



Planck-Scale Unification  
and  
Dynamical Symmetry Breaking

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**Abstract**

We explore the possibility of unification of gauge couplings near the Planck scale in models of extended technicolor. We observe that models of the form  $G \times SU(3)_c \times SU(2)_L \times U(1)_Y$  cannot be realized, due to the presence of massless neutral Goldstone bosons (axions) and light charged pseudo-Goldstone bosons; thus, unification of the known forces near the Planck scale cannot be achieved. The next simplest possibility,  $G \times SU(4)_{PS} \times SU(2)_L \times U(1)_{T_{3R}}$ , cannot lead to unification of the Pati-Salam and weak gauge groups near the Planck scale. However, superstring theory provides relations between couplings at the Planck scale without the need for an underlying grand-unified gauge group, which allows unification of the  $SU(4)_{PS}$  and  $SU(2)_L$  couplings.

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The standard model of the strong and electroweak interactions is based on the gauge group  $SU(3)_c \times SU(2)_L \times U(1)_Y$ , with  $SU(2)_L \times U(1)_Y$  spontaneously broken to  $U(1)_{EM}$  at the weak scale,  $(\sqrt{2}G_F)^{-1/2} = 246$  GeV. Although the coupling strengths of the three gauge forces are apparently unrelated at ordinary energies, it is attractive to hypothesize that, as a result of their evolution, they are related at some higher energy [1]. One realization of this conjecture is grand unification, in which the standard gauge group is embedded in a larger gauge group, which is spontaneously broken at one or more scales above the weak scale [2]. The simplest example is minimal  $SU(5)$  [2], which nearly succeeds in unifying the known gauge forces at a scale of around  $10^{15}$  GeV [1], far above the weak scale.

A well-known difficulty with attempts at grand unification is the enormous disparity between the weak scale and the grand-unified scale. It is not natural for such a hierarchy of scales to occur if the gauge symmetries are broken by the vacuum-expectation values of fundamental scalar fields [1, 3]. Furthermore, a hierarchy based on fundamental scalar fields is unstable due to quadratic divergences in the renormalization of the parameters of the scalar-field potential [3]. A generic means to stabilize this hierarchy is to invoke low-energy supersymmetry (SUSY) [4]. Supersymmetry itself must be softly broken, but at a scale not far above the weak scale if it is to protect the hierarchy.

The introduction of supersymmetry requires the existence of the superpartners of the standard particles, with masses of order the SUSY breaking scale, as well as an additional Higgs doublet and its superpartner. These additional particles influence the evolution of the three gauge couplings [5]. As is well known, minimal SUSY  $SU(5)$  succeeds in unifying the three known gauge forces, at a scale of about  $10^{16}$  GeV [6]. This is often considered to be indirect evidence of the fundamental correctness of both  $SU(5)$  grand unification and supersymmetry.

The other known force, gravity, is not a gauge interaction. At ordinary energies, gravity is described by a classical field theory. The scale at which quantum gravity becomes relevant is  $(8\pi G_N)^{-1/2} \approx 2.4 \times 10^{18}$  GeV, which we will refer to as the Planck scale.<sup>1</sup> It is compelling to hypothesize that this is a fundamental scale of physics, and that unification of the four known forces should occur there. The fact that the minimal  $SU(5)$  grand-unified scale is close to the Planck scale also suggests that gravity and unification are related [1].

Despite the success of the minimal SUSY  $SU(5)$  grand-unified scenario, we wish to explore models of Planck-scale unification based on dynamical symmetry breaking [3, 7, 8]. There are several motivations for doing so. First, dynamical symmetry breaking is the only other known generic mechanism besides supersymmetry to maintain the hierarchy between the Planck scale (or grand-unified scale) and the weak scale [1, 3]. Thus it is the only realistic alternative to the SUSY grand-unified sce-

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<sup>1</sup>The energy  $G_N^{-1/2} = 1.22 \times 10^{19}$  GeV is usually called the Planck scale. The factor  $8\pi$  comes from the Einstein field equation,  $G^{\mu\nu} = 8\pi G_N T^{\mu\nu}$ .

nario. Second, it *explains* why these scales are so enormously different [3]. Third, the  $SU(3)_c$  and  $SU(2)_L$  couplings merge at about  $10^{17}$  GeV in the standard model, close to the Planck scale, if the Higgs doublet is removed from the evolution equations.<sup>2</sup> This suggests replacing the Higgs sector with some other electroweak-symmetry-breaking mechanism. Fourth, superstring theory predicts relations between couplings at the Planck scale *without the need for an underlying grand-unified gauge group* [9]. This opens up the possibility of Planck-scale unification with dynamical symmetry breaking, which may be impossible in a grand-unified approach [8][10]-[19].

Since we are attempting to relate physics at the weak scale to physics at the Planck scale, we must consider models of dynamical symmetry breaking that account for the generation of fermion masses as well as the weak-boson masses. One such class of models is extended technicolor (ETC) [20, 21].<sup>3</sup> These models have several well-known potential problems: large flavor-changing neutral currents [20, 22, 23], large contributions to low-energy precision electroweak phenomena [24], and relatively light pseudo-Goldstone bosons [20, 21]. We will not address these problems, but simply assume they may be obviated via fixed-point or walking technicolor [25], or some other mechanism. The lack of any realistic model is another difficulty with extended technicolor.

There is one potential problem with extended-technicolor models which cannot be ignored: the presence of massless neutral Goldstone bosons (weak-scale axions [27]) and light charged pseudo-Goldstone bosons, of mass  $\mathcal{O}(\alpha M_Z) \sim 5$  GeV [20, 28]. The necessary and sufficient conditions on the ETC representation for the avoidance of these particles were derived long ago by Eichten and Lane [20]. They showed that there may be at most one irreducible representation (irrep) of  $SU(2)_L$  doublets,  $\mathcal{D}_L$ , and at most two (inequivalent) irreps of  $SU(2)_L$  singlets,  $\mathcal{D}_{u^c}$  and  $\mathcal{D}_{d^c}$ , with  $SU(2)_L$  singlet leptons belonging to one or both of these.<sup>4</sup>  $SU(2)_L$  may or may not commute with the extended-technicolor group.

Using these conditions, it is easy to enumerate the grand-unified models based entirely on dynamical symmetry breaking which are potentially realistic. There can be at most one irrep of the (simple) grand-unified gauge group, since more than one irrep would produce an ETC representation which violates the above conditions [20]. This irrep must be complex to avoid unification-scale masses [29]. In order for the grand-unified group to break itself via tumbling [30], the coupling must become strong as one descends from the unification scale, so the theory must be asymptotically free. The only anomaly-free, irreducible, complex representations of simple groups which are also asymptotically free are the 16, 126, and 144-dimensional representations of

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<sup>2</sup>This is with two-loop evolution and the strong coupling  $\alpha_3(M_Z) = .115$ .

<sup>3</sup>For a discussion of gauge- and Yukawa-coupling unification in a SUSY top-quark-condensate model, see Ref. [26].

<sup>4</sup>In Ref. [20],  $\mathcal{D}_L$ ,  $\mathcal{D}_{u^c}$ , and  $\mathcal{D}_{d^c}$  are called  $\mathcal{D}_L^S$ ,  $\mathcal{D}_{u^c}^S$ , and  $\mathcal{D}_{d^c}^S$ , respectively ( $S$  = "sideways"). We have chosen to work with left-handed fermions.

SO(10); the 64-dimensional representation of SO(14); the 256-dimensional representation of SO(18); and the 27-dimensional representation of  $E_6$  [31]. The group SO(10), of rank 5, is not large enough to accommodate the standard gauge group, of rank 4, and a technicolor group. The 27-dimensional representation of  $E_6$  can accommodate only one generation of fermions. The 64-dimensional representation of SO(14) can accommodate only four generations, which is not enough to support a non-Abelian technicolor group<sup>5</sup>. This leaves the 256-dimensional spinor representation of SO(18). A grand-unified technicolor model based on this group and representation has been considered in Refs. [12, 13], and more recently in Ref. [19]; see also Ref. [18]. The group SO(10)  $\times$  SO(10), with a discrete symmetry equating the couplings and the representation (16,16), is also a candidate since as many as 22 16-dimensional representations are allowed by asymptotic freedom [31]. A model based on this group and representation has been considered in Ref. [14]. A model based on this group and the reducible representation (16, 10)  $\oplus$  (10, 16), which is asymptotically free, has been considered in Refs. [15, 17]; however, it suffers from light color-singlet Goldstone bosons.

One need not insist that the breaking of the grand-unified gauge group be dynamical. As long as this breaking occurs near the Planck scale, it may be produced by the vacuum-expectation value of a fundamental scalar field without requiring an unnatural hierarchy of scales.<sup>6</sup> It is only the breaking of the electroweak interaction which must proceed dynamically in order to produce and stabilize a hierarchy of scales [1, 3]. Thus we need not insist that the irrep of the grand-unified group be asymptotically free. Nevertheless, the restriction to an anomaly-free, irreducible, complex representation of the grand-unified gauge group is a severe constraint. Only complex representations of  $E_6$  and spinor representations of SO( $4N + 2$ ) [ $N \geq 2$ ] are generically allowed. For SU( $N$ ), the lowest-dimensional anomaly-free, irreducible, complex representation is the 374,556-dimensional representation of SU(6) [31].

Rather than pursuing grand-unified technicolor models from the top down any further, we will instead consider such models from the bottom up. Another consequence of the representation content of extended technicolor models is that quarks and leptons cannot reside in separate representations. This implies that SU(3)<sub>c</sub> and U(1)<sub>Y</sub> cannot survive as ununified groups above the ETC scale [20]. Thus, *the observed fact that SU(3)<sub>c</sub>  $\times$  SU(2)<sub>L</sub>  $\times$  U(1)<sub>Y</sub> (nearly) unify at around  $10^{15}$  GeV is an accident if nature is described by an extended-technicolor model.* Put another way, in extended-technicolor theories one necessarily loses the successful prediction of the weak mixing angle [1, 6]. In searching for Planck-scale unification of the low-energy forces, one must therefore consider groups which contain SU(3)<sub>c</sub> and U(1)<sub>Y</sub> as

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<sup>5</sup>For a two-generation model based on SO(14), with SU(2) technicolor, see Ref. [8]. See the second note added to that paper.

<sup>6</sup>However, the small observed value of the cosmological constant remains a mystery.

subgroups.<sup>7</sup> The simplest manner to achieve this, and one that is often employed in model building [32, 22][33]-[37][14]-[16], is to embed  $SU(3)_c \times U(1)_Y$  in a Pati-Salam group [38],  $SU(4)_{PS} \times U(1)_{T_{3R}}$ , where the  $U(1)_{T_{3R}}$  quantum numbers are chosen such that the standard particles have the correct hypercharge when  $SU(4)_{PS} \times U(1)_{T_{3R}}$  is broken. Alternatively,  $U(1)_{T_{3R}}$  may be the diagonal subgroup of an  $SU(2)_R$  group. Quarks and leptons reside in the four-dimensional representation of  $SU(4)_{PS}$ , with the leptons providing the fourth “color” [38]. We will pursue models of the form  $G \times SU(4)_{PS} \times SU(2)_L \times U(1)_{T_{3R}}$ , where  $G$  contains the ETC group, and attempt to unify the Pati-Salam and weak couplings near the Planck scale.

The bound  $BR(K_L \rightarrow \mu e) < 3.3 \times 10^{-11}$  (Ref. [39]) implies that  $M_{PS}/g_{PS} \gtrsim 10^6$  GeV (Ref. [22, 40]). The contribution of the broken Pati-Salam generators to the mass of the axion and the charged pseudo-Goldstone boson is therefore  $\lesssim 1$  GeV (Ref. [22]). This may be increased in walking technicolor by as much as  $M_{ETC}/\Lambda_{TC}$  [25]. Assuming  $M_{ETC}/\Lambda_{TC} \lesssim 10^3$ , the allowed range of  $M_{PS}$  is therefore about  $10^6$ - $10^7$  GeV.

The model we study has the representation content  $(G, SU(4)_{PS}, SU(2)_L, U(1)_{T_{3R}})$  of [32]

$$\begin{aligned} \mathcal{D}_L &= (N_g, 4, 2, 0) \\ \mathcal{D}_{u^c} &= (N_g, \bar{4}, 1, -\frac{1}{2}) \\ \mathcal{D}_{d^c} &= (N_g, \bar{4}, 1, +\frac{1}{2}) \end{aligned} \tag{1}$$

We leave  $G$  unspecified, since we only need the dimension of the representations, i.e., the number of generations of fermions and technifermions,  $N_g$ .  $G$  need not be simple, and may contain groups other than extended technicolor. This is the unique representation which is free of  $SU(4)_{PS} \times U(1)_{T_{3R}}$  anomalies and contains no exotic representations.  $G$  anomalies may be canceled by adding representations which are  $SU(4)_{PS} \times SU(2)_L \times U(1)_{T_{3R}}$  singlets, if needed. Such representations may also be needed to break the extended-technicolor group dynamically [32].

The one-loop renormalization-group evolution equation for the couplings is

$$\frac{1}{\alpha_n(\mu)} - \frac{1}{\alpha_n(\mu_0)} = -\frac{b_n}{2\pi} \ln \frac{\mu}{\mu_0} \tag{2}$$

where  $\alpha_n = g_n^2/4\pi$ , and  $b_n$  is the one-loop beta-function coefficient,

$$b_n = -\frac{11}{3}C_2(G) + \frac{2}{3} \sum_R T(R) \tag{3}$$

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<sup>7</sup>In Ref. [11], a class of grand-unified technicolor models of the form  $SU(N) \rightarrow SU(n)_{TC} \times SU(3)_c \times SU(2)_L \times U(1)_Y$  are ruled out based on anomaly cancellation and asymptotic freedom. Such models do not provide fermion masses, so we do not consider them.

where  $C_2(G)$  is the quadratic Casimir of the group, and  $T(R)$  is the Dynkin index of the (chiral) representation  $R$ . We equate the Pati-Salam and weak couplings at the unification scale,  $M_U$ , and evolve the couplings down to the Pati-Salam scale,  $M_{PS}$ , using the beta-function coefficients

$$b_4 = -\frac{44}{3} + \frac{4}{3}N_g \quad (4)$$

$$b_2 = -\frac{22}{3} + \frac{4}{3}N_g. \quad (5)$$

At  $M_{PS}$ ,  $SU(4)_{PS} \times U(1)_{T_3R}$  breaks down to  $SU(3)_c \times U(1)_Y$ . The strong coupling,  $\alpha_3$ , equals the Pati-Salam coupling,  $\alpha_4$ , at this scale and evolves down to the weak scale with the beta-function coefficient

$$b_3 = -11 + \frac{4}{3}N_g. \quad (6)$$

At the scale  $\Lambda_{TC}$  the technicolor force becomes strong and breaks  $SU(2)_L \times U(1)_Y$  to  $U(1)_{EM}$ . Scaling from QCD and  $SU(N)$  technicolor in the large  $N$  limit, one finds [8]

$$\Lambda_{TC} = \frac{(\sqrt{2}G_F)^{-1/2}}{f_\pi} \Lambda_{QCD} \left(\frac{3}{N}\right)^{1/2} \frac{1}{r^{1/2}} \approx (520 \text{ GeV}) \left(\frac{3}{N}\right)^{1/2} \frac{1}{r^{1/2}}$$

for  $r$  technidoublets. For one technigeneration ( $r = 4$ ) and  $N \geq 2$  one finds  $\Lambda_{TC} \leq 300$  GeV. Technifermions acquire a dynamical mass of this order, and decouple from the renormalization-group evolution below this scale. Pseudo-Goldstone bosons lighter than  $\Lambda_{TC}$  do contribute to the beta-function coefficients, but the uncertainty in their masses does not permit us to include them. Since  $\Lambda_{TC}$  is not far above  $M_Z$ , where the couplings are known, neglecting the contributions of the pseudo-Goldstone bosons introduces only a small error.<sup>8</sup> Thus, below  $\Lambda_{TC}$  we evolve the couplings down to  $M_Z$  with the beta-function coefficients,  $b_n^{SM}$ , of the three known generations of quarks and leptons.

Putting it all together yields a relation between the couplings at  $M_Z$ :<sup>9</sup>

$$\frac{1}{\alpha_2(M_Z)} - \frac{1}{\alpha_3(M_Z)} = \frac{11}{6\pi} \left[ 2 \ln \frac{M_U}{\Lambda_{TC}} - \ln \frac{M_{PS}}{\Lambda_{TC}} + \ln \frac{\Lambda_{TC}}{M_Z} \right]. \quad (7)$$

Note that  $N_g$  has canceled out; the fermions do not contribute to the relative evolution of  $\alpha_2$  and  $\alpha_3$ , nor  $\alpha_2$  and  $\alpha_4$ . Using

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<sup>8</sup>We have verified this by including the pseudo-Goldstone bosons of a one-technigeneration model, with the masses estimated in Ref. [8].

<sup>9</sup>We are neglecting the fact that  $m_t > M_Z$ . For  $m_t < 200$  GeV, this introduces only a small error.

$$\begin{aligned}\alpha_3(M_Z) &= .115 \pm .010 \\ \alpha_2(M_Z) &= \frac{\alpha(M_Z)}{\sin^2 \theta_W(M_Z)} = \frac{1}{29.7}\end{aligned}\tag{8}$$

it is easy to show that Eq. (7) cannot be satisfied for any value of  $M_{PS}$  between  $10^6$ – $10^7$  GeV and  $M_U$  between  $10^{14}$ – $10^{18}$  GeV. Thus  $SU(4)_{PS}$  and  $SU(2)_L$  cannot be unified into a larger group near the Planck scale. The reason for this observation is simple. In the standard model with no Higgs doublet, the  $SU(3)_c$  and  $SU(2)_L$  couplings meet at about  $10^{17}$  GeV, not far from the Planck scale. When  $SU(3)_c$  is subsumed by  $SU(4)_{PS}$  at  $M_{PS}$ , the beta-function coefficient decreases by  $-11/3$ , driving the Pati-Salam coupling much lower than the  $SU(2)_L$  coupling near the Planck scale.

Faced with the failure to grand-unify the Pati-Salam and weak gauge groups near the Planck scale, we turn to string unification of gauge couplings. Superstring theory is the leading candidate for a quantum theory of gravity. Although supersymmetry is necessary for a consistent string theory, it need not survive to low energies, and may be broken at the Planck scale.<sup>10</sup> A generic feature of superstring theory is tree-level relations between couplings at the string-unification scale, without the need for a grand-unified gauge group.<sup>11</sup> These relations follow from the need to embed the gauge symmetry into a unitary, modular-invariant conformal field theory [9]. The relations are of the form

$$k_n g_n^2 = g_{\text{string}}^2\tag{9}$$

where  $k_n$  is the level of the Kac-Moody algebra associated with the gauge group with coupling  $g_n$  at the string-unification scale, and  $g_{\text{string}}$  is the string coupling. The levels are positive integers for non-Abelian groups. The higher the level, the larger the allowed representations of the gauge group (e.g., for  $SU(N)$  the Dynkin labels of the representations must sum to less than or equal to  $k_n$ ). The levels for Abelian groups may take any rational value. String unification not only allows a more liberal condition for relating couplings near the Planck scale, it also frees one from the constraint that the fermions must form a single irrep of the grand-unified gauge group in extended technicolor. Even if superstring theory should ultimately prove not to be realized in nature, it provides an existence proof of Planck-scale unification other than grand unification.

The string scale,  $M_{\text{string}}$ , is related to the Planck scale,  $M_P = (8\pi G_N)^{-1/2}$ , by

$$M_{\text{string}} = g_{\text{string}} M_P\tag{10}$$

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<sup>10</sup>However, the fact that the cosmological constant vanishes in an exactly supersymmetric theory can be used to argue that SUSY should survive to low energies [41].

<sup>11</sup>For a review, see Refs. [42, 43].

at tree level. An estimate of the effect of Planck-scale physics (threshold effect) on the scale  $M_U$  at which the couplings most closely satisfy Eq. (9) is [44]

$$M_U = \frac{e^{(1-\gamma)/23-3/4}}{\sqrt{2\pi}} M_{\text{string}} \approx .2 M_{\text{string}} . \quad (11)$$

Due to the uncertainty in this estimate, we will vary the unification scale  $M_U$  between  $10^{17}$ – $10^{18}$  GeV. The fact that the minimal SUSY SU(5) grand-unification scale is about  $10^{16}$  GeV may be construed as a deficiency of the model from the perspective of string theory [45, 46, 47, 43].

Relating the  $SU(4)_{PS}$  and  $SU(2)_L$  couplings at the unification scale via Eq. (9) and evolving the couplings as before yields the relation

$$\frac{1}{k_4 \alpha_3(M_Z)} - \frac{1}{k_2 \alpha_2(M_Z)} = \frac{1}{2\pi} \left[ \left( \frac{b_4}{k_4} - \frac{b_2}{k_2} \right) \ln \frac{M_U}{\Lambda_{TC}} - \frac{(b_4 - b_3)}{k_4} \ln \frac{M_{PS}}{\Lambda_{TC}} - \left( \frac{b_2^{SM}}{k_2} - \frac{b_3^{SM}}{k_4} \right) \ln \frac{\Lambda_{TC}}{M_Z} \right] . \quad (12)$$

For  $k_2 = k_4 = 1$ , Eq. (12) reduces to Eq. (7). Thus *unification of the Pati-Salam and weak couplings cannot be achieved with unit Kac-Moody levels*. From Eq. (12) we see that this statement is true for  $k_2 = k_4$  in general.

It is possible to construct string models with different groups realized at different levels [48, 49]. Equation (12) may be solved for  $k_4/k_2$ , varying  $M_{PS}$  between  $10^6$ – $10^7$  GeV and  $M_U$  between  $10^{17}$ – $10^{18}$  GeV. The variation of  $\alpha_3$  within the range of Eq. (8) is a small effect. We find the values of  $k_4/k_2$  given in Table 1 for various choices of  $N_g$ . Only  $N_g = 8$  yields a model (nearly) consistent with  $k_4 = 2$ ,  $k_2 = 1$ . If  $SU(4)_{PS}$  and  $SU(2)_L$  are realized at different levels, they cannot be subgroups of the same group (such as  $SO(10)$ ). For  $N_g \geq 10$ , the  $SU(2)_L$  coupling blows up before  $10^{17}$  GeV. The Pati-Salam coupling is asymptotically free for  $N_g \leq 10$ .

We may also evolve the  $U(1)_{T_{3R}}$  coupling,  $\alpha_{1R}$ , up to the Planck scale and find its relation to  $\alpha_2$  and  $\alpha_4$ . The hypercharge generator is related to the  $U(1)_{T_{3R}}$  generator by<sup>12</sup>

$$Y = T_{3R} + \sqrt{\frac{2}{3}} P_{15} \quad (13)$$

where  $P_{15}$  is the  $SU(4)_{PS}$  generator  $P_{15} = 1/\sqrt{24} \text{diag}(1, 1, 1, -3)$ . The coupling  $\alpha_{1R}$  is related to the hypercharge coupling,  $\alpha_1$ , at  $M_{PS}$  by

$$\alpha_1 = \frac{\alpha_{1R} \alpha_3}{\alpha_3 + \frac{2}{3} \alpha_{1R}} . \quad (14)$$

Evolving the couplings as above, and using  $\alpha_1(M_Z) = \alpha(M_Z)/\cos^2 \theta_W = 1/98.2$ , yields the values of  $k_4/k_{1R}$  given in Table 1. The value  $k_4/k_{1R} = 1$  would suggest that

<sup>12</sup>The hypercharge generator is normalized such that  $Q = T_{3L} + Y$ .

$U(1)_{T_{3R}}$  and  $SU(4)_{PS}$  are subgroups of  $SO(10)$ , broken at  $M_U$ ; this value is (nearly) obtained for  $N_g \leq 9$  (the lower end of the range corresponds to  $M_{PS} = 10^6$  GeV,  $M_U = 10^{17}$  GeV). In a specific model, one could also evolve the ETC coupling up to the unification scale and see if it has a simple relation to the other couplings.

Table 1

$N_g$	$k_4/k_2$	$k_4/k_{1R}$
5	1.37–1.49	1.06–1.23
6	1.50–1.65	1.07–1.28
7	1.65–1.89	1.08–1.37
8	2.04–2.59	1.11–1.54
9	3.58–5.48	1.16–2.00

Although  $k_4/k_2$  may take any rational value in principle, the fact that the fermions lie in the fundamental representations of the gauge groups suggests that the levels are small. Furthermore, in specific models the levels are restricted by other considerations[49], such as the fact that the central charges of the Kac-Moody factors must sum to  $\leq 22$ . For example, consider  $N_g = 8$  with  $SU(8)$  extended technicolor, and with  $SU(4)_{PS} \times U(1)_{T_{3R}}$  as subgroups of  $SO(10)$ , broken at  $M_U$ . The central charge of a level  $k_n$  Kac-Moody algebra of the group  $G$  is

$$c_n = \frac{k_n \dim(G)}{k_n + C_2(G)}. \quad (15)$$

For  $SU(8)$  realized at level 1,  $c_8 = 7$ . For  $SU(2)_L$  realized at level 1,  $c_2 = 1$ . Thus  $c_{10}$  must be  $\leq 14$ , which implies  $k_{10} \leq 3$ .

The above analysis is accurate to one-loop order. Attempts to refine it must deal with several issues besides the extension of the beta functions to two loops. We have already mentioned the pseudo-Goldstone-boson contribution to the beta-function coefficients. The proper treatment of the threshold due to the dynamical technifermion mass is more complicated than the simple step function we used. The technicolor force influences the evolution of the other couplings at two loops, and may have a significant effect, especially if it “walks”, i.e., remains strong over an order of magnitude or more in energy.

In extended technicolor, one has in mind that there are several symmetry-breaking scales above the weak scale, and that these are ultimately responsible for the hierarchy of the masses of the three known generations of fermions. However, it is not implausible that the weak force remains ununified up to the Planck scale. We have seen that this cannot be the case for  $SU(3)_c \times U(1)_Y$ ; however, it is possible for  $SU(4)_{PS} \times U(1)_{T_{3R}}$ .

It is striking that the known fermions form representations of the group  $SU(5)$  (and also  $SO(10)$ ); this alone is compelling support for  $SU(5)$  (and perhaps even

SO(10)) grand unification. Since SU(5) is eschewed in our string-unified model (and also SO(10), from the perspective of SU(4)<sub>PS</sub> and SU(2)<sub>L</sub> unification), this may be regarded as a deficiency of this approach. However, the hypercharge quantum numbers of the known fermions may be fixed by the requirement of anomaly cancellation alone (including the mixed gravitational anomaly), without recourse to grand unification [50]. Perhaps the quantum numbers of the known fermions reflect something other than SU(5) or SO(10) grand unification.

As we remarked in the introduction, the SU(3)<sub>c</sub> and SU(2)<sub>L</sub> couplings merge at about 10<sup>17</sup> GeV, close to the Planck scale, if the Higgs doublet is removed from the standard model. Our attempt to implement this by replacing the Higgs doublet with a generation of technifermions was foiled by the need to break the chiral flavor symmetry in order to generate fermion masses. In the minimal SUSY SU(5) grand-unified model, it is actually the addition of a second Higgs doublet and the superpartners of both Higgs doublets which are responsible for the unification of the couplings; the superpartners of the other particles (in particular, the gauginos) merely increase the unification scale [5]. This again suggests that it is the electroweak-symmetry-breaking sector which is responsible for producing a successful unification of the couplings. We will never be confident of our extrapolations up to the Planck scale until we understand the electroweak- and flavor-symmetry-breaking mechanisms.

In this letter we have remarked that SU(3)<sub>c</sub> and U(1)<sub>Y</sub> cannot survive as ununified groups up to the Planck scale in extended-technicolor models, so the observed (near) unification of SU(3)<sub>c</sub> × SU(2)<sub>L</sub> × U(1)<sub>Y</sub> near the Planck scale in minimal SU(5) cannot be realized in these models. The simplest models, based on embedding SU(3)<sub>c</sub> × U(1)<sub>Y</sub> into SU(4)<sub>PS</sub> × U(1)<sub>T<sub>3R</sub></sub>, cannot unify the Pati-Salam and weak gauge groups near the Planck scale. However, superstring theory provides relations between couplings at the Planck scale without the need for an underlying grand-unified gauge group, which allows unification of the SU(4)<sub>PS</sub> and SU(2)<sub>L</sub> couplings.

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### References

- [1] H. Georgi, H. Quinn, and S. Weinberg, Phys. Rev. Lett. **33**, 451 (1974).
- [2] H. Georgi and S. Glashow, Phys. Rev. Lett. **32**, 438 (1974).

- [3] L. Susskind, Phys. Rev. D **20**, 2619 (1979).
- [4] M. Veltman, Acta Phys. Polon. B **12**, 437 (1981); L. Maiani, *Proceedings of the Summer School on Particle Physics*, Gif-Sur-Yvette, 1979, (Paris, 1980), p. 1; E. Witten, Nucl. Phys. **B188**, 513 (1981).
- [5] S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Rev. D **24**, 1681 (1981); M. Einhorn and D. R. T. Jones, Nucl. Phys. **B196**, 475 (1982); W. Marciano and G. Senjanovic, Phys. Rev. D **25**, 3092 (1982).
- [6] J. Ellis, S. Kelley, and D. Nanopoulos, Phys. Lett. **260B**, 131 (1991); U. Amaldi, W. de Boer, and H. Fürstenau, Phys. Lett. **260B**, 447 (1991); P. Langacker and M. Luo, Phys. Rev. D **44**, 817 (1991).
- [7] S. Weinberg, Phys. Rev. D **13**, 974 (1976); D **19**, 1277 (1979).
- [8] E. Fahri and L. Susskind, Phys. Rep. **74**, 277 (1981).
- [9] P. Ginsparg, Phys. Lett. **197B**, 139 (1987).
- [10] E. Fahri and L. Susskind, Phys. Rev. D **20**, 3404 (1979).
- [11] P. Frampton, Phys. Rev. Lett. **43**, 1912 (1979).
- [12] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, eds. P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315.
- [13] F. Wilczek and A. Zee, Phys. Rev. D **25**, 553 (1982).
- [14] A. Davidson, P. Mannheim, and K. Wali, Phys. Rev. Lett. **45**, 1135 (1980).
- [15] A. Davidson, P. Mannheim, and K. Wali, Phys. Rev. Lett. **47**, 149 (1981); Phys. Rev. D **26**, 1133 (1982); F. del Aguila and A. Mendez, Z. Phys. C **13**, 347 (1982).
- [16] S. King, Phys. Lett. **184B**, 49 (1987).
- [17] S. King, Nucl. Phys. **B320**, 487 (1989).
- [18] T. Appelquist and J. Terning, in *Proceedings of the International Workshop on Electroweak Symmetry Breaking*, Hiroshima, 1991, eds. W. Bardeen, J. Kodaira, and T. Muta (World Scientific, Singapore, 1992), p. 68.
- [19] G. Giudice and S. Raby, Nucl. Phys. **B368**, 221 (1992).
- [20] E. Eichten and K. Lane, Phys. Lett. **90B**, 125 (1980).
- [21] S. Dimopoulos and L. Susskind, Nucl. Phys. **B155**, 237 (1979).
- [22] S. Dimopoulos, S. Raby, and G. Kane, Nucl. Phys. **B182**, 77 (1981).

- [23] S. Dimopoulos and J. Ellis, Nucl. Phys. **B182**, 505 (1981).
- [24] M. Peskin and T. Takeuchi, Phys. Rev. Lett. **65**, 964 (1990); B. Holdom and J. Terning, Phys. Lett. **247B**, 88 (1990); M. Golden and L. Randall, Nucl. Phys. **B361**, 3 (1991).
- [25] B. Holdom, Phys. Rev. D **24**, 1441 (1981); Phys. Lett. **150B**, 301 (1985); K. Yamawaki, M. Bando, and K. Matumoto, Phys. Rev. Lett. **56**, 1335 (1986); T. Appelquist, D. Karabali, and L. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986); T. Appelquist and L. Wijewardhana, Phys. Rev. D **36**, 568 (1987).
- [26] C. Wagner, MPI-Ph/93-25 (1993), to appear in the proceedings of *Properties of SUSY Particles*, Erice, 1992.
- [27] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).
- [28] S. Dimopoulos, Nucl. Phys. **B168**, 69 (1980); M. Peskin, Nucl. Phys. **B175**, 197 (1980); J. Preskill, Nucl. Phys. **B177**, 21 (1981).
- [29] H. Georgi, Nucl. Phys. **B156**, 126 (1979).
- [30] S. Raby, S. Dimopoulos, and L. Susskind, Nucl. Phys. **B169**, 373 (1980).
- [31] E. Eichten, K. Kang, and I.-G. Koh, J. Math Phys. **23**, 2529 (1982).
- [32] S. Dimopoulos, S. Raby, and P. Sikivie, Nucl. Phys. **B176**, 449 (1980).
- [33] P. Binétruy, A. Chadha, and P. Sikivie, Phys. Lett. **107B**, 425 (1981).
- [34] B. Holdom, Phys. Rev. D **23**, 1637 (1981); D **24**, 157 (1981); Phys. Rev. Lett. **57**, 2496 (1986); Phys. Lett. **143B**, 227 (1984); **246B**, 169 (1990).
- [35] S. Dimopoulos and S. Raby, Nucl. Phys. **B192**, 353 (1981).
- [36] H. Georgi, in *Architecture of Fundamental Interactions at Short Distances*, Les Houches, 1985, eds. P. Ramond and R. Stora (North-Holland, Amsterdam, 1987), p. 339.
- [37] T. Appelquist and L. Wijewardhana, Phys. Rev. D **36**, 568 (1987).
- [38] J. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974).
- [39] K. Arisaka *et al.*, Phys. Rev. Lett. **70**, 1049 (1993).
- [40] N. Deshpande and R. Johnson, Phys. Rev. D **27**, 1193 (1983); J. Ritchie and S. Wojcicki, DOE-ER40757-016, CPP-93-16 (1993).

- [41] M. Dine, R. Rohm, N. Seiberg, and E. Witten, *Phys. Lett.* **156B**, 55 (1985).
- [42] L. Ibáñez, CERN-TH.5982/91, lectures presented at the 1990 CERN School of Physics, Mallorca, Spain, 1990; CERN-TH.6342/91, talk given at the Workshop on Electroweak Physics beyond the Standard Model, Valencia, Spain, 1991.
- [43] S. Weinberg, in *Proceedings of the XXVI International Conference on High-Energy Physics*, Dallas, TX, 1992, ed. J. Sanford (American Institute of Physics, New York, 1993), p. 346.
- [44] V. Kaplunovsky, *Nucl. Phys.* **B307**, 145 (1988); erratum **B382**, 436 (1992).
- [45] L. Ibáñez, *Phys. Lett.* **126B**, 196 (1983); L. Ibáñez, D. Lüst, and G. Ross, *Phys. Lett.* **272B**, 251 (1991).
- [46] I. Antoniadis, J. Ellis, S. Kelley, and D. Nanopoulos, *Phys. Lett.* **272B**, 31 (1991); S. Kelley, J. Lopez, and D. Nanopoulos, *Phys. Lett.* **278B**, 140 (1992).
- [47] D. Bailin and A. Love, *Phys. Lett.* **278B**, 125 (1991); **280B**, 26 (1992); **292B**, 315 (1992).
- [48] D. Lewellen, *Nucl. Phys.* **B337**, 61 (1990).
- [49] A. Font, L. Ibáñez, and F. Quevedo, *Nucl. Phys.* **B345**, 389 (1990).
- [50] A. Font, L. Ibáñez, and F. Quevedo, *Phys. Lett.* **228B**, 79 (1989); C. Geng and R. Marshak, *Phys. Rev. D* **39**, 693 (1989); **D 41**, 717 (1990); J. Minahan, P. Ramond, and R. Warner, *Phys. Rev. D* **41**, 715 (1990); K. Babu and R. Mohapatra, *Phys. Rev. D* **41**, 271 (1990); X.-G. He, G. Joshi, and R. Volkas, *Phys. Rev. D* **41**, 278 (1990).