Fermi National Accelerator Laboratory

FERMILAB-Pub-93/181-A
astro-ph/9307025
submitted to *Physical Review D*

EFFECT OF FINITE NUCLEON MASS ON PRIMORDIAL NUCLEOSYNTHESIS

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ABSTRACT

We have modified the standard code for primordial nucleosynthesis to include the effect of finite nucleon mass on the weak-interaction rates as calculated by Seckel [1]. We find a small, systematic increase in the ^{4}He yield, $\Delta Y \simeq 0.0057\,Y$, which is insensitive to the value of the baryon-to-photon ratio η and slightly larger than Seckel's estimate. The fractional changes in the abundances of D, ^{3}He, and ^{7}Li range from 0.08% to 3% for $10^{-11} \leq \eta \leq 10^{-8}$.

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1 Introduction

Primordial nucleosynthesis is one of the cornerstones of the hot big-bang cosmology. The agreement between its predictions for the abundances of D, ^{3}He, ^{4}He and ^{7}Li and their inferred primordial abundances provides its earliest, and perhaps most, stringent test. Further, big-bang nucleosynthesis has been used to provide the best determination of the baryon density [2, 3] and to test particle-physics theories, e.g., the stringent limit to the number of light neutrino species [4].

The scrutiny of primordial nucleosynthesis, both on the theoretical side and on the observational side, has been constant: Reaction rates have been updated and the effect of their uncertainties quantified [5], finite-temperature corrections have been taken into account [6], the effect of inhomogeneities in the baryon density explored [7], and the slight effect of the heating of neutrinos by $e^{\pm}$ annihilations has been computed [8]; the primordial abundance of ^{7}Li has been put on a firm basis [9], the production and destruction of D and ^{3}He have been studied carefully [10], and astrophysicists now argue about the third significant figure in the primordial ^{4}He abundance [11].

A measure of the progress in this endeavour is provided by the shrinking of the "concordance region" of parameter space. The predicted and measured primordial abundances agree provided: the baryon-to-photon ratio lies in the narrow interval $3 \times 10^{-10} \lesssim \eta \lesssim 4 \times 10^{-10}$ and the equivalent number of light neutrino species $N_\nu \lesssim 3.3$ [3]. The shrinking of the concordance interval motivates the study of smaller and smaller effects.

Seckel [1] has recently calculated the corrections to the weak-interaction rates that arise from taking account of the finite nucleon mass (in the standard code these rates are computed in the infinite-nucleon-mass limit). The corrections involve terms of order m_e/m_N, T/m_N, and Q/m_N, which are all of the order of 0.1%. Here m_e is the electron mass, m_N is the nucleon mass, $T \sim \mathcal{O}(\,\mathrm{MeV})$ is the temperature during the epoch of nucleosynthesis, and $Q = m_n - m_p = 1.293\,\mathrm{MeV}$ is the neutron-proton mass difference. The weak interaction rates govern the neutron-to-proton ratio and thereby are crucial to the outcome of nucleosynthesis; e.g., the mass fraction of ^{4}He produced is roughly twice the neutron fraction at the time nucleosynthesis commences ($T \sim 0.07\,\mathrm{MeV}$).

The net effect of the finite-nucleon-mass corrections is to decrease the weak rates by about 1% around the time of nucleosynthesis. Based upon a simple code that follows the neutron fraction Seckel estimated that the cumulative effect of all the corrections increase the mass fraction of ^{4}He synthesized by $\Delta Y \approx 0.0012$. Because the third significant figure of the primordial ^{4}He abundance is now very relevant, we decided to incorporate the finite-nucleon-mass corrections to the weak-interaction rates into the standard nucleosynthesis code [12]. In the next Section we describe the modifications we made; we finish with a discussion of our results for the change in the yield of ^{4}He, which is slightly larger than Seckel's estimates, and for the changes in the yields of the other light elements.

2 Modifications to the Standard Code

2.1 Role of weak interactions

The weak interactions that interconvert neutrons and protons, $n \leftrightarrow p+e+\nu$, $n+e \leftrightarrow p+\nu$, and $n+\nu \leftrightarrow p+e$, play a crucial role as they govern the neutron fraction, and the neutron fraction ultimately determines the amount of nucleosynthesis that takes place. (Here and throughout we use e to indicate electron or positron, and ν to indicate electron neutrino or antineutrino; the appropriate particle or antiparticle designation follows from charge and lepton number conservation.)

The weak-interaction rate per nucleon is very roughly $\Gamma_{n\to p} \sim \Gamma_{p\to n} \sim G_F^2 T^5$, while the expansion rate of the Universe $H \sim T^2/m_{\rm Pl}$; here $G_F = 1.1664 \times 10^{-5}\,{\rm GeV}^{-2}$ is the Fermi constant and $m_{\rm Pl} = 1.22 \times 10^{19}\,{\rm GeV}$ is the Planck mass. At temperatures greater than about 0.8 MeV, Γ is greater than H, and the neutron-to-proton ratio tracks its equilibrium value

$$\left(\frac{n}{p}\right)_{\rm EQ} = (m_n/m_p)^{3/2} e^{-Q/T}. \tag{1}$$

Two comments are in order. First, in the infinite-nucleon-mass limit the prefactor is unity; taking account of this factor tends to increase the equilibrium ratio by about 0.2%, suggesting that the neutron abundance and final ^{4}He abundance should be correspondingly larger. Second, at a temperature of about 1 MeV, the neutrino and photon temperatures begin to deviate as neutrinos decouple from the electromagnetic plasma ($e^\pm$, γ) and $e^\pm$ annihilations begin to heat the photons relative to the neutrinos as $e^\pm$ pairs transfer their entropy to the photons. When this happens the equilibrium value of the neutron-to-proton ratio is no longer given by such a simple formula.

When the temperature of the Universe drops below about 0.8 MeV the weak-interaction rate are no longer greater than the expansion rate and the neutron-to-proton ratio ceases to track its equilibrium value, and is said to "freeze out." Until nucleosynthesis begins in earnest ($T \sim 0.07\,\mathrm{MeV}$) the neutron-to-proton ratio decreases slowly due to weak interactions (especially neutron decay). For $\eta \sim 3 \times 10^{-10}$ it decreases from about 1/6 at freeze out to about 1/7 when nucleosynthesis begins, finally resulting in a mass fraction of ^{4}He equal to about 25%. For a detailed discussion of the physics of primordial nucleosynthesis see Refs. [13, 14].

In the infinite-nucleon-mass limit the rates (per particle) for the six reactions that interconvert neutrons and protons are given by [13]

$$\begin{aligned}
\lambda(n+\nu \to p+e) &= \mathcal{C}\int_Q^\infty \frac{p\,E\,(E-Q)^2\,dE}{[e^{(E-Q)/T_\nu}+1]\,[e^{-E/T}+1]};\\
\lambda(n+e \to p+\nu) &= \mathcal{C}\int_{m_e}^\infty \frac{p\,E\,(E+Q)^2\,dE}{[e^{-(E+Q)/T_\nu}+1]\,[e^{E/T}+1]};\\
\lambda(n \to p+e+\nu) &= \mathcal{C}\int_{m_e}^Q \frac{p\,E\,(E-Q)^2\,dE}{[e^{(E-Q)/T_\nu}+1]\,[e^{-E/T}+1]};\\
\lambda(p+e \to n+\nu) &= \mathcal{C}\int_Q^\infty \frac{p\,E\,(E-Q)^2\,dE}{[e^{(Q-E)/T_\nu}+1]\,[e^{E/T}+1]};\\
\lambda(p+\nu \to n+e) &= \mathcal{C}\int_{m_e}^\infty \frac{p\,E\,(E+Q)^2\,dE}{[e^{(E+Q)/T_\nu}+1]\,[e^{-E/T}+1]};\\
\lambda(p+e+\nu \to n) &= \mathcal{C}\int_{m_e}^Q \frac{p\,E\,(E-Q)^2\,dE}{[e^{(Q-E)/T_\nu}+1]\,[e^{E/T}+1]}; \qquad (2)
\end{aligned}$$

where T denotes the photon temperature, T_ν the neutrino temperature, and the common factor

$$\mathcal{C} = \frac{G_F^2 \cos^2\theta_C (1+3c_A^2)}{2\pi^3}\, f_{\mathrm{EM}}.$$

Here θ_C is the Cabibbo angle ($\cos\theta_C = 0.975$), $c_A = 1.257$ is the ratio of the axial vector to vector coupling of the nucleon, and $f_{\mathrm{EM}} \sim 1.08$ quantifies the radiative and Coulomb corrections, which for our purposes here are not important (for more details see Ref. [6]).

2.2 Finite-mass corrections

Seckel has calculated the corrections to the weak rates due to finite nucleon mass [1]. He has grouped them into three categories; the corrections due to: (i) thermal motion of the target nucleon; (ii) the recoil energy of the outgoing nucleon; and (iii) weak magnetism. The net effect of these corrections is to reduce the weak rates by about 1% around the time the neutron-to-proton ratio freezes out. This in turn causes freeze out to occur slightly earlier, at a higher value of the neutron-to-proton ratio, resulting in an increase in ^{4}He production.

The corrections to the rates for neutron decay and inverse decay are the simplest, just the factor that accounts for time dilation,

$$\lambda(n \leftrightarrow p + e + \nu) \longrightarrow \left(1 - \frac{3}{2}\frac{T}{m_N}\right) \lambda(n \leftrightarrow p + e + \nu). \tag{3}$$

Since the neutron lifetime is used to normalize all the weak rates, the recoil effect is automatically taken into account.

The corrections to the weak interactions that involve $2 \leftrightarrow 2$ scatterings are organized in a way such that the integrands in the previous expressions, cf. Eqs. (2), are simply multiplied by a correction factor,[1]

$$1 - \delta_n + \tilde{\gamma}_{\rm wm} + \tilde{\gamma}_{\rm rec} + \tilde{\gamma}_{\rm th} + \tilde{\gamma}_{\rm etc}, \tag{4}$$

where $\delta_n = -0.00201$ "uncorrects" the neutron lifetime for recoil effects, which are taken into account in the term $\tilde{\gamma}_{\rm rec}$, $\tilde{\gamma}_{\rm wm}$ is the weak magnetism correction, $\tilde{\gamma}_{\rm th}$ is the thermal correction, and $\tilde{\gamma}_{\rm etc}$ is the sum of three smaller (by a factor of 100) and less important terms, which we have included in our computations, but which have negligible effect.

The weak magnetism, recoil, and thermal corrections are given by [1]

$$\begin{aligned}
\tilde{\gamma}_{\rm wm} &= \frac{2c_A f_2}{1+3c_A^2}\,\frac{E_3 k_1^2 + E_1 k_3^2}{m_N E_1 E_3}; && (5)\\
\tilde{\gamma}_{\rm rec} &= \frac{\tilde{\gamma}_{\rm wm}}{f_2} - \frac{2E_1 k_3^2 + E_3(k_1^2+k_3^2)}{2m_N k_3^2}\,\frac{m_1^2 - m_3^2 - Q^2}{2(1+3c_A^2)m_N E_3} \\
&\quad + \frac{c_A^2}{1+3c_A^2}\,\frac{6E_1^2E_3 - 6E_1E_3^2 - 3E_1k_1^2 - 4E_3k_1^2 - E_1k_3^2}{2m_N E_1 E_3}; && (6)\\
\tilde{\gamma}_{\rm th} &= \left(\frac{T}{m_N}\right)\left[\frac{3E_1^2 + 2k_1^2}{2E_1E_3} + \frac{3k_1^2E_1 + 3E_1^2E_3 + 2k_1^2E_3}{2E_1k_3^2} - \frac{k_1^2E_3^2}{2k_3^4}\right]; && (7)
\end{aligned}$$

[1]Our notation differs very slightly from that of Ref. [1]; $\tilde{\gamma}_i = \gamma_i/\gamma_0$.

subscript 1 (3) refers to the incoming (outgoing) lepton, subscript 2 (4) refers to the incoming (outgoing) nucleon, E_i is the energy of the ith particle, k_i is its momentum, and $f_2 = \pm 3.62$ where + applies to reactions with leptons and − to reactions with antileptons. For the three smaller terms embodied in $\tilde{\gamma}_{\rm etc}$ we refer the reader to Ref. [1].

Three small points; the limits of integration are those in the infinite-nucleon-mass limit, cf. Eqs. (2), with one exception (see below). In Eqs. (2) the integrals are performed over the energy of the electron; the neutrino energy is just that of the electron plus or minus Q. The integral of the final term in $\tilde{\gamma}_{\rm th}$, which is proportional to k_3^{-4}, diverges for the reaction $p+\nu \rightarrow n+e$ for the infinite-nucleon-mass limits because $k_3 \rightarrow 0$ at threshold. For this term, and only this term, the lower limit of integration involves the finite-nucleon-mass kinematics: the minimum electron momentum is: $(k_3)_{\rm min} = (m_e/m_N)^{1/2}(Q+m_e)$ [15]. Because essentially all of the integral accumulates near $(k_3)_{\rm min}$ this term can be integrated analytically:

$$\left(\frac{T}{m_N}\right)\int \frac{p\,E\,(E+Q)^2\,dE}{[e^{(E+Q)/T_\nu}+1][e^{-E/T}+1]}\frac{k_1^2E_3^2}{2k_3^4} = \frac{(T/m_N)(Q+m_e)^3 m_e^{3/2} m_N^{1/2}}{2[e^{(Q+m_e)/T_\nu}+1][e^{-m_e/T}+1]}. \tag{8}$$

Finally, we mention that Seckel [1] has derived linear fits to the perturbed weak-interaction scattering rates:

$$\lambda(n\rightarrow p) \rightarrow \left[1-\delta_n-0.00185-0.01032\left(\frac{T}{m_N}\right)\right]\lambda(n\rightarrow p); \tag{9}$$

$$\lambda(p\rightarrow n) \rightarrow \left[1-\delta_n+0.00136-0.01067\left(\frac{T}{m_N}\right)\right]\lambda(p\rightarrow n); \tag{10}$$

where the first linear correction applies to the $2 \leftrightarrow 2$ reactions that convert neutrons to protons, $n+\nu \rightarrow p+e$ and $n+e \rightarrow p+\nu$, and is valid for $2\,{\rm MeV} \gtrsim T \gtrsim 0.3\,{\rm MeV}$, and the second applies to the $2 \leftrightarrow 2$ reactions that convert protons to neutrons, $p+\nu \rightarrow n+e$ and $p+e \rightarrow n+\nu$, and is valid for $2\,{\rm MeV} \gtrsim T \gtrsim 0.7\,{\rm MeV}$.

3 Results and Conclusions

We have modified the integrands in the subrountines that compute the weak-interaction rates in the standard nucleosynthesis code [12] as outlined above.

For comparison, we have also modified the standard code just using Seckel's linear fit, which is much easier to implement since the usual rate is multiplied by a factor outside the integral. Our results, which were obtained by taking three massless neutrino species and a mean neutron lifetime of 889 sec, are shown in Figs. 1 and 2.

In Fig. 1 the change in the mass fraction of ^{4}He and 0.0057 times the ^{4}He yield are shown. While over the range $\eta = 10^{-11}$ to 10^{-8} ΔY varies from about 0.0004 to 0.0015, ΔY is remarkably close to $0.0057\,Y$. We also computed the change in ^{4}He yield using Seckel's linear fit; for the values of η above, $\Delta Y/Y$ was slightly lower, by about 0.05%. Neglecting the subdominant terms ($\tilde{\gamma}_{\rm etc}$) decreases $\Delta Y/Y$ from about 0.57% to about 0.53%.

In Fig. 2 we show the fractional changes in the abundances of D, ^{3}He, and ^{7}Li (relative to H). These changes, over the same interval in baryon-to-photon ratio, range from 0.08% to almost 3%. Since the inferred primeval abundances of these elements are no where near as well known as that of ^{4}He, these changes are of little relevance at present.

We thank David Seckel for many helpful conversations. This work was supported in part by the DOE (at Chicago and Fermilab), by the NASA through NAGW-2381 (at Fermilab), and GG's NSF predoctoral fellowship.

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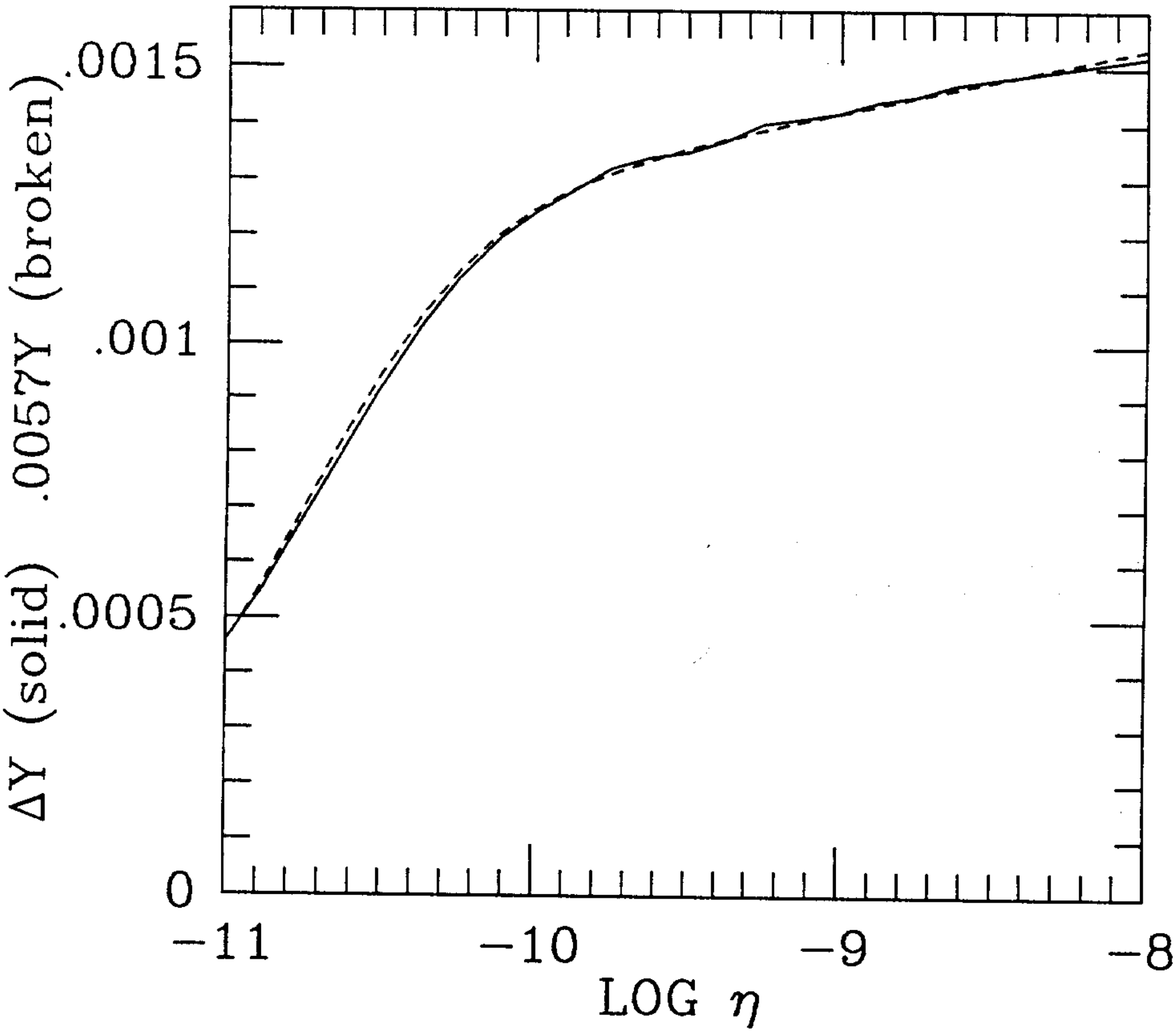
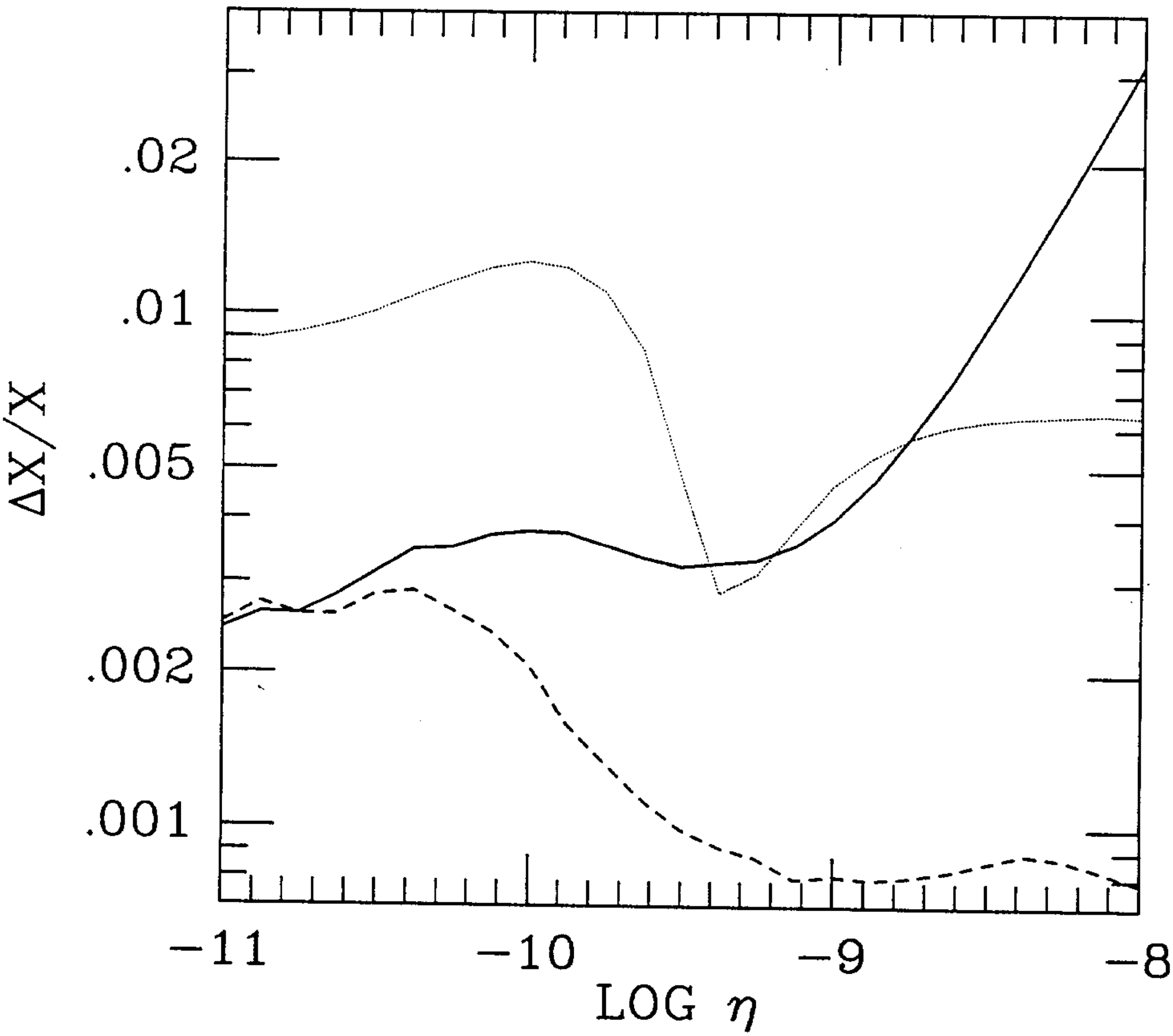
FIGURE CAPTIONS

Figure 1: The change in the yield of ^{4}He as a function of the baryon-to-photon ratio η, and, for comparison, 0.0057 times the ^{4}He mass fraction.

Figure 2: The fractional changes in the yields of D (solid curve), ^{3}He (broken curve), and ^{7}Li (dotted curve) as a function of the baryon-to-photon ratio η.



-FIG 1-



- FIG 2 -