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Dimensional regularization and on-shell double-box problem

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Abstract

The analytic expression for massless double-box diagram in the form of two-fold parametric integral on reduced three-point function is presented. The representation of this three-point function in the form of Feynman parameter integral permits to analyse the behaviour of on- shell double-box diagram.

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I.

It is well known, that calculation of radiative corrections to elementary processes is very important in applications of perturbative QCD. One of the problem of perturbative QCD is the infrared divergences. The most convenient extraction of the divergences has been carried out through a dimensional regularization procedure. The Feynman integrals that we faced with in perturbation theory can be decomposed in a sum of scalar integrals, among which also those appear which correspond to a ϕ^3 theory. Double-box diagram in ϕ^3 theory is relevant to a number of important physical processes, but its evaluation presents a difficult technical problem.

In [1] has been proposed an approach, which makes it possible to evaluate double-box diagram with massless internal particles and arbitrary external momenta. The consideration has been carried out in four-dimensional space-time and has been based on such remarkable property as the correspondence of the off-shell expressions for three- and four-point ladder massless diagrams in four-dimensional space. Besides that the use of the four-dimensional space has given us the possibility to apply the analytical regularization to the considered diagrams and "uniqueness method" of evaluation [2][3] [4]. This approach involves also the using the Mellin-Barnes expansions at the intermediate stages.

For on-shell massless double-box diagram, when we need to use the dimensional regularization, the problem is far from trivial. We unfortunately cannot use the property of the correspondence between three- and four-point functions. We can only use the intermediate expansions and the "uniqueness method", as in our approach [1].

One-loop massless on-shell box diagram in ϕ^3 theory has been considered in [5]. We'll present here the analytic expression for on-shell double-box diagram $D^{(2)}(s,t,k_i^2)$, shown in Fig. 1, for two cases: when all $k_i^2 = 0 (i = 1,2,3,4)$ and when at least one of these external invariants k_i^2 isn't equal to zero.

II.

We use the standard notation $(k_1+k_2)^2 = s$, $(k_1+k_4)^2 = t$. Let $n = 4+2\varepsilon$ is the space-time dimension. Each line of a diagram ($m = 0$) carries a power-like factor $1/(x^2 - i\varepsilon)^\alpha$ in coordinate space, that is pictured as a line with the mark α. We do the calculations either in momentum or in coordinate space. We mention also the formula for propagator transformation from momentum to coordinate space:

$$\frac{1}{(k^2)^\alpha} \Longrightarrow \frac{\Gamma(\mu-\alpha)}{\Gamma(\alpha)} \cdot \frac{1}{(x^2)^{\mu-\alpha}}, \tag{2.1}$$

$2\mu = n$ being the space-time dimension.Not to complicate the formulae, we omit henceforth the factors, that are powers of 2,π,i. These factors can be easily restored in the final results. If $k_1^2 = 0$, $k_4^2 = 0$, $D^{(2)}(s,t,0,k_2^2,k_3^2,0)$ can be reduced to the three-point diagram $\Gamma^{(2)}(q_1^2,q_2^2,q_3^2)$ (see Fig.2):

$$D^{(2)}(s,t,0,k_2^2,k_3^2,0) = \int_0^1 dx \int_0^1 dy\, \Gamma^{(2)}(q_1^2,q_2^2,q_3^2), \tag{2.2}$$

where $q_1^2 = (k_2 + k_1 x_1)^2$, $q_2^2 = (k_3 + k_4 y_1)^2$, $q_3^2 = t x_2 y_2$. In coordinate space the diagram of Fig.2 corresponds to the diagram of Fig.3. Using the Feynman parameter method for the diagram in Fig.4 we get

$$\frac{\Gamma(1+2\varepsilon)}{\Gamma^3(1+\varepsilon)} \int \frac{\prod d\alpha_i \delta(1-\sum \alpha_i)(\alpha_1\alpha_2\alpha_3)^\varepsilon}{\{(x-z)^2\alpha_1\alpha_3 + (y-z)^2\alpha_2\alpha_3 + (x-y)^2\alpha_1\alpha_2\}^{1+2\varepsilon}}. \tag{2.3}$$

This integral can also be presented in terms of a two-fold Mellin-Barnes integral

$$\frac{1}{\Gamma(1-\varepsilon)\Gamma^3(1+\varepsilon)} \int_{-i\infty}^{i\infty} da \int_{-i\infty}^{i\infty} db \frac{\Gamma(a)\Gamma(b)\Gamma(1+2\varepsilon-a-b)}{\{(x-z)^2\}^a \{(y-z)^2\}^b \{(x-y)^2\}^{1+2\varepsilon-a-b}} \tag{2.4}$$

$$\cdot \Gamma(a-\varepsilon)\Gamma(b-\varepsilon)\Gamma(1+\varepsilon-a-b),$$

where the integration contours are chosen so to separate the "right" and "left" series of poles of gamma functions in the integrand (see, e.g. ref. [6]).

For diagram of Fig.3 we have the expression corresponding to diagram of Fig.5, which in momentum space corresponds to diagram of Fig.6. Thus for three-point diagram $\Gamma^{(2)}(q_1^2,q_2^2,q_3^2)$ we get the Feynman parameter integral

$$\Gamma^{(2)}(q_1^2,q_2^2,q_3^2) = \int \frac{\prod d\alpha_i \delta(1-\sum\alpha_i)\psi(\alpha_i)\alpha_1^{1-\varepsilon}\alpha_2^{1-\varepsilon}\alpha_3}{\{q_1^2\alpha_1\alpha_3 + q_2^2\alpha_2\alpha_3 + q_3^2\alpha_1\alpha_2\}^{3-2\varepsilon}}, \tag{2.5}$$

where

$$\psi(\alpha_i) = \frac{1}{\Gamma(1-\varepsilon)} \int_{-i\infty}^{i\infty} da \int_{-i\infty}^{i\infty} db\, \frac{\Gamma(a-\varepsilon)\Gamma(b-\varepsilon)\Gamma(1+\varepsilon-a-b)}{\alpha_1^{a-\varepsilon}\alpha_2^{b-\varepsilon}\alpha_3^{1+\varepsilon-a-b}} \tag{2.6}$$

$$\cdot \frac{\Gamma(a)\Gamma(\varepsilon)}{\Gamma(a+\varepsilon)} \cdot \frac{\Gamma(b)\Gamma(\varepsilon)}{\Gamma(b+\varepsilon)} = \int_0^1 d\xi\, (1-\xi)^{\varepsilon-1} \int_0^1 \frac{d\tau\, (1-\tau)^{\varepsilon-1}}{\{\alpha_1\xi + \alpha_2\tau + \alpha_3\xi\tau\}^{1-\varepsilon}}.$$

For the case of all $k_i^2 = 0$ (i=1,2,3,4) we have $q_1^2 = s x_1$, $q_2^2 = s y_1$, $q_3^2 = t x_2 y_2$.

The integral

$$\int_0^1 dx \int_0^1 \frac{dy}{\{s\alpha_1\alpha_3 x_1 + s\alpha_2\alpha_3 y_1 + t x_2 y_2 \alpha_1\alpha_2\}^{3-2\varepsilon}} \tag{2.7}$$

can be calculated. The obtained expression contains the double pole at $\alpha_3 = 0$ between other singularities in α_i. From the representation for $\psi(\alpha_i)$ we can see, that $\psi(\alpha_i)|_{\alpha_s=0} \neq 0$.

So this example shows that between the diagrams of the high order of perturbation theory such diagrams may be contained, which structure and high degree of singularities (in our example four external invariants $k_i^2 = 0(i = 1,2,3,4)$) don't allow to get the finite result for the diagram if we use only the dimensional regularization. The similar analysis of the representation for $\Gamma^{(2)}(q_1^2, q_2^2, q_3^2)$, when at least one of the external invariants k_i^2 isn't equal to zero, shows that for such double-box diagram the coefficients for the expansion into the series with respect to regularization parameter ε are definite functions.

III.

For the case $k_2^2 = \lambda \neq 0$ it is convenient to make a number of transformations. Using the uniqueness identity (see references in [2][3][4]) and the formula for convolution of lines in coordinate space (see Fig.7) we can reduce the initial diagram in such a form as shown in Fig.8, where

$$a_1(\varepsilon) = \frac{\Gamma(1+2\varepsilon)}{\Gamma(\varepsilon)\Gamma(1-\varepsilon)\Gamma(1+\varepsilon)},$$

$$a_2(\varepsilon) = \frac{\Gamma(1+2\varepsilon)}{\Gamma^3(1+\varepsilon)\Gamma(1-\varepsilon)},$$

$$a_3(\varepsilon) = 2\varepsilon\frac{\Gamma(1+2\varepsilon)\Gamma(2-\varepsilon)}{\Gamma^2(1+\varepsilon)\Gamma^2(1-\varepsilon)}.$$

At the end of reducing we get two-loop diagram for which the internal vertex is semiunique and the left triangle is semiunique too. We can use (see [3]) such identity for semiunique triangle as depicted in Fig.9.

In this way we reduce the initial triangle to the sum of three terms (see Fig.10), where

$$b_1 = 2, \quad b_2 = \frac{\Gamma(1+2\varepsilon)}{\Gamma^3(1+\varepsilon)\Gamma(1-\varepsilon)}, \quad b_3 = 2\varepsilon^2\frac{\Gamma(1+2\varepsilon)}{\Gamma^2(1+\varepsilon)\Gamma(1-\varepsilon)}.$$

For the second term we used the uniqueness identity such as in Fig.11 to get the final result .

The main idea of the representation of the initial diagram into such form consists in the possibility to reduce in the considered case the initial two-loop diagram to the sum of two one-loop diagrams and one two-loop diagram, but singularity of so obtained two-loop diagram concerning to parameter ε is weeker than the singularity of the initial diagram.

Then we transform slightly the terms in Fig.10 (with the help of uniqueness identity) . So the final expression for further analysis of the ε-expansion is defined by the diagrams of the form of Fig.12, where

$$c_1 = 3\cdot 2\varepsilon\frac{\Gamma(1+2\varepsilon)\Gamma(2-2\varepsilon)}{\Gamma^2(1+\varepsilon)\Gamma(1-2\varepsilon)\Gamma(1-\varepsilon)},$$

$$c_2 = 3\varepsilon\frac{\Gamma(1+2\varepsilon)\Gamma(2-2\varepsilon)}{\Gamma^2(1+\varepsilon)\Gamma(1-2\varepsilon)\Gamma(1-\varepsilon)},$$

$$c_3 = 2\varepsilon^2\Gamma(1+\varepsilon).$$

In momentum representation see Fig.13,where

$$d_1 = \frac{2}{\varepsilon}\frac{\Gamma^2(1+\varepsilon)\Gamma(1-\varepsilon)}{\Gamma(1+2\varepsilon)},$$

$$d_2 = \frac{2}{\varepsilon}\frac{\Gamma^2(1+\varepsilon)\Gamma(2-\varepsilon)}{\Gamma(1+2\varepsilon)},$$

$$d_3 = 2\varepsilon.$$

Then

$$\Gamma^{(2)}(q_1^2,q_2^2,q_3^2) = d_1F_1(q_1^2,q_2^2,q_3^2) - d_2F_2(q_1^2,q_2^2,q_3^2) - d_3F_3(q_1^2,q_2^2,q_3^2)$$

and $q_1^2 = \lambda x_2 + sx_1, q_2^2 = sy_1, q_3^2 = x_2y_2t$.

For $d_1F_1(q_1^2,q_2^2,q_3^2)$ we have

$$\frac{2\cdot 3\cdot\Gamma^3(1+\varepsilon)}{\Gamma(1+3\varepsilon)\Gamma(1+2\varepsilon)(q_1^2)^{1-2\varepsilon}(q_2^2)^{1-\varepsilon}}\int\frac{\prod d\alpha_i\delta(1-\sum\alpha_i)\alpha_1\alpha_2^{-\varepsilon}\alpha_3^{3\varepsilon-1}}{\{q_1^2\alpha_1\alpha_3+q_2^2\alpha_2\alpha_3+q_3^2\alpha_1\alpha_2\}^{1+\varepsilon}} = \qquad (3.1)$$

$$= \frac{2}{\varepsilon}\frac{\Gamma^2(1+\varepsilon)\Gamma(1-2\varepsilon)}{\Gamma(1+2\varepsilon)q_1^2(q_3^2)^{1-2\varepsilon}}\int_0^\infty\frac{d\zeta\zeta^{\varepsilon-1}}{(q_2^2+q_1^2\zeta)(1+\zeta)^{3\varepsilon}}+\Delta,$$

where Δ leads to term proportional to $1/\varepsilon$ in the final result.

For $d_2F_2(q_1^2,q_2^2,q_3^2)$ we have similarly

$$\frac{2}{\varepsilon}\frac{\Gamma^2(1+\varepsilon)\Gamma(1-2\varepsilon)}{\Gamma(1+2\varepsilon)q_1^2(q_3^2)^{1-2\varepsilon}}\int_0^\infty\frac{d\zeta\zeta^{2\varepsilon-1}}{(q_2^2+q_1^2\zeta)(1+\zeta)^{3\varepsilon}}+\Delta'.$$

Then

$$\int_0^1 dx \int_0^1 dy(d_1F_1 - d_2F_2) =$$

$$= \frac{3}{\varepsilon^3}\frac{f_1}{t^{1-2\varepsilon}s^{1-\varepsilon}}\int_0^1 \frac{dx x_2^{2\varepsilon-1}}{(\lambda x_2 + sx_1)^{1+\varepsilon}} - \frac{1}{\varepsilon^3}\frac{f_2}{t^{1-2\varepsilon}s^{1-2\varepsilon}}\int_0^1 \frac{dx x_2^{2\varepsilon-1}}{(\lambda x_2 + sx_1)^{1+2\varepsilon}} + O(\frac{1}{\varepsilon}),$$

where

$$f_1 = \frac{\Gamma^4(1+\varepsilon)\Gamma(1-\varepsilon)\Gamma(1-2\varepsilon)}{\Gamma(1+3\varepsilon)},$$

$$f_2 = \frac{\Gamma^2(1+2\varepsilon)\Gamma^2(1+\varepsilon)\Gamma^2(1-2\varepsilon)}{\Gamma(1+4\varepsilon)},$$

$$\int_0^1 \frac{dx x_2^{2\varepsilon-1}}{(\lambda x_2 + sx_1)^{1+\varepsilon}} = \frac{\Gamma(1+2\varepsilon)\Gamma(1-\varepsilon)}{2\varepsilon\Gamma(1+\varepsilon)\lambda^{2\varepsilon}s^{1-\varepsilon}} - \varepsilon\int_0^\infty \frac{d\zeta \ln(1+\zeta)}{\zeta(\lambda\zeta + s)} =$$

$$= \frac{\Gamma(1+2\varepsilon)\Gamma(1-\varepsilon)}{2\varepsilon\Gamma(1+\varepsilon)\lambda^{2\varepsilon}s^{1-\varepsilon}} - \varepsilon\int_{-i\infty}^{i\infty} \frac{da\Gamma^2(a)\Gamma^2(1-a)}{a\lambda^a s^{1-a}} =$$

$$= \frac{\Gamma(1+2\varepsilon)\Gamma(1-\varepsilon)}{2\varepsilon\Gamma(1+\varepsilon)\lambda^{2\varepsilon}s^{1-\varepsilon}} - \frac{\varepsilon}{s}(Li_2(1) - Li_2(\frac{\lambda - s}{\lambda})),$$

$$\int_0^1 \frac{dx x_2^{2\varepsilon-1}}{(\lambda x_2 + sx_1)^{1+2\varepsilon}} = \frac{1}{2\varepsilon s\lambda^{2\varepsilon}}.$$

To calculate $\int_0^1 dx \int_0^1 dy F_3(q_1^2, q_2^2, q_3^2)$ we first make such transformation of diagram at Fig.12(3) as depicted at Fig.14, where $g_1 = 1 + O(1/\varepsilon)$, $g_2 = 1 + O(1/\varepsilon)$. It is convenient to make such transformation since the integration of the resulting expression for such diagram over y is carried out immediatly.

The Feynman parameter representation for resulting two-loop diagram in momentum space has the form (the proof is similarly to that for diagram of Fig.3)

$$\int_0^1 dx \int_0^1 dy\, d_3F_3(q_1^2, q_2^2, q_3^2) =$$

$$= 6\varepsilon\int_0^1 dx \int_0^1 dy \frac{1}{(q_1^2)^{1-2\varepsilon}} \int \frac{\prod d\alpha_i \delta(1 - \sum\alpha_i)\alpha_1^{2\varepsilon}\alpha_2^{1-\varepsilon}\alpha_3^{1+2\varepsilon}\psi(\alpha_i)}{\{q_1^2\alpha_1\alpha_3 + q_2^2\alpha_2\alpha_3 + q_3^2\alpha_1\alpha_2\}^2} + O(\frac{1}{\varepsilon}), \quad (3.2)$$

where

$$\psi(\alpha_i) = \int_0^\infty \frac{dt}{(1+t)^{1-\varepsilon}} \int_0^\infty \frac{d\tau\tau^{3\varepsilon-1}}{(1+\tau)^{2\varepsilon}\{\alpha_1 t + \alpha_2\tau + 1\}^{1+2\varepsilon}} = -\frac{\alpha_2^{-3\varepsilon}\ln\alpha_1}{3\varepsilon(1-\alpha_1)} + O(1). \quad (3.3)$$

Then because $q_2^2 = sy_1$, $q_3^2 = x_2y_2t$, we have

$$\int_0^1 dx \int_0^1 dy d_3F_3(q_1^2, q_2^2, q_3^2) =$$

$$= -2\frac{1}{(q_1^2)^{1-2\varepsilon}}\int_0^1 dx \int \frac{\prod d\alpha_i \delta(1-\sum\alpha_i)\alpha_1^{2\varepsilon-1}\alpha_2^{1-4\varepsilon}\alpha_3^{2\varepsilon}\ln\alpha_1}{(1-\alpha_1)(q_1^2\alpha_1+s\alpha_2)(q_1^2\alpha_3+tx_2\alpha_2)} + O(\frac{1}{\varepsilon}) = \tag{3.4}$$

$$= \frac{1}{2\varepsilon^2 s^{1-2\varepsilon}}\int_0^1 d\beta\int_0^1 dx\frac{1}{q_1^2(tx_2\beta+q_1^2(1-\beta))} + O(\frac{1}{\varepsilon}) =$$

$$= \frac{1}{2\varepsilon^2 s^{2-2\varepsilon}}\int_{-i\infty}^{i\infty}\frac{da\Gamma^2(a)\Gamma^2(1-a)}{a\lambda^a t^{1-a}} + O(\frac{1}{\varepsilon}) =$$

$$= \frac{1}{2\varepsilon^2 s^{2-2\varepsilon}t}(Li_2(1) - Li_2(\frac{\lambda-t}{\lambda})) + O(\frac{1}{\varepsilon}).$$

So

$$D^{(2)}(s,t,0,\lambda,0,0) = \int_0^1 dx\int_0^1 dy\Gamma^{(2)}(q_1^2,q_2^2,q_3^2) =$$

$$= \frac{1}{\varepsilon^4\lambda^{2\varepsilon}t^{1-2\varepsilon}s^{2-2\varepsilon}}(\frac{3}{2}f_1\frac{\Gamma(1+2\varepsilon)\Gamma(1-\varepsilon)}{\Gamma(1+\varepsilon)} - \frac{1}{2}f_2) -$$

$$-\frac{3}{\varepsilon^2 s^{2-\varepsilon}t^{1-2\varepsilon}}(Li_2(1)-Li_2(\frac{\lambda-s}{\lambda})) - \frac{1}{2\varepsilon^2 s^{2-2\varepsilon}t}(Li_2(1)-Li_2(\frac{\lambda-t}{\lambda})) + O(\frac{1}{\varepsilon}), \tag{3.5}$$

where polylogarithms $Li_N(z)$ are defined as (see[7]):

$$Li_N(z) = \frac{(-1)^N}{(N-1)!}\int_0^1 d\xi\frac{\ln^{N-1}(\xi)}{\xi - z^{-1}}. \tag{3.6}$$

IV.

It is well known that the calculation of the next to the leading order contribution in different physical processes is rather intricate. In the present paper the approach has been presented which makes it possible to evaluate on-shell double-box diagram in the case when one of the external invariants k_i^2 isn't equal to zero. The use of the "uniqueness method" and such intermediate expansions as Mellin-Barnes expansions essentially simplified the procedure of calculating the double-box diagram and allowed to solve this problem up to the next to the leading order.

V.

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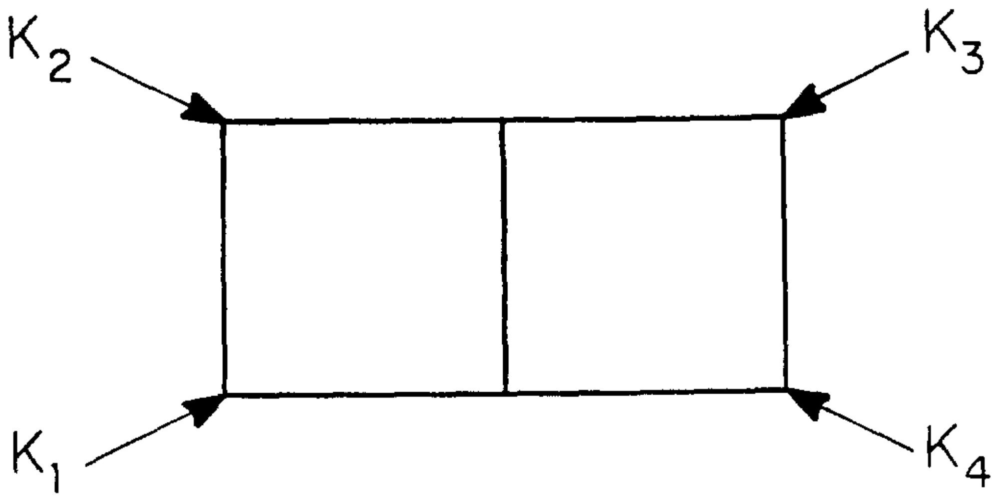


Fig. 1

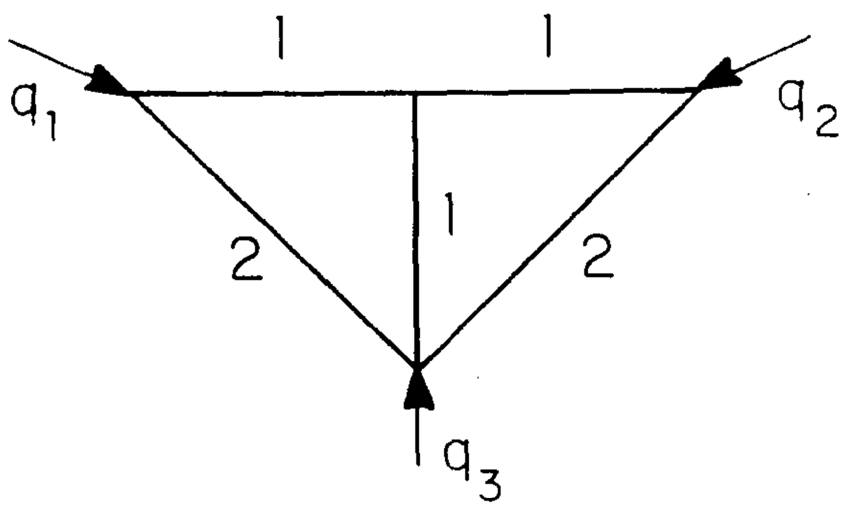


Fig. 2

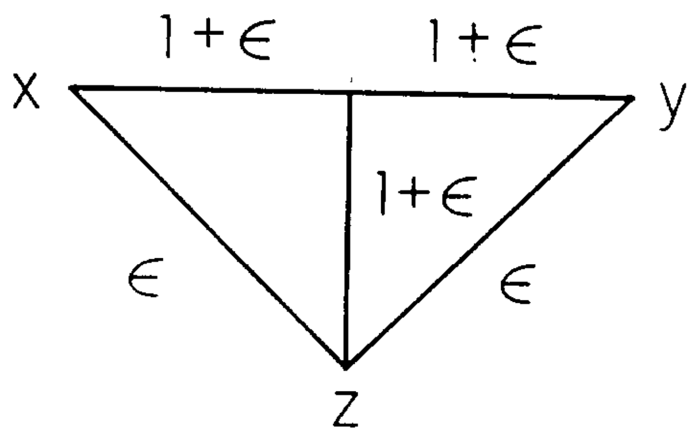
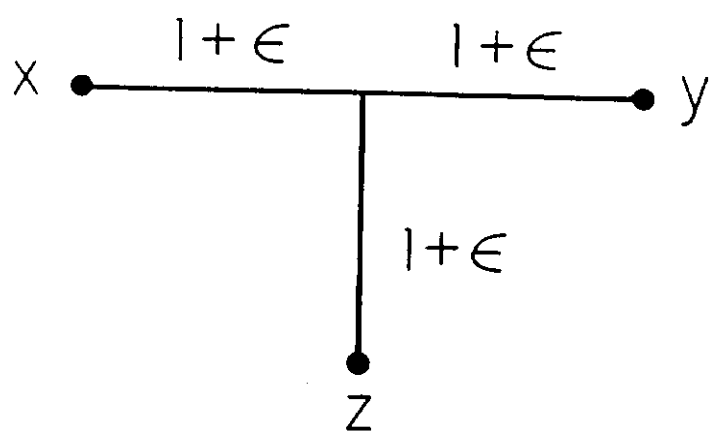


Fig. 3



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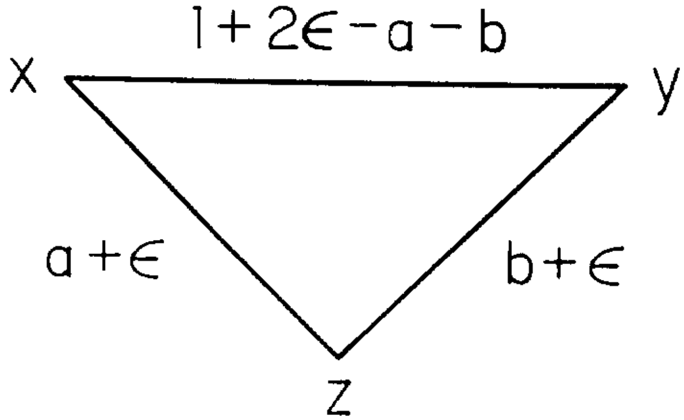

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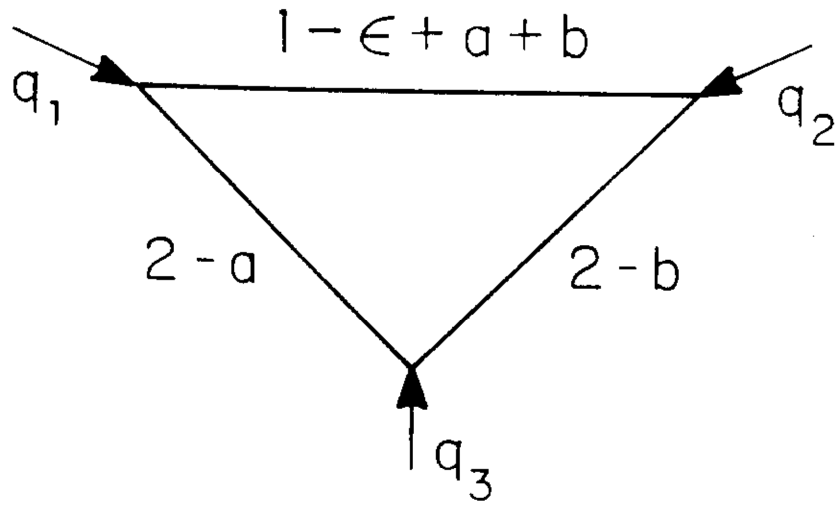

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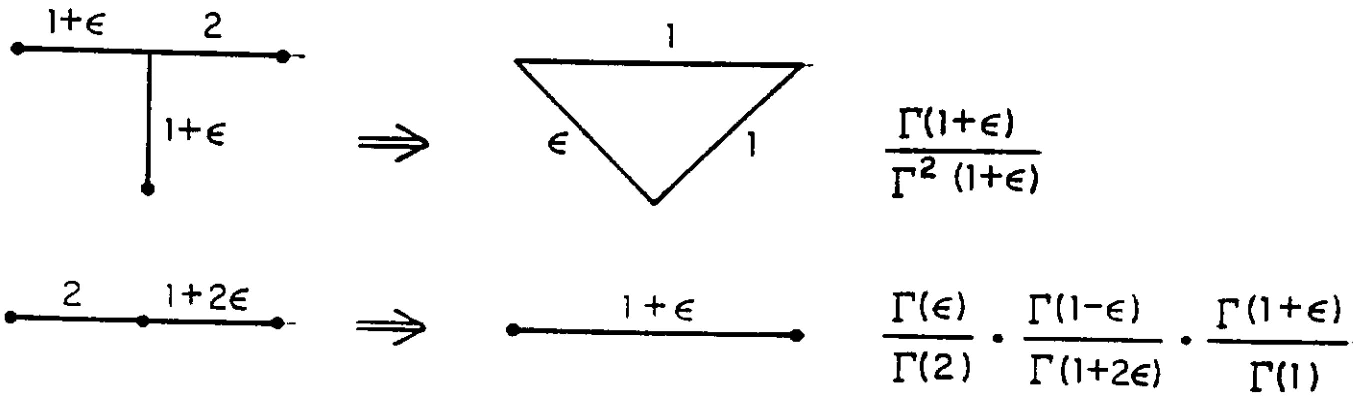
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Fig. 7

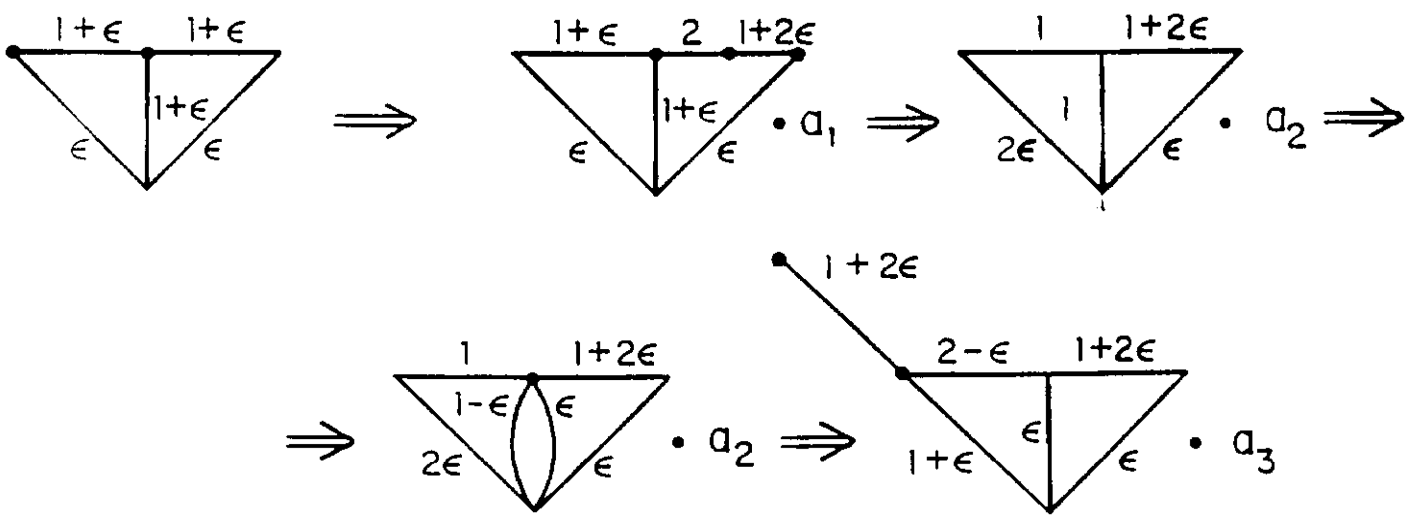
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Fig. 8

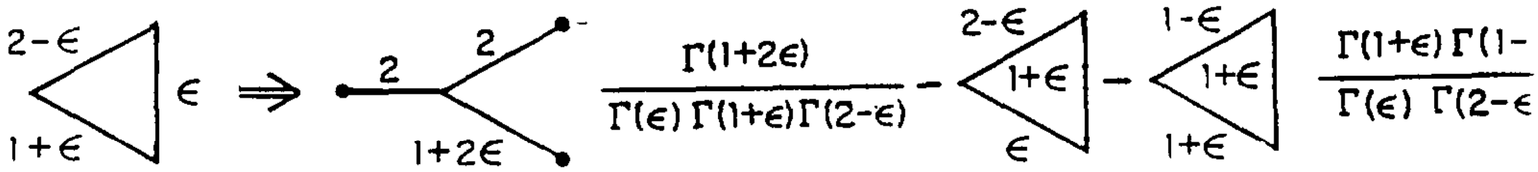
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Fig. 9

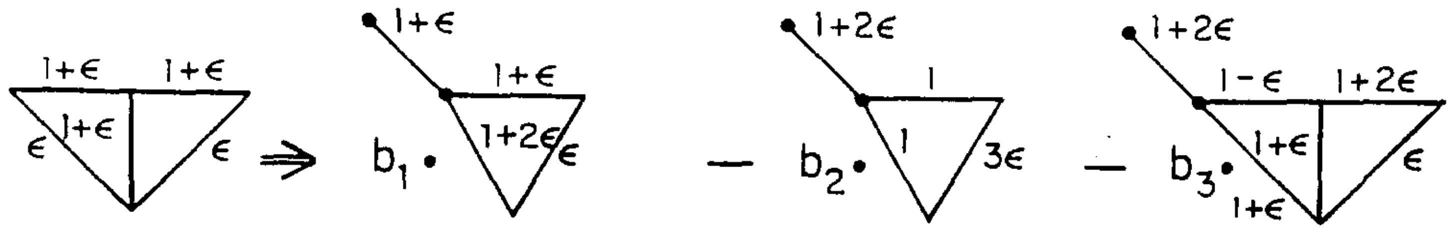
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Fig. 10

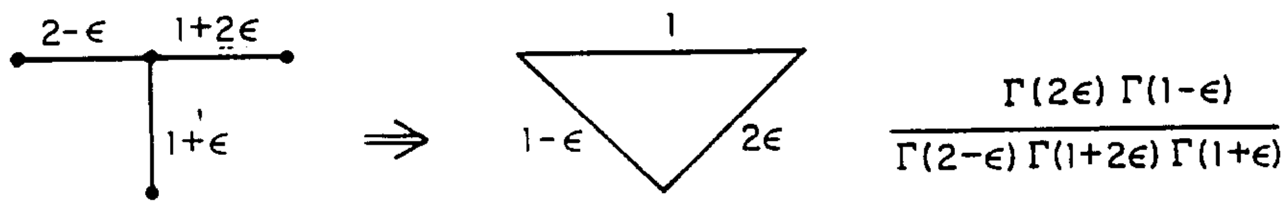
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Fig. 11

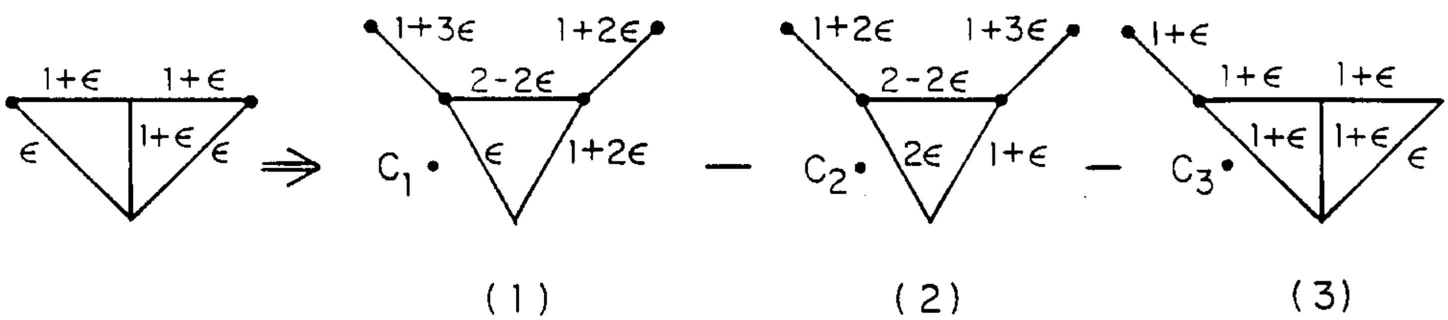
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Fig. 12

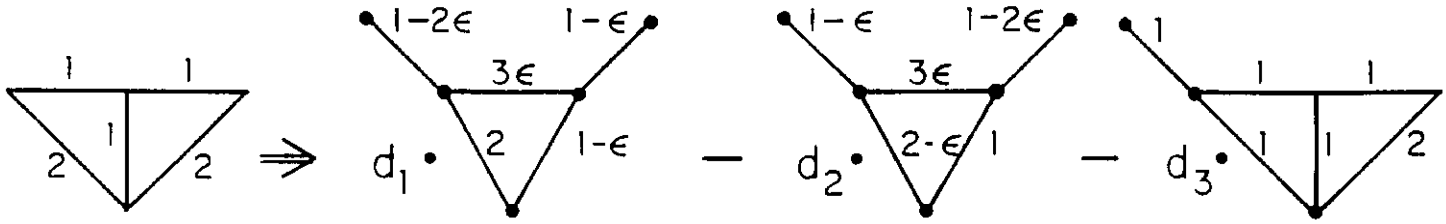
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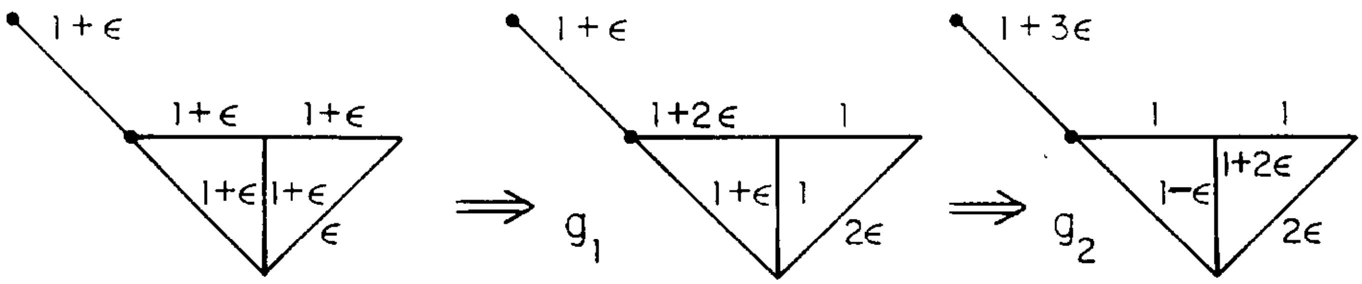
Fig. 13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1−ϵ
1+2ϵ
2ϵ

Fig. 14