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Plasmon decay to $\nu\bar{\nu}$ in a relativistic plasma

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Abstract

The plasmon decay rate to neutrino-antineutrino pairs is calculated assuming a non-vanishing magnetic moment for the neutrino. Since we are interested in the ultrarelativistic regime ($T > m_e c^2$), an appropriate covariant formalism is introduced to treat collective plasma excitations. We show that this process could result in an important contribution to the production of wrong-helicity neutrinos in the early Universe.

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I. Introduction

The present interest in the electromagnetic properties of neutrinos is driven by several intriguing hypothesis. Perhaps the best motivated is the observation that (at least some aspects of) the solar neutrino puzzle[1] can be solved if the electron neutrino has a magnetic moment of about $10^{-11}\mu_B$ (μ_B is the Bohr magneton).[2] Although the required magnitude of the magnetic moment is very nearly in conflict with the present experimental limit, $\mu_{\nu_e} < 2 \times 10^{-10}\,\mu_B$,[3] the idea has attracted a host of attempts to explain such a relatively large magnetic moment in particle models. Another interesting hypothesis regarding the role of neutrino magnetic moments has been proposed by Melott and Sciama.[4] They suggest that the dark matter present in our Galaxy is made prevalently of τ-neutrinos with mass near 30 eV and a lifetime of about 10^{-24}s. The γ-rays generated by the decay $\nu_\tau \to \nu_i\,\gamma$, where ν_i is a lighter neutrino, could be the source of energy necessary to reionize the Universe. A third proposal is the very recent suggestion of Giudice that τ-neutrinos with mass between 1 and 35 MeV and a magnetic moment near the present experimental limit[5] (i.e., $\mu_{\nu_\tau} \simeq 10^{-6}\mu_B$) could play a relevant role in cosmology.[6] In fact, he showed that relic neutrinos with these properties behave like cold dark matter. This would supply a nice solution to the problem of structure formation.

Since laboratory experiments do not yet provide enough data to confirm or to reject any of these hypotheses, astrophysics and cosmology could give some directions. In fact, constraints on the electromagnetic properties of neutrinos can be obtained by considering their effects on the evolution of stars and on primordial nucleosynthesis. The study of such effects, which take place under the very peculiar conditions of very high temperatures and density, require the solution to subtle theoretical problems. One of these problems is neutrino pair creation by plasmon decay. Our aim is to calculate the rate of this process in an ultrarelativistic plasma, such as would have permeated the primordial Universe.

Adams et al., in 1963, calculated the energy loss rate of stars caused by neutrino emission.[7] They observed that radiative corrections lead to a small electromagnetic coupling for neutrinos. This allows their production by the process *plasmon* $\rightarrow \nu\bar{\nu}$. This process is found to be the most efficient mechanism to radiate neutrinos in very hot and dense stars ($T \gtrsim 10^7$K, $\rho \gtrsim 10^5$g cm^{-3}). Under these conditions the plasma is semi-relativistic, and thermal quantum effects complicate the dispersion relation of plasmons.[8] In their work the authors were able to find an approximate relation for the neutrino emission rate considering only plasmons with wavenumber $k \ll m_e$ and frequency $\omega \ll 2m_e$, where m_e is the electron mass. This is adequate for stellar plasmas where the temperature is usually well below the electron mass ($T < 5 \times 10^9$K). However, this is not the case if one wishes to study the effects of the same process on primordial nucleosynthesis, i.e., in the range of temperatures $10^9 < T < 10^{12}$K.

Of particular interest for primordial nucleosynthesis is the possibility that interactions mediated by the neutrino magnetic moment would populate the "sterile," wrong-chirality neutrinos. Since we are interested in Dirac neutrinos, there are four degrees of freedom for each neutrino field. For a massive neutrino, the helicity eigenstates (ν_+, ν_-) do not coincide with the chirality eigenstates (ν_L, ν_R). It is of course the chirality eigenstates that are the eigenstates of the standard weak interactions. A relativistic ν_- (ν_+) has order unity projection onto ν_L (ν_R), with a small admixture of order m_ν/E_ν onto ν_R (ν_L).

The effects of a neutrino magnetic moment on primordial nucleosynthesis have already been considered. In 1981 Morgan showed that right-handed neutrinos produced by the processes $e\nu_L \rightarrow e\nu_R$ and $e^-e^+ \rightarrow \nu\bar{\nu}$ can destroy the successful prediction of the standard nucleosynthesis calculation[9] if the magnetic moment of a light neutrinos species ($m_\nu \lesssim 1$ MeV) is greater than $10^{-11}\mu_B$.[10] However, recently Giudice showed that for $m_\nu \gtrsim 1$ MeV and $\mu_\nu \sim 10^{-6}\mu_B$ (experimental limits make this possible only for the τ-neutrino), $\nu\bar{\nu} \rightarrow e^-e^+$ annihilations can reduce the wrong helicity neutrino density sufficiently to be compatible with observational limits.[6] We are going to shown that, a priori, the process *plasmon* $\rightarrow \nu\bar{\nu}$ should not discarded in this consideration.

In Sec. II we introduce the correct formalism to treat plasmon decay in a relativistic plasma. Since a rest frame for longitudinal plasmons is not well defined, this problem needs some attention. Furthermore, we show that plasmon decay into fermion-antifermion pairs is the main dispersion process. In Sec. III we calculate the plasmon decay rate into neutrino–antineutrino pairs due to a neutrino magnetic moment. The final section contains our conclusion.

II. Electromagnetic waves in a relativistic plasma

In vacuum, photon decay to massive pairs is forbidden. In fact, gauge invariance implies a zero photon mass, and a rest frame for the photon is not defined. In a plasma, the situation is altered by the interaction of electromagnetic waves with the sea of charged particles through which the photon moves. In analogy with electrons moving in a solid, where the waves acquire an effective mass by the interaction with the lattice, photons in a plasma are no longer massless. They should be considered as collective excitations known as *plasmons*. The decay of plasmons to massive particles is no longer forbidden.

The free photon propagator is modified by the interaction with high temperature matter. The polarization tensor in the presence of matter can be written [11]

$$\pi_{\mu\nu} = \pi_{\mu\nu}^{mat} + \pi_{\mu\nu}^{vac}, \tag{2.1}$$

where the first term takes into account the interaction with matter (essentially electrons and positrons) and the second is defined by

$$\pi_{\mu\nu}^{vac} = \lim_{T,\mu\to 0} \pi_{\mu\nu}\,. \tag{2.2}$$

The one-loop contribution to the photon self-energy is

$$\pi_{\mu\nu} = \frac{ie^2}{(2\pi)^4}\int d^4p\; Tr\left[\gamma_\mu S_F(p+q)\gamma_\nu S_F(p)\right]. \tag{2.3}$$

The finite-temperature fermion propagator (using the real-time formalism) is[12]

$$S_F(p) = (\not p + m)\left(\frac{1}{p^2 - m^2 + i\delta} + \frac{2\pi i\delta(p^2 - m^2)}{e^{p\cdot u/T} + 1}\right), \tag{2.4}$$

where $\not p \equiv \gamma_\mu p^\mu$ and u_α is the 4-velocity of the plasma. We should note that u_α is not defined in vacuum. Hereafter, we assume the chemical potential to be zero.

The dispersion relation of plasmons is obtained from the poles of the plasmon propagator. They are given by the condition

$$\det\left(q^2 g_{\mu\nu} - \pi_{\mu\nu}\right) = 0\,. \tag{2.5}$$

Gauge invariance implies that

$$q^{\mu}\pi_{\mu\nu} = 0\,. \tag{2.6}$$

For an isotropic plasma the general solution is

$$\pi_{\mu\nu}(\omega,\mathbf{q}) = \pi_T(\omega,\mathbf{q})P_{\mu\nu} + \pi_L(\omega,\mathbf{q})Q_{\mu\nu} \tag{2.7}$$

where, following Adams et al.,[6]

$$P_{ij} = \delta_{ij} - \frac{q_i q_j}{|\mathbf{q}|^2}; \qquad P_{00} = P_{0i} = P_{i0} = 0;$$

$$Q_{\mu\nu} = \eta_{(3)\,\mu}\eta_{(3)\,\nu}; \qquad \eta_{(3)\,\mu} = \frac{1}{(q^2)^{1/2}}(|\mathbf{q}|,0,0,\omega)\,. \tag{2.8}$$

Here, we have assumed that the wavevector $\mathbf{q}$ is directed along the z axis. Then, from (2.5) we obtain

$$\left[(q^2 - \pi_{00})(q^2 + \pi_{33}) + \pi_{03}^2\right](q^2 + \pi_{11})^2 = 0\,. \tag{2.9}$$

We see that there exist three kinds of collective excitations of the plasma. Two correspond to transverse modes and have the same dispersion relation

$$q^2 + \pi_{11} = 0\,. \tag{2.10}$$

The third corresponds to a longitudinal mode with

$$\mathrm{q}^2 + \pi_{00} = 0\,. \tag{2.11}$$

Longitudinal plasmons correspond to charge-density (electrostatic) waves. For a non-relativistic plasma, their dispersion relation is $\omega = \omega_P + \mathcal{O}(\vec{q}^2 v^2)$, where ω_P is the plasma frequency (see below) and v^2 is the mean-square electron velocity. It is evident that this expression is not Lorentz invariant.[13] Physically, this means that by performing a Lorentz transformation, longitudinal modes can acquire a transverse component. This creates a problem in the calculation of the decay rate of a plasmon. In fact, fixing a rest frame for plasmons is now an ambiguous operation.

This problem can be solved only by appealing to an appropriate covariant formalism, developed by Weldon in 1982.[14] Weldon made it clear that the correct frame to study collective excitations of the plasma is the plasma rest frame. In fact, the plasma 4-velocity u_α provides a physical distinction between time-like and space-like directions, so that it becomes possible to treat separately longitudinal and transverse plasma excitations. In the Weldon treatment, π_T and π_L, which depend only on $q_\alpha q^\alpha$ at $T=0$, are functions of the scalars

$$\omega = q^\alpha u_\alpha \tag{2.12.a}$$

and

$$|\mathbf{q}| = \left[(u_\alpha q^\alpha)^2 - q^2\right]^{1/2} \tag{2.12.b}$$

separately at $T \neq 0$, as the result of the lack of Lorentz invariance. In our opinion, the treatment of Tsytovich and Adams et al. is fully meaningful only in this framework.

The electric permittivity ϵ and magnetic permeability μ are related to π_T and π_L by the relations

$$\epsilon = 1 - \frac{\pi_L}{q^2} \qquad \frac{1}{\mu} = 1 + \frac{q^2\pi_T - \omega^2\pi_L}{|\mathbf{q}|^2 q^2} \ . \tag{2.13}$$

When $T = 0$, $\pi_T = \pi_L$, so that we have the usual relation $\epsilon = 1/\mu$.

The expressions for the real parts of π_T and π_L for a relativistic plasma have been calculated by Weldon in the limit $T \gg \omega$ and $T \gg |\mathbf{q}|$. They are

$$Re\pi_L(\omega, |\mathbf{q}|) = \frac{e^2T^2}{3}\left(1 - \frac{\omega^2}{|\mathbf{q}|^2}\right)\left[1 - \frac{\omega}{2|\mathbf{q}|}\ln\left|\frac{\omega + |\mathbf{q}|}{\omega - |\mathbf{q}|}\right|\right] \tag{2.14.a}$$

$$Re\pi_T(\omega, |\mathbf{q}|) = \frac{e^2T^2}{6}\left[\frac{\omega^2}{|\mathbf{q}|^2} + \left(1 - \frac{\omega^2}{|\mathbf{q}|^2}\right)\frac{\omega}{2|\mathbf{q}|}\ln\left|\frac{\omega + |\mathbf{q}|}{\omega - |\mathbf{q}|}\right|\right]. \tag{2.14.b}$$

In contrast to Adams et al., we are interested in the very high temperature limit, that is $\omega > 2m_e$ and $|\mathbf{q}| > m_e$. In this regime the dispersion relation for transverse and longitudinal waves can be calculated analytically only in the opposite limits of short and long wavelengths. The real parts of (2.10,11) now read

$$\omega^2 = |\mathbf{q}|^2 + Re\,\pi_T \simeq |\mathbf{q}|^2 + m_P^2, \qquad \text{for } \omega \simeq |\mathbf{q}| \gg T; \tag{2.15.a}$$

$$\omega^2 \simeq \omega_P^2 + \frac{6}{5}|\mathbf{q}|^2, \qquad \text{for } |\mathbf{q}| \ll \omega < T \tag{2.15.b}$$

for transverse modes; and for longitudinal modes

$$\omega^2 = |\mathbf{q}|^2 + 4|\mathbf{q}|^2\exp\Big(-\frac{|\mathbf{q}|^2}{m_P^2}\Big), \qquad \text{for } \omega \simeq |\mathbf{q}| \gg T; \tag{2.16.a}$$

$$\omega^2 \simeq \omega_P^2 + \frac{3}{5}|\mathbf{q}|^2; \qquad \text{for } |\mathbf{q}| \ll \omega < T. \tag{2.16.b}$$

The plasma frequency ω_P is defined as the frequency of plasmons at $|\mathbf{q}| = 0$, that is

$$\omega_P^2 = Re\,\pi_T(\omega, |\mathbf{q}| = 0) = Re\,\pi_L(\omega, |\mathbf{q}| = 0) = \frac{e^2T^2}{9}. \tag{2.17}$$

The imaginary parts of the dispersion relations give the damping constants of the plasma. They depend on the nature of the damping process. The damping of electromagnetic waves in a relativistic plasma can take place by means of two mechanisms: the inverse Cerenkov effect (Landau damping) and e^+e^- pair formation. Of course e^+e^- pair production is possible only if $q^2 > 4m_e^2 > 0$, i.e., $\epsilon < 1$. Tsytovich[8] showed that the real part of the dielectric constant of a relativistic plasma is greater than one if $\omega \gg 2|\mathbf{p}|$. Thus, in this limit, Cerenkov dissipation is the dominant effect. However, in a ultrarelativistic plasma the mean value of the 3-momentum of the electrons (or positrons) is equal to $\langle|\mathbf{p}|\rangle \simeq 3.15T$. Assuming that plasmons are in thermal equilibrium, the mean value of their frequency is $\langle\omega\rangle \simeq 0.60T$, if they behave as non-relativistic particles ($\omega \sim \omega_P$), and $\langle\omega\rangle \simeq 2.7T$ in the opposite case. In both these cases, the limit studied by Tsytovich can not be applied here. It seems more realistic that the plasmons are well described by the large wavelength limit of the dispersion relations. In this limit $\omega^2/|\mathbf{q}|^2 > 1$, i.e. the phase velocity is greater than the velocity of light. This means that the damping arises prevalently by the decay of the $|\mathbf{q}| \simeq 0$ plasmons into real fermion-antifermion pairs.

In the next section we are going to estimate the contribution to the damping of electromagnetic waves due to the process *plasmon* $\rightarrow \nu\bar{\nu}$.

III. Plasmon decay to $\nu\bar{\nu}$ pairs

A neutrino magnetic moment coupling to the electromagnetic field generates a term in the effective Hamiltonian

$$H_I = \frac{1}{2}\mu_\nu \bar{\nu}\sigma_{\alpha\beta}F^{\alpha\beta}\nu, \tag{3.1}$$

where μ_ν is the neutrino magnetic moment, $\sigma_{\alpha\beta} = (i/2)\left[\gamma_\alpha, \gamma_\beta\right]$ and $F^{\alpha\beta} = \partial_\alpha A_\beta - \partial_\alpha A_\beta$ is the electromagnetic tensor. The decay amplitudes of transverse plasmons and longitudinal plasmons in neutrino-antineutrino pairs have to be calculated separately. In fact, the vector potential of the electromagnetic modes A^α, takes different values for each kind of excitation. According to Adams et al.[6]

$$A_T^\alpha = \left(\frac{1}{2\pi}\right)^{3/2}\int \frac{d^3\mathbf{q}}{\left[\omega\left(2\epsilon_T + \omega\frac{\partial\epsilon_T}{\partial\omega}\right)\right]^{1/2}}\sum_{i=1}^{2}\left[\eta_{(i)}^\alpha(q)a_i(q)e^{-i(\omega t-\mathbf{q}\cdot\mathbf{x})} + \eta_{(i)}^\alpha(q)a_i^\dagger(q)e^{i(\omega t-\mathbf{q}\cdot\mathbf{x})}\right] \tag{3.2}$$

$$A_L^\alpha = \left(\frac{1}{2\pi}\right)^{3/2}\int \frac{d^3\mathbf{q}}{\left(\omega^2\frac{\partial\epsilon_L}{\partial\omega}\right)^{1/2}}\left[\eta_{(3)}^\alpha(q)a_3(q)e^{-i(\omega t-\mathbf{q}\cdot\mathbf{x})} + \eta_{(3)}^\alpha(q)a_3^\dagger(q)e^{i(\omega t-\mathbf{q}\cdot\mathbf{x})}\right] \tag{3.3}$$

where $\eta_{(\lambda)}^\alpha(q)$ are the polarizations vectors of a plasmon with 4-momentum q. Because the plasmons are massive vector fields, the polarization vectors must satisfy[15]

$$\eta_{(\lambda)}(q)\cdot\eta_{(\lambda')}(q) = -\delta_{\lambda\lambda'} \tag{3.4.a}$$

$$\eta_{(\lambda)}\cdot q = 0 \tag{3.4.b}$$

$$\sum_{\lambda=1}^{3}\eta_{(\lambda)}^\mu(q)\eta_{(\lambda)}^\nu(q) = -\left(g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) \tag{3.4.c}$$

The vectors that fulfil these relations are

$$\eta_{(1)}^\mu = (0,1,0,0) \qquad \eta_{(2)}^\mu = (0,0,1,0) \qquad \eta_{(3)}^\mu = \frac{1}{(q^2)^{1/2}}(|\mathbf{q}|,0,0,\omega). \tag{3.4.d}$$

The dielectric constants of transverse and longitudinal plasmons are, respectively

$$\epsilon_T(\omega, |\mathbf{q}|) = 1 - \frac{Re\,\pi_T(\omega, |\mathbf{q}|)}{\omega^2} \tag{3.5.a}$$

$$\epsilon_L(\omega, |\mathbf{q}|) = 1 - \frac{Re\,\pi_L(\omega, |\mathbf{q}|)}{q^2}. \tag{3.5.b}$$

The decay amplitudes, generated by the Hamiltonian in eq. (3.1), of transverse and longitudinal plasmons in neutrino-antineutrino pairs, are

$$M_T = \frac{i}{(2\epsilon_T + \omega\partial\epsilon_T/\partial\omega)^{1/2}}\ \mu_\nu\,\bar{\nu}(p_1)\,\sigma_{\alpha\beta}q^\beta\nu(p_2)\ \eta^\alpha_{(k)}(q) \qquad k = 1, 2 \tag{3.6}$$

$$M_L = \frac{i}{(\omega\partial\epsilon_L/\partial\omega)^{1/2}}\ \mu_\nu\,\bar{\nu}(p_1)\,\sigma_{\alpha\beta}q^\beta\nu(p_2)\ \eta^\alpha_{(3)}(q)\ . \tag{3.7}$$

Using equations (3.4) after some kinematics, we obtain

$$\sum_{spin} |M_L|^2 = \frac{\mu_\nu^2}{\omega\partial\epsilon_L/\partial\omega}\,8(p_1\cdot\eta_{(3)})^2 q^2. \tag{3.8}$$

We wish to find the square of the decay amplitude for transverse plasmons by subtracting $|M_L|^2$ from the square of the total decay amplitude (summed over spins and averaged over polarizations):

$$\sum_{spin} \overline{|M_{tot}|^2} = \mu_\nu^2(q^2 - 4m_\nu^2)q^2. \tag{3.9}$$

This will be done in an appropriate frame.

From the previous section we know that ϵ and μ are Lorentz invariant quantities. This allows us to calculate them in the rest frame of the plasmon. *This frame is now well defined as the frame where* $|\mathbf{q}| = 0$. In this frame $q^2 = \omega_P^2$ (see eq. (2.12.b)). From the definition of plasma frequency we know

$$Re\pi_L(\omega, 0) = Re\pi_T(\omega, 0) = \omega_P^2. \tag{3.10}$$

Thus, according to eqs. (3.5), we obtain

$$\epsilon_L(\omega, 0) = \epsilon_T(\omega, 0) = 1 - \frac{\omega_P^2}{\omega^2} \,. \tag{3.11}$$

This coincides with the well known expression for the electric permittivity of a nonrelativistic plasma in the limit of long wavelength. Although, eq. (3.11) is a well known formula, we stress that in the works of Adams et al. and Tsytovich, it should be regarded as an approximate formula. The possibility to treat the decay in the plasmon rest frame, allows us to use it for a ultrarelativistic plasma as an exact formula *for every plasmon* avoiding the condition $\omega \ll 2m$ that Adams et al. needed to impose.

Now we are able to find

$$\sum_{spin} |M_L|^2 = \mu_\nu^2 \omega_P^2 (\omega_P^2 - 4m_\nu^2) \cos^2\theta \tag{3.12}$$

and

$$\sum_{spin} |M_T|^2 = \mu_\nu^2 \omega_P^2 (\omega_P^2 - 4m_\nu^2) \sin^2\theta \tag{3.13}$$

where θ is the angle between $\mathbf{p_1}$ and $\mathbf{q}$.

The total decay rate is:

$$\Gamma = \frac{1}{16\pi} \mu_\nu^2 \, (\omega_P^2 - 4m_\nu^2)^{3/2} \,. \tag{3.14}$$

The exponent $3/2$ reveals the p-wave character of the decay. Plasmon decay into $\bar{\nu}\nu$ pairs is possible only if $\omega_P \geq 2m_\nu$; using (2.17), this implies $T \geq 19.81\, m_\nu$.

We assume the plasma to be comoving with the space-time. This means that its 4-velocity is $u_\mu = (R^{-1}, 0, 0, 0)$, where R is the scale factor of the FRW metric. Hereafter, we define ω and $\mathbf{q}$ to be the comoving values, ω/R and $\mathbf{q}/R$.

Since we are interested in an ultrarelativistic plasma, we must take into account the effects of time dilation on the decay rate. This can be made multipling Γ by the factor

$$\gamma = m_P \langle 1/E_P \rangle_T = \frac{m_P}{T} \frac{\zeta(2)}{2\zeta(3)} = 0.08, \tag{3.15}$$

where $\langle 1/E_P \rangle_T$ is the thermal average of the inverse plasmon energy and $\zeta(n)$ is the Riemann zeta function.

IV. Conclusions

In the previous section we have calculated the neutrino production rate by plasmon decay in a ultrarelativistic plasma. Our treatment differs from the one of Adams, et al. in the use of a fully covariant formalism to describe plasma waves. This allows us to define, without ambiguity, a common rest frame for transverse and longitudinal modes. Then the usual procedure can be applied to calculate the plasmon decay rate into neutrino-antineutrino pairs via a magnetic moment vertex.

Since this process produces neutrinos with both helicity states, it could influence primordial nucleosynthesis.[9] Although this is not the aim of our paper, it is useful to compare the rate from equation (3.14) with other neutrino magnetic moment induced processes which can play a role during primordial nucleosynthesis. Assuming a vanishing neutrino mass we have

$$n_\gamma \Gamma_{P\to\nu\bar{\nu}} = \frac{\zeta(3)}{\pi^2}\, 3T^3\, \frac{\mu_\nu^2}{16\pi}\, m_P^3 \simeq 1\times 10^{-3}\ \alpha\mu_\nu^2 T^6 \tag{4.1}$$

$$\langle n_\nu^2 \sigma_{e^-e^+\to\nu\bar{\nu}}\rangle = \left(\frac{3}{4}\frac{\zeta(3)}{\pi^2}\, 2T^3\right)^2 \frac{\alpha\mu_\nu^2}{6} \simeq 5\times 10^{-3}\ \alpha\mu_\nu^2 T^6 \tag{4.2}$$

$$\langle n_\nu^2 \sigma_{e\nu_L\to e\nu_R}\rangle = \left(\frac{3}{4}\frac{\zeta(3)}{\pi^2}\, 2T^3\right)^2 \alpha\mu_\nu^2 \ln\left(\frac{q_{max}}{q_{min}}\right)^2 \simeq 5\times 10^{-2}\ \alpha\mu_\nu^2 T^6 \tag{4.3}$$

Also assuming a non-vanishing neutrino mass, if the magnetic moment is small enough, wrong helicity neutrinos decouple before the quark-hadron phase transition ($T \sim 200$ MeV) and their energy density will be diluted during this transition. In this case, again one has taken into account corrections due to neutrino mass, and the neutrino magnetic moment upper limit obtained by Morgan should not be considerably modified by the process *plasmon* $\to \nu\bar{\nu}$.

The situation is more complex if we consider neutrino magnetic moments near $10^{-6}\ \mu_B$. Giudice[6] showed that τ-neutrinos with mass in the range $1 < m_{\nu_\tau} < 35$ MeV and magnetic moment near $10^{-6}\mu_B$, can annihilate efficiently via $\nu_\tau\bar{\nu}_\tau \to e^+e^-$ so as to reduce the wrong helicity neutrino abundance enough to not compromise successful standard nucleosynthesis predictions without additional neutrino species.

As long as parity violating weak interactions are not important [this is true for $\mu_\nu \gtrsim 10^{-10}(m_\nu/1\ \mathrm{MeV})\ \mu_B$] the scattering $e\nu \leftrightarrow e\nu$ provides only kinetic (not chemical) equilibrium. Thus, this process does not enter into the neutrino Boltzmann equation.[6,18,19] On the other hand, the neutrino number changing process *plasmon* $\to \nu\bar{\nu}$ needs, a priori, to be taken into account. In fact, the rate of this decay can be of the same order as the annihilation rate. Although threshold effects can reduce the importance of this process, for a neutrino mass close to 1 MeV plasmon decay contribution to neutrino production must be estimated carefully. Furthermore, since neutrinos with a mass of about 1 MeV and a magnetic moment close to $10^{-6}\ \mu_B$ decouple when they are semirelativistic ($T_D \sim m_\nu$), a numerical treatment is necessary to integrate the Boltzmann equation and to evaluate the energy density of the decoupled neutrinos.[17,19] The precise weight of the plasmon decay process can be only established in this framework.[20]

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References

1. R. Davis et al., proc. Neutrino '88 Boston conference (J. Schneps, T. Kafka, W. Mann and P. Nath eds.), World Scientific (1989).

2. A. Cisneros, *Astr. Space Sci.* **10**, 87 (1979); L. Okun, *Sov. Journal Nucl. Phys.* **44**, 546 (1986); M. Voloshin, M. Vysotsky and L. Okun, *Sov. Journal Nucl. Phys.* **44**, 440 (1986).

3. F. Reines et al., *Phys. Rev. Lett.* **37**, 315 (1976); A. Kyuldjiev, *Nucl. Phys.* **B243**, 387 (1984).

4. A.L. Melott and D.W. Sciama, *Phys. Rev. Lett.* **49**, 1369 (1981).

5. H. Grotch and R.W. Robinett, *Z. Phys.* **C39**, 553 (1988).

6. G.F. Giudice, *Phys. Lett.* **B251**, 460 (1990).

7. J.B. Adams, A.M. Ruderman and C.H. Woo, *Phys. Rev.* **129**, 1383 (1963).

8. V.N. Tsytovich, *Sov. Phys.*JEPT **13**, 1249 (1961).

9. E.W. Kolb and M.S. Turner, *The Early Universe* Addison-Wesley, Redwood City, (1989).

10. J.A. Morgan, *Phys.Lett.* **102B**, 247 (1981).

11. J.I. Kapusta, *Finite Temperature field theory*, Cambridge University Press, (1989).

12. A.L. Fetter and J.D. Walecka, *Quantum Theory of Many Particle Systems* McGraw-Hill, New York, (1971).

13. V.L. Ginzburg, *Propagation of Electromagnetic Waves in Plasma* North-Holland, Amsterdam, (1961).

14. H.A. Weldon, Phys. Rev. **D26**, 1394 (1982).

15. C. Itzykson and J.B. Zuber, *Quantum field theory*, McGraw-Hill New York, (1980).

16. M. Fukugita and S. Yazaki, *Phys. Rev.* D36, 3817 (1987).

17. E.W. Kolb, M.S. Turner, A. Chakravorty and D. Schramm, *Phys. Rev. Lett.* 67, 533 (1991).

18. P. Gondolo and G. Gelmini, *Nucl. Phys.* B360, 145 (1991).

19. L.H. Kawano et al., *Phys. Lett.* B275, 487 1992.

20. D. Grasso and E.W. Kolb, work in progress.