Fermi National Accelerator Laboratory

FERMILAB-PUB-93/144-T
June 1993

LOW x_B NEWS for EXPERTS

E. Levin

Fermi National Accelerator Laboratory, P.O.Box 500,Batavia,Il 60510, USA
and
St. Petersburg Nuclear Physics Institute,188350 , Gatchina, Russia

Review talk at UK Phenomenology Workshop " HERA - the new frontier for QCD ",
Durham, March 1993.

Abstract

This is a status report on the recent theoretical development in the region of small x_B in deeply inelastic scattering.

1 Introduction

I am viewing this talk as a summary of the recent theoretical developments in the region of small x_B. I hope to convince you that we made considerable progress in our understanding of the theoretical basis and the physical meaning of the new phenomena that we anticipate in the region of small x_B. In the talk I am going to cover the following topics:

1. Anomalous dimension of high twist operators and a new evolution equation for the deep inelastic structure function.
2. Energy conservation in Gribov - Levin - Ryskin (GLR) equation.
3. Large rapidity gaps in deeply inelastic scattering.
4. Nonperturbative (*instanton*) contribution to deeply inelastic scattering.
5. Scale of the shadowing (screening) correction (SC).

Operated by Universities Research Association Inc. under contract with the United States Department of Energy

2 Anomalous dimension of high twist operators and a new evolution equation.

Let me start by recalling the main steps of our theoretical approach to deeply inelastic scattering:

1. We introduce the moments of the deep inelastic structure function,namely

$$M(\omega,r) \;=\; \int_0^1 x_B^{N-1} dx_B\, x_B G(x_B,Q^2) \;=\; \int_0^\infty e^{\omega y} dy [x_B G(x_B,Q^2)] \;, \tag{1}$$

where $\omega \;=\; N-1$, $y = \ln(1/x_B)$ and $r = \ln(Q^2/Q_0^2)$.

2. Each moment is given as Wilson Operator Product Expansion in the form:

$$M(\omega,r) \;=\; C_2(\omega,r)\langle p|O^{(2)}|p\rangle \;+\; \frac{1}{Q^2} C_4(\omega,r)\langle p|O^{(4)}|p\rangle \;+\; \dots \frac{1}{Q^{i-2}} C_i(\omega,r)\langle p|O^{(i)}|p\rangle \dots \tag{2}$$

where C_i is the coefficient function and $\langle p|O^{(i)}|p\rangle$ is the matrix element of the twist i operator (see ref.[1] for details).

3. It is well known from the renormalization group approach that a coefficient function C_i behaves as

$$C_i \;\propto\; e^{\gamma_i(\omega) r} \tag{3}$$

where γ_i is the anomalous dimension of the twist i operator [1].

4. Now we neglected all high twist contributions (all terms in eq. (1) except the first one) assuming that they are small at large value of Q^2 due to the factor $\frac{1}{Q^{i-2}}$ in front.

5. The anomalous dimension of the leading twist contribution can be calculated using GLAP evolution equation [2] and it is equal to

$$\gamma_2(\omega) \;=\; \frac{N_c \alpha_s}{\pi \omega} \quad at \quad \omega \rightarrow 0 \tag{4}$$

6. The specific contribution to the value of the anomalous dimension of high twist operator that originates from the exchange of many 'leading twist ladders' in t- channel was found in the GLR paper [3] . It gives

$$\gamma_{2n}(\omega) \;=\; n \gamma_1(\frac{\omega}{n}) \;. \tag{5}$$

7. Recently Bartels [4] and Levin,Ryskin and Shuvaev [5] have performed the next step in understanding the high twist contribution to eq. (1) and both groups calculated the

[1] For simplicity we consider here the case of fixed α_s.

be able to help us for twist $2n > 4$ operator. However we are able to learn more about our system of partons in solving this problem, in particular we can answer the question: what could be the scenario for the system of bosons (gluons) interacting with the attractive forces besides a collapse.

2. to find the generalization of nonlinear GLR evolution equation taking into account both the arbitrary initial condition and the correct value of the anomalous dimension for high twist operator ($\gamma_{2n} \neq n\,\gamma(\frac{\omega}{n}) = \frac{n^2\bar{\alpha}_s}{\omega}$ that has been used in GLR equation). Here I would like to emphasize that GLR equation was proved within additional hypothesis that gluons have no correlations except the fact that they are distributed in the hadron disc of the radius R. The correct value of the anomalous dimension for twist four operator means that the correlation radius between two gluons increases at $x_B \rightarrow 0$. and gluons create a more compact system than the hadron. In ref. [8] you can find the evolution equation that describes the fact that the correlation radius increases and this growth will be stopped due to shadowing corrections.

Solutions:

1. The value of the anomalous dimension of the high twist operators has been found in ref. [9] by E.Laenen,E.Levin,M. Ryskin and A.Shuvaev. The main idea was to reduce the complicated problem of the gluon - gluon interaction to interaction of colourless gluon - " ladders" (Pomerons) in the t-channel. It was shown in refs. [4] [5] that this idea works for the case of the anomalous dimension of the twist four operator. The fact that we can consider the rescattering of n - pomerons to find the anomalous dimension γ_{2n} really means that we are dealing with a quantum mechanical problem: the calculation of the energy of the ground state for n - particle system where the interactions are attractive and given by a four particle contact term (λ). We can calculate the value of λ in QCD. It turns out that

$$\lambda = 4\,\bar{\alpha}_s\,\delta\ . \tag{7}$$

This observation considerably simplifies the problem and will enable us to reduce it to solving the Nonlinear Schrodinger Equation for n-Pomerons in t - channel. It is very important to mention that the effective theory is two dimendsional one or in other words the Schrodinger equation can be written for n - particles moving only in one dimension. It is well known (see refs. [10] for details) that this problem can be solved exactly.

The answer for the energy of the ground state translated into the value of the anomalous dimension of the twist $2n$ operator is the following:

$$\gamma_{2n} = \frac{\bar{\alpha}_s n^2}{\omega}\{1 + \frac{\delta^2}{3}(n^2 - 1)\}\ , \tag{8}$$

anomalous dimension of the twist four gluon operator, using completely different techniques. It turns out that the value of the anomalous dimension is equal to

$$\gamma_4 \;=\; 2\gamma_2(\frac{\omega}{2})[\,1\,+\,\delta^2\,] \;=\; \frac{4N_c\alpha_s}{\pi\omega}\cdot[1\,+\,\delta^2] \tag{6}$$

where $\delta^2 \sim (\frac{1}{N_c^2-1})^2 \approx 10^{-2}$ is very small.

The most important outcome of this calculation is the fact that we **cannot trust the GLAP evolution equation** in the region of small ω (or large $\ln(1/x_B)$). Indeed for $\omega < \omega_{cr}$ the twist four contribution in eq. (1) becomes larger than the leading twist one. The value of ω_{cr} can be found from the equation

$$\gamma_2(\omega_{cr}) \;=\; \frac{N_c\alpha_s}{\pi\omega_{cr}} \;=\; -\,1\,+\,\gamma_4(\omega_{cr}) \;=\; -\,1\,+\,\frac{4N_c\alpha_s}{\pi\omega_{cr}}[1+\delta^2]\;.$$

Of course we could arrive at the same conclusion using the GLR approach but now we proved this statement considering the whole set of Feynman diagrams instead of the two ladder contribution that the GLR paper took into account [3].

I would like to discuss first several lessons that we have learned from this calculation:

1. Eq. (6) confirms the main hypothesis of ref. [3] which is that the small x_B behaviour of the deep inelastic structure function is determined by the exchange of many Pomerons in t-channel and their interactions.

2. The smallness of δ mentioned above reflects the smallness of pomeron - pomeron interaction which is nonplanar and proportional to $\frac{1}{N_c^2-1}$.

3. Strictly speaking the pomeron-pomeron interaction was not taken into account in the GLR - equation.However the good news is the fact that the correction to the GLR equation is so small that it gives a noticeable contribution only at ultra high energies.

Bad news:

At first sight the main theoretical conclusion from this exercise looks rather pessimistic because it was shown that QCD cannot cure the old problem of the reggeon approach that was pointed out in ref. [6], namely, the fact that pomeron cannot be the correct first approximation to high energy interaction of virtual photon with a hadron at least in perturbative QCD. In other words the pomeron - pomeron interactions turns out to be attractive and the system of many pomerons cannot be stable.

Problems:

The above comments clearly show that we urgently need to solve two problems:

1. to find the value of anomalous dimension of higher than four twist operators. I have to admit that I do not see any reason why a specific coherence effects in QCD will where $\delta = (N_c^2 - 1)^{-1}$. We can trust the answer only when $\frac{\delta^2}{3} \ll 1$. So first we need to solve the second problem to clear up what value of n really is important for the deep inelastic structure function using the above result for the anomalous diomension and only after the solution we have to go back to the calculation of the value of the anomalous dimension. The point is that our all perturbation series are asymptotic ones, so I know only one practical way how we can operate with such series, namely to we find (if possible) the analytical function with the same seria and treat this function as a solution to our problem. Expanding this function we are able to study what value of typical n works in the series and to consider the question whether we can trust our answer. If the value of the typical n will be of the order of 1 we can claim that we have solved our problem, if not we have to go back and try to find a more general expression for the value of the anomalous dimension of high twist operators that is valid for any large n.

Another conclusion that we can get from eq. (8) is the fact that the widespread opinion that the number of colours N_c is sufficiently good numerical parameter to use in the perturbation series is not justified at least for the value of anomalous dimension. Indeed from eq. (8) we see that at n of the order of N_c the $\frac{1}{N_c}$ corrections become important.

2. E.Laenen and E.Levin [11] have gotten the generalization of GLR evolution equation, taking into account both the arbitrary initial condition and the exact value of the anomalous dimension (see eq. (8)). Since the contribution of the high twist operators become essential in the region of small x_B we have to consider the whole series (2) or better to say the sum of all twist contribution to the deep inelastic structure function:

$$x_B G(x_B, Q^2) \;=\; x_B G^{(1)}(x_B, Q^2) + \frac{1}{Q^2} x_B^2 G^{(2)} \ldots + \ldots \frac{1}{Q^{2(n-1)}} x_B^n G^{(n)}(x_B, Q^2) \ldots \qquad (9)$$

To get the equation we need to introduce the generating function

$$g(x_B, Q^2, \eta) \;=\; \sum_{n=1}^{\infty} e^{n\eta} g^{(n)} \;, \qquad (10)$$

where $g^{(n)} = x_B^n G^{(n)}(x_B, Q^2)$. Comparing eq. (10) with eq. (9) we see that the deep inelastic function is equal to

$$x_B G(x_b, Q^2) \;=\; Q^2\, g(x_B, Q^2, \eta = -\ln Q^2) \;. \qquad (11)$$

The new evolution equation looks as follows:

$$\frac{\partial^2 g(x_B.Q^2, \eta)}{\partial \ln \frac{1}{x_B} \partial \ln Q^2} \;=\; \bar{\alpha}_s g''_{\eta\eta} + \frac{\bar{\alpha}_s \delta^2}{3} (g''''_{\eta\eta\eta\eta} - g''_{\eta\eta}) - \gamma\, e^{-\ln Q^2} e^{-\eta} (g'_\eta - g) \;, \qquad (12)$$

where $g'_\eta = \frac{\partial g}{\partial \eta}$ and γ is the vertex for one " ladder" to two "ladders" splitting that has been calculated by Mueller and Qiu (see ref. [7] for details):

$$\gamma \;=\; \alpha_s^2 \cdot \frac{9N_c^2}{2R^2(N_c^2-1)}. \tag{13}$$

To solve the above equation we need to put some initial and boundary conditions which are the price we must pay for the relative simplicity of the equation. The boundary condition looks very simple,namely :

$$\eta = fixed; \;\ln\frac{1}{x_B} = fixed; \;\ln Q^2 \to \infty \;\; g(x_B,Q^2,\eta) \to e^\eta\, g_{LLA}(x_B,Q^2)\;, \tag{14}$$

where g_{LLA} is the solution of usual GLAP evolution equation.

However the initial condition is much more complicated problem since we need to know the fuction $g(x_B = x_{B0}, Q^2, \eta)$, while experimentally we are only able to measure the structure function. So we need more detail information about the structure of a hadron in the region $x_B \sim 1$. To start, we suggest the initial condition in the form:

$$g(x_{B0},Q^2,\eta) \;=\; \sum_{n=1}^{\infty} e^{n\eta}\frac{(-1)^n}{n!}\cdot[\,g_{LLA}(x_{B0},Q^2)\,]^n \;=\; 1 \;-\; exp(-e^\eta g_{LLA}(x_{B0},Q^2))\;. \tag{15}$$

In favour of the above formula we can say that it is simple, has very transparent physical meaning, namely,it reflects the assumption that there is no correlation between gluons with $x_B \sim 1$ except the fact that they are distributed in the hadron disc of the radius R. In the case of the nucleus such an approach can be proved and corresponds to so called Glauber Theory of shadowing correction. In the case of the deeply inelastic scattering the formula of this type was discussed by A.Mueller in ref. [12] and we use formulas from his paper to establish the exact relationship with g_{LLA} in eq. (15).

We are only in the begining of finding of the solution to eq. (12). At the moment we can claim that we found how eq. (12) transforms to nonlinear GLR equation if we neglect the second term in r.h.s. of eq. (12) and assume the eikonal initial condition of eq. (15). We also solve eq. (12) with eq. (15) in the oversimplified case only remaining the second term in r. h. s. of eq.(12). The result looks very encouraging since the effective n that works in the series of eq.(9) turns out to be of the order of 1. However we certainly have to consider this result as very preliminary since we need to understand the general solution of eq. (12) better.

3 Energy conservation in GLR equation.

Now let me descend from heaven to earth and discuss more practical questions in low x_B physics. They illustrate in a direct way our difficulties related to the restricted leading log approximation of perturbative QCD that we used to estimate the behaviour of the structure function especially in the region of low x_B. I would like to discuss here only one of such questions, nanely the energy conservation in the nonlinear equation.

The simplest form of GLR equation looks as follows:

$$\frac{\partial x_B G(x_B, Q^2)}{\partial \ln \frac{1}{x_B} \partial \ln Q^2} = \bar{\alpha}_s x_B G(x_B, Q^2) - \frac{\gamma}{Q^2} \cdot [x_B G(x_B, Q^2)]^2 . \qquad (16)$$

The first term describes the low x_B behaviour of the GLAP evolution equation while the second one takes into account the SC orignated from parton - parton annihilation in the parton cascade. It is easy to show that even if we neglect the nonlinear contribution the equation does not conserve energy. Indeed energy conservation means that

$$\int_0^1 dx_B x_B \{q(x_B.Q^2) + \bar{q}(x_B, Q^2) + G(x_B, Q^2)\} = \int_0^1 dx_B \Sigma(x_B, Q^2) = 1 . \qquad (17)$$

Integrating explicitly eq. (16) one can see that eq. (17) is violated. However in the linear case we know how to cure this problem: we need only to go out of so called double log approximation of perturbative QCD and use the correct kernel of the GLAP equation [2]. For energy conservation in the GLAP equation is very important to know the behaviour of the kernel not only at small value of x_B but also at $x_B \sim 1$. So the fact that we have a problem with the energy conservation in the nonlinear equation does not look surprising since we can derive this equation only in $\ln \frac{1}{x_B}$ approximation and cannot quarantee the kernel in the second term of the equation in the region of moderate x_B. However for practical use this very important problem seriously diminishes the predictive power of the equation for accessible region of small x_B.

Here I would like to suggest some generalization of eq. (16) that gives the enegy conservation. The main idea of the generalization is based on a beautiful property of the "fan" diagrams that were summed in the nonlinear term of the equation (see ref. [3] for details),namely they do not contribute to any even integral moment of the deep inelastic structure function. The proof of this statement is very simple [2] and can be done as straightforward generalization of so called Gribov Reggeon Calculus [13] for the case of deep inelastic structure function and related to correct calculation of the real part of the

[2]It will be published soon elsewhere.

Compton amplitude. Let me remind you that the deep inelastic structure function is the imaginary part of the same amplitude. It means that the correct form of the nonlinear term cannot contribute to the second monents of the deep inelastic structure function which is the integral (17).

Finally the new form of the GLR equation reads as follows:

$$\omega \frac{\partial M(\omega, \ln Q^2)}{\partial \ln Q^2} = \bar{\alpha}_s M(\omega, \ln Q^2) - \frac{\gamma}{Q^2} \cdot \int \frac{d\omega'}{2\pi i} cos^2(\frac{\pi\omega}{2}) M(\omega', Q^2) M(\omega - \omega', Q^2) \,. \quad (18)$$

One can see that the second term does not contribute to the second moment ($\omega = 1$). The equation can be rewritten in $\ln \frac{1}{x_B} = y$ representation,namely

$$\frac{\partial x_B G(x_B, Q^2)}{\partial \ln \frac{1}{x_B} \partial \ln Q^2} = \bar{\alpha}_s x_B G(x_B, Q^2) - \frac{\gamma}{Q^2} \cdot cos^2(\frac{\pi}{2} \frac{\partial}{\partial \ln \frac{1}{x_B}}) [x_B G(x_B, Q^2)]^2 \,. \quad (19)$$

Now the second term looks more complicated but in practice the high derivatives with respect to $\ln \frac{1}{x_B}$ are very small so it should be easy to solve the new equation. Unfortunately I have no numerical solution by now.

4 Large Rapidity Gap (LRG) in deeply inelastic scattering.

In this section I would like to describe briefly the result that I got for the survival probability of the LRG in the deep inelastic scattering [14]. I am viewing on deeply inelastic scattering as a good laboratory to study the structure of the parton cascade to develop Bjorken's ideas [15] on the LRG physics. Unfortunately we have no theory to describe hadron - hadron interaction with guaranteed theoretical accuracy and we have to use a model approach to discuss so called "soft" hadron physics. That is the reason why I prefer to discuss the deeply inelastic processes for better understanding and generalization of Bjorken's formula since we can use QCD as the theory of the parton cascade and we can arrive at a definite conclusion with respect to the survival probability of the LRG.

To understand the problems that we face discussing the LRG physics let me consider the production of a Higgs particle via WW fusion in $\gamma*(Q^2, x_B)p$ collision as was suggested by Bjorken [15]. The cross section of this reaction can be described by a simple factorized formula due to AGK cutting rules [16] and / or the factorization theorem [17].

$$f(y - \Delta y, y_H, p_{Ht}) = \frac{d\sigma}{dy_H dp^2_{Ht}} = \quad (20)$$

$$\int \phi\left(Q^2, x_B, q_{1t}^2, x_1\right)\phi\left(q_{2t}^2, x_2\right)\cdot d^2q_{1t}d^2q_{2t}\cdot\sigma_{hard}\left(q_1^2, q_2^2, x_1x_2S\right)(q_1+q_2\rightarrow q_1+q_2+H),$$

where

$$y = \ln\frac{1}{x_B},;\quad \Delta y \;=\; y_1 \;-\; y_2\;, \tag{21}$$

while y_1 (y_2) is rapidity of produced quark. In eq. (20) we used the so called transverse momentum factorization approach [18]. Furthermore $\sigma_{\rm hard}$ is the cross section for the hard subprocess:

$$\bar{q}_1(x_1, q_{1t}) + q_2(x_2, q_{2t}) \rightarrow \bar{q}_1(x_1, p_{1t}) + q_2(x_2, p_{2t}) + H\;, \tag{22}$$

whereas the function ϕ is closely related to the deep inelastic structure function namely

$$\alpha_s\left(x\,F(x, q^2)\right) \;=\; \int^{q^2}\alpha_s(q'^2)\;\phi(q'^2, x)dq'^2 \tag{23}$$

We do not need to know the exact formula for the hard cross section in eq. (20), for us it is only important to know that the rapidity gap between the two produced quarks with transverse momenta $p_{1t} =\sim p_{2t}$ is large enough.

At first sight this mechanism of Higgs production has an excellent signature for the experimental detection. The event topology is very remarkable: two collimated ($p_{1t} \approx -p_{2t}$) jets with rapidities y_1 and y_2 and no hadrons between them except the Higgs boson and the secondary particles from its decay. However, the formula of eq. (20) can not give the value of the cross section for the event with such a striking signature. Indeed each parton with $x > x_1$ can interact with a parton with $x < x_2$. and such an interaction generally speaking produces a lot of partons (hadrons) with rapidities between y_2 and y_1. It should be stressed that in the case of deeply inelastic scattering such an additional interaction can contribute only because of the shadowing correction in this process that I have discussed in the previous sections of the talk.

Thus, to calculate the cross section of Higgs production with a rapidity gap we need to multiply the value calculated by eq. (20) by factor $< S^2 >$ which gives the survival probability of the rapidity gap. $< S^2 >$ is equal 1 in the deeply inelastic process if we neglect the contribution of the shadowing correction.

Bjorken suggested a formula [15] that allows one to calculate $< S^2 >$. However this formula was based on the eikonal approach which oversimplifies the structure of the parton interactions, reducing the complicated parton cascade with a rich variety of different parton interactions to an interaction between the fastest parton and the slowest one in the Breit frame for the deep inelastic scattering. It should be stressed that the eikonal approach is in obvious contradiction with the structure of QCD cascade that has been described by eq. (16).

The correct formula for the survival probability $< S^2 >$ taking into account the structure of the parton cascade in the deep inelastic scattering looks as follows (see ref. [14] for details):

$$< S^2 >= \frac{1}{\bar{\phi}_{LLA}(y - y_1, q^2)} \cdot \cdot \int d^2 b_t \cdot T(b_t) \frac{\bar{\Psi}(y, b_t, q^2)}{\bar{\Psi}(y_1, b_t, q^2)} e^{-\Psi(y, b_t, q^2) + \Psi(y_1, b_t, q^2)} , \quad (24)$$

where

$$T(b_t) = \int d^2 b'_t F(b_t - b'_t) F(b'_t) .$$

$F(b_t)$ is introduced as

$$\Psi(b_t, x_B, Q^2) = F(b_t) \phi(x_B, Q^2) , \quad (25)$$

where in LLA of QCD we can claim that

$$\int d^2 e^{i(\mathbf{Q}_t, \mathbf{b}_t)} F(b_t) = G(Q_t^2) . \quad (26)$$

$G(Q_t^2)$ is the electromagnetic form factor of a hadron. $\bar{\phi}_{LLA}$ is the solution of the linear evolution equation,namely eq.(16) without the nonlinear term. This contribution appears in eq. (24) since in the expression for the inclusive cross section all shadowing corrections between partons with $y > y_1$ and with $y < y_2$ cancel due to AGK - cutting rules.

Formula (24) looks quite different from Bjorken's formula for $< S^2 >$ and the next step will be make the numerical estimate to clarify in what kinematical region this difference will be important.

5 Instantons in deep inelastic scattering.

J.Balitsky and V. Braun [19] found that the instanton contribution could be sizable in the deep inelastic structure function. This contribution is a pure nonperturbative effect which can be large only in the region of small x_B. The main observation is that the quark - antiquark pair of the transverse size $\propto \frac{1}{Q^2}$ can interact with the instanton inside the proton. The size of the instanton fluctuation is equal to

$$\rho^2 = \frac{1}{Q^2 (\frac{\alpha_s(Q^2)}{4\pi})^2 \cdot (\frac{1}{\xi^2})^2} . \quad (27)$$

Practically $\rho^2 \sim 2Gev^{-2}$ in the kinematical region of HERA,so this size is not too small. The instanton contribution to

$$F_2^{instanton}(x_B, Q^2) \;=\; C \int dx' x' G(x', p^2) \cdot \pi^{\frac{13}{2}} [\frac{2\pi}{\alpha_s(p^2)}]^{\frac{21}{2}} \cdot e^{\frac{4\pi}{\alpha_s(p^2)} S(\frac{x_B}{x'})} \,. \qquad (28)$$

The value of C is of the order of $10^{-4} - 10^{-5}$ and function S is equal

$$S \;=\; 1 \;-\; \frac{6}{\xi^2} \;\; and \;\; \xi \;=\; 2\,\frac{1+x}{1-x} \,. \qquad (29)$$

The biggest contribution comes from the region of $x_B/x' \sim 0.25$ and $Q^2 \sim 400GeV^2$ and for this Q^2:

$$\frac{F_2^{instanton}}{F_2} \;\sim\; (2 \;-\; 5)\%.$$

The cross section for instanton production is pretty small but the signature of the event is quite clear: a lot of particles (multiplicity of the order of $\frac{2\pi}{\alpha_s}$) within the rapidity interval of the order of 1 and with transverse momentum of the order of $1GeV$ are produced together with the current jet. I firmly believe that this quite interesting possibility to study the properties of nonperturbative QCD but we need time to check the calculation and understand better the main characteristics of these striking events.

6 Scale of Shadowing Correction.

One of the problems that we face in our estimates of the valus of SC is the fact that we do not know the value of radius R in the equations (13) and (16).The physical meaning of R is that it is the correlation radius between two gluons in a hadron, the value of which we cannot calculate in the framework of perturbative QCD. The spectrum of the estimates covers the values from the radius of hadron ($R \sim 1Fm$) to the radius of the constituent quark ($R_Q \sim 0.2 - 0.3Fm$). For the first value we get small SC in the HERA kinematical region while for the second we expect sufficiently big SC.

The right strategy to calculate R is to use some methods of nonperturbative QCD such as QCD sum rules or lattice calculation. I am very happy that very recently the estimates from QCD sum rules have appeared in the market, namely V.Braun,P.Gornicki,L.Mankiewicz and A.Shafer [20] applied the sum rules of QCD to calculate the correlation radius between gluons. The result is very encouraging, they got R = 0.3 - 0.35 Fm. So this radius is smaller than the radius of proton. Of course, it is only the first estimate,we need time to understand how reliable this estimates,but I think it is the right direction approach to the selfconsistent theoretical description of the deep inelastic scattering process.

7 Conclusions.

I hope I have convinced you that an important step has been made toward the understanding of the main properties of QCD at low x_B. I had no time to discuss such problems as diffraction dissociation processes, AGK cutting rules and the factorization theorem that should be reconsidered in the light of the new development in the theory at low x_B considered in the first section of the talk. I hope that you have gotten a taste of the difficult and interesting problems that we are fighting with in this field of high energy physics. Certainly all of us believe that the coming experimental data from HERA will shed the light on the discussed problems as well as stimulate the appearence of new ones.

Acknowlegements: My talk is the collection of new results that mostly have not been published. I am very grateful to J.Balitsky, V.Braun, E.Laenen, M.Ryskin, A.Shuvaev and M.Wuesthoff who worked with me or told me their new results, for encouraging optimism and hot discussions on the subject. I also thank all participants of the Durham workshop and especially J. Bartels, J. Kwiecinski, A.Martin and A. Mueller for numerous and useful exchanges of ideas. I would like to thank specially N.Govel and V.von Schlippe for kind hospitality that gave me the great possibility to feel the taste of good old England during my first visit to this country.

References

[1] J. C. Collins: *Renormalization* , Cambridge Univ. Press, Cambridge,1984.

[2] V.N.Gribov and L.N.Lipatov:*Sov. J. Nucl. Phys.*15 (1972) 438,L.N. Lipatov: *Yad.Fiz.* **20** (1974) 181, G.Altarelli, G.Parisi:*Nucl.Phys.* **B 126** (1977) 298.

[3] L.V.Gribov,E.M.Levin and M.G.Ryskin:*Phys.Rep.* **100** (1983) 1.

[4] J.Bartels: *Phys. Letters* **B 298** (1993) 204.

[5] E.M. Levin,M.G. Ryskin and A.G. Shuvaev: *Nucl.Phys.* **B 387** (1992) 589.

[6] B.M. Mc Coy and T.T. Wu:*Phys. Rev.* **D12** (1975) 546,577, S.G.Matinian and A.G.Sedrakian:*JETP letter* **23** (1976) 588,**24** (1976) 240, *Sov. J. Nucl. Phys.* **24** (1976) 844.

[7] A.H. Mueller and J. Qiu: *Nucl. Phys.* **B268** (1986) 427.

[8] E. Levin: FERMILAB - CONF -92/386 - T (DPF92 talk) December, 1992.

[9] E. Laenen, E. Levin, M.G. Ryskin and A.G. Shuvaev: (in preparation).

[10] N.M.Bogolubov,A.G. Izergin and V.E. Korepin: *Lecture Notes in Physics* **242** Springer -Verlag, (1985) 221; J.B. McGuire: *J. Math. Phys.* **5** (1964) 622.

[11] E. Laenen and E. Levin: (in preparation).

[12] A.H. Mueller:*Nucl.Phys¿* **B 335** (1990) 115,*Nucl.Phys.* **B 317** (1989) 573, *Nucl.Phys.* **B 307** 34.

[13] V.N.Gribov: *JETP* **53** (1967) 654.

[14] E. Levin: FERMILAB - PUB - 93/012 - T, January 1993.

[15] J.D. Bjorken: *Phys. Rev.* **D 47** (1993) 563.

[16] V.A.Abramovski, V.N.Gribov and O.V.Kancheli, *Sov. J. Nucl. Phys.* **18** (1973) 308.

[17] J.C. Collins,D.E.Soper and G.Sterman: *Nucl. Phys.* **B 261** (1985) 104, **B 308** (1988) 833.

[18] S.Catani, M,Ciafoloni and F.Hautmann: *Phys.Lett.* **B 242** (1990) 97, *Nucl. Phys.* **B 366** (1991) 135; E.M. Levin, M.G.Ryskin, Yu. M. Shabelski and A.G. Shuvaev: *Sov. J. Nucl. Phys.* **53** (1991) 657; J.C. Collins and R.K. Ellis: *Nucl. Phys.* **B 360** (1991) 3.

[19] J.Balitsky and V. Braun: MPI - Ph /92 - 89, PSU/TH/115 Octorber 1992, (in preparation).

[20] V.Braun, P. Gornicki, L. Mankievicz and A. Schafer: CERN-TH.6673/92, Oct.1992.