Fermi National Accelerator Laboratory

FERMILAB–PUB–93/316–T

OSU Preprint 282

October 1993

New Approach for the Construction of Fermion Mass Matrices in SO(10) SUSY GUTS

Carl H. ALBRIGHT

Department of Physics, Northern Illinois University, DeKalb, Illinois 60115*

and

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510†

and

Satyanarayan NANDI

Department of Physics, Oklahoma State University, Stillwater, Oklahoma 74078‡

Abstract

A new procedure is developed which enables one to start from the quark and lepton mass and mixing data at the low scale and construct mass matrices which exhibit simple SO(10) structure at the SUSY GUT scale. This approach is applied to the present data involving quark and charged lepton masses, the CKM mixing matrix and the MSW solar neutrino and atmospheric neutrino depletion effects. In terms of just 12 model parameters suggested by the procedure for the 5 mass matrices, we can reproduce 15 masses and 8 mixing parameters remarkably consistent with the input starting values.

PACS numbers: 12.15.Ff, 11.30Ly

*Permanent address

†Electronic address: ALBRIGHT@FNALV

‡Electronic address: PHYSSNA@OSUCC

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

The apparent success[1] of the minimal supersymmetric standard model to predict unification of the gauge couplings at a scale of 10^{16} GeV has also spurred renewed interest in supersymmetric grand unified models of quark and lepton mass matrices based on SO(10). Various model constructions have been proposed[2] and explored in detail[3] which exhibit a maximum number of texture zeros (zeros in the upper triangular parts of the up and down quark mass matrices), since they may arise naturally from discrete symmetries present at the SUSY GUT scale. Other models have been devised[4] based on a minimal number of SO(10) Higgs representations, such as one **10** and one **126**. Still others[5] propose that additional SO(10) representations be present due to higher-dimensional operators in the Yukawa Lagrangian.

In this Letter, we propose yet another approach which involves the construction of mass matrices from all the known or presumed-known low-energy data. Our procedure has the desired feature that as more of the low-energy mass and mixing data becomes better pinned down, the construction of the mass matrices can be repeated and the symmetry-breaking character of the grand unified theory more clearly illuminated. For purposes of illustration, we apply the general method to masses and mixings consistent with the observed MSW[6] solar neutrino and atmospheric neutrino depletion effects.[7-8] The model constructed has a number of interesting features and is able to reproduce 15 mass and 8 mixing parameters with just 12 model input parameters.

The new approach proposed for the construction of quark and lepton mass matrices consists of the following steps:

- Start from the known and/or presumed-known quark and lepton masses, m_q's, m_ℓ's and m_ν's; and quark and lepton mixing matrices, V_{CKM} and V_{LEPT}, at the low scales.
- Evolve the masses and mixing matrices to the SUSY GUT scale using the appropriate

renormalization group equations (RGEs) for the minimal supersymmetric standard model (MSSM).

- Construct complex symmetric M^U, M^D, M^E, and $M^{N_{eff}}$ matrices for the up and down quarks, charged leptons and light neutrinos using a modified procedure of Kusenko[9] described later. Two parameters x_q and x_ℓ allow one to adjust the diagonal- off-diagonal nature of the quark and lepton mass matrices.

- Vary x_q and x_ℓ systematically over their support regions while searching for as many pure **10** or pure **126** $SO(10)$ contributions to the matrix elements as possible.

- For the "best" choice of x_q and x_ℓ, construct a simple model of the mass matrices with as many texture zeros as possible.

- Evolve the mass eigenvalues and mixing matrices determined from the model at the SUSY GUT scale to the low scale and compare the results with the starting input data.

In order to illustrate this procedure with a concrete example, we shall apply it to the reasonably well-known quark data[10] and assume that the lepton data is best described by the non-adiabatic MSW resonant oscillation depletion[6-7] of the solar electron-neutrino flux and by the depletion of the atmospheric muon-neutrino flux[8] through oscillation into tau-neutrinos. To our knowledge, no SO(10) seesaw model has been constructed which explains both observations with these interpretations.

With $m_t^{phys} \sim 160$ GeV, we adopt as starting input the following quark masses[10,11]

$$\begin{aligned} m_u(1\text{GeV}) &= 5.1 \text{ MeV}, & m_d(1\text{GeV}) &= 8.9 \text{ MeV} \\ m_c(m_c) &= 1.27 \text{ GeV}, & m_s(1\text{GeV}) &= 175 \text{ MeV} \\ m_t(m_t) &= 150 \text{ GeV}, & m_b(m_b) &\simeq 4.25 \text{ GeV} \end{aligned} \tag{1a}$$

which are evaluated at the 1 GeV scale for the light quarks and at the running scale for the heavy quarks, together with the CKM mixing matrix[10] at the weak scale

$$V_{CKM} = \begin{pmatrix} 0.9753 & 0.2210 & (-0.283 - 0.126i) \times 10^{-2} \\ -0.2206 & 0.9744 & 0.0430 \\ 0.0112 - 0.0012i & -0.0412 - 0.0003i & 0.9991 \end{pmatrix} \qquad (1b)$$

For the leptons, we use the central values in the non-adiabatic MSW solar neutrino conversion region:[7] $\delta m^2_{12} \sim 5 \times 10^{-6}$ eV^2, $\sin^2 2\theta_{12} \sim 8 \times 10^{-3}$; as well as the central values singled out in the muon-neutrino atmospheric depletion region:[8] $\delta m^2_{23} \sim 2 \times 10^{-2}$ eV^2, $\sin^2 2\theta_{23} \sim 0.5$. Here we are assuming the "conventional" interpretation that solar electron-neutrinos undergo resonant conversion into muon-neutrinos in the sun, while muon-neutrinos oscillate into tau-neutrinos in traveling through the atmosphere. We then take for the lepton input

$$\begin{aligned} m_{\nu_e} &= 0.5 \times 10^{-6}\ \text{eV}, & m_e &= 0.511\ \text{MeV} \\ m_{\nu_\mu} &= 0.224 \times 10^{-2}\ \text{eV}, & m_\mu &= 105.3\ \text{MeV} \\ m_{\nu_\tau} &= 0.141\ \text{eV}, & m_\tau &= 1.777\ \text{GeV} \end{aligned} \qquad (2a)$$

and

$$V_{LEPT} = \begin{pmatrix} 0.9990 & 0.0447 & (-0.690 - 0.310i) \times 10^{-2} \\ -0.0381 - 0.0010i & 0.9233 & 0.3821 \\ 0.0223 - 0.0030i & -0.3814 & 0.9241 \end{pmatrix} \qquad (2b)$$

We have simply assumed a value for the electron-neutrino mass and constructed the lepton mixing matrix[12] by making use of the unitarity conditions with the same phase in (1b) and (2b).

We now evolve the low energy data to the SUSY GUT scale, $\Lambda_{SGUT} = 1.2 \times 10^{16}$ GeV, using numbers taken from the work of Naculich.[13] For this purpose and in most cases the one-loop RGEs will suffice, so one can use analytic expressions for the running variables. We adjust m_b and $\tan\beta = v_u/v_d$, the ratio of the up quark to the down quark VEVs, so that complete Yukawa unification is achieved at Λ_{SGUT}, i.e., $\bar{m}_\tau = \bar{m}_b = \bar{m}_t/\tan\beta$. This is accomplished by choosing $m_b(m_b) = 4.09$ GeV at the running b quark mass scale[14] and $\tan\beta = 48.9$. The evolved masses at Λ_{SGUT} are then found to be

$$\begin{aligned}
\bar{m}_u &= 1.098 \text{ MeV}, & \bar{m}_d &= 2.127 \text{ MeV} \\
\bar{m}_c &= 0.314 \text{ GeV}, & \bar{m}_s &= 42.02 \text{ MeV} \\
\bar{m}_t &= 120.3 \text{ GeV}, & \bar{m}_b &= 2.464 \text{ GeV} \\
\bar{m}_{\nu_e} &= 0.581 \times 10^{-6} \text{ eV}, & \bar{m}_e &= 0.543 \text{ MeV} \\
\bar{m}_{\nu_\mu} &= 0.260 \times 10^{-2} \text{ eV}, & \bar{m}_\mu &= 111.9 \text{ MeV} \\
\bar{m}_{\nu_\tau} &= 0.164 \text{ eV}, & \bar{m}_\tau &= 2.464 \text{ GeV}
\end{aligned} \tag{3a}$$

The following V_{CKM} and V_{LEPT} mixing matrix elements also evolve to

$$\begin{aligned}
\bar{V}_{ub} &= (-0.2163 - 0.0963i) \times 10^{-2}, & \bar{V}_{13} &= (-0.634 - 0.285i) \times 10^{-2} \\
\bar{V}_{cb} &= 0.0329 & \bar{V}_{23} &= 0.3508 \\
\bar{V}_{td} &= 0.0086 - 0.0009i & \bar{V}_{31} &= 0.0205 - 0.0028i \\
\bar{V}_{ts} &= -0.0315 - 0.0002i & \bar{V}_{32} &= -0.3502
\end{aligned} \tag{3b}$$

while the other mixing matrix elements receive smaller corrections which can be neglected.

In order to construct the quark mass matrices at the SUSY GUT scale from the above information, we use a procedure due to Kusenko[9] modified for our purposes. One expresses the unitary CKM mixing matrix in terms of one Hermitian generator by writing $V_{CKM} = U_L^{\prime} U_L^{\dagger} = \exp(i\alpha H)$, where

$$i\alpha H = \sum_{k=1}^{3} (\log v_k) \frac{\prod_{i\neq k}(V_{CKM} - v_i I)}{\prod_{i\neq k}(v_k - v_i)}$$

and the v_j are the eigenvalues of the unitary mixing matrix. The transformation matrices from the weak to the mass bases are given by

$$U_L^{\prime} = \exp(i\alpha H x_q), \qquad U_L = \exp\left[i\alpha H(x_q - 1)\right]$$

where for $x_q = 0$ the up quark mass matrix is diagonal, while for $x_q = 1$ the down quark mass matrix is diagonal. Finally, the complex-symmetric mass matrices are determined by

$$M^U = U_L'^{\dagger} D^U U_L'^{\dagger T}, \qquad M^D = U_L^{\dagger} D^D U_L^{\dagger T}$$

where D^U and D^D are the diagonal matrices in the mass bases with entries taken from (3a). It suffices to expand V_{CKM}, U_L' and U_L to third order in α in order to obtain accurate expressions for the mass matrices. The lepton mass matrices, M^E for the charged leptons and $M^{N_{eff}}$ for the light neutrinos, are constructed in a similar fashion with the parameter x_q replaced by x_ℓ.

The SO(10) Yukawa interaction Lagrangian for the non-supersymmetric fermions is given by

$$\mathcal{L}_Y = -\sum_i \bar{\psi}^{c(16)} f^{(10_i)} \psi^{(16)} \phi^{(10_i)} - \sum_j \bar{\psi}^{c(16)} f^{(126_j)} \psi^{(16)} \bar{\phi}^{(126_j)} + \text{h.c.} \tag{4a}$$

where the f's represent Yukawa coupling matrices and we assume just **10** and **126** contributions which are symmetric.[15] The mass matrices are given by

$$\begin{aligned} M^U &= \textstyle\sum_i f^{(10_i)} v_{ui} + \sum_j f^{(126_j)} w_{uj} \\ M^D &= \textstyle\sum_i f^{(10_i)} v_{di} + \sum_j f^{(126_j)} w_{dj} \\ M^{N_{Dirac}} &= \textstyle\sum_i f^{(10_i)} v_{ui} - 3\sum_j f^{(126_j)} w_{uj} \\ M^E &= \textstyle\sum_i f^{(10_i)} v_{di} - 3\sum_j f^{(126_j)} w_{dj} \end{aligned} \tag{4b}$$

where v_{ui} and w_{uj} are the **10** and **126** VEV contributions to the up quark and Dirac neutrino matrices, and similarly for the down quark and charged lepton contributions. The equations in (4b) can be inverted to determine the sum of the **10** and sum of the **126** contributions separately. At this stage we do not know how many **10** and **126** representations of each type are necessary.

By varying the x_q and x_ℓ parameters over the unit square support region and by allowing all possible signs to appear in the diagonal matrix entries of D^U, D^D, D^E and $D^{N_{eff}}$, we can search for a set of mass matrices which have the simplest **10** - **126** structure for as many matrix elements as possible. Such a preferred choice is found with $x_q = 0$ and $x_\ell = 0.88$, where the observed structure for M^U is diagonal and

$$M^D \sim M^E \sim \begin{pmatrix} 10,126 & 10,126 & 10 \\ 10,126 & 126 & 10 \\ 10 & 10 & 10 \end{pmatrix} \qquad (5a)$$

with M^D_{11}, M^E_{12} and M^E_{21} anomalously small. We shall assume these elements, in fact, exhibit texture zeros and also assume that the same **10** and **126** contribute, respectively, to the 33 and 22 diagonal elements of M^U and M^D. Hence

$$M^U \sim M^{N_{Dirac}} \sim diag(10, 126;\ 126;\ 10) \qquad (5b)$$

If we require as simple a structure as possible for the 11 elements of the four matrices, we are led numerically to the following choices for the Yukawa coupling matrices at Λ_{SGUT}

$$\begin{aligned} f^{(10)} &= diag(0,\ 0,\ f^{(10)}_{33}), & f^{(126)} &= diag(f^{(126)}_{11},\ f^{(126)}_{22},\ 0) \\ f^{(10')} &= \begin{pmatrix} f^{(10')}_{11} & f^{(10')}_{12} & f^{(10')}_{13} \\ f^{(10')}_{12} & 0 & f^{(10')}_{23} \\ f^{(10')}_{13} & f^{(10')}_{23} & 0 \end{pmatrix}, & f^{(126')} &= \begin{pmatrix} 0 & f^{(126')}_{12} & 0 \\ f^{(126')}_{12} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \end{aligned} \qquad (6)$$

The model requires two **10**'s and two **126**'s of SO(10) with **10**$'$ and **126**$'$ having no VEVs in the up direction. There are four texture zeros in the M^U and M^D matrices taken together. The four matrices assume the simple textures

$$M^U = f^{(10)}v_u + f^{(126)}w_u = diag(F',\ E',\ C') \qquad (7a)$$

$$M^{N_{Dirac}} = f^{(10)}v_u - 3f^{(126)}w_u = diag(-3F',\ -3E',\ C') \qquad (7b)$$

$$M^D = f^{(10)}v_d + f^{(126)}w_d + f^{(10')}v'_d + f^{(126')}w'_d$$

$$= \begin{pmatrix} 0 & A & D \\ A & E & B \\ D & B & C \end{pmatrix} \tag{7c}$$

$$M^E = f^{(10)}v_d - 3f^{(126)}w_d + f^{(10')}v'_d - 3f^{(126')}w'_d$$

$$= \begin{pmatrix} F & 0 & D \\ 0 & -3E & B \\ D & B & C \end{pmatrix} \tag{7d}$$

with only D complex and the following relations holding

$$\begin{aligned} C'/C &= v_u/v_d, & E'/E &= w_u/w_d \\ f^{(10')}_{11}v'_d &= -f^{(126)}_{11}w_d = \tfrac{1}{4}F, & f^{(126)}_{11}w_u &= F' \\ f^{(10')}_{12}v'_d &= 3f^{(126')}_{11}w'_d = \tfrac{3}{4}A \end{aligned} \tag{7e}$$

from which we obtain the constraint, $4F'/F = -E'/E$.

With $F' = -\bar{m}_u,\ E' = \bar{m}_c,\ C' = \bar{m}_t$

$$\begin{aligned} C &= 2.4607, & &\text{so} \quad v_u/v_d = \tan\beta = 48.9 \\ E &= -0.3830 \times 10^{-1}, & &\text{hence} \quad w_u/w_d = -8.20 \\ F &= -0.5357 \times 10^{-3}, & B &= 0.8500 \times 10^{-1} \\ A &= -0.9700 \times 10^{-2}, & D &= (0.4200 + 0.4285i) \times 10^{-2} \end{aligned} \tag{8}$$

the masses and mixing matrices are calculated at Λ_{SGUT} by use of the projection operator technique of Jarlskog[16] and then evolved to the low scales. The following low-scale results emerge for the quarks:

$$\begin{aligned} m_u(1\text{GeV}) &= 5.10 \text{ MeV}, & m_d(1\text{GeV}) &= 9.33 \text{ MeV} \\ m_c(m_c) &= 1.27 \text{ GeV}, & m_s(1\text{GeV}) &= 181 \text{ MeV} \\ m_t(m_t) &= 150 \text{ GeV}, & m_b(m_b) &= 4.09 \text{ GeV} \end{aligned} \tag{9a}$$

$$V_{CKM} = \begin{pmatrix} 0.9753 & 0.2210 & (0.2089 - 0.2242i) \times 10^{-2} \\ -0.2209 & 0.9747 & 0.0444 \\ 0.0078 - 0.0022i & -0.0438 - 0.0005i & 0.9994 \end{pmatrix} \qquad (9b)$$

These results are in excellent agreement with the input in (1a,b), aside from the unknown CP phase, with[10,11] $|V_{ub}/V_{cb}| = 0.069$ and $m_s/m_d = 19.4$.

For the leptons we observe that the heavy righthanded Majorana neutrino mass matrix can be computed at Λ_{SGUT} from the approximate seesaw mass formula[17]

$$M^R = -M^{N_{Dirac}}(M^{N_{eff}})^{-1}M^{N_{Dirac}} \qquad (10a)$$

and numerically can be approximated by the nearly geometric form

$$M^R = \begin{pmatrix} F'' & -\frac{2}{3}\sqrt{F''E''} & -\frac{1}{3}\sqrt{F''C''}e^{i\phi_{D''}} \\ -\frac{2}{3}\sqrt{F''E''} & E'' & -\frac{2}{3}\sqrt{E''C''}e^{i\phi_{B''}} \\ -\frac{1}{3}\sqrt{F''C''}e^{i\phi_{D''}} & -\frac{2}{3}\sqrt{E''C''}e^{i\phi_{B''}} & C'' \end{pmatrix} \qquad (10b)$$

where $E'' = \frac{2}{3}\sqrt{F''C''}$ and $\phi_{B''} = -\phi_{D''}/3$; moreover, the structure in (10b) can be separated into two parts with coefficients 2/3 and 1/3 which suggests they may arise again from two different **126** contributions. Such geometric textures have been studied at some length by Lemke.[18]

With $C'' = 0.6077 \times 10^{15}$, $F'' = 0.1745 \times 10^{10}$ and $\phi_{D''} = 45°$, M^R is reproduced exceedingly well at Λ_{SGUT} with the resulting heavy Majorana neutrino masses $M_{R_1} = 0.249 \times 10^9$ GeV, $M_{R_2} = 0.451 \times 10^{12}$ GeV and $M_{R_3} = 0.608 \times 10^{15}$ GeV. At the low scales we find

$$\begin{aligned} m_{\nu_e} &= 0.534 \times 10^{-5}\ \text{eV}, & m_e &= 0.504\ \text{MeV} \\ m_{\nu_\mu} &= 0.181 \times 10^{-2}\ \text{eV}, & m_\mu &= 105.2\ \text{MeV} \\ m_{\nu_\tau} &= 0.135\ \text{eV}, & m_\tau &= 1.777\ \text{GeV} \end{aligned} \qquad (11a)$$

and

$$V_{LEPT} = \begin{pmatrix} 0.9990 & 0.0451 & (-0.029 - 0.227i) \times 10^{-2} \\ -0.0422 & 0.9361 & 0.3803 \\ 0.0174 - 0.0024i & -0.3799 - 0.0001i & 0.9371 \end{pmatrix} \qquad (11b)$$

The agreement with our starting input is remarkably good, especially since only 12 model parameters have been introduced in order to explain 15 masses and 8 effective mixing parameters. Although we need two **10** and two **126** Higgs representations for the up, down, charged lepton and Dirac neutrino matrices with one or two additional **126**'s for the Majorana matrix, pairs of irreducible representations more naturally emerge in the superstring framework than do single Higgs representations. We have thus demonstrated by the model constructed that all quark and lepton mass and mixing data (as assumed herein) can be well understood in the framework of a simple SUSY GUT model based on SO(10) symmetry.

While we have gone into some detail about the model which has been constructed based on the low energy data involving the solar neutrino and atmospheric neutrino depletions, we wish to emphasize that the same approach can be carried out for other starting points. In a paper[19] to be published elsewhere, we shall elaborate on the numerical details leading to the solution presented here and consider alternative scenarios for the lepton masses and mixing matrix.

The authors gratefully acknowledge the Summer Visitor Program and warm hospitality of the Fermilab Theoretical Physics Department, which enabled the initiation of this research. We thank Joseph Lykken for his continued interest and comments during the course of this work. The research of CHA was supported in part by Grant No. PHY-9207696 from the National Science Foundation, while that of SN was supported in part by the U.S. Department of Energy, Grant No. DE-FG05-85ER 40215.

References

[1] U. Amaldi, W. de Boer and H. Funstenau, Phys. Lett. B **260**, 447 (1991); J. Ellis, S. Kelley and D. V. Nanopoulos, Phys. Lett. B **260**, 131 (1991); P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991).

[2] J. A. Harvey, P. Ramond, and D. B. Reiss, Phys. Lett **92B**, 309 (1980); S. Dimopoulos, L. J. Hall and S. Raby, Phys. Rev. Lett. **68**, 1984 (1992); Phys. Rev. D **45**, 4192 (1992); **46**, R4793 (1992); **47**, R3702 (1993); G. F. Giudice, Mod. Phys. Lett. A **7**, 2429 (1992); H. Arason, D. J. Castaño, E.-J. Piard and P. Ramond, Phys. Rev. D **47**, 232 (1993); P. Ramond, R. G. Roberts and G. G. Ross, Nucl. Phys. **B406**, 19 (1993).

[3] V. Barger, M. S. Berger, T. Han and M. Zralek, Phys. Rev. Lett. **68**, 3394 (1992); Phys. Rev. D **47**, 2038 (1993).

[4] K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993); L. Lavoura, Carnegie-Mellon University preprint 93-0473, June 1993.

[5] G. W. Anderson, S. Raby, S. Dimopoulos, L. J. Hall and G. D. Starkman, Lawrence Berkeley Laboratory preprint LBL-33531, August 1993.

[6] S. P. Mikheyev and A. Yu Smirnov, Yad Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1986)]; Zh. Eksp. Teor. Fiz. **91**, 7 (1986) [Sov. Phys. JETP **64**, 4 (1986)]; Nuovo Cimento **9C**, 17 (1986); L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); **20**, 2634 (1979).

[7] R. Davis et al., Phys. Rev. Lett. **20**, 1205 (1968); in *Neutrino '88*, ed. J. Schnepp et al. (World Scientific, 1988); K. Hirata et al., Phys. Rev. Lett. **65**, 1297, 1301 (1990);

A. I. Abazov et al., Phys. Rev. Lett. **67**, 3332 (1991); Phys. Rev. C **45**, 2450 (1992); P. Anselmann et al., Phys. Lett. B **285**, 376, 390 (1992).

[8] K. S. Hirata et al., Phys. Lett. B **205**,416 (1988); D. Casper et al., Phys. Rev. Lett. **66**, 2561 (1991);

[9] A. Kusenko, Phys. Lett. B **284**, 390 (1992).

[10] Review of Particle Properties, Phys. Rev. D **45**, No. 11, Part II, June (1992).

[11] J. Gasser and H. Leutwyler, Phys. Rep. C **87**, 77 (1982).

[12] C. H. Albright, Phys. Rev. D **45**, R725 (1992); Zeit. f. Phys. C **56**, 577 (1992); A. Yu. Smirnov, Institute for Advanced Study and ICTP preprint, IASSNS-AST-93-14.

[13] S. Naculich, Johns Hopkins University preprint JHU-TIPAC-930002, January 1993.

[14] The two loop RGEs would lead to somewhat different $\tan\beta$ and $m_b(m_b)$.

[15] We neglect possible **120** Higgs contributions which would give rise to antisymmetric mass matrices.

[16] C. Jarlskog, Phys. Rev. D **35**, 1685 (1987); **36**, 2138 (1987); C. Jarlskog and A. Kleppe, Nucl. Phys. **B286**, 245 (1987).

[17] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supersymmetry*, edited by P. Van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, Prog. Theor. Phys. **B 315**, 66 (1978).

[18] E. H. Lemke, Mod. Phys. Lett. A **7**, 1175 (1992).

[19] C. H. Albright and S. Nandi, in preparation.