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The Strength of String Nonperturbative Effects and Strong-Weak Coupling Duality

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ABSTRACT

A strong-weak coupling duality symmetry of the string equations of motion has been suggested in the literature. This symmetry implies that vacua occur in pairs. Since the coupling constant is a dynamical variable in string theory, tunneling solutions between strong and weak coupling vacua may exist. Such solutions would naturally lead to non-perturbative effects with anomalous coupling dependence. A highly simplified example is given.

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Matrix model descriptions[1] of strings in low dimensions provide tractable generating functionals of string perturbation theory. At large orders, the perturbation theory for these strings grows at order k as $(2k)!$ rather than the usual $k!$ found in most field theories. Shenker[2] has presented arguments based on properties of the moduli spaces of Riemann surfaces with k handles that this growth is generic. However, a spacetime interpretation of the origins of the nonperturbative effect underlying this growth is not known. This is the issue we address here.

In field theory, the growth of perturbation theory is related to the coupling constant dependence of nonperturbative effects[3]. Starting with $Z(g) \equiv \int \mathrm{D}\psi \mathrm{e}^{-S} \equiv \sum g^k Z_k$, Lipatov's argument[4] applies saddle point analysis to $Z_k = \oint \mathrm{d}g g^{-k-1} Z(g)$ for large k. For $S[\psi, g] = \int \mathrm{d}^D x \left[(\partial\psi)^2 + g^{n-1}\psi^{2n}\right]$, the saddle point equations for g_s, ψ_s are:

$$\begin{aligned} &\frac{k+1}{g_s} + (n-1) g_s^{n-2} \int \psi_s^{2n} = 0, \\ &-\partial^2 \psi_s + n g_s^{n-1} \psi_s^{2n-1} = 0. \end{aligned} \tag{1}$$

Rescaling $\sqrt{-g}\psi = \phi$, one finds $Z_k \sim g_s^{-(k+1)} e^{-S(\phi_s, 1)/g_s}$, where $S(\phi_s, 1)$ is independent of k,

$$g_s = \frac{1}{k+1} \int \phi_s \partial^2 \phi_s \frac{n-1}{n}, \tag{2}$$

and the l.h.s. of this equation is negative definite if ϕ_s is real. Eq. (2) implies that Z_k grows as $k!$, up to factors of the form $k^\alpha A^k$ which come from taking into account fluctuations about, and zero modes of, the saddle point configuration. (Of course, Z_k is only nonzero in this example when $k = 0 \bmod (n-1)$.) For a more general potential the argument goes through similarly. The above explicitly shows that ϕ_s, the saddlepoint solution, is independent of k, so the growth in the perturbation series is due to the explicit $1/g$ in front of the rescaled action.

In the case of an effective action, write $\psi \equiv \tilde{\psi} + \xi$, and integrate out the fluctuations ξ above some mass scale to get $Z(g) = \int \mathrm{D}\tilde{\psi} \mathrm{e}^{-S_{\mathrm{eff}}(\tilde{\psi})}$. Its saddle point is at $\tilde{\psi}$ such that $\delta S_{\mathrm{eff}}/\delta\tilde{\psi} = 0$. Schematically then, ignoring zero modes,

$$\frac{k+1}{g_s} = -\frac{\partial S_{\mathrm{eff}}}{\partial g}\Big|_{g_s, \tilde{\psi}_s}, \tag{3}$$

may be expected to reproduce the same $k!$ growth as discussed above. However, quantum effects (due to the fluctuations that have been integrated out) may induce a change in the vacuum structure[5] and thus allow for new solutions with anomalous g dependence. A

recent example, due to Bhattacharya *et al.*[6], shows in the context of an $SU(N)$ gauge theory at high temperature that such a change in vacuum structure due to radiative corrections does indeed lead to anomalous g dependence of the interface tension of domain walls between quantum vacua. Calling the string coupling constant κ, the $(2k)!$ growth in string theory would follow from an instanton action scaling as κ. One sees from the above that explicit terms of $O(\kappa)$ in the action may not be necessary to produce this.

We suggest that a strong-weak coupling duality in string theory[1] may lead to nonperturbative effects with anomalous κ dependence, as follows. This duality interchanges strong and weak coupling, and the rôles of particles and solitons. It leaves the equations of motion invariant, but not the action. Thus solutions to the equations of motion, *e.g.*, vacua of string theory, are related by this symmetry and generically appear in pairs. Because the duality involves the loop counting parameter, the multiple vacua will only become evident in the equations of motion when quantum corrections are included. The coupling constant in string theory is a dynamical variable, so there can be field configurations that interpolate between such pairs of vacua. The actions of such configurations will scale anomalously with what we interpret in (one of) the weak coupling vacuum(vacua) as the coupling.

The concepts of tunneling and anomalous scaling must be made precise for these configurations, since they involve changing the dilaton field whose expectation value determines the coupling. In this form of duality, physics in the strong coupling vacuum has a description in terms of weakly coupled solitons. A complete tunneling process should be described as tunneling from the weak coupling vacuum (κ small) until a point in field space where $\kappa = O(1)$, after which the appropriate description is one of tunneling into the strong coupling vacuum (κ^{-1} small), described in terms of the weakly coupled dual theory. In a given weak coupling vacuum, the effective action is a series in some small parameter. Nonperturbative effects usually have actions $\sim O(1/\kappa^2)$. Here, in a given perturbative vacuum one can identify the coupling. What is meant by anomalous scaling is that the actions of the instantons will not be $\sim O(1/\kappa^2)$.

This can be demonstrated in a toy model, putting the system in finite volume and neglecting gravity. The scaling is anomalous and depends upon the details of the potential. The duality symmetry appears in the equations of motion for the action in the Einstein

[1] This has recently been discussed in several string backgrounds [7][8][9][10][11][12]. It generalizes a duality found in supergravity, for extensive references see [11].

basis, where the metric has been rescaled so that the coupling does not appear out in front of the action. Leaving out gravity and other fields, the action is

$$S = \int \mathrm{d}^{D-1}x\mathrm{d}t \left[(\frac{\partial\kappa}{\kappa})^2 - V(\kappa) \right] \tag{4}$$

In finite spatial volume of size L^{D-1} a time dependent classical solution to the equations of motion has finite action[2]. Searching for a solution $\kappa = \kappa(t)$ and using energy conservation,

$$\left(\frac{1}{\kappa} \frac{\mathrm{d}\kappa}{\mathrm{d}t} \right)^2 + V(\kappa) = E \ , \tag{5}$$

choose $V(\kappa) = g(\kappa)^2, g(\kappa) = g(1/\kappa)$. The extrema of V are at $2g(\kappa)g'(\kappa) = 0$. Consider a path starting at κ_0 and ending at $1/\kappa_0$, where g, κ_0 obey $g(\kappa_0) = g(1/\kappa_0) = V(\kappa_0) = E = 0$. Then from conservation of energy, for this classical solution, $\mathrm{d}\kappa/\mathrm{d}t = \kappa\sqrt{-V(\kappa)}$ and

$$S_{cl} = -\int \mathrm{d}^{D-1}x dt \ 2V(\kappa) = -L^{D-1} \int \mathrm{d}\kappa \ \frac{\mathrm{d}t}{\mathrm{d}\kappa} \ 2V(\kappa) = 2iL^{D-1} \int_{\kappa_0}^{1/\kappa_0} \frac{\mathrm{d}\kappa}{\kappa} g(\kappa) \tag{6}$$

For a potential $g(\kappa) = \sum a_n(\kappa^n + \kappa^{-n})$,

$$S_{cl} = 4L^3 i \left[\sum_{n>0} \frac{a_n}{n} (\kappa_0^{-n} - \kappa_0^n) - 2a_0 \ln \kappa_0 \right] \tag{7}$$

The simplest potential with an extremum away from $\kappa = \pm 1$ is $V = (\kappa + 1/\kappa - a)^2$. The minima (couplings in the perturbative vacua) are $\kappa_\pm = (a/2)\left[1 \pm \sqrt{1 - 4/a^2}\right]$. The corresponding action, when a is large, scales to leading order as $1/\kappa_-$, that is, in theories where the coupling κ_- is small. The coupling constant dependence of the action evaluated at this solution receives corrections from the measure due to zero modes, altering *e.g.* the logarithmic dependence on the coupling. Solutions to the equations of motion which keep the dilaton fixed have actions that scale as κ^{-2} as usual.

If strong-weak coupling duality is generic to string theory, an understanding of it should be based on a stringy formulation (such as can be done for $R \leftrightarrow 1/R$ duality, and its generalizations, in string field theory[13]). So far, supporting arguments for this duality have been presented that are specific to particular string backgrounds[7][8][11][12]. A coupling constant duality has been conjectured[7] in ten dimensional string–five-brane duality[14][7]. In the case of compactification of the heterotic string on a six dimensional

[2] JDC thanks J. Polchinski for discussions about this.

torus, it was found that dyonic solutions to the equations of motion were duality invariant[10], and Sen[15] showed that this duality is a full symmetry of the equations of motion (of the low energy weak coupling lagrangian). He also showed that the spectrum of the charges of the particles and the solitons is consistent with this duality[16][17].

Jevicki[18] has also found a duality between the solitons and particles in the $c = 1$ collective field theory of matrix models with no matrix model potential[3]. It is a symmetry of the equations of motion but not of the action. It is not clear whether this is a strong-weak coupling duality because the spacetime interpretation is unknown.

A calculable example is required to see the consequences of nonperturbative effects due to the pairing of vacua caused by strong-weak coupling duality. It is necessary to have a tractable description of the theory for the regime where $\kappa \sim O(1)$. One difficulty with computations in the known examples is the requirement that one explicitly express the fields in the dual vacua in terms of each other, $\Phi_{dual} = \Phi_{dual}[\Phi]$. This must be done, when $\kappa \sim O(1)$, in order to match tunneling solutions coming out of/going in to the dual vacua. The examples with $N = 4$ supersymmetry have strongly constrained radiative corrections and may allow calculations of tunneling effects. Sen[17] points out that the tree-level spectrum of the strongly coupled theory may be preserved in some cases due to supersymmetry nonrenormalization theorems. Ref. [8] pointed out several analogies between $R \leftrightarrow 1/R$ duality and strong-weak coupling duality. For $N = 1$ supergravity in four dimensions they wrote down the most general superpotential for the field $S = e^{\phi} + ia$ (ϕ is the dilaton, a is the axion) which is without singularities for finite values of S. It may be possible to have more general duality invariant couplings for the kinetic terms and couplings to other fields. Neglecting gravity and other fields besides the axion, and putting the system in a finite volume, these potentials can be treated as above (or by using [20]). For time dependent solutions interpolating between a supersymmetry preserving minimum at S_{min} and its dual, the dependence upon S_{min} goes as $|e^{i\theta} W(S_{min})(1 - e^{i\beta} S_{min})|$ times the volume. Here β, θ are phases, and θ is chosen to maximize the action. Ref. [8] points out that duality symmetry may prevent an arbitrarily weak coupling expansion from being valid, see also Ref. [21].

It is not clear how to connect strong-weak coupling duality to the source of nonperturbative effects which has been identified[2] in solvable matrix models of string theory.

[3] This theory maps[19] to the $c = 1$ collective field theory with a matrix model potential and a time dependent chemical potential.

The bare coupling constant is N^{-2}. The models reduce to the study of the interactions of N eigenvalues. The leading nonperturbative effects are tunnelings of individual eigenvalues[22][2] in the matrix model potential. Since $1/N$ of the degrees of freedom are involved, the motion results in an instanton action $\sim e^{-S_{inst}N^2} \sim e^{-(\text{const.})N}$. There have been some attempts to describe these in the string field theory for the $c = 1$ matrix model[4]. There are many nonperturbative definitions of the $c = 1$ theory[27]. One possibility, if there exist boundary conditions which respect this duality symmetry, is to require[5] duality symmetry to be respected by the nonperturbative definition of the model.

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[4] Ref. [23] describes the $c = 1$ matrix model tunneling in the collective field theory[24], emphasizing that the coupling must appear explicitly in the action. Ref. [25] uses a minimal phase space volume argument to constrain the instanton size. Ref. [26] shows that a quantum wavepacket has some support in the classically forbidden region.

[5] We thank M. Douglas for suggesting this.

References

[1] E. Brezin, V. Kazakov, *Phys. Lett.* **B236** (1990) 144; M. Douglas, S.H. Shenker, *Nucl. Phys.* **B335** (1990) 635; D. Gross, A. Migdal, *Phys. Rev. Lett.* **64** (1990) 127

[2] S.H. Shenker, in *Random surfaces and quantum gravity*, ed. O. Alvarez et al., (New York, Plenum, 1990)

[3] J. C. LeGuillou, J. Zinn-Justin, *Large Order Behavior of Perturbation Theory* (Amsterdam, North Holland, 1990)

[4] L.N. Lipatov, *Pis'ma Zh. Eksp. Teor. Fiz.* **25** (1977) 116

[5] For a recent review of related topics see E. Weinberg, "Vacuum Decay in Theories with Symmetry Breaking by Radiative Corrections", CU-TP-577, hep-ph/9211314 (Nov. 1992)

[6] T. Bhattacharya, A. Gocksch, C.P. Korthals Altes and R.D. Pisarski, *Phys. Rev. Lett.* **66** (1991) 998

[7] A. Strominger, *Nucl. Phys.* **B343** (1990) 167

[8] A. Font, L. E. Ibanez, D. Lust, F. Quevedo, *Phys. Lett.* **B249** (1990) 35

[9] M. J. Duff and J. X. Lu, *Nucl. Phys.* **B354** (1991) 129, **B354** (1991) 141, *Phys. Rev. Lett.* **66** (1991) 1402; C. Callan, J. Harvey, A. Strominger, *Nucl. Phys.* **B367** (1991) 60

[10] A. Shapere, S. Trivedi, F. Wilczek, *Mod. Phys. Lett.* **A6** (1991) 2677

[11] A. Sen, "Electric Magnetic Duality in String Theory", TIFR-TH-92-41, IC-92-171, hep-th/9207053 (July 1992)

[12] J. Maharana and J.H. Schwarz, *Nucl. Phys.* **B390** (1993) 3; J. Schwarz, "Dilaton-Axion Symmetry", CALT-68-1815, hep-th@xxx/9209125 (Sept. 1992)

[13] T. Kugo, B. Zwiebach, "Target space duality as a symmetry of string field theory", IASSNS-HEP-92/3 (Jan. 1992)

[14] M. Duff, *Class. Quantum Grav.* **5** (1988) 189

[15] A. Sen, "SL(2,R) Duality and Magnetically Charged Strings", TIFR-TH-93-03, hep-th/9302038 (Feb. 1993)

[16] A. Sen, "Quantization of Dyon Charge and Electric-Magnetic Duality in String Theory", TIFR/TH/92-46, hep-th@xxx/9209016 (Sept. 1992)

[17] A. Sen, "Magnetic Monopoles, Bogomol'nyi bound and SL(2,Z) Invariance in String Theory", NSF-ITP-93-29, TIFR-TH-93-07, hep-th/9303057 (March 1993)

[18] A. Jevicki, *Nucl. Phys.* **B376** (1992) 75

[19] J. Avan, A. Jevicki, *Phys. Lett.* **B266** (1991) 35

[20] M. Cvetic, F. Quevedo, S-J Rey, *Phys. Rev. Lett.* **67** (1991) 1836; M. Cvetic, S. Griffies, S-J Rey, *Nucl. Phys.* **B389** (1993) 3

[21] M. Dine, N. Seiberg, *Phys. Lett.* **162B** (1985) 299

[22] F. David, *Nucl. Phys.* **B348** (1991) 507

[23] R. Brustein, B. Ovrut, "Nonperturbative Interactions in String Theory", UPR-524T, hep-th/9209033 (Aug. 1992); "Nonperturbative Effects in 2-D String Theory or Beyond the Liouville Wall", UPR-523T, hep-th/9209081 (Sept. 1992); "Stringy Instantons", UPR-523T, hep-th/9209045 (Sept. 1992)

[24] S. Das, A. Jevicki, *Mod. Phys. Lett.* **A5** (1990) 1639; J. Polchinski, *Nucl. Phys.* **B346** (1990) 253

[25] J. Lee and P. Mende, "Semiclassical Tunneling in (1+1) Dimensional String Theory", Brown-HET-880, hep-th/9211049 (Oct 1992)

[26] A. Dhar, G. Mandal, S. Wadia, "A Time Dependent Classical Solution of c=1 String Field Theory and Nonperturbative Effects", TIFR-TH-92-40A, hep-th@xxx/9212027 (Dec. 1992)

[27] G. Moore, R. Plesser, S. Ramgoolam, *Nucl. Phys.* **B377**(1992) 143