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Inhomogeneous Field Configurations
and the
Electroweak Phase Transition

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Abstract

We investigate the effects of inhomogeneous scalar field configurations on the electroweak phase transition. For this purpose we calculate the leading perturbative correction to the wave function correction term $Z(\varphi, T)$, i.e., the kinetic term in the effective action, for the electroweak Standard Model at finite temperature and the top quark self–mass. Our finding for the fermionic contribution to $Z(\varphi, T)$ is infra–red finite and disagrees with other recent results. In general, neither the order of the phase transition nor the temperature at which it occurs change, once $Z(\varphi, T)$ is included. But a non–vanishing, positive (negative) $Z(\varphi, T)$ enhances (decreases) the critical droplet surface tension and the strength of the phase transition. We find that in the range of parameter space, which allows for a first–order phase transition, the wave function correction term is negative — indicating a weaker phase transition — and especially for small field values so large that perturbation theory becomes unreliable.

I Introduction

The understanding of the electroweak phase transition has matured rapidly during recent years. The original work on restoration of gauge symmetries at high temperatures [1] and systematic, partial summation of perturbation theory [2] has attracted a lot of attention in the framework of the Standard Model, since the suggestion of a mechanism for baryo–genesis at the electroweak scale [3]. Already in the early eighties it became clear that non–abelian gauge theories contain massless degrees of freedom in perturbation theory at high temperatures [4]. Therefore, infrared problems complicate the straightforward setup of perturbation theory and a considerable effort was made recently to resolve them [5]-[17]. Up to Higgs masses of about 80 GeV, these difficulties may be cured by introducing improved propagators (c.f. e.g., [14]-[17]). In the improved perturbation theory self–energy and self–mass corrections are included in the action from the beginning. They are determined by solving appropriate special cases of the Dyson–Schwinger equations, the gap equations [2, 16, 17]. Keeping so–called "hard thermal loops" and summing over "soft thermal loops" the leading infrared singularities can be shown to cancel out [18].

The order of the phase transition depends crucially on the value of the magnetic mass of the gauge bosons, whose calculation requires non–perturbative techniques. Although different approaches suggest the same order of magnitude [16, 19], the determination of its value to within more then 10% accuracy would be desirable. As long as the magnetic mass is smaller than $0.07\,T$, where T denotes the temperature of the heat bath, the phase transition appears to be of first order in the range of Higgs masses accessible to perturbation theory. However, it is still an open question, whether the phase transition within the electroweak standard model is strong enough to account for the observed baryon asymmetry of the universe [20]. Due to its weakness the phase transition may even proceed via formation of sub–critical droplets [21].

All the above results have been extracted purely from the effective potential $V_{\text{eff}}(\varphi, T)$, i.e., the effective action for vanishing derivative terms $\partial_\mu \varphi = 0$ of the scalar field φ, which plays the role of the order parameter. In this paper we investigate whether the full effective action is indeed dominated by homogeneous field configurations and whether it is justified to neglect quantum corrections which give rise to derivative terms in the effective action. We wish, however, to point out from the beginning that, as a matter of fact, a consistent one–loop treatment of the phase transition requires the inclusion of the wave function correction term whose leading contribution occurs at this level in perturbation theory. Indeed, it was argued in refs. [22]-[25] that depending on the theory under consideration the expansion of the effective action around $\partial_\mu \varphi = 0$ might break down and non–perturbative effects become important. In addition we will see that quantities essential for the mechanism of critical droplet nucleation such as the droplet surface tension and the strength of the phase transition are modified by the

presence of higher derivative terms. Although, in this paper, we will perform a perturbative calculation there are techniques of averaging the action over a range of momenta without expanding it in terms of derivatives [26].

The outline of the paper is as follows: In section II we will introduce some essential quantities for the description of first–order phase transitions, review their derivation for homogeneous scalar field configurations and extend the analysis to inhomogeneous field configurations. In particular, we explain the impact of the wave function correction factor $Z(\varphi, T)$ on droplet nucleation rate and surface tension for a first–order phase transition. In section III we discuss the self–energy corrections for the various particles in the theory in the presence of a plasma which help to cut off infrared divergences. A general method to derive the kinetic term in the effective action developed in [27] is reviewed in section IV where we explicitly calculate the different contributions to the wave function correction term for the Standard Model. Our results are described in section V, and our conclusions are presented in section VI. A discussion of the top quark self–mass and several useful integrals with their high–temperature expansions have been relegated to two appendices.

II The wave function correction term

A Decay of metastable states

The entire dynamics of the phase transition is contained in the quantity

$$Z_\beta[\varphi] = \int [\mathcal{D}\hat{\varphi}][\mathcal{D}W_\mu][\mathcal{D}\psi] \exp\left\{S_\beta[\varphi + \hat{\varphi}, W_\mu, \psi]\right\} . \tag{II.1}$$

The path integral is performed over fluctuations $\hat{\varphi}$ around a classical field configuration $\varphi(x)$, over vector and ghost fields, whose measure is collectively denoted by $[\mathcal{D}W_\mu]$ and over fermion fields with measure $[\mathcal{D}\psi]$. The exponent contains the classical action S_β at finite temperature $T = \beta^{-1}$. An additional term that vanishes for stationary field configurations has been neglected.

Following the approach of [16], in a first step we integrate out all vector, ghost and fermion fields (but not the scalar field fluctuation $\hat{\varphi}$) to arrive at an effective (coarse-grained) finite-temperature action $\Gamma_\beta[\varphi]$ defined via

$$Z_\beta[\varphi] =: \int [\mathcal{D}\hat{\varphi}] \exp\left\{\Gamma_\beta[\varphi + \hat{\varphi}]\right\} , \tag{II.2}$$

which may be expanded in powers of derivatives of φ:

$$\Gamma_\beta[\varphi] = \int_\beta \left\{ \frac{1}{2}(\partial\varphi)^2 - V(\varphi, T) \right\} + \int_\beta \sum_{n=2}^\infty \frac{1}{n!} Z_n(\varphi, T)(\partial\varphi)^n . \qquad (\text{II.3})$$

Each Z_n may in addition be expanded in a power series in the coupling constants of the theory.

The remaining integration over $\dot{\varphi}$ in (II.2) will be carried out in the saddle point approximation. We collectively denote all possible Lorentz invariant derivative terms of φ with n derivatives[3] by $(\partial\varphi)^n$ and use the shorthand notation

$$\int_\beta = \int_0^\beta d\tau \int d^3x . \qquad (\text{II.4})$$

Throughout this paper we will use the imaginary time formalism [28, 29]. Therefore boson and fermion fields satisfy periodic and anti–periodic boundary conditions in the imaginary time $\tau = ix_0$, respectively. In momentum space this leads to integrals

$$\sum_\omega \frac{1}{\beta} \int \frac{d^3k}{(2\pi)^3} \qquad (\text{II.5})$$

where $\omega = 2\pi i n T$ ($\omega = (2n+1)\pi i T$) for bosonic (fermionic) fields and n runs over all integers. The first term in (II.3) contains the effective potential $V(\varphi, T)$ and the classical kinetic term, whereas the second summand incorporates derivative terms due to quantum corrections.

For stationary fields, $\partial_\tau \varphi = 0$, the effective action $\Gamma_\beta[\varphi]$ plays the role of the free energy

$$F[\varphi, T] = -\frac{1}{\beta}\Gamma_\beta[\varphi] , \qquad (\text{II.6})$$

and φ is the order parameter of the phase transition. Usually first–order phase transitions are studied under the assumption that the effective action $\Gamma_\beta[\varphi]$ is dominated by homogeneous field configurations, i.e., the derivative terms proportional to Z_n in (II.3) are neglected. Then the remaining path integral in (II.2) is carried out in a saddle point approximation around configurations $\overline{\varphi}(\vec{x})$ which extremize the classical free energy

$$F[\varphi, T] = \int d^3x \left\{ V(\varphi, T) + \frac{1}{2}\left|\vec{\nabla}\varphi\right|^2 \right\} . \qquad (\text{II.7})$$

Isotropic configurations $\overline{\varphi}(r \equiv |\vec{x}|)$ obey the differential equation

$$\frac{d^2\overline{\varphi}}{dr^2} + \frac{2}{r}\frac{d\overline{\varphi}}{dr} - \frac{\partial V}{\partial \varphi}(\overline{\varphi}) = 0 \qquad (\text{II.8})$$

---

[3]For simplicity we have not taken into account the fact that for given n different combinations of derivatives may have different prefactors Z_n.

which is obtained by varying (II.7) w.r.t. φ.

The generic potential for a first–order phase transition and for $T_b < T < T_c$ has two local minima (c.f. Fig. 1) one of which is metastable. Here T_c denotes the critical temperature, where the minima are degenerate, and T_b is the so–called barrier temperature at which the potential barrier between the minima vanishes. We choose the metastable minimum to occur at $\varphi = 0$ and denote the position of the global one by $\varphi = \varphi_{\min}(T)$. The appropriate boundary conditions for the tunneling solution, which interpolates between the symmetric ($\varphi = 0$) and the broken ($\varphi \neq 0$) phase, are $\overline{\varphi}'(r = 0) = 0$ and $\overline{\varphi}(r \to \infty) = 0$ [30]. In the thin wall approximation [31] one finds

$$\overline{\varphi}(r) = \frac{1}{2}\varphi_{\min}\left[1 - \tanh\left(\frac{r - R(T)}{d}\right)\right] \tag{II.9}$$

where

$$R(T) = \frac{2\sigma}{\Delta V(T)} \tag{II.10}$$

is the droplet radius at temperature T. It depends on the surface tension $\sigma(T)$ which may be evaluated at the critical temperature T_c

$$\sigma = \int_0^{\varphi_{\min}(T_c)} d\varphi\sqrt{2V(\varphi, T_c)} \tag{II.11}$$

and the potential difference between the two minima

$$\Delta V(T) = V(0, T) - V(\varphi_{\min}, T) . \tag{II.12}$$

The thickness d of the droplet wall depends on the detailed shape of the potential. For a polynomial of the form

$$V(\varphi, T) = \frac{1}{2}m^2(T)\varphi^2 - ET\varphi^3 + \frac{1}{4}\lambda\varphi^4 \tag{II.13}$$

one obtains

$$d = \sqrt{\frac{2}{\lambda}}\frac{2}{\varphi_{\min}} . \tag{II.14}$$

By inserting (II.9) into (II.7) we find the free energy in the thin wall approximation as a sum of a surface and a volume term:

$$F_{\mathrm{TW}}[\overline{\varphi}, T] = 4\pi R^2(T)\sigma - \frac{4\pi}{3}R^3(T)\,\Delta V(T) = \frac{4\pi}{3}\sigma R^2(T) . \tag{II.15}$$

Once the thin wall approximation breaks down, $\Delta V(T)$ becomes larger than the barrier between the minima, and (II.8) has to be solved numerically. Unfortunately, in the Standard Model the thin wall approximation is only marginally applicable for $m_H \simeq 80\,\mathrm{GeV}$ [6, 16].

To second order in the saddle point approximation of (II.2) we need to perform a Gaussian path integral over the fluctuations $\hat{\varphi}$ around the tunneling solution $\overline{\varphi}$. This correction appears as a pre–exponential factor A in the nucleation rate of critical droplets

$$\Gamma(T) = AT^4 \exp\left\{ -\beta \left(F[\overline{\varphi}, T] - F[0, T] \right) \right\} . \tag{II.16}$$

It has been shown to be of order one for the electroweak phase transition [6, 16]. In the following we will therefore only be concerned with the saddle point configuration $\overline{\varphi}(r)$.

The temperature T_e which marks the end of the phase transition may be defined as the temperature for which the droplet nucleation rate becomes larger than the expansion rate of the universe, i.e., $\Gamma(T_e) \gtrsim H^4(T_e)$, where $H(T)$ denotes the Hubble function. Using (II.16) this leads to the rough estimate $\beta_e F[\overline{\varphi}, T_e] \lesssim 145$.

B   Inhomogeneous field configurations

Higher order derivative corrections will in general alter the results of the last section. Some recent publications [22, 23] indicate that depending on the parameters of the theory under consideration they may even dominate the leading terms and the expansion breaks down altogether. But even in the domain where the perturbative results seem to signal that this is not the case, non–perturbative considerations yield results which are different from perturbative calculations for the effective potential alone [24, 25]. In this paper we will restrict ourselves to the first term of the derivative expansion, i.e., $Z(\varphi, T) := Z_2(\varphi, T)$, that corrects the scalar wave function. Here we will merely discuss possible physical consequences of a non–vanishing $Z(\varphi, T)$ and leave the explicit calculations within the Standard Model to the forthcoming sections. We would like to point out that a consistent one–loop analysis of the electroweak phase transition indeed requires the inclusion of $Z(\varphi, T)$, since its leading perturbative contribution occurs at the one–loop level.

To check whether homogeneous field configurations are important for the dynamics of the phase transition we wish to investigate how $Z(\varphi, T) \neq 0$ affects the characteristic quantities of the phase transition. For this purpose we perform the scalar field transformation

$$\check{\varphi}(r) = \int d\varphi \sqrt{1 + Z(\varphi, T)} \tag{II.17}$$

under which the free energy

$$F[\check{\varphi}, T] = \int d^3x \left\{ \check{V}(\check{\varphi}, T) + \frac{1}{2} (\vec{\nabla}\check{\varphi})^2 \right\} \tag{II.18}$$

has the same form as for $Z(\varphi,T) = 0$. The new potential $\tilde{V}(\check{\varphi},T) = V(\varphi(\check{\varphi}),T)$ is locally rescaled. Since $\frac{\partial\check{\varphi}}{\partial\varphi} = \sqrt{1+Z} > 0$ there is a one–to–one correspondence between $\check{\varphi}$ and φ, i.e., (II.17) amounts to a local rescaling of the φ–axis in Fig. 1. Minima and maxima still have the same potential energy $\tilde{V}(\check{\varphi}_{\min},T) = V(\varphi_{\min},T)$ and as a consequence the critical temperature T_c, at which the two minima are degenerate, does not change either. Also neither the height of the barrier nor the amount of supercooling $\Delta\tilde{V}(T) = \Delta V(T)$ change. Thus, once the thin wall approximation is valid for homogeneous field configurations it survives the incorporation of the wave function correction term.

However, the new surface tension

$$\tilde{\sigma} = \int_0^{\check{\varphi}_{\min}} d\check{\varphi}\sqrt{2\tilde{V}(\check{\varphi},T)} = \int_0^{\varphi_{\min}} d\varphi\sqrt{2\left[1 + Z(\varphi,T)\right]V(\varphi,T)} \tag{II.19}$$

may be substantially different from the surface tension σ without wave function correction term. Hence, a possible measure for the effect of the wave function correction term on the dynamics of the phase transition is given by

$$\delta_Z F := \frac{F[\check{\varphi},T] - F[\overline{\varphi},T]}{F[\overline{\varphi},T]} = \frac{\tilde{\sigma}^3 - \sigma^3}{\sigma^3} \tag{II.20}$$

where $\overline{\varphi}$ denotes the stationary isotropic solution which extremizes the corrected free energy (II.18). For a constant $Z = 0.25$, for instance, one would obtain the significant relative deviation of $\delta_Z F \simeq 0.4$.

Another important consequence of a non–negligible Z–factor is the end temperature of the phase transition. Suppose that $Z(\varphi,T) > 0$; this would result in an increased surface tension and the universe would have to supercool further to complete the phase transition: $\tilde{T}_e < T_e$. Therefore, the strength of the phase transition would increase, since $\frac{\varphi_{\min}(\tilde{T}_e)}{\tilde{T}_e} > \frac{\varphi_{\min}(T_e)}{T_e}$, and could be more favorable for baryogenesis than anticipated without wave function corrections. A rough estimate based on the potential (II.13) and the assumption of an approximately constant Z yields

$$\frac{\check{\varphi}_{\min}(\tilde{T}_e)}{\tilde{T}_e} = \left\{1 + \frac{1}{6}\left[(1+Z)^{\frac{3}{4}} - 1\right]\right\}\frac{\varphi_{\min}(T_e)}{T_e}, \tag{II.21}$$

where we assumed that $\frac{\lambda m^2(T_e)}{9ET_e} \ll 1$. Eqn. (II.21) shows that a positive wave function correction factor Z would enhance the strength of a first–order phase transition whereas a negative Z would decrease it.

III Self–energy corrections and improved propagators

Finite–temperature field theory is known to develop infrared singularities once massless degrees of freedom are present. Since in the symmetric phase all masses are essentially zero, straightforward perturbation theory breaks down. This problem can be avoided by using an improved perturbation theory with propagators that include finite–temperature self–energy and self–mass corrections [16]–[18], [32] generically denoted by $\Sigma_n(x)$ in this paper. In general, the full propagator $\mathcal{D}_n(k)$ is determined by the Dyson–Schwinger equation

$$\mathcal{D}_n^{-1}(k) = \mathcal{D}_{n,0}^{-1}(k) + i\Sigma_n(k) , \tag{III.1}$$

where n labels the different fields in the theory and $\mathcal{D}_{n,0}(k)$ is the tree–level propagator. The Dyson–Schwinger equations may be solved perturbatively. To the order to which we are calculating, the vertex functions can be replaced by the tree level couplings g_i and we are left with a one–loop diagram for Σ_n.

In the following we will simply state the known self–energies $\Sigma_n = \Pi_s(k)$ for scalars and $\Sigma_n = -\Pi_v(k)$ for vectors in the electroweak Standard Model. The top quark self–mass $\Sigma_n = \Sigma_t(k)$ is evaluated in appendix A in the limit of small k.

The starting point for our explicit analysis is the simplified $SU_L(2)$ Standard Model (SM) with vanishing hypercharge gauge coupling g'. In this approximation the $SU_L(2)$ gauge bosons are degenerate in mass and the Weinberg angle Θ_W is neglected. The corresponding Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{fermion}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{ghost}} \tag{III.2}$$

with

$$\mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{Higgs}} = -\frac{1}{4}F_{\mu\nu}^a F^{a,\mu\nu} + (D_\mu\Phi)^\dagger (D^\mu\Phi) - \mu^2 \left(\Phi^\dagger\Phi\right) - \lambda \left(\Phi^\dagger\Phi\right)^2 , \tag{III.3}$$

$\mu^2 < 0$ and

$$D_\mu = \partial_\mu - ig\frac{\tau^a}{2}W_\mu^a , \tag{III.4}$$

where the τ^a, $a = 1,2,3$, denote the three Pauli matrices. The field Φ is an $SU_L(2)$ doublet parameterized by four real scalar fields:

$$\Phi(x) = \frac{1}{\sqrt{2}} \left(\begin{array}{c} \chi_1(x) + i\chi_2(x) \\ \varphi(x) + h(x) + i\chi_3(x) \end{array} \right) , \tag{III.5}$$

where h is the Higgs field, χ_a, $a = 1,2,3$, are the three Goldstone bosons and φ is a real background field.

In the fermionic part of the Lagrangian we neglect all lepton and quark Yukawa couplings compared to the top quark Yukawa coupling f_t. Hence

$$\mathcal{L}_{\text{fermion}} = \overline{\psi}_L i D_\mu \gamma^\mu \psi_L + f_t \left(\overline{t}_L, \overline{b}_L \right) (i\tau_2 \Phi^*) t_R + \text{h.c.} \tag{III.6}$$

The gauge fixing and ghost Lagrangians are given by

$$\begin{aligned}
\mathcal{L}_{\text{GF}} &= -\frac{1}{2\xi} \mathcal{F}_a \mathcal{F}^a \\
\mathcal{L}_{\text{ghost}} &= \overline{c}_a \mathcal{M}^{ab} c_b
\end{aligned} \tag{III.7}$$

with $\mathcal{F}_a(W)$ and $\mathcal{M}$ defined via

$$\begin{aligned}
\mathcal{M}^{ab} &= \left. \frac{\delta}{\delta \omega_a} \mathcal{F}^b(W^\omega) \right|_{\omega=0} \\
\mathcal{F}_a &= \partial_\mu W_a^\mu - \frac{1}{2} g \xi \varphi \chi_a
\end{aligned} \tag{III.8}$$

where W^ω denotes the result of an infinitesimal gauge transformation $U(\omega) = 1 - iT_a \omega^a$ on the gauge field W. Throughout this paper we will work in Landau gauge $\xi = 0$.

To one–loop the full scalar propagator can easily be read off from (III.1):

$$\mathcal{D}_{\varphi,\chi}(k) = \frac{i}{k^2 - m_{\varphi,\chi}^{(0)2} - \Pi_{\varphi,\chi}(k)} \tag{III.9}$$

with $m_\varphi^{(0)2} = \lambda(3\varphi^2 - v^2)$, $m_\chi^{(0)2} = \lambda(\varphi^2 - v^2)$ being the tree–level scalar masses. Due to the breakdown of Lorentz invariance in the presence of a heat bath the full vector propagator $(m_W^{(0)} = g\varphi/2)$

$$\begin{aligned}
\mathcal{D}^{\mu\nu}(k) &= \frac{-i}{k^2 - m_W^{(0)2} - \Pi_L(k)} P_L^{\mu\nu} \\
&+ \frac{-i}{k^2 - m_W^{(0)2} - \Pi_T(k)} P_T^{\mu\nu} + \frac{-i\xi}{k^2 - \xi m_W^{(0)2} - \Pi_G(k)} P_G^{\mu\nu}
\end{aligned} \tag{III.10}$$

involves projections onto the direction k_μ of propagation $(P_G^{\mu\nu})$, onto the component of the heat bath flow u_μ perpendicular to k_μ $(P_L^{\mu\nu})$ and onto the remaining two directions $(P_T^{\mu\nu})$. For explicit definitions and properties c.f. [16]. In terms of these projectors the self–energy tensor

$$\Pi^{\mu\nu}(k) = \Pi_L(k) P_L^{\mu\nu} + \Pi_T(k) P_T^{\mu\nu} + \Pi_G(k) P_G^{\mu\nu} + \Pi_S(k) S^{\mu\nu} \tag{III.11}$$

also involves a traceless projector S which does not contribute to $\mathcal{D}_{\mu\nu}$ in Landau gauge. Note that we keep the third term in (III.10) although at first sight it seems to vanish for our gauge

choice $\xi \to 0$. Eventually one encounters singularities in the gauge fixing contribution of $Z(\varphi, T)$ for $\xi \to 0$ which are precisely canceled by this term.

In the limit $k \to 0$ the self–energies give rise to the plasma mass corrections

$$
\begin{aligned}
\Pi_{\varphi,\chi}(0) &= \delta m_{\varphi,\chi}^2 = \left(\frac{3}{16}g^2 + \frac{1}{2}\lambda + \frac{1}{4}f_t^2\right) T^2 \\
\Pi_L(0) &= \delta m_{W,L}^2 = \frac{11}{6}g^2 T^2 \\
\Pi_T(0) &= \delta m_{W,T}^2 = \frac{1}{9\pi^2}\gamma^2 g^4 T^2 \;.
\end{aligned}
\tag{III.12}
$$

The magnetic gauge boson plasma mass $\delta m_{W,T}$ of order g^2 is a non–perturbative feature of the theory which was first predicted in [4] and derived in e.g. [16, 17]. The factor γ is expected to be of order one as confirmed by lattice simulations and other non–perturbative methods [19].

Though fermionic contributions to the wave function correction term are not expected to introduce new infrared singularities, the top quark self–mass Σ_t is evaluated in appendix A. We do not find any plasma mass correction to first loop–order. This is however not surprising, since at least in the symmetric phase $m_t = 0$ is protected to any finite order in perturbation theory by chiral invariance. Instead we find a chemical potential which is non–vanishing in both phases of the theory. We will however neglect this one–loop correction to the tree–level propagator. In the next section we will find, that the (unimproved) top quark contribution to $Z(\varphi, T)$ is indeed infrared–finite and furthermore negligible compared to other contributions. Therefore there is no need to improve perturbation theory by including this plasma induced effective chemical potential into the top quark propagator.

The improved propagators for the bosonic degrees of freedom are now given in terms of the full masses

$$
m_i^2 = m_i^{(0)\,2} + \delta m_i^2 \;.
\tag{III.13}
$$

The tree–level connection between the couplings and zero–temperature masses is given by $g = 2m_W(T = 0)/v$, $f_t = \sqrt{2}m_t(T = 0)/v$ and $\lambda = m_H^2(T = 0)/2v^2$ where $v = \sqrt{-\mu^2/\lambda} \simeq 246\,\mathrm{GeV}$.

IV   Local momentum expansion

Previous work on terms quadratic in spatial derivatives in the effective action include [27] and [33]–[43]. We will follow the approach of Moss et. al. which is based on the local momentum space method of [38].

We start with a short review of the methods developed in [27] and generalize them by

including plasma masses. The calculation of the one–loop contribution to the wave function correction term in the effective action only requires the part of (III.2) which is bilinear in the quantum fluctuations[4] h, χ_i, W, c and t:

$$\mathcal{L}_2 = \frac{1}{2}\delta_{ab}W^a_\mu\Delta^{\mu\nu}_W W^b_\nu + \delta_{ab}\bar{c}^a\Delta_c c^b + \frac{1}{2}h\Delta_\varphi h + \frac{1}{2}\sum_j \chi_j\Delta_\chi\chi^j + \bar{t}\Delta_t t \qquad \text{(IV.1)}$$

with

$$\begin{aligned}
\Delta^{\mu\nu}_W(x) &= \left(\partial^\rho\partial_\rho + m^2_W\right)g^{\mu\nu} - \left(1 - \frac{1}{\xi}\right)\partial^\mu\partial^\nu \\
&\quad - \frac{2}{\xi}(N^\mu\partial^\nu - N^\nu\partial^\mu) + \frac{2}{\xi}(\partial^\nu N^\mu) - \frac{4}{\xi}N^\mu N^\nu \\
\Delta_c(x) &= -\left(\partial^\rho\partial_\rho + \xi m^2_W + 2N^\mu\partial_\mu\right) \\
\Delta_{\varphi,\chi}(x) &= -\partial_\mu\partial^\mu - m^2_{\varphi,\chi} \\
\Delta_t(x) &= i\partial\!\!\!/ + m_t
\end{aligned} \qquad \text{(IV.2)}$$

and $N_\mu = \partial_\mu\ln\varphi$. The one–loop corrections to the effective action may then be written as

$$\Gamma^{(1)} = -\sum_n a_n\,\text{tr}\left(\ln\Delta_n(x)\right) \qquad \text{(IV.3)}$$

where n runs over all fields in the theory. As seen above, Δ_n is an operator of the general form $\Delta_n(x) = D_n + M_n(x)$ with D_n being a differential operator and $M_n(x)$ a mass term[5]. The coefficients a_n are fixed by the statistics and the number of degrees of freedom of the corresponding field. In particular, we have $a_\varphi = a_{\chi_j} = \frac{1}{2}$, $a_W = \frac{3}{2}$ and $a_t = -1$.

The effects of the plasma are taken into account by using the inverse propagators[6]

$$\Delta_n = (D_n + M_n + \Sigma_n) - \Sigma_n \equiv \Delta'_n - \Sigma_n(x)\,, \qquad \text{(IV.4)}$$

where Σ_n denotes all plasma corrections for the inverse propagator, and may in general depend on x via φ. Therefore, to one–loop, (IV.3) is equivalent to the modified relation

$$\Gamma^{(1)}_n = -a_n\,\text{tr}\left[\left(1 - \Sigma_n\frac{\partial}{\partial\Sigma_n}\right)\ln\Delta'_n\right]\,. \qquad \text{(IV.5)}$$

The situation for gauge fields is slightly more complicated due to the existence of different plasma masses for transverse and longitudinal degrees of freedom. According to (III.10), (III.11)

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[4]Of course, h and χ_i are exactly the fluctuations which are collectively denoted by $\hat{\varphi}$ in (II.1).

[5]Note that $M_n(x)$ actually corresponds to a mass term only for fermions but to a mass *squared* term for gauge bosons and scalars.

[6]One may equivalently use improved (inverse) propagators from the beginning in (IV.1). In this case one has to add appropriate plasma mass counter terms to (IV.1) in order to compensate for these corrections [16].

the operator

$$\Delta_W = \left(\Delta'_{W,L} - \delta m^2_{W,L} P_L\right) + \left(\Delta'_{W,T} - \delta m^2_{W,T} P_T\right) + \Delta_{W,G} + \dots \, , \qquad (IV.6)$$

where the dots indicate terms depending on N_μ, splits up into three contributions:

$$
\begin{aligned}
\Delta'_{W,L} &= \left(\partial^\rho \partial_\rho + m^2_{W,L}\right) P_L \\
\Delta'_{W,T} &= \left(\partial^\rho \partial_\rho + m^2_{W,T}\right) P_T \\
\Delta_G &= \frac{1}{\xi}\left(\partial^\rho \partial_\rho + \xi m^{(0)2}_W\right) P_G \, .
\end{aligned}
\qquad (IV.7)
$$

In Landau gauge the final result does not depend on the last term and thus we need no counter terms for Δ_G.

The first step in evaluating (IV.5) is the definition of a (finite–temperature) Greens function $G_n(x, x')$ for each Δ'_n via

$$\Delta'_n G_n(x, x') = \delta_{x_0, x'_0} \delta(\vec{x} - \vec{x}') \, . \qquad (IV.8)$$

If we furthermore assume that M_n may be written as $M_n(x) = m_n + \overline{M}_n(x)$, where m_n is independent of x and $\overline{M}_n$ is independent of m_n, we arrive at

$$\frac{\partial \Gamma^{(1)}_n}{\partial m_n} = -a_n \int_\beta \mathrm{tr}\left[\left(1 - \Sigma_n \frac{\partial}{\partial \Sigma_n}\right) G_n(x, x)\right] \, , \qquad (IV.9)$$

where the functional trace is accounted for by the integral. Hence the knowledge of the $G_n(x, x')$ will enable us to derive the one–loop contribution $\Gamma^{(1)}_n$ to the effective action via integration w.r.t. m_n.

Since we only need to know the[7] $G(x, x')$ in the limit $x \to x'$, we expand the functions $M'(x) := M(x) + \Sigma(x)$ into a Taylor series around $x = x'$:

$$M'(x) = M'(x') + \sum_{l=1}^{\infty} \frac{1}{l!} M'_{\mu_1 \dots \mu_l}(x') y^{\mu_1} \dots y^{\mu_l} \qquad (IV.10)$$

with $y = x - x'$ and

$$M'_{\mu_1 \dots \mu_l}(x') = \left.\frac{\partial^l M'(x)}{\partial x^{\mu_1} \dots \partial x^{\mu_l}}\right|_{x=x'} \, . \qquad (IV.11)$$

Inserting this into (IV.8) and Fourier transforming w.r.t. x (denoted by a tilde)

$$G(x, x') = \sum_{k_0} \frac{1}{\beta} \int \frac{d^3 k}{(2\pi)^3} e^{i(k_0 y_0 - \vec{k}\vec{y})} \tilde{G}(k, x') \, , \qquad (IV.12)$$

---

[7]For simplicity we will suppress the index n from now on. It is clear that the following steps have to be carried out for each degree of freedom labeled by n, separately.

we arrive at

$$\left(\tilde{D} + M'(x') + \sum_{l=1}^{\infty} \frac{i^l}{l!} M'_{\mu_1 \ldots \mu_l}(x') \frac{\partial^l}{\partial k_{\mu_1} \ldots \partial k_{\mu_l}} \right) \tilde{G}(k, x') = 1 . \qquad (\text{IV.13})$$

Note that the sum in (IV.12) may be over bosonic ($k_0 = 2\pi i n T$) or fermionic ($k_0 = (2n+1)\pi i T$) Matsubara frequencies, depending on the type of inverse propagator Δ' used to define $G(x, x')$. We may now expand $\tilde{G}(k, x')$ and $M'(x')$ in a series w.r.t. the number of derivatives[8]:

$$\begin{aligned} M'(x') &= \sum_{l=0}^{\infty} i^l M'^{(l)}(x') \\ \tilde{G}(k, x') &= \sum_{j=0}^{\infty} \tilde{G}^{(j)}(k, x') . \end{aligned} \qquad (\text{IV.14})$$

One immediately obtains

$$\tilde{G}^{(0)}(k, x') = \frac{1}{\tilde{D}(k) + M'(x')} \qquad (\text{IV.15})$$

and

$$\tilde{G}^{(j)}(k, x') = -\tilde{G}^{(0)}(k, x') \sum_{s=1}^{j} \sum_{l=0}^{s} \frac{i^s}{l!} M'^{(s-l)}_{\mu_1 \ldots \mu_l}(x') \frac{\partial^l}{\partial k_{\mu_1} \ldots \partial k_{\mu_l}} \tilde{G}^{(j-s)}(k, x') . \qquad (\text{IV.16})$$

This iterative solution of (IV.8) may now be used to determine the Greens functions and subsequently the effective action via (IV.9) order by order in the derivative expansion (IV.14). The $\tilde{G}^{(0)}(k, x')$ yield the effective potential whereas the $\tilde{G}^{(2)}(k, x')$ will give the desired kinetic term in the one-loop effective action:

$$\Gamma^{(1)}_{\text{kin}} = -\sum_{n} a_n \int dm \int_{\beta} \lim_{y \to 0} \sum_{k_0} \frac{1}{\beta} \int \frac{d^3 k}{(2\pi)^3} e^{i(k_0 y_0 - \vec{k}\vec{y})} \, \text{tr} \left[\left(1 - \Sigma_n \frac{\partial}{\partial \Sigma_n} \right) \tilde{G}_n^{(2)}(k, x') \right] . \qquad (\text{IV.17})$$

Although (IV.16) and (IV.17) provide a well-defined prescription for calculating the wave function correction, it still requires rather lengthy calculations. We have used the symbolic manipulation package *Mathematica* to accelerate the computation. All that is needed are the (improved) zeroth order propagators $\tilde{G}_n^{(0)}(k, x)$ as given in (III.9), (III.10) as well as the standard tree level top quark propagator, and the space-time dependent pieces $M_n'^{(l)}(x)$ which may be read off directly from the results of section III. As explained in section II we will only take into account stationary fields, i.e., $\partial_0 \varphi = 0$, and use standard integration techniques for the finite-temperature momentum integrals. All contributions split up into a T-independent and a T-dependent piece as is generally the case in thermal field theory.

---

[8]For scalar and fermion fields the derivative expansion of M' is trivial. Eqn. (IV.2) however shows that it is non-trivial for vector and ghost fields.

The temperature dependent scalar contribution turns out to be

$$Z_{\text{scalar}}^{T\neq 0}(\varphi, T) = \frac{\lambda^2 T \varphi^2}{16\pi} \left(\frac{9\delta m_\varphi^2}{2m_\varphi^5} + \frac{3}{m_\varphi^3} + \frac{3\delta m_\chi^2}{2m_\chi^5} + \frac{1}{m_\chi^3} \right) .$$ (IV.18)

The corresponding T–independent part is UV–finite and given by

$$Z_{\text{scalar}}^{T=0}(\varphi) = \frac{\lambda^2 \varphi^2}{16\pi^2} \left(\frac{3}{m_\varphi^2} + \frac{3\delta m_\varphi^2}{m_\varphi^4} + \frac{1}{m_\chi^2} + \frac{\delta m_\chi^2}{m_\chi^4} \right) .$$ (IV.19)

Since it is subleading in the high–T expansion compared to $Z_{\text{scalar}}^{T\neq 0}(\varphi, T)$ we will neglect it in the following. The T–dependent gauge boson one–loop contribution turns out to be

$$Z_{\text{vector}}^{T\neq 0}(\varphi, T) = -\frac{3g^2 T}{4\pi} \left(\frac{m_W^{(0)4}}{32m_L^5} - \frac{5m_W^{(0)2}}{96m_L^3} + \frac{5m_W^{(0)4}}{16m_T^5} - \frac{41m_W^{(0)2}}{48m_T^3} + \frac{1}{m_T} \right) .$$ (IV.20)

The zero–temperature contribution is UV–divergent and has to be renormalized. It can be seen to be of the form [27]

$$Z_{\text{vector}}^{T=0}(\varphi) \sim \ln \left(\frac{m_W}{m_{\text{ren}}} \right) ,$$ (IV.21)

where m_{ren} denotes the renormalization scale. Compared to (IV.20) this part of the wave function correction term is again subleading in the high–T expansion and may therefore be neglected.

We now turn to the one–loop top quark contribution $Z_{\text{fermion}}(\varphi, T)$. In the high–temperature expansion its leading order T–dependent piece is given by:

$$Z_{\text{fermion}}^{T\neq 0}(\varphi, T) = \frac{f_t^2}{24\pi^2} \left(1 + 3\gamma_E + 3\ln \frac{m_t}{\pi T} \right) ,$$ (IV.22)

which is logarithmically IR–divergent. However, in this case the renormalized–temperature contribution which could be neglected for the other fields comes to the rescue. It is given by

$$Z_{\text{fermion}}^{T=0}(\varphi) = \frac{f_t^2}{32\pi^2} \left(1 - 4\ln \frac{m_t}{m_{\text{ren}}} \right) ,$$ (IV.23)

and therefore cancels exactly the IR–divergence of the T–dependent part. The full fermionic contribution is of the form

$$Z_{\text{fermion}}(\varphi, T) = \frac{f_t^2}{96\pi^2} \left(7 + 12\gamma_E + 12\ln \frac{m_{\text{ren}}}{\pi T} \right) .$$ (IV.24)

We would like to point out that our results for the bosonic as well as the T–independent fermionic contributions to $Z(\varphi, T)$ agree with the findings of [27] for vanishing plasma masses.

However, the fermionic T-dependent piece (IV.22) disagrees with that of [27] where a $1/m_t$ infrared singularity (which can not be canceled by the T-independent part) was found. The reason for this discrepancy appears to be that in [27] the kinetic energy was calculated following (IV.17) by summing over *bosonic* frequencies k_0 instead of *fermionic* frequencies as required by the Fourier transformation (IV.12) of a fermionic Greens function. Our results show that all infrared singularities in the symmetric phase are removed by the plasma masses to leading order in the high-T and the loop-expansion.

The full wave function correction term is now given by

$$Z(\varphi, T) = Z_{\text{vector}}(\varphi, T) + Z_{\text{fermion}}(\varphi, T) \ . \tag{IV.25}$$

Note that here Z_{scalar} is not included. We wish to emphasize that it would be inconsistent to add the scalar contribution for the calculation of physical quantities like the free energy, the droplet tension or droplet nucleation rate, following the methods described in section II.A. The scalar contributions to the effective action are fully taken into account by solving the bosonic path integral (II.2) in the saddle point approximation. However, for checking the range of validity of the derivative expansion (II.3) one might as well integrate out the scalars in the same way as the other fields and consider the quantity

$$\tilde{Z}(\varphi, T) = Z_{\text{scalar}}(\varphi, T) + Z_{\text{vector}}(\varphi, T) + Z_{\text{fermion}}(\varphi, T) \ . \tag{IV.26}$$

V Results

A short look at the various contributions to $Z(\varphi, T)$ shows that it is completely dominated by its gauge boson contribution $Z_{\text{vector}}(\varphi, T)$ for temperatures close to the critical temperature T_c. This is demonstrated in Fig. 2, where Z and all its contributions are plotted for physically realistic values of the parameters $m_H(T = 0)$, $m_t(T = 0)$ and γ at $T = T_c$. We also note that an inclusion of Z_{scalar} would be numerically insignificant.

Since the perturbative expansion of the wave function correction term starts at the one-loop level, the size of $Z(\varphi, T)$ is a direct measure for the convergence of perturbation theory. Furthermore, for values of φ and T for which $|Z(\varphi, T)| \gtrsim 1$ or $\left| \tilde{Z}(\varphi, T) \right| \gtrsim 1$, we have to expect that other higher derivative terms of the effective action in (II.3) could also become important and may no longer be neglected. It is already known [16], that there is only a small window of parameters close to the current experimental bound on the Higgs mass, where perturbation theory can be trusted as far as a calculation of the effective potential is concerned. In particular, for Higgs masses above approximately 85 GeV higher order loop-contributions to

the effective potential become dominant and perturbative statements on the phase transition are questionable. A first order phase transition can reliably be found for $m_H \lesssim 85\,\mathrm{GeV}$ and small γ. It is altogether excluded for $\gamma \gtrsim 2$ taking into account the current lower bound on the Higgs mass of $m_H \gtrsim 63.5\,\mathrm{GeV}$ [44].

In Fig. 3 we have plotted the curve which divides γ–φ–space into two regions. Above it $|Z(\varphi, T)| < 1$ and the wave function correction term is smaller than the classical kinetic term of the effective action. Below this curve $|Z(\varphi, T)| > 1$, perturbation theory breaks down and higher derivative terms might dominate the dynamics of the phase transition. We have used $|Z(\varphi, T)| \simeq |Z_{\mathrm{vector}}(\varphi, T)|$, which is a good approximation for all temperatures relevant for the phase transition. We find that for the whole range of magnetic masses which permit a first–order phase transition (taking into account the experimental lower bound on the Higgs mass), there is an interval of small φ–values for which the wave function correction dominates over the classical kinetic term and is furthermore negative. Therefore, in this range the field transformation (II.17) yields an imaginary scalar field $\tilde{\varphi}$ and a complex effective potential $\tilde{V}(\tilde{\varphi}, T)$. This in turn renders the methods of section II inapplicable. However, a rough estimate, using an averaged (constant) wave function correction factor $< Z(\varphi, T) >= \frac{1}{\varphi_{\min}} \int_0^{\varphi_{\min}} d\varphi\, Z(\varphi, T)$, gives e.g. for $\gamma = 1$, $m_H = 85\,\mathrm{GeV}$ and $m_t = 160\,\mathrm{GeV}$ the value $< Z(\varphi, T) > \approx -0.4$. This in turn yields an uncertainty of the derivative expansion of approximately 50% (c.f. (II.20)) and a weakening of the phase transition strength $\frac{\varphi(T_c)}{T_c}$ by 3%.

At the one–loop level, we are thus left with the following situation: For small magnetic masses ($\gamma \lesssim 2$), i.e., for the range of γ which, in combination with the current experimental lower bound on the Higgs–mass, allows for a first–order phase transition, neither perturbation theory nor the derivative expansion seem to be reliable for small scalar field values. In particular, for those values of φ which are important for the determination of the dynamics of the phase transition, $0 \le \varphi \le \varphi_{\min}$, the one–loop wave function correction term dominates over the classical kinetic term in the effective action. Therefore, in this range of parameter space, it is not possible to make physically reliable predictions at the one–loop level. In order to clarify the dynamics of a first–order phase transition, clearly, the calculation of higher loop–corrections to $Z(\varphi, T)$ as well as higher derivative terms of the effective action are necessary. However, since the one–loop result for $Z(\varphi, T)$ is negative for the whole range of $0 \le \varphi/T \le 1$, we expect that its inclusion will *weaken* the first–order electroweak phase transition, as explained in section II.

For larger values of the magnetic mass, $\gamma \gtrsim 2$, the perturbative calculation of the wave function correction term appears to be reliable. On the other hand such magnetic masses correspond to a part of parameter space, where the phase transition is expected to be of second order [16], and baryo–genesis can not appear at the electroweak scale within the mechanism of

[3], [20].

VI   Conclusions

We have investigated the impact of the scalar field wave function correction term $Z(\varphi, T)$ on the electroweak phase transition. Since $Z(\varphi, T)$ is the quantum correction to the classical scalar field kinetic term in the effective action, its knowledge is crucial for an understanding of the influence of inhomogeneous scalar field configurations on the dynamics of this transition.

It turned out that generally a positive Z–factor would increase the strength of a possible first–order transition whereas a negative Z would decrease it. A calculation of $Z(\varphi, T)$ within the Standard Model to one–loop revealed that it is fully dominated by its gauge boson contribution for temperatures close to the critical temperature T_c and well above the barrier temperature T_b. Furthermore we find that the one–loop result for $Z(\varphi, T)$ is negative for the whole range $0 \leq \varphi/T \leq 1$ and $T \simeq T_c$. However, for small magnetic gauge boson masses, $\gamma \lesssim 2$, which would allow for a first–order phase transition for appropriate Higgs masses, we find $|Z(\varphi, T)| \gtrsim 1$ for small φ. We interpret this fact as an indication that higher loop–corrections as well as higher derivative terms in the effective action might be crucial for an understanding of the dynamics of the phase transition. For larger magnetic masses we obtain $|Z(\varphi, T)| \lesssim 1$, and conclude that in this part of parameter space perturbation theory seems to be reliable. However, for such values of the magnetic gauge boson masses the phase transition is presumably of second order.

Our negative one–loop result for $Z(\varphi, T)$ for the range $0 \leq \gamma \lesssim 2$ seems to indicate a decrease of the strength of the first–order phase transition. A further clarification of this point would however require the knowledge of higher loop–corrections to $Z(\varphi, T)$ and perhaps also higher derivative terms in the effective action.

Appendix

A   The top quark self–mass

In this appendix we evaluate the top quark self–mass Σ_t. Among the various one–loop Feynman diagrams contributing to Σ_t the dominant contribution comes from the gluon loops[9]. The corresponding contribution to Σ_t has been calculated in [45] for the symmetric phase. For $SU_c(3)$ one obtains

$$\Sigma_t^{\text{gluon}}(k_0, 0) = \frac{1}{6} \frac{g_s^2 T^2}{k_0} \gamma_0 \,. \tag{A.1}$$

Hence the gluon loops do not induce a plasma mass but an effective chemical potential $\mu(k_0, \vec{k} = 0) = \frac{1}{6} \frac{g_s^2 T^2}{k_0}$. The absence of a plasma mass in the symmetric phase is not surprising, because we do not expect chiral invariance to be broken to any finite order in perturbation theory. For the same reason the $SU_L(2)$ gauge boson loop–corrections to Σ_t should not induce a plasma mass. Since furthermore $g \ll g_s$, they will be neglected in the following, and we will concentrate on the scalar loop–corrections to Σ_t.

The Higgs boson loop–contribution is given by

$$\Sigma_t^{\text{Higgs}}(k_0, \vec{k}) = \frac{1}{4} f_t^2 \sum_{p_0} T \int \frac{d^3 p}{(2\pi)^3} \frac{-i(\slashed{p} + m_t)}{p^2 - m_t^2} \frac{i}{(p - k)^2 - m_\varphi^2} \,, \tag{A.2}$$

where $p_0 = (2n + 1)i\pi T$ and $k_0 = (2m + 1)i\pi T$ are fermionic Matsubara frequencies. The determination of a plasma mass or chemical potential requires the knowledge of $\Sigma_t(k_0, 0)$:

$$\Sigma_t^{\text{Higgs}}(k_0, 0) = \frac{1}{4} f_t^2 \left[m_t + \frac{\gamma_0}{2k_0} \left(m_t^2 - m_\varphi^2 + k_0^2 \right) \right] I_1 - \frac{1}{8k_0} f_t^2 \gamma_0 I_2 \tag{A.3}$$

with

$$
\begin{aligned}
I_1 &= \sum_{n=0}^{\infty} T \int \frac{d^3 p}{(2\pi)^3} \frac{1}{(p^{(-)2} - m_t^2)(p^{(+)2} - m_\varphi^2)} \\
I_2 &= \sum_{n=0}^{\infty} T \int \frac{d^3 p}{(2\pi)^3} \left(\frac{1}{p^{(-)2} - m_t^2} - \frac{1}{p^{(+)2} - m_\varphi^2} \right) \,.
\end{aligned}
\tag{A.4}
$$

Fermionic four–momenta are denoted by $p^{(-)} = (p_0^{(-)}, \vec{p})$ whereas bosonic momenta are denoted as $p^{(+)} = (p_0^{(+)}, \vec{p})$ with Matsubara frequencies $p_0^{(-)} = (2n + 1)i\pi T$, $p_0^{(+)} = 2ni\pi T$. The evaluation of I_2 in the high–temperature expansion is straightforward, whereas I_1 is somewhat

---

[9] For a very heavy top quark with $m_t \simeq 200 \, \text{GeV}$ one expects the Higgs loop to give a comparable contribution, since then $f_t \simeq g_s$.

more involved and given in appendix B. Using these results we obtain to leading order in f_t, $y_b = m_\varphi/T$ and $y_f = m_t/T$

$$\Sigma_t^{\text{Higgs}}(k_0, 0) = \frac{f_t^2}{32\pi} C y_f T + \frac{f_t^2}{32\pi} \gamma_0 \left[\frac{1}{2} C k_0 - k_0 \frac{y_b}{\pi} - \frac{T^2}{k_0} \left(\frac{1}{2} + \frac{y_b}{\pi} \right) \right] \tag{A.5}$$

with $C = -1 + \gamma_E + \ln 2$.

The physical degrees of freedom are found by determining the poles of the propagator $S_F(k_0, \vec{k}) = [\slashed{k} - m_t - \Sigma_t(k_0, \vec{k})]^{-1}$ for $\vec{k} = 0$. A straightforward calculation reveals four different zeros of S_F^{-1}. In the symmetric phase, i.e., for $y_f = y_b = 0$, the pole positions are degenerate and given by $k_0 = \pm f_t T/8$, which is in perfect agreement with the findings of [45, 46][10].

The form of $\Sigma_t(k_0, 0)$ shows an additional plasma correction to the effective chemical potential as well as a plasma mass for the top quark of the form

$$\mu_t = \frac{f_t^2}{32\pi} \left(\frac{1}{2} - \frac{y_b}{\pi} \right) \frac{T^2}{k_0} - \frac{f_t^2}{32\pi} \left(\frac{1}{2} C - \frac{y_b}{\pi} \right) k_0 \, , \quad \delta m_t = \frac{f_t^2}{32\pi} C y_f T \, . \tag{A.6}$$

As expected, δm_t vanishes in the symmetric phase. Furthermore one easily sees that it is exactly canceled by the neutral Goldstone boson loop–contribution to Σ_t, which may be calculated together with the charged Goldstone contributions analogously to Σ_t^{Higgs}. The final result is

$$\begin{aligned}
\Sigma_t(k_0, 0) = {} & \frac{1}{6} \frac{g_s^2 T^2}{k_0} \gamma_0 + \frac{f_t^2}{32\pi} \gamma_0 \left[C k_0 - \frac{1}{\pi} y_b k_0 - \frac{T^2}{k_0} \left(1 + \frac{1}{\pi} y_b \right) \right] \frac{1 + \gamma_5}{2} \\
& + \frac{f_t^2}{32\pi} \gamma_0 \left[2 C k_0 - \frac{1}{\pi} y_b k_0 - \frac{T^2}{k_0} \left(2 + \frac{1}{\pi} y_b \right) \right] \frac{1 - \gamma_5}{2} \, .
\end{aligned} \tag{A.7}$$

B Some useful integrals

In this appendix we provide the high–temperature expansion of the integral I_1 defined in appendix A and for some related integrals also containing fermionic as well as bosonic momenta[11]. Converting the Matsubara frequency summation into a contour integral and reading off the residues, one finds after evaluation of the spatial angular integral

$$I_1 = \frac{1}{8\pi^2} \left\{ \left(\Delta y^2 - \pi^2 \right) I_3^{(b)}(y_b, z_b) - \left(\Delta y^2 + \pi^2 \right) I_3^{(f)}(y_f, z_f) \right\} \tag{B.1}$$

---

[10]Actually the degeneracy already occurs for $y_f = 0$ but $y_b \neq 0$. Furthermore it is lifted even in the symmetric phase for $\vec{k} \neq 0$ [45].

[11]Note that $p_0^{(-)} - k_0^{(-)}$ is bosonic, though $p_0^{(-)}$ and $k_0^{(-)}$ are fermionic.

with $\Delta y^2 = y_b^2 - y_f^2$ and $z_{b/f}^2 = \pm\frac{1}{2}\Delta y^2 + \frac{1}{4\pi^2}\left(\Delta y^2\right)^2 + y_b^2$ and

$$I_n^{(b/f)}(y,z) := \int_0^\infty \frac{x^{n-1}}{\left(\frac{\pi^2}{4} + z^2\right)\sqrt{x^2 + y^2}\left(1 \mp \exp\sqrt{x^2 + y^2}\right)} \ . \tag{B.2}$$

The integrals $I_n^{(b/f)}(y,z)$ may be evaluated in the high–temperature expansion, i.e., for $y, z \ll 1$, as follows: In a first step one determines $I_1^{(b/f)}$. Subsequently $I_{1+2k}^{(b/f)}$ is calculated for $y = 0$ and positive k. The leading y–dependence for arbitrary k may then be recovered by applying

$$I_n^{(b/f)}(y,z) = -\frac{1}{ny}\frac{\partial}{\partial y}I_{n+2}^{(b/f)}(y,z) \ . \tag{B.3}$$

In this way one obtains

$$
\begin{aligned}
I_1^{(b)} &= \frac{1}{\pi} - \frac{2}{\pi^2}\left(2 + \ln\frac{y}{4\pi}\right) - \frac{2}{\pi}\left(1 - \frac{4}{\pi^2}z^2\right)\frac{1}{y} \\
&\quad + \left\{-\frac{4}{\pi^3} - \frac{1}{\pi^2} + \frac{4}{\pi^4}\left(7 - \gamma_C + 2\ln\frac{y}{4\pi}\right)\right\}z^2 \\
I_1^{(f)} &= -\frac{1}{\pi} - \frac{2}{\pi^2}\ln\frac{y}{2\pi} + \left\{\frac{4}{\pi^3} - \frac{1}{\pi^2} + \frac{4}{\pi^4}\left(1 + 2\gamma_C + 2\ln\frac{y}{2\pi}\right)\right\}z^2 \\
&\quad + \left\{-\frac{2}{\pi^3} + \frac{1}{\pi^2} + \frac{1}{\pi^4}\left(-2 - 8\gamma_C - 4\ln\frac{y}{2\pi} + \frac{7}{4}\zeta(3)\right)\right\}y^2 \\
I_3^{(b)} &= 1 - \frac{\pi}{2} - \frac{1}{2}\gamma_E + \frac{\pi}{4} - \frac{1}{2}\ln 2 + \frac{2}{\pi}y_b \\
&\quad + \left\{\frac{1}{4} + \frac{1}{\pi^2}\left(2\gamma_C - 3\right)\right\}z^2 + \left\{-\frac{1}{2\pi} + \frac{1}{2\pi^2}\left(2\ln\frac{y}{4\pi} + 3\right)\right\}y^2 \\
I_3^{(f)} &= \frac{\pi}{4} - \frac{1}{2}\gamma_E - \frac{1}{2}\ln 2 \\
&\quad + \left\{\frac{1}{4} - \frac{1}{\pi^2}\left(2\gamma_C + 1\right)\right\}z^2 + \left\{\frac{1}{2\pi} + \frac{1}{2\pi^2}\left(2\ln\frac{y}{2\pi} - 1\right)\right\}y^2 \\
I_5^{(b)} &= \frac{\pi^2}{8}\left(\gamma_E + \ln 2 - \frac{10}{3} + \frac{\pi}{2}\right) \\
&\quad + \frac{1}{2}\left\{\frac{\pi}{2} - \frac{\pi^2}{8} + \left(\gamma_E - \gamma_C + \ln 2 - \frac{1}{2}\right)\right\}z^2 + \frac{3}{4}\left\{\frac{\pi}{2} + \gamma_E + \ln 2 - 2\right\}y^2 \\
I_5^{(f)} &= \frac{\pi^2}{8}\left\{\frac{2}{3} + \gamma_E - \frac{\pi^2}{2} + \ln 2\right\} \\
&\quad + \frac{1}{2}\left\{\frac{1}{2} + \gamma_E + \gamma_C - \frac{\pi}{2} - \frac{\pi^2}{8} + \ln 2\right\}z^2 + \frac{3}{4}\left\{\gamma_E - \frac{\pi}{2} + \ln 2\right\}y^2 \ ,
\end{aligned}
\tag{B.4}
$$

where $\gamma_E \simeq 0.577$ denotes Euler's and $\gamma_c \simeq 0.916$ Catalan's constant.

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Figure 1: *The qualitative shape of the effective potential is plotted for three different temperatures: $T = T_c$, $T_b < T < T_c$ and $T = T_b$. Here T_b denotes the barrier temperature for which the barrier between the two minima vanishes. For $T_b < T < T_c$ we have also indicated the position of $\varphi_{\min}$ and the potential difference $\Delta V(T)$ between the two minima.*

Figure 2: *The total wave function correction factor $Z(\varphi, T)$ (2.a), its scalar (2.b) and vector (2.c) field contribution are plotted for $m_H(T = 0) = 85\,\text{GeV}$, $m_t(T = 0) = 160\,\text{GeV}$ and $\gamma = 0, 1, 2$ at the corresponding critical temperature $T = T_c$. Note that T_c is a function of γ, so that also $Z_{\text{scalar}}(\varphi, T_c)$ is implicitly γ-dependent.*

Figure 3: *The plotted boundary curve divides γ-φ space into two regions. Above the boundary $|Z(\varphi, T)| \simeq |Z_{\text{vector}}(\varphi, T)| < 1$. Below it $|Z(\varphi, T)| > 1$ and the wave function correction term dominates over the classical kinetic term of the effective action.*
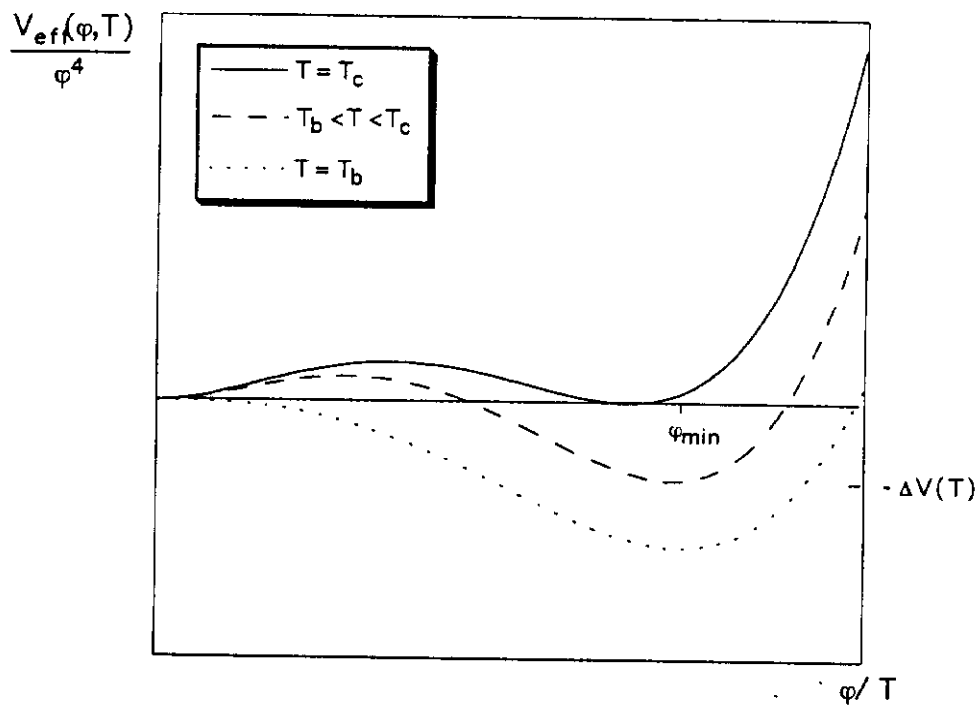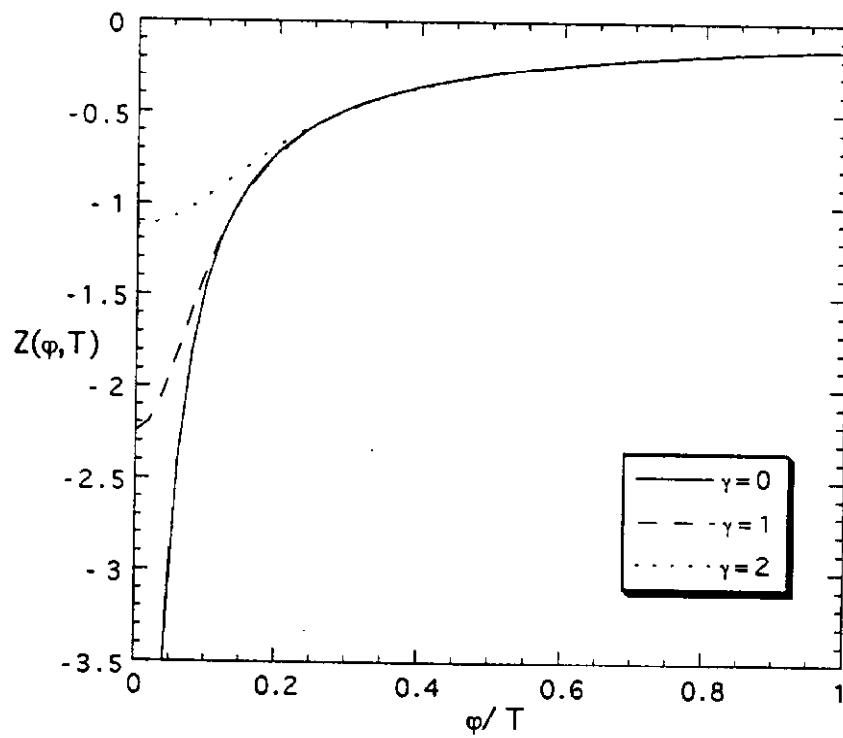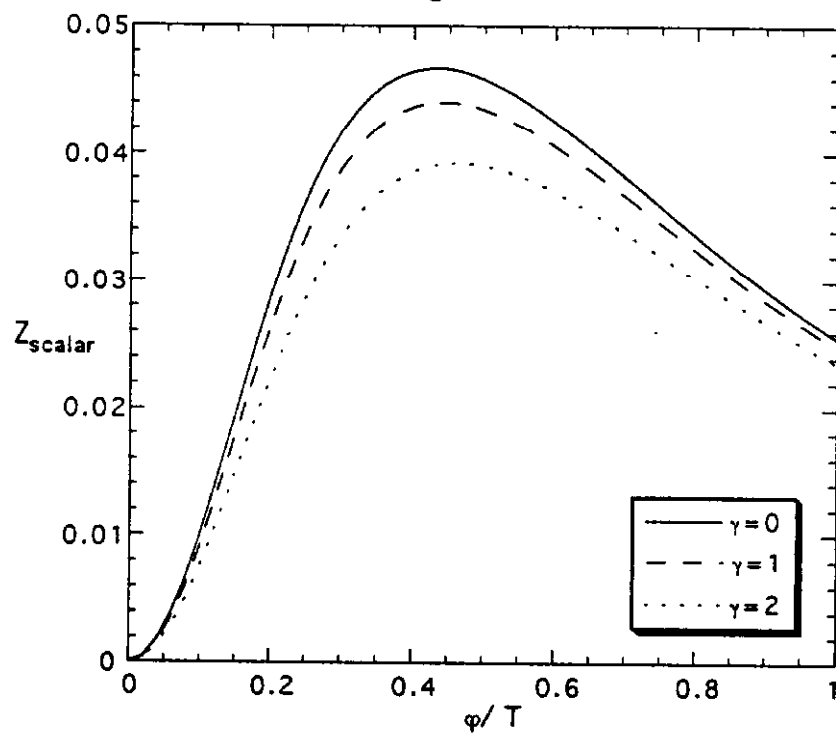
Figure 1

Figure 2.a

Figure 2.b
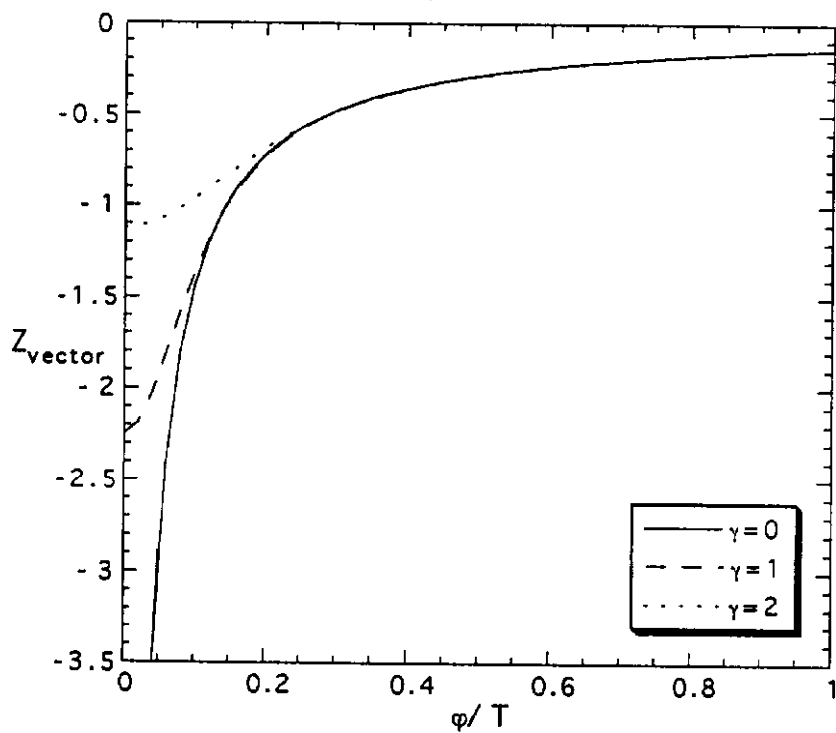

Figure 2.c

Figure 3