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ABSTRACT

The small dispersion in the absolute magnitude of type Ia supernovae at maximum light make them potentially useful extragalactic distance indicators. Their extreme brightness has allowed their detection at redshifts as large as $z = 0.46$. At redshifts this large, observations of several supernovae could be used to estimate the deceleration parameter, q_0. In this paper, we explore factors that contribute to the systematic error in measuring q_0 using type Ia supernovae. We propose methods for reducing the systematic error to a fraction of the statistical uncertainty and estimate the number of supernovae required to determine q_0 with a given accuracy. We find that about 50 supernovae at a redshift of $z = 0.3$ would allow a measurement of q_0 to within 0.12, if the dispersion in type Ia absolute magnitudes is 0.3. These methods allow a measurement of q_0 even if the type Ia luminosity function is not a perfect delta function, nor even a perfect Gaussian.

1. Introduction

Since the identification of the type Ia as a distinguishable subclass of supernovae, there has been growing evidence that they may be useful as "standard candles" for distance measurement (Minkowski 1964; Kowal 1968; Arnett et al. 1985; Cadonau et al. 1985). Their extreme brightness ($M_B \approx -19$) make them particularly valuable for measuring the deceleration parameter, q_0. This may be done using the relation between apparent broad-band magnitude m and the redshift z:

$$m = 5\log\left(\frac{c}{q_0^2}\left(1 - q_0 + q_0 z + (q_0 - 1)\sqrt{2q_0 z + 1}\right)\right) + \\ 25 + M - 5\log H_0 + K, \tag{1}$$

where M is the absolute magnitude at maximum luminosity; $M - 5\log H_0$ may be determined by observations of nearby type Ia supernovae (SNe Ia). K is the correction that must be added to account for the redshifting of the part of the spectrum that passes through the broad-band filter.

Two high-redshift supernovae have already been discovered: one at $z = 0.31$ (Nørgaard-Nielsen et al. 1989) and one at $z = 0.458$ (Perlmutter et al. 1993). The second was found in a systematic search that soon should be finding tens of such high-redshift SNe Ia. It is therefore important to understand how accurately one can measure q_0 based on the comparison of a sample of distant SNe Ia with a sample of nearby SNe Ia.

There is, of course, some dispersion in the measured SNe Ia absolute magnitudes at maximum which could seriously affect the accuracy of a q_0 measurement. Elias et al. (1985) suggested that the dispersion is less than 0.2 mag based on the infrared light curves of four SNe Ia. More recently, Leibundgut and Tammann (1990) identified a group of six SNe Ia with light curves that could define a template with a dispersion of less than $\sigma_{M_B} = 0.18$ magnitudes. A more comprehensive study of 53 SNe Ia by Miller and Branch (1990) found that although there is a fraction of the SNe Ia that are significantly fainter than the majority, these SNe occurred in highly inclined spiral galaxies and thus were likely to have had their light absorbed by host galaxy dust. In a later paper Branch and Miller (1993) showed that the $B - V$ colors of SNe Ia at peak support this explanation; they found that the reddest SNe Ia are the subluminous ones. These subluminous SNe Ia could thus be identified by their color and eliminated from a sample used to estimate q_0.

Recently, a few subluminous supernovae have been identified that cannot be explained solely by reddening. In particular, three subluminous, unusually red supernovae have been differentiated from the approximately 30 well-observed SNe Ia. The spectra of these subluminous supernovae generally resemble each other, but are different from the other SNe Ia, and they can be excluded from a sample of normal SNe Ia either by color or spectrum. A single supernova, 1991T, with an unusual premaximum spectrum has been discussed as a possible superluminous SN Ia by Filippenko et al. (1992). However, Phillips et al. (1992) pointed out that its distance is somewhat uncertain and that once this is taken into account its peak luminosity is consistent with that of other SNe Ia.

In this paper we study the accuracy of a measurement of q_0 using high-redshift supernovae. In particular, we wish to quantify and propose ways to minimize some important sources of bias due to deviations of the distribution of Type Ia absolute magnitudes from a perfect delta function, and imperfect search efficiencies. We begin with the simplest deviation from the delta-function distribution, a Gaussian, and then investigate the effect of subluminous or absorbed supernovae, magnitude-limited samples, and the time interval between search observations. In all cases, we estimate the number of supernovae necessary to measure q_0 to a given precision.

One problem not addressed here is the error introduced because of uncertainties in the K-correction. The K-correction can be calculated if the supernova's spectrum is well known (e.g. Leibundgut 1990); alternatively, explicit calculation of the correction can be avoided by using broadband filters whose transmission functions are redshifted from those of standard filters to match the redshift of the supernova (e.g. Perlmutter et al. 1993). Our analysis therefore assumes the K-correction has been made either observationally or analytically. For further discussion of the uncertainties in the K-correction, see Perlmutter, Goobar, and Kim 1993.

2. A Gaussian Model for the SN Ia Luminosity Function

The solid histogram of Fig. 1 shows the distribution of absolute B magnitudes at maximum light from Branch et al. (1993) for the SNe Ia that have measurement errors less than 0.4 magnitudes, are not spectroscopically peculiar or unusually red; the three faintest show evidence of extinction. The unextincted supernovae show a narrow distribution with a dispersion of approximately 0.2 mag. The intrinsic dispersion of SNe Ia absolute magnitude may be less than this, since the error can be completely accounted for by the errors in estimating the distance to the supernovae ($\sigma_{\text{dist}} = 0.3$–$0.4$ mag) and errors in measuring the apparent magnitude at maximum ($\sigma_B \approx 0.2$). If this is an accurate reflection of the actual SNe Ia luminosity function then a Gaussian model is a reasonable approximation to use.

For a single distant supernova drawn from this Gaussian distribution, with peak apparent magnitude measured with negligible error, the statistical uncertainty in q_0 is easy to calculate from Eq. (1). We find

$$\delta q_0 \approx \frac{\partial q_0}{\partial M} \delta M \approx \frac{\delta M}{z}. \tag{2}$$

where δM is the dispersion in SNe Ia peak absolute magnitude distribution. For a dispersion of $\delta M = 0.2$ and a redshift of 0.5, $\delta q_0 = 0.4$. However, if N SNe Ia are observed at a given redshift, and if their maximum absolute magnitudes are normally distributed, the resulting

q_0 values could be averaged to get a more accurate estimate. The uncertainty in q_0 is then

$$\delta q_0 \approx \frac{\delta M}{z\sqrt{N}}. \tag{3}$$

Given a sample of 50 SNe Ia the uncertainty in q_0 is only 0.06.

3. A Magnitude-limited Sample and the Malmquist Bias

The analysis above gives a good estimate of the statistical uncertainty. However, if the observed supernovae do not uniformly sample the underlying Gaussian luminosity function, the resulting q_0 value will be biased. A magnitude limited supernova search preferentially finds intrinsically bright supernovae, which biases q_0 toward larger values. The biasing caused by a magnitude limited sample is sometimes called the Malmquist effect. There are two ways to manage the Malmquist bias. If the underlying luminosity function were well understood, one could explicitly calculate the effect of the bias. However, this requires a more detailed understanding of the supernova luminosity function than is currently feasible. Without this detailed understanding, we impose a redshift limit on the supernovae used to estimate q_0 so that they sample the entire luminosity function.

Any realistic supernova search scheme is naturally magnitude-limited, so imposing an additional redshift limit requires that some fraction of the discovered supernovae be discarded. Typically, $m_{\rm lim}$ is chosen to maximize the number of supernovae found, trading off sky coverage with survey depth. Then, $z_{\rm lim}$ is chosen so that the Malmquist bias is small compared to the statistical uncertainty. The "discarded" supernovae would still be useful in testing spectra and light curves for evolution effects, but they could not be used to estimate q_0.

For our analysis we assumed the distribution of SNe Ia maximum absolute magnitudes was a Gaussian with a width of $\sigma_M = 0.3$ as shown by the smooth curve in Fig. 1. Judging from the histogrammed Branch et al. data, this is a very conservative estimate. We chose to be conservative since other studies (Miller and Branch 1990; Tammann and Leibundgut 1990) have implied larger dispersions, but as has already been noted, the larger dispersion may be due to uncertainties in the supernovae distances or photometric error. In a recent paper, Phillips (1993) argues that the intrinsic dispersion could be larger. After removing three SNe Ia labeled as peculiar by Branch and Miller (1993) he estimates a dispersion of ~ 0.3 mag in both B and V. In any case, the results we present can be scaled using Eq. (1) to obtain δq_0 for any dispersion in M. For our analysis, it doesn't matter whether the 0.3 magnitude spread is due to inherent dispersion in the supernova luminosity function, or whether it is due to measurement error.

4. Redshift Distribution of Supernovae

The estimate given by Eq. (3) assumes all of the supernovae have the same redshift. In order to investigate the systematic error in q_0 for a real search we used Monte Carlo techniques to pick supernovae from a realistic supernova redshift distribution. The analysis consisted of two steps. First, a file containing 10^5 supernova events was generated. Each record in the file consisted of an apparent magnitude and redshift for a single supernova. The apparent magnitude depends on the filter band used by the supernova search. For the purposes of this analysis we use apparent magnitudes for a typical filter; for an actual experiment these apparent magnitudes would have to be adjusted by the K-correction for the filter and the target redshifts. The apparent magnitude was computed using Eq. (1) with the peak absolute magnitude picked from the distribution described in Sec. 3. The redshift was picked from one of the two distributions discussed below. Second, the file was divided into sets of from 10 to 100 supernovae that were then fit to Eq. (1), yielding a value for q_0 for each set. The median of the fit q_0's was compared to the input q_0 value to check for a bias in the fit. The standard deviation of the fit values gave the uncertainty in the q_0 measurement.

The appropriate supernova redshift probability distribution depends on the procedure used to find the supernovae. We used redshift distributions appropriate to two different techniques: a subtraction technique and a brightness monitoring technique. The subtraction approach finds supernovae by subtracting a reference image from a newly acquired image. Supernovae are detected by looking for objects that appear above the detection limit in the difference image. The brightness monitoring technique works by cataloging all the objects in the reference image. Possible supernovae are found by watching for changes in the brightness of the cataloged objects. In principle, the subtraction method finds all supernovae in the search area that have a maximum luminosity above the detection threshold. The brightness monitoring procedure finds supernovae only in galaxies that are visible in the reference image. The brightness monitoring technique finds fewer supernovae per square degree than the subtraction technique, but fewer of the detected supernovae are eliminated by the redshift cut. (We refer to the subtraction method as the inclusive search technique since, in principle, it finds *all* supernovae brighter than the detection threshold.) For simplicity, we begin with the case of $q_0 = 0.5$ and later consider cases with q_0 between 0.1 and 0.9.

4.1. Supernova Redshift Distribution for an Inclusive Search

The supernova redshift probability distribution for the subtraction technique was derived assuming the rest frame supernova rate does not evolve. This is certainly not true for galaxies at very high redshift which are undergoing their first generation of star formation, but for deep supernova searches limited to $z \leq 0.5$, these evolution effects are not significant. Time dilation does however give a redshift dependence to the supernova rate and this was included. We also assumed a uniform distribution of galaxies. This does not account for galaxy clusters, but clustering would only increase the number of supernovae found at a given redshift and not significantly alter the result of a q_0 measurement. Making these assumptions and using the density of sources per unit z per unit solid angle for a matter-dominated Friedmann universe (Weinberg 1972) gives the supernova redshift probability distribution

$$\frac{dP}{dz} = k\frac{(zq_0 + (q_0 - 1)(\sqrt{2q_0 z + 1} - 1))^2}{(1+z)^4\sqrt{2q_0 z + 1}}, \tag{4}$$

where k is a constant.

The results of the simulation using this distribution with an input $q_0 = 0.5$, $m_{\rm lim} = 23$, $z_{\rm lim} = 0.45$, and $\sigma_M = 0.3$ are shown in Fig. 2 and 3. Fig. 2 shows the median of the fit q_0 values vs. the number of supernovae per fit. The error bars represent the standard deviation of the fit q_0 values. Note that the median fit q_0 recovers the input q_0 of 0.5 with only a slight Malmquist bias. The points labeled "Inclusive Search" in Fig. 3 show the standard deviation of the fit q_0 values. They represent the uncertainty in a given q_0 measurement vs. the number of supernovae, N, in the fit. Note that the uncertainty drops roughly as $1/\sqrt{N}$, approximating the result of Eq. (3) if we had observed a sample of SNe all at redshift $z = 0.31$ (hence we call this the "effective redshift"). In order to characterize the bias we define

$$\frac{\Delta q_0^{(50)}}{\sigma_{q_0}^{(50)}} = \frac{q_0^{(\rm fit)} - q_0^{(\rm input)}}{\sigma_{q_0}^{(50)}}, \tag{5}$$

where $q_0^{(\rm input)}$ is the value of q_0 input into the Monte Carlo; $q_0^{(\rm fit)}$, and $\sigma_{q_0}^{(50)}$ are the mean and standard deviation of the fit q_0 values for 50 supernovae in the fit.

For the parameters above the bias was small (see Table 1). This is because, with these parameters, the entire supernova luminosity function is uniformly sampled. The limiting magnitude was 1.5 standard deviations fainter than the average apparent supernova magnitude at the $z = 0.45$ redshift limit. At the $z = 0.31$ effective redshift, the limiting magnitude is approximately four standard deviations fainter than the average. For $z_{\rm lim} = 0.5$ the bias becomes significant, $\Delta q_0^{(50)}/\sigma_{q_0}^{(50)} = 0.69$ for $N = 50$. In general, for $N = 50$ we found the bias to be acceptable, $\Delta q_0^{(50)}/\sigma_{q_0}^{(50)} \leq 0.5$, if the redshift limit was chosen such that all the supernovae brighter than approximately $m_{\rm lim} - \sigma_M$ were included in the sample. As the number of supernovae in the sample increases, the redshift limit must be reduced to keep the bias an acceptable fraction of the statistical uncertainty.

4.2. Supernova Redshift Distribution for a Visible Galaxy Search

In computing the distribution for a visible galaxy search, we again ignored galaxy evolution and clustering. In this case the redshift probability distribution is

$$\frac{dP}{dz} = \sum_{L_g} r(z) L_g \frac{dN_g}{dz}, \tag{6}$$

where $r(z)$ is the redshift dependent supernova rate per unit luminosity, L_g is the galaxy luminosity and dN_g/dz is the number of visible galaxies of luminosity L_g per unit redshift per unit solid angle. In order to compute dN_g/dz the apparent magnitude of each galaxy of luminosity L_g at given redshift and with the appropriate K-correction was computed using a galaxy luminosity distribution and K-corrections taken from Ellis (private communication). The relative number of galaxies with apparent R magnitudes between 20 and 23 was then computed to yield dN_g/dz. In general, these magnitude limits should be chosen to match the search strategy. Finally, if a galaxy's brightness was over 10 times the brightness of a typical SN Ia, it was excluded. This reflects the difficulty in finding supernovae on top of a very high background signal.

Fig. 4 compares the visible galaxy and inclusive supernova redshift distributions. They are normalized to include approximately the same number of supernovae. The visible galaxy distribution initially climbs more steeply than the inclusive distribution as more energy is redshifted into the R filter wavelength band. Beyond $z = 0.5$ the inclusive distribution begins to fall as most of the galaxies begin to exceed the 23 mag limit.

Using the visible galaxy redshift distribution $q_0 = 0.5$, $m_{\rm lim} = 23$, $\sigma_M = 0.3$ and $z_{\rm lim} = 0.45$, we obtain the results shown in Fig. 3 and Table 1. The bias is small and σ_{q_0} falls as $1/\sqrt{N}$ with an effective redshift of 0.32. In this case, 31% of the supernovae are above the detection limit but beyond the redshift limit. Since current searches find supernovae by monitoring visible galaxy brightness, we will use this distribution for the rest of our analysis.

5. Dependence on q_0

All of the results discussed so far assumed an input value of $q_0 = 0.5$. However, it is apparent from Table 1 that both the statistical uncertainty, $\sigma_{q_0}^{(50)}$, and the bias, $\Delta q_0^{(50)}/\sigma_{q_0}^{(50)}$, depend on q_0. The statistical uncertainty is smaller for smaller values of q_0 because the effective redshift, $z_{\rm eff}$, increases with decreasing q_0 (see Table 1). This occurs because the supernova redshift distribution (Eq. 4) rises much more steeply for small values of q_0, so proportionally more supernovae are found at high redshift. Since the bias parameter $\Delta q_0^{(50)}/\sigma_{q_0}^{(50)}$ and the statistical uncertainty are inversely proportional, a reduced statistical uncertainty also increases the bias parameter.

Not all of the increase in the bias parameter at small q_0 is due to a reduced statistical uncertainty. A larger effect arises because reducing q_0 reduces the spread between the magnitude limit and the apparent magnitude of the supernova at the redshift limit. Reducing the spread pushes supernovae on the faint side of the luminosity function beyond the magnitude limit and therefore increases the Malmquist bias. Given the redshift limit of $z = 0.45$, the peak of the supernova luminosity function would be at 22.57 and 22.76 mag for $q_0 = 0.5$ and 0.1, respectively. For $m_{\rm lim} = 23$ and $q_0 = 0.5$, the spread is 0.43 mag or

$1.43\sigma_M$; for $q_0 = 0.1$ the spread is only 0.24 mag or $0.80\sigma_M$.

Table 1 shows the results of simulations with input values of q_0 of 0.1, 0.3, 0.5, 0.7, and 0.9. For simulations using $z_{\rm lim} = 0.45$, the bias falls from 0.56 for $q_0 = 0.1$ to 0.06 for $q_0 = 0.9$. Since the bias is most severe for small values of q_0, small values must be assumed when setting the redshift limit.

6. Surveillance Time

The analysis so far has assumed that all supernovae above the detection limit will be found. However, a practical supernova search might monitor the sky every two weeks to a month. Any supernova that appeared above the detection limit for less than one or two weeks respectively would probably be missed. Intrinsically bright supernovae will remain above the detection threshold longer than intrinsically faint ones which again leads to a biased sampling of the supernova luminosity function and a biased q_0 estimate.

In order to incorporate this effect into our simulation, we first derived an expression for the surveillance time or the amount of time a supernova's brightness is above the detection threshold. During the first month, a parabola fits the light curve quite well so that the surveillance time is

$$t_{\rm surv} = 26(1+z)\sqrt{m_{\rm lim} - m} \quad \text{days.} \tag{7}$$

The probability for detection is

$$P(m,z) = \begin{cases} \frac{t_{\rm surv}(m,z)}{\Delta t}, & t_{\rm surv} \le \Delta t \\ \\ 1, & \text{otherwise,} \end{cases} \tag{8}$$

where Δt is the time between observations. This probability was incorporated in simulations using the visible galaxy distribution with $q_0 = 0.1$, $m_{\rm lim} = 23$, $\sigma_M = 0.3$ and with both 14 day and 30 day intervals between observations. As might be expected, the 30 day intervals produced the largest bias (see Table 1). For $z_{\rm lim} = 0.45$, $\Delta q_0^{(50)}/\sigma_{q_0}^{(50)} = 1.33$ for 30 day intervals and 0.77 for 14 day intervals. The bias was much smaller for $q_0 = 0.5$. The results are summarized in Table 1.

7. Subluminous Supernovae

It is obvious from Fig. 1 that some SNe Ia do not fit into the Gaussian distribution used so far. As noted in Sec. 1. most of these supernovae are obscured by gas and dust in the host galaxy, but there is some evidence that perhaps 10% are truly subluminous. We argued that both types of supernovae could be excluded by their colors or spectra. However, it is possible that some of these supernovae could leak into sample. This might happen, for example, if a supernova could not be observed well enough or if its colors and spectra were not significantly different from ordinary SNe Ia.

The effect of subluminous SNe Ia would be to bias the fit q_0 to a lower value. This is because the average absolute magnitude computed including subluminous supernovae would be less than the peak of the luminosity function. Therefore, by Eq. (1), q_0 would be underestimated. Note that the Malmquist bias has the opposite effect.

Extremely subluminous supernova could strongly bias the fit. If one in ten supernovae discovered was one magnitude dimmer than typical SNe Ia, then the average maximum absolute magnitude would be 0.1 mag below the Gaussian peak. If the effective redshift of the sample was 0.3, then by Eq. 2 the measurement of q_0 would be biased by $\Delta q_0 = -0.3$, regardless of the number of supernovae discovered. However, even if such a subluminous supernova could not be identified by its color or spectrum, it would be over three standard deviations too dim and could easily be identified, provided enough supernovae were discovered to sufficiently define the peak of the luminosity function.

A more insidious bias is caused by slightly subluminous supernovae that leak into the sample. We investigated the effect of these subluminous supernovae by modeling extinction in the host galaxy, and then incorporating the model into the Monte Carlo simulation. We adopted the model suggested by Miller and Branch (1990). Elliptical galaxies were assumed to be relatively free of gas and dust, and thus cause a negligible reduction in the observed luminosity of the embedded supernovae. Spiral galaxies were modeled as a thin sheet of obscuring dust, with no extinction in the bulge and halo. The extinction is then a simple function of the galaxy's inclination; $A(i) = A\sec(i)$. We assumed an extinction $A = 0.80$ mag for face-on galaxies. Including extinction in this way is equivalent to adding separate population of supernovae that is peaked 0.8 magnitudes below the main peak, so this simulation effectively tests the effect of intrinsically subluminous supernova as well.

The extinction simulation program proceeded as before except that after it picked an absolute magnitude from the Gaussian supernova luminosity function it used an input extinction probability, $p_{\rm ext}$, to determine whether or not the supernova suffered extinction. If extinction was called for, an inclination angle was chosen assuming that all galaxy orientations were equally likely. The program then computed the absolute magnitude after extinction and then continued as before. In order to isolate the effect of subluminous SNe Ia from the Malmquist bias and the surveillance time effect, we ran these simulations without the surveillance time effect and with a larger magnitude limit; $m_{\rm lim} = 23.5$. The fainter magnitude limit with a redshift limit of 0.45 eliminated the Malmquist bias entirely.

Given that roughly 27% of the supernovae in the sample occur in ellipticals and assuming that half of the supernovae in spirals occur on the near side of the galaxy and so suffer no extinction, the maximum number of absorbed supernovae is about 37%. If we assume that $^1/_4$of these supernovae leak into the fitting sample, then $p_{\rm ext} = 0.10$ and our simulation gives the results presented in Table 2. These results show that even if 10% of a sample of 50 SNe Ia are slightly subluminous the bias in q_0 is still less than the statistical uncertainty. Table 2 summarizes the results of simulations run with larger and smaller extinction probabilities. Although having 10% of the sample composed of subluminous SNe Ia may be acceptable, the results for larger p_{ext} emphasize the importance of follow-up observations in order to either correct for the host galaxy extinction or eliminate subluminous SNe Ia from the sample. If half of the absorbed supernovae are not screened out ($p_{ext} = 0.20$), the bias is large; $\Delta q_0^{(50)}/\sigma_{q_0}^{(50)} = -1.31$ and -1.63 for $q_0 = 0.1$ and 0.5 respectively.

The bias produced by subluminous supernova is potentially more serious for samples of more than 50 supernovae, because the systematic error they produce becomes a larger fraction of the statistical uncertainty. However, for larger numbers of supernovae the luminosity function can be inferred from the sample and the subluminous supernovae easily identified.

8. Conclusion

In this paper, we studied many factors that contribute to the systematic error in measuring q_0 from SNe Ia. In each case, we have introduced a method for choosing a set of useful supernovae that reduces the systematic error to a fraction of the statistical error. The dependence of σ_{q_0} on the number of supernovae in the sample was roughly the same for all simulations, falling as $1/\sqrt{N}$ (see Fig. 3.). The results of all of the simulations can then be characterized by Eq. (3) with an effective redshift $z_{eff} = 0.3$ to 0.4. The small range of z_{eff} is expected since changing the simulation parameters only eliminated or included a few more high redshift supernovae so the average redshift stayed approximately the same.

Table 1 summarizes our results for a sample of 50 useful supernovae from a magnitude-limited supernova search with $m_{lim} = 23$. With the exception of contamination by a large number of subluminous supernovae, we expect that, given the assumptions outlined in the paper, 50 supernovae closer than $z = 0.45$ will allow a determination of q_0 with an uncertainty of approximately 0.12. The uncertainty is nearly independent of the value of q_0, the surveillance time, and the redshift distribution of supernovae. Of course, all of these simulations were done assuming $\sigma_M = 0.3$. It is easy to see from Eq. (3) that if we assume a narrower distribution then fewer SNe Ia are needed to obtain the same uncertainty in q_0. For example, if we use the dispersion from Branch et al. ($\sigma_M \approx 0.2$) then only about 22 supernovae would be needed to measure q_0 with an uncertainty $\sigma_{q_0} = 0.12$. However, Phillips (1993) argues that the dispersion could be much larger (if one includes the SNe Ia labeled as peculiar by Branch and Miller). Clearly, more photometry is needed to resolve this question.

Two sources of systematic error, the Malmquist bias and the surveillance time effect, can be eliminated by choosing an appropriate redshift limit. For $m_{\rm lim} = 23$ and $N = 50$, we found the bias to be acceptable ($\Delta q_0^{(50)}/\sigma_{q_0}^{(50)} < 0.5$) if the redshift limit was chosen such that all the supernovae brighter than $m_{\rm lim} - \sigma_M$ were included in the sample. For larger samples of supernovae, the redshift limit must be reduced further to keep the bias less than the improved statistical error.

It is likely that most intrinsically subluminous supernovae could be eliminated from a sample used to estimate q_0 on the basis of either their spectra or color. However, our simulations show that for a sample of 50 supernovae, even if 10% of the sample comes from a distribution that is 0.8 mag subluminous (and escaped detection by spectra or light curves), the resulting bias in q_0 is still less than the statistical uncertainty in q_0. Table 2 summarizes these results.

We have not discussed the effect of superluminous SNe Ia because the evidence for them is weak. However, the analysis used for subluminous supernova could be applied to superluminous SNe Ia. Given the current distribution of well-observed SN Ia shown in Fig.

1, it is unlikely that there are more than 1 in 30 superluminous SN Ia. For the example in which 1 in 30 SNe Ia is in fact one magnitude brighter than a typical SN Ia, *and* if this superluminous supernova is not distinguished by color or spectrum, the average absolute magnitude would be shifted 0.03 mag brighter than the peak of the luminosity function. If the effective redshift of the sample was 0.3, then, by Eq. 2, q_0 would be biased by $\delta q_0 = 0.11$.

All of the results presented in this paper are based on our current understanding of SNe Ia, aspects of which are uncertain. The effects of the distribution width and the subluminous tail have already been discussed but the fit q_0 value is very sensitive to the assumed value of the peak of the distribution as well. The systematic error produced in q_0 given an error in position of the peak of the luminosity function can be estimated using Eq. (2). An error of only 0.15 magnitudes in the position of the peak results in an systematic error in q_0 of 0.5 for $z_{\rm eff} = 0.3$. Further photometry of nearby SNe Ia is therefore necessary to confirm the current measurement of the position of this peak, and thus obtain an unbiased estimate of q_0.

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Table 1

Monte Carlo Simulation Results

Redshift Distribution	q_0	$z_{\rm lim}$	% beyond $z_{\rm lim}$	$z_{\rm eff}$	$\sigma_{q_0}^{(50)}$	$\Delta q_0^{(50)}/\sigma_{q_0}^{(50)}$
Inclusive	0.5	0.45	33	0.31	0.12	0.20
Inclusive	0.5	0.50	18	0.38	0.11	0.69
Visible Galaxy	0.5	0.50	16	0.36	0.11	0.90
q_0 Dependence						
Visible Galaxy	0.1	0.45	21	0.42	0.10	0.56
Visible Galaxy	0.3	0.45	27	0.37	0.11	0.38
Visible Galaxy	0.5	0.45	31	0.34	0.12	0.20
Visible Galaxy	0.7	0.45	36	0.31	0.13	0.08
Visible Galaxy	0.9	0.45	39	0.28	0.14	0.06
Surveillance Time Effect[a]						
Visible Galaxy 14 day	0.1	0.45	20	0.42	0.10	0.77
Visible Galaxy 14 day	0.5	0.45	29	0.35	0.12	0.28
Visible Galaxy 30 day	0.1	0.45	15	0.41	0.10	1.33
Visible Galaxy 30 day	0.1	0.40	31	0.39	0.11	0.60
Visible Galaxy 30 day	0.5	0.45	24	0.34	0.15	0.79

[a]Simulation run with a 14 day or 30 day surveillance time.

Table 2

The Effect of Host Galaxy Extinction

$p_{\rm ext}$[a]	q_0	$\sigma_{q_0}^{(50)}$	$\Delta q_0^{(50)}/\sigma_{q_0}^{(50)}$
0.05	0.1	0.11	−0.41
0.10	0.1	0.11	−0.72
0.15	0.1	0.12	−0.84
0.20	0.1	0.12	−1.31
0.05	0.5	0.13	−0.49
0.10	0.5	0.14	−0.84
0.15	0.5	0.15	−1.24
0.20	0.5	0.15	−1.63

[a]Fraction of supernovae that suffer host galaxy extinction. Because approximately 27% of supernovae occur in ellipticals and half that occur in spirals suffer minimal extinction, the maximum $p_{\rm ext}$ is 0.37.

REFERENCES

Arnett, W. D., Branch, D., and Wheeler, J. C. 1985, Nature, 314, 337

Branch, D., and Miller, D. L. 1993, ApJ, 405, L5

Branch, D., et al. 1993 in preparation

Cadonau, R., Sandage, A., and Tammann, G. A. 1985, in Supernovae as Distance Indicators, ed. N. Bartel (Berlin, Springer), Lecture Notes in Physics, No.224, p. 151

Elias, J. H., Matthews, K., Neugebauer, G., and Persson, S. E. 1985, ApJ, 296, 379

Filippenko, A. V., et al. 1992, ApJ, 384, L15

Kowal, C. T. 1968, AJ, 73, 1021

Leibundgut, B. 1990, A&A, 229, 1

Leibundgut, B., and Tammann, G. A. 1990 A&A, 230, 81

Miller, D. L., and Branch, D. 1990, AJ, 100, 530

Minkowski, R. 1964, ARA&A, 2, 247

Nørgaard-Nielsen, H. U., Hansen, L., Jorgensen, H. E., Salamanca, A. A., Ellis, R. S., and Couch, W. J. 1989, Nature, 339, 523

Perlmutter, S., et al. 1993 ApJ, submitted

Perlmutter, S., Goobar, A., and Kim A. G. 1993 LBL Report, LBL-34498

Phillips, M. M. 1993, ApJ, 413, L105

Phillips, M. M., Wells, L. A., Suntzeff, N. B., Hamuy, M., Leibundgut, B., Kirshner, R. P., Foltz, C. B. 1992, AJ, 103, 1632

Tammann, G. A., and Leibundgut, B. 1990, A&A, 236, 9

Weinberg, S. 1972, in Gravitation and Cosmology, (New York, Wiley), p. 486

Fig. 1.— The absolute B magnitude distribution of SNe Ia at maximum light. The data in the histogram are from Branch et al. (1993) and represent supernovae with absolute magnitude uncertainties of less than 0.4 mag. The dashed bars represent supernovae that are spectroscopically or photometrically peculiar. The smooth curve over the main peak is a Gaussian with $\overline{M} = -18.82$ (for $H_0 = 75$ km sec^{-1} Mpc^{-1}) and a width of $\sigma_M = 0.3$. The secondary curve represents our model distribution of supernovae that suffer host galaxy extinction. Note that the three subluminous supernovae that are not in some way peculiar show evidence of host galaxy extinction (reddening or high inclination angle of the host galaxy).

Fig. 2.— The median of fit q_0 values vs. the number of supernovae per fit. The error bars represent the standard deviation of the fit q_0 values. This simulation was run using the inclusive galaxy distribution with $m_{\rm lim} = 23$ and $z_{\rm lim} = 0.45$.

Fig. 3.— The uncertainty in a measurement of q_0 vs. the number of supernovae in the fit. The solid lines represent Eq. (3) with $\sigma_M = 0.3$. The data for both the visible and inclusive supernova redshift distributions were obtained using $m_{\rm lim} = 23$ and $z_{\rm lim} = 0.45$.

Fig. 4.— A comparison of the visible galaxy and inclusive supernova redshift distributions. The distributions are normalized to include approximately the same total number of supernovae.

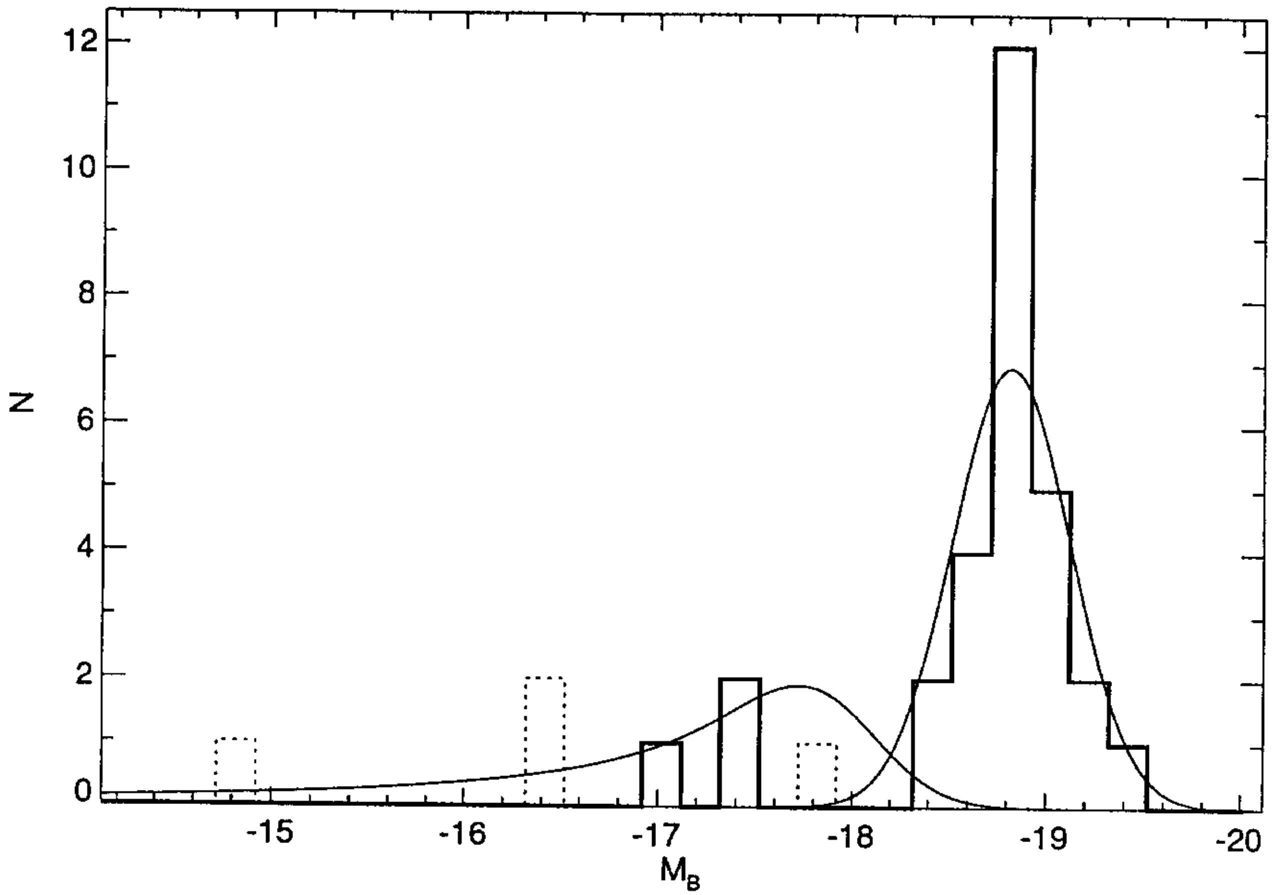


Fig. 1.

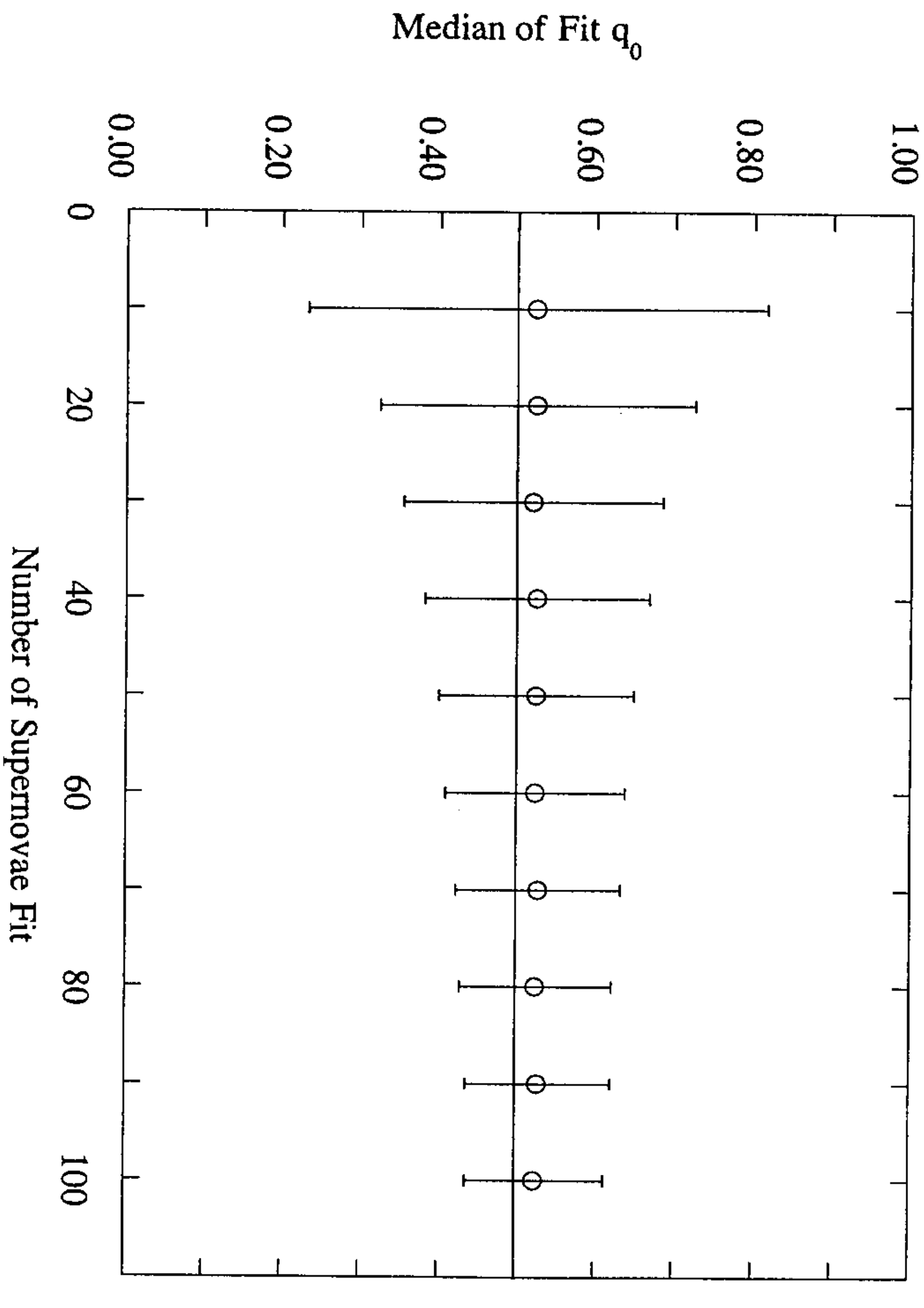
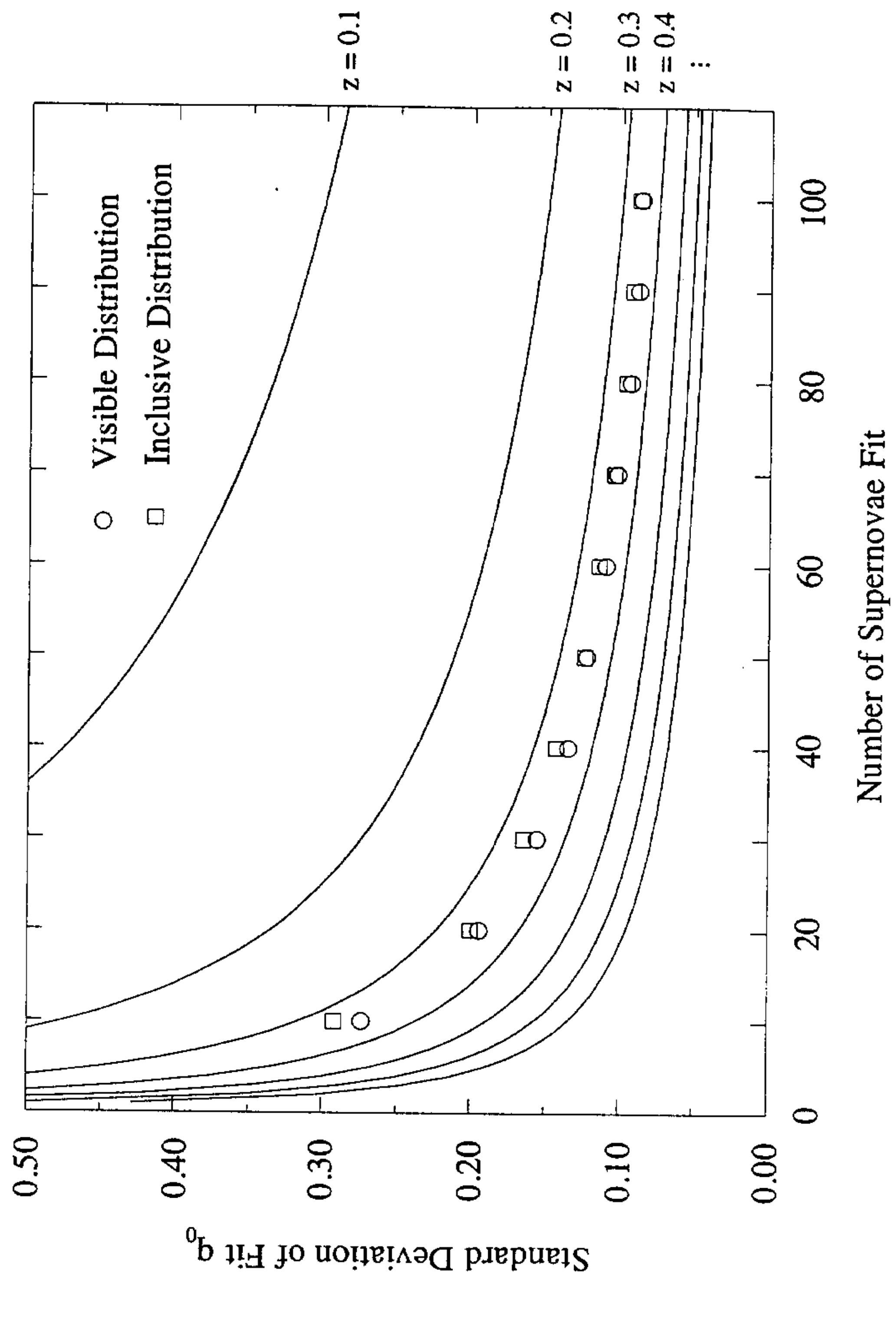

Median of Fit q0
0.00
0.20
0.40
0.60
0.80
1.00
0
20
40
60
80
100
Number of Supernovae Fit

Fig. 2.



Visible Distribution
Inclusive Distribution
z = 0.1
z = 0.2
z = 0.3
z = 0.4
0
20
40
60
80
100
Number of Supernovae Fit
0.50
0.40
0.30
0.20
0.10
0.00
Standard Deviation of Fit q_0

Fig. 3.

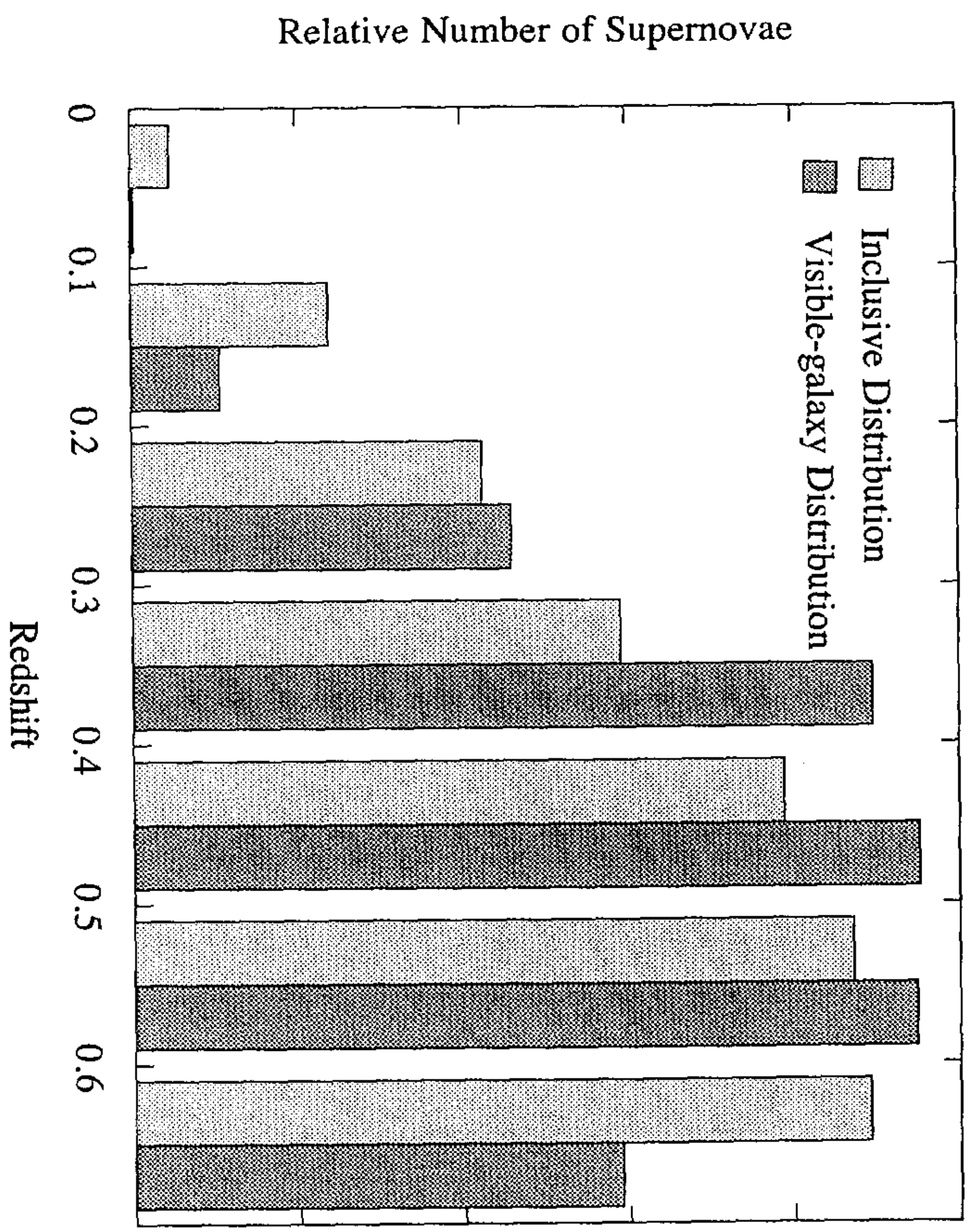

Relative Number of Supernovae
Inclusive Distribution
Visible-galaxy Distribution
0
0.1
0.2
0.3
0.4
0.5
0.6
Redshift

Fig. 4.