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Reconstructing the inflaton potential— perturbative reconstruction to second-order

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One method to reconstruct the scalar field potential of inflation is a perturbative approach, where the values of the potential and its derivatives are calculated as an expansion in departures from the slow-roll approximation. They can then be expressed in terms of observable quantities, such as the square of the ratio of the gravitational wave amplitude to the density perturbation amplitude, the deviation of the spectral index from the Harrison–Zel'dovich value, etc. Here, we calculate complete expressions for the second-order contributions to the coefficients of the expansion by including for the first time corrections to the standard expressions for the perturbation spectra. As well as offering an improved result, these corrections indicate the expected accuracy of the reconstruction. Typically the corrections are only a few percent.

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I. INTRODUCTION

An intriguing prospect raised by recent large-scale structure observations, particularly those of the Cosmic Background Explorer (COBE) satellite [1], is that observations may soon provide rather detailed information regarding the nature of the vacuum energy driving inflation [2,3,4]. In most models this vacuum energy is identified as the self-interaction potential of a scalar *inflaton* field, and its precise form is determined by some particle physics model. Since there currently exist many possible models [5], it is of great interest to investigate whether observations can select which, if any, of these models is correct. As well as having detailed implications for the initial conditions for structure formation in the Universe, the energy scale of inflation provides a link with particle physics at high energies, and may be a useful method of probing models of unification.

The prime observational consequences of inflation derive from the stochastic spectra of density (scalar) perturbations and gravitational wave (tensor) modes generated during inflation. Each stretches from scales of order centimeters to scales well in excess of the size of the presently observable Universe. Once within the Hubble radius, gravitational waves redshift away and so their main influence is on the large-scale microwave background anisotropies, such as those probed by COBE [6]. Advanced gravitational wave detectors such as the proposed beam-in-space experiments may be able to detect the gravitational waves on a much shorter (about 10^{14}cm) wavelength range [7]. The density perturbations are thought to lead to structure formation in the Universe. They produce microwave background anisotropies across a much wider range of angular scales than do the tensor modes, and constraints on the scalar spectrum are also available from the clustering of galaxies and galaxy clusters, peculiar velocity flows and a host of other measurable quantities [4].

Recently, we provided a formalism which allows one to reconstruct the inflaton potential $V(\phi)$ directly from a knowledge of these spectra [8,9]. This developed an original but incomplete analysis by Hodges and Blumenthal [10]. An important result that follows from our formalism is that knowledge of the scalar spectrum alone is insufficient for a unique reconstruction. Reconstruction from only the scalar spectrum leaves an arbitrary integration constant, and since the reconstruction is nonlinear, different choices of this constant lead to different functional forms for the potential. A minimal knowledge of the tensor spectrum, say its amplitude at a single wavelength, is sufficient to lift this degeneracy. With further information the problem becomes overdetermined, providing powerful consistency relations which would exclude inflation if not satisfied.

The most ambitious aim of reconstruction is to employ observational data to deduce the complete functional form of the inflaton potential over the range corresponding to large-scale structure. The observational situation is some way from providing the quality of data that this would require, and at present a more realistic approach is to attempt a reconstruction of the potential about a single point ϕ_0 [9].

For this one requires such information as the amplitudes of both scalar and tensor modes and also the spectral index of the scalar perturbations at a single scale. It is possible that such information can be deduced from a combination of microwave background experiments that span a range of angular scales [11]. In this paper it is our aim to provide an improved calculation of the coefficients of such a perturbative reconstruction.

To some extent all inflationary calculations rely on the use of the slow-roll approximation. In the form we present here, the slow-roll approximation is an expansion in terms of quantities defined from derivatives of the Hubble parameter H. In general there are an infinite hierarchy of these which can in principle all enter at the same order in an expansion. However, to calculate $V(\phi_0)$ one needs only the first derivative of H, for $V'(\phi_0)$ one needs up to the second derivative of H, and so on. These parameters can be converted into more observationally related quantities, as we shall see.

The slow-roll approximation arises in two separate places. The first is in simplifying the classical inflationary dynamics of expansion, and the lowest-order approximation ignores the contribution of the inflaton's kinetic energy to the expansion rate. The second is in the calculation of the perturbation spectra; the standard expressions are true only to lowest-order in slow-roll. In our earlier work [8,9], we utilized the Hamilton-Jacobi approach [12] to treat the dynamical evolution exactly, but were forced for analytic tractability to retain the lowest-order approximation for the perturbation spectra. Technically therefore, the results were accurate only to lowest-order, though in models close to the power-law inflation limit this hybrid approach offers substantial improvements for certain quantities.

Until recently further improvements have not been possible, but a very elegant calculation of the perturbation spectra to next order in slow-roll has now been provided by Stewart and Lyth [13]. This does not permit analytic progress in functional reconstruction, but their results can be combined with the Hamilton-Jacobi approach to generate the complete second-order term in perturbative reconstruction. The purpose of this work is to calculate this correction. This serves two useful purposes. Firstly, the results allow a more accurate reconstruction to be performed, and secondly the relative size of lowest-order and second-order contributions provides a useful (though not rigorous) measure of the theoretical error in reconstruction. As we shall see, even the lowest-order results are typically accurate to within a few percent.

II. TO SECOND-ORDER IN SLOW-ROLL

We shall use the notation and philosophy of our earlier paper [9], except that the perturbation spectra shall be given by expressions improving on Eq. (3.4) of that work. The Hamilton-Jacobi equations arise when one uses the scalar field ϕ as a time

variable, and writes the Hubble parameter $H = \dot{a}/a$, where a is the scale factor, as a function of ϕ. The field equations are [12]

$$[H'(\phi)]^2 - \frac{3}{2}\kappa^2 H^2(\phi) = -\frac{1}{2}\kappa^4 V(\phi)\,, \tag{2.1}$$

$$\kappa^2\dot{\phi} = -2H'\,, \tag{2.2}$$

where dots are time derivatives, primes are ϕ derivatives, $\kappa^2 = 8\pi/m_{Pl}^2$ and m_{Pl} is the Planck mass. Without loss of generality we may assume $\dot{\phi} > 0$, so that $H'(\phi) < 0$. Where square roots appear later this choice is used to fix the sign of the prefactor.

The slow-roll approximation can be specified by parameters defined from derivatives of $H(\phi)$. There are in general an infinite number of these as each derivative is independent, but usually only the first few enter into any expressions. We shall require the first three, which are all of the same order when defined by[1]

$$\begin{aligned}
\epsilon(\phi) &= \frac{2}{\kappa^2}\left[\frac{H'(\phi)}{H(\phi)}\right]^2\,,\\
\eta(\phi) &= \frac{2}{\kappa^2}\frac{H''(\phi)}{H(\phi)} = \epsilon - \frac{\epsilon'}{\sqrt{2\kappa^2\epsilon}}\,,\\
\xi(\phi) &= \frac{2}{\kappa^2}\frac{H'''(\phi)}{H'(\phi)} = \eta - \frac{2\eta'}{\sqrt{2\kappa^2\epsilon}}\,.
\end{aligned} \tag{2.3}$$

The slow-roll approximation applies when these slow-roll parameters are small in comparison to unity. The condition for inflation, $\ddot{a} > 0$, is precisely equivalent to $\epsilon < 1$.

The lowest-order expressions for the scalar (A_S) and tensor (A_G) amplitudes assume $\{\epsilon,\ \eta,\ \xi\}$ are negligible compared to unity. Improved expressions for the scalar and tensor amplitudes for finite but small $\{\epsilon,\ \eta,\ \xi\}$ were found by Stewart and Lyth [13]:

$$A_S \simeq -\frac{\sqrt{2}\kappa^2}{8\pi^{3/2}}\frac{H^2}{H'}\left[1 - (2C+1)\epsilon + C\eta\right]\,, \tag{2.4}$$

$$A_G \simeq \frac{\kappa}{4\pi^{3/2}} H\left[1 - (C+1)\epsilon\right]\,, \tag{2.5}$$

where $C = -2 + \ln 2 + \gamma \simeq -0.73$ is a numerical constant, $\gamma \approx 0.577$ being the Euler constant. The right hand sides of these expressions are evaluated when the scale in question crosses the Hubble radius during inflation, $2\pi/\lambda = aH$. The spectra can equally well be considered to be functions of wavelength or of the scalar field value. Eq. (2.7) below allows one to move from one to the other.

The standard results to lowest-order are given by setting the square brackets to unity. Historically it has been common even for this result to be written as only

[1] Let us stress that our choice $\dot{\phi} > 0$ implies $\sqrt{\epsilon} = -\sqrt{2/\kappa^2}\,H'/H$; one needs to be careful with the signs to reproduce our results.

an approximate equality (the ambiguity arising primarily because of a vagueness in defining the precise meaning of the density perturbation), though the precise normalization to lowest-order was established some time ago by Lyth [14] (see also the discussion in [4]).

The improved expressions for the spectra in Eqs. (2.4) and (2.5) are accurate in so far as ϵ and η are sufficiently slowly varying functions that they can be treated adiabatically as constants while a given scale crosses outside the Hubble radius. Corrections to this would enter at next order. This differs from the usual situation in which H is treated adiabatically. For the standard calculation to be strictly valid H must be constant, but provided it varies sufficiently slowly (characterized by small ϵ and $|\eta|$), it can be evaluated separately at each epoch. This injects a scale dependence into the spectra. There is a special case corresponding to power-law inflation for which ϵ and η are precisely constant and equal to each other. In this case the above expressions for the perturbation spectra are exact [13,15]. Furthermore, the corrections to each spectrum are the same and they cancel when the ratio is taken. In the general case ϵ and η may be treated as different constants if it is assumed that the timescale for their evolution is much longer than the timescale for perturbations to be imprinted on a given scale. This assumption worsens as η is removed from ϵ, which would be characterized by the next order terms becoming large.

Throughout we shall be quoting results which feature a leading term and a correction term linear in the slow-roll parameters. We shall utilize the symbol "$\simeq$" to indicate this level of accuracy throughout. The correction terms shall be placed in square brackets, so the lowest-order equations can always be obtained by setting the square brackets equal to one. A useful relationship can be obtained from Eqs. (2.3)–(2.5):

$$\epsilon \simeq \frac{A_G^2}{A_S^2}\left[1 - 2C(\epsilon - \eta)\right] . \tag{2.6}$$

As we shall see, η is encoded in the spectral index of the scalar perturbations, whose deviation from unity must also be small for slow-roll to apply.

A key equation in Refs. [8,9] is the consistency equation, which connects the scalar spectrum to the tensor spectrum and its derivative. The spectra as given in Eqs. (2.4) and (2.5) are functions of the value of ϕ when the fluctuations crossed the Hubble radius during inflation. This is converted into a dependence on wavelength λ with the relation [9]

$$\frac{d\lambda}{d\phi} = \lambda \frac{H}{H'} \frac{\kappa^2}{2}\left[1 - \epsilon\right] . \tag{2.7}$$

Differentiation of Eq. (2.5) with respect to ϕ implies that

$$\frac{d \ln A_G}{d\phi} \simeq -\sqrt{\frac{\kappa^2}{2}} \frac{A_G}{A_S}\left[1 + (C+2)\epsilon - (C+2)\eta\right] , \tag{2.8}$$

and it follows that

$$\frac{\lambda}{A_G}\frac{dA_G}{d\lambda} = \frac{A_G^2}{A_S^2}\left[1 + 3\epsilon - 2\eta\right] . \tag{2.9}$$

As expected this agrees with the expansion of the corresponding expression in Ref. [9] for the special case $\epsilon = \eta$. This equation is interesting in its own right, but for perturbative reconstruction its use is restricted to the removal of derivatives of the tensor spectrum.

III. PERTURBATIVE RECONSTRUCTION TO SECOND ORDER

The aim now is to obtain expressions for the potential and its derivatives about a single point ϕ_0, given information regarding the spectra at the scale λ_0 which left the horizon at $\phi = \phi_0$. The four main quantities of observational interest are the amplitudes and spectral indices of the two spectra. However, in view of the consistency equation, Eq. (2.9), only three of these are independent. We shall concentrate on the two amplitudes and the scalar spectral index, since these are probably the easiest to measure. The scalar spectral index n is defined by

$$1 - n = \frac{d\ln A_S^2(\lambda)}{d\ln\lambda} . \tag{3.1}$$

To lowest-order in slow-roll one can show that the spectral index is given by [9]

$$1 - n = 4\epsilon - 2\eta , \tag{3.2}$$

which provides the route to determining η. Conceptually we are passing from these three observables to the three parameters that describe the potential, which are the slow-roll parameters ϵ and η, and the overall normalization. In terms of the observables, therefore, the slow-roll approximation amounts to an expansion in both A_G^2/A_S^2 and $(1-n)$, which give corrections to the same order. We shall see that the correction term for $V''(\phi_0)$ requires the introduction of the third slow-roll parameter ξ, requiring a new independent observable to determine it.

One obtains directly from the field equation Eq. (2.1) and the definitions of the spectra in Eqs. (2.4) and (2.5) an expression for the amplitude of the potential:

$$V(\phi_0) \simeq \frac{48\pi^3}{\kappa^4}A_G^2(\lambda_0)\left[1 + \left(\frac{5}{3} + 2C\right)\frac{A_G^2(\lambda_0)}{A_S^2(\lambda_0)}\right] , \tag{3.3}$$

$$\simeq \frac{48\pi^3}{\kappa^4}A_G^2(\lambda_0)\left[1 + 0.21\frac{A_G^2(\lambda_0)}{A_S^2(\lambda_0)}\right] . \tag{3.4}$$

In Refs. [8,9], we gave the numerical factor on the second-order term as $-1/3$, which incorporated only the dynamical slow-roll corrections. In fact, the spectral corrections

to $V(\phi_0)$ dominate the dynamical ones for any inflation model, reversing the sign of the correction, which may be significant if the tensors are important. However the relative contribution of the scalar and tensor modes to large angle microwave anisotropies with our spectral normalization is given approximately by R [9], where

$$R = \frac{2A_S^2}{25A_G^2} \,. \tag{3.5}$$

Even if the contributions to the anisotropies from scalar and tensor modes are equal, the correction term in the potential is only 2%. This is a powerful indication that even the lowest-order perturbative reconstruction promises to be very accurate.

To obtain $V'(\phi_0)$ we need the scalar spectral index at λ_0, denoted by n_0. Differentiation of the potential with respect to ϕ, followed by some straightforward algebra gives

$$\begin{aligned} V'(\phi_0) &\simeq -\frac{96\pi^3}{\sqrt{2}\kappa^3}\frac{A_G^3(\lambda_0)}{A_S(\lambda_0)}\left[1+(C+2)\epsilon+(C-1/3)\eta\right] , \\ &\simeq -\frac{96\pi^3}{\sqrt{2}\kappa^3}\frac{A_G^3(\lambda_0)}{A_S(\lambda_0)}\left[1+1.27\epsilon-1.06\eta\right] , \\ &\simeq -\frac{96\pi^3}{\sqrt{2}\kappa^3}\frac{A_G^3(\lambda_0)}{A_S(\lambda_0)}\left[1-0.85\frac{A_G^2(\lambda_0)}{A_S^2(\lambda_0)}+0.53(1-n_0)\right] . \end{aligned} \tag{3.6}$$

Note that for power-law inflation, which has [6]

$$(1-n_0) \simeq \frac{25}{4\pi}\frac{A_G^2(\lambda_0)}{A_S^2(\lambda_0)} , \tag{3.7}$$

the corrections in the square brackets nearly cancel, but other models [16,17] can feature larger corrections, *e.g.* 16 % for natural inflation with $n_0 = 0.7$.

The calculation for $V''(\phi_0)$ is much more involved. One can show a precise relationship

$$\frac{V''(\phi_0)}{H^2(\phi_0)} = 3(\epsilon+\eta) - \left(\eta^2+\epsilon\xi\right) . \tag{3.8}$$

A new observable will be needed to determine ξ, the easiest example being the rate of change of the scalar spectral index. This would be substantially harder to measure, and it is fortunate that it only enters at second-order. [It would however enter at leading order in $V'''(\phi_0)$]. From Eqs. (2.5) and (3.8), we can obtain the second-order correction to $V''(\phi_0)$ in terms of the slow-roll parameters

$$V''(\phi_0) \simeq \frac{48\pi^3}{\kappa^2}A_G^2(\lambda_0)\,(\epsilon+\eta)\left[1+(2C+2)\epsilon-\frac{\eta^2+\epsilon\xi}{3(\epsilon+\eta)}\right] . \tag{3.9}$$

It is however harder to convert the prefactor into observables, because there are now lowest-order terms in both ϵ and η. To generate the correct second-order term, it is

not enough to use the first-order expression for η in terms of the spectral index. One must instead use the second-order result, as given by Stewart and Lyth [13]

$$1 - n \simeq 4\epsilon - 2\eta + 8(1+C)\epsilon^2 - (6+10C)\epsilon\eta + 2C\epsilon\xi\,, \tag{3.10}$$

(our η being the negative of their δ), which leads to

$$\begin{aligned} \epsilon + \eta &\simeq 3\epsilon\left[1 + \frac{4C+4}{3}\epsilon - \frac{5C+3}{3}\eta + \frac{C}{3}\xi\right] - \frac{1-n_0}{2}\,, \\ &\simeq 3\frac{A_G^2}{A_S^2}\left[1 + \frac{4-2C}{3}\epsilon + \frac{C-3}{3}\eta + \frac{C}{3}\xi\right] - \frac{1-n_0}{2}\,. \end{aligned} \tag{3.11}$$

Substituting this into Eq. (3.9) yields

$$\begin{aligned} V''(\phi_0) &\simeq \frac{144\pi^3}{\kappa^2}\frac{A_G^4(\lambda_0)}{A_S^2(\lambda_0)}\left[1 + \frac{4C+10}{3}\epsilon + \frac{C-3}{3}\eta + \frac{C}{3}\xi\right] \\ &\quad - \frac{24\pi^3}{\kappa^2}A_G^2(\lambda_0)\,(1-n_0)\,[1+(2C+2)\epsilon] - \frac{16\pi^3}{\kappa^2}A_G^2(\eta^2+\epsilon\xi)\,, \end{aligned} \tag{3.12}$$

where the last term is entirely second-order. Note that there are two lowest-order terms. An interesting case is $\eta = -\epsilon$, corresponding to $H \propto \phi^{1/2}$, for which the lowest-order term vanishes identically and the final term of Eq. (3.9) is the only one to contribute. The second derivative of the potential is the lowest derivative at which it is possible for the expected lowest-order term to vanish.

The final step is to convert the second-order terms into the observables. As they are already second-order, one only needs the lowest term in their expansion to convert. From the expression for the spectral index, one finds to lowest-order that

$$\xi \simeq \frac{1}{2\epsilon}\left.\frac{dn}{d\ln\lambda}\right|_{\lambda_0} + 5\eta - 4\epsilon\,. \tag{3.13}$$

Note that the derivative of the spectral index is of order ϵ^2. To lowest-order we also have

$$\eta \simeq 2\epsilon - \frac{1}{2}(1-n_0); \qquad \epsilon \simeq \frac{A_G^2}{A_S^2}\,. \tag{3.14}$$

Progressively substituting all these into Eq. (3.12) yields

$$\begin{aligned} V''(\phi_0) &\simeq \frac{16\pi^3}{\kappa^2}A_G^2(\lambda_0)\left[9\frac{A_G^2(\lambda_0)}{A_S^2(\lambda_0)} - \frac{3}{2}(1-n_0) + (36C+2)\frac{A_G^4(\lambda_0)}{A_S^4(\lambda_0)}\right. \\ &\quad -\frac{1}{4}(1-n_0)^2 - (12C-6)\frac{A_G^2(\lambda_0)}{A_S^2(\lambda_0)}(1-n_0) \\ &\quad \left. + \frac{3C-1}{2}\left.\frac{dn}{d\ln\lambda}\right|_{\lambda_0}\right]\,, \end{aligned} \tag{3.15}$$

where the first two terms are lowest-order and the remainder are second-order.[2]

For power-law models, the last term is zero and the remaining correction terms nearly cancel each other, though they are not small individually. For natural inflation models the correction terms are all individually small, with an overall correction of about 4% at $n_0 = 0.7$. (In natural inflation, $(dn/d\ln\lambda)|_{\lambda_0} \simeq \eta^2 \simeq (1-n_0)^2/16$ [13].)

IV. DISCUSSION AND CONCLUSIONS

To conclude, we have calculated the full second-order corrections to the perturbative reconstruction of the inflaton potential. The first-order terms agree with those we found previously [8,9], while the second-order terms offer an improvement. They serve to quantify the expected errors in the perturbative reconstruction, and in general these errors are small. Even in cases where tensors provide a substantial contribution to the large angle microwave background anisotropies and/or the spectral index deviates significantly from unity, the corrections are typically only a few percent. Consequently, example figures based on plausible data sets that we presented in our earlier papers remain valid. One then has some degree of confidence that one can use our lowest-order expressions as was done in [18].

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