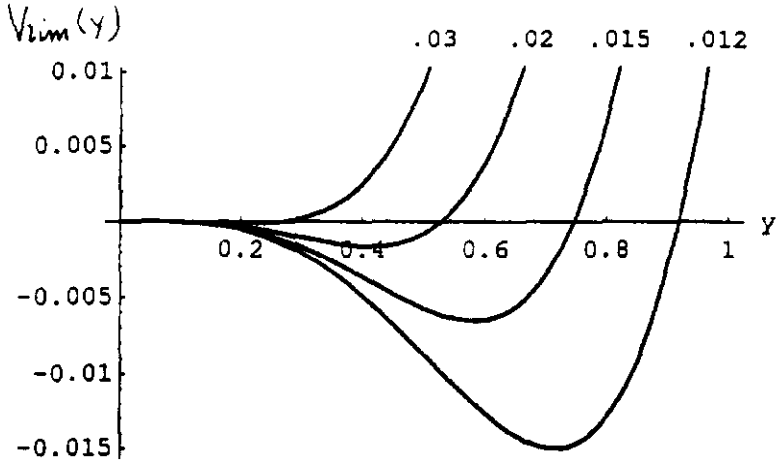

Figure 1.

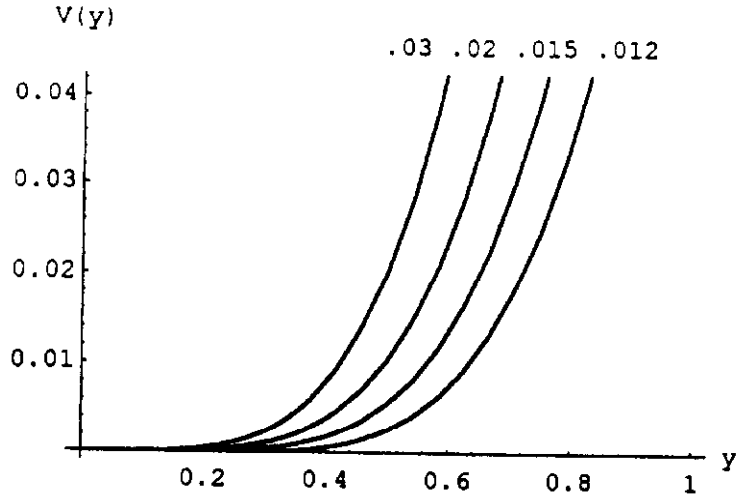

Figure 2.

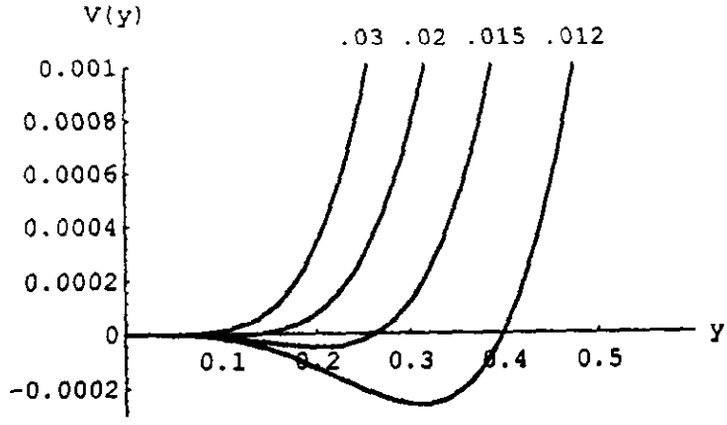
V(y)
.03 .02 .015 .012
0.001
0.0008
0.0006
0.0004
0.0002
0
-0.0002
0.1
0.2
0.3
0.4
0.5
y

Figure 3.