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**“ HARD” POMERON APPROACH to “SOFT”  
PROCESSES at HIGH ENERGY**

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**Abstract**

An attempt was made to give a consistent description of high energy hadron interactions starting with the physical assumption that only “hard” processes contribute to the Pomeron structure. Using the general properties of a “hard” Pomeron in perturbative QCD an equation for shadowing corrections is suggested and solved. It allows one to develop the new approach to high energy hadron collisions which generalizes the so called eikonal approximation widely used to describe the shadowing corrections for both hadron and nucleus scattering at high energy. New formulas are also suggested for the large rapidity gap survival probability which crucially differ from the eikonal ones.



# 1 Introduction

Large-cross - section physics at high energy is usually regarded as dirty one since there is widespread delusion that it is impossible to develop any theoretical approach to such processes based on our microscopic theory - QCD. It is widely believed that the gap between current phenomenological models for high energy hadron and/or nucleus scattering and QCD is so big that it is difficult to see any interrelation between them. The main goal of this paper is to develop an approach that is based on QCD and establishes very transparent relationship between high energy “soft” scattering and our microscopic theory.

Of course, in order to do this we need to make a hypothesis. Our key assumption is that only “hard” processes contribute to the Pomeron structure. It means that we can describe the Pomeron in the framework of the perturbative QCD. Let me list the arguments that show that the assumption is not so crazy as it seems to be at first sight.

1. In any attempt to fit the experimental data the slope of the Pomeron trajectory ( $\alpha'$ ) turns out to be very small, at least not bigger than  $\alpha' = 0.25 \text{ GeV}^{-2}$  [1] [2] [3]. We use the following notation for the Pomeron trajectory  $\alpha_P(t = -q_t^2) = 1 + \Delta + \alpha't$ .

2. The experimental slope of the diffractive dissociation in the system of secondary hadrons with large mass is approximately two times smaller than the slope for the elastic scattering. In terms of Pomeron phenomenology this fact results in the small proper size of so called triple Pomeron vertex ( $G_{3P}$ ). In first approximation we can assign no slope for the triple Pomeron vertex to describe the experimental data on diffraction dissociation.

3. The idea that gluons inside a hadron are confined in the volume of smaller radius ( $R_G \approx 0.1 \text{ Fm} \ll R_h \sim 1 \text{ Fm}$ ) is still a working hypothesis which helps to describe the experimental data ( see ref. [1] for the details ).

4. The discovery so called “semihard” processes in QCD [4] which are responsible for the total inclusive cross section of hadron interaction at high energy and lead to the value of the total cross section compatible with the geometrical size of the hadron makes the assumption is the most probable and natural way to provide the matching between “hard” and “soft” processes.

5. All previous experience in multiperipheral models shows that one could describe the global features of the “soft” interaction at high energy but only if the main transverse momentum of produced hadrons is large enough ( of the order of  $1 \text{ GeV}$ ).

6. In the eikonal approach the QCD - Pomeron is able to describe the current experimental data on total and elastic cross section and the slope (see ref. [9] for

details).

I hope the above arguments convince you that a hard Pomeron should not be bad first approximation to high energy scattering. This hypothesis has at least three big advantages: simplicity, natural matching with QCD at small distances and the obvious possibility to check it experimentally.

Now I am ready to discuss the general strategy of the approach. The first step is a review the main properties of the QCD Pomeron. It will be shown in section 2 that the QCD Pomeron has no slope ( $\alpha' = 0$ ) and can be considered as an exchange with definite impact parameter  $b_t$ . Moreover in the leading log approximation the interaction between Pomerons cannot change  $b_t$ . This fact allows us to regenerate the old Reggeon Field Theory [5] for the interaction of hard Pomerons in section 3. In this section the new equation for the shadowing (screening) corrections will be discussed as well as solutions to these new equations. Physical applications are collected in section 4 where we discuss such important problems as the behaviour of the inclusive cross section and the large rapidity gap survival probability. The results and the their physical meaning will be discussed in the Conclusions.

## 2 “Hard” Pomeron in QCD

As has been discussed in the introduction we assume that only hard processes contribute to the structure of the Pomeron. It means that we believe in some natural cut off in momentum ( $Q_0$ ) and that only production of quarks and gluons with transverse momenta  $k_t > Q_0$  are dominant in the Pomeron. Since we assume that the value of  $Q_0$  is so large that  $\alpha_s(Q_0^2) \ll 1$  we can use perturbative QCD to calculate Pomeron exchange in the leading log approximation (LLA) considering the following parameters as small ones:

$$\alpha_s(Q_0^2) \ll 1, \quad \alpha_s(Q_0^2) \ln \frac{k_t^2}{Q_0^2} \ll 1, \quad \text{but} \quad \alpha_s(Q_0^2) \ln s \gg 1.$$

The answer for the scattering amplitude in LLA is the summation of the perturbative series:

$$f(s, t; k^2, Q_0^2) = \sum_n C_n (\alpha_s(Q_0^2) \ln s)^n + O(\alpha_s(Q_0^2); \alpha_s(Q_0^2) \ln \frac{k^2}{Q_0^2}), \quad (1)$$

where  $k^2$  and  $Q_0^2$  are virtualities of the scattering partons (quarks or gluons).

During the last two decades the answer for the sum of eq. (1) has been studied in great detail (see original papers [7] [6] or several reviews [4] [8] [10]). I would like to outline the solution to this problem, using a little bit different technique which will be very convenient for further presentation.

1. *The Pomeron in LLA of the perturbative QCD at  $t=0$ .*

In leading *logs* approximation ( LLA ) we can reduce the problem of summation of the perturbative series of eq. (1) to the solution of the so called 'ladder' equation (see Fig.1.). Namely,

$$\phi_n(y, q^2) = \frac{N_c \alpha_s}{\pi} \int dy' \int \frac{d^2 q'}{\pi} K(q^2, q'^2) \phi_{n-1}(y', q'^2), \quad (2)$$

where  $y$  ( $y'$ ) is equal  $y = \ln s/q^2$  ( $y' = \ln s'/q'^2$ ),  $q^2$  and  $q'^2$  are virtualities of two slowest particles (see Fig.1). The function  $\phi$  is closely related to the gluon structure function in deep inelastic scattering, since

$$\alpha_s(q^2) x_B G(x_B, q^2) = \int^{q^2} \alpha_s(q'^2) \phi(y = \ln \frac{1}{x_B}, q'^2). \quad (3)$$

To solve eq.(2) it is very convenient to introduce the auxillary function

$$\Psi(y, r = \ln q^2, x) = \sqrt{q^2} \cdot \sum_n \phi_n(y, r) x^n. \quad (4)$$

It is easy to see from eq. (4) that the amplitude of gluon - gluon scattering or in other words the gluon structure function can be reduce to

$$\phi(y, q^2) = \sum_n \phi_n(y, r) = \frac{1}{\sqrt{q^2}} \cdot \Psi(y, r, x = 1). \quad (5)$$

The partial cross section ( $\phi_n$ ) can be calculated as

$$\phi_n(y, r) = \frac{1}{\sqrt{q^2}} \cdot \frac{1}{n!} \cdot \frac{\partial^n \Psi(y, r, x)}{\partial x^n} \Big|_{x=0}. \quad (6)$$

We can also write down simple expressions for different correlators through the function  $\Psi$ , such as:

$$\begin{aligned} \langle n \rangle &= \frac{\partial \Psi}{\partial x} \Big|_{x=1}, \\ \langle n(n-1) \rangle &= \frac{\partial^2 \Psi}{\partial x^2} \Big|_{x=1}. \end{aligned} \quad (7)$$

We can rewrite eq. (2) as the equation for  $\Psi$  which is very simple if we assume that  $r - r' \ll r$  and adopt the following expansion (as was first done in ref. [7]):

$$\Psi(y, r', x) = \Psi(y, r, x) + \frac{\partial \Psi(y, r, x)}{\partial r} (r' - r) + \frac{1}{2} \frac{\partial^2 \Psi(y, r, x)}{\partial r^2} (r' - r)^2 + \dots \quad (8)$$

Finally the equation for  $\Psi$  looks as follows:

$$\frac{\partial \Psi(y, r, x)}{\partial y} = \omega_0 x \Psi(y, r, x) + \delta x \frac{\partial^2 \Psi(y, r, x)}{\partial r^2}, \quad (9)$$

where (see ref. [7] for details)

$$\omega_0 = \frac{4N_c \alpha_s}{\pi} \ln 2; \quad \delta = \frac{N_c \alpha_s}{\pi} 14\zeta(3).$$

Eq. (9) can be solved by going to a Laplace representation and noting that  $\Psi$  depends on  $z = xy$ :

$$\Psi(z, r) = \int \frac{d\omega df}{(2\pi)^2} e^{(\omega z + fr)} \psi(\omega, f). \quad (10)$$

For  $\psi$  the equation reads as follows:

$$\omega = \omega_0 + \delta f^2, \quad (11)$$

which leads to the answer:

$$\Psi(y, r, x) = \int e^{[(\omega_0 + \delta f^2)x + fr]} \psi(f) \frac{df}{2\pi i}. \quad (12)$$

Starting with the initial condition namely  $\Psi(y = 0, r, x) = \delta(r - r_0)$  we can easily get the famous diffusion solution of the equation, namely:

$$\Psi(y, r, x) = \frac{1}{2\sqrt{xy\delta}} e^{\omega_0 xy - \frac{(r-r_0)^2}{4\delta xy}}. \quad (13)$$

Eq. (13) gives the solution that allows one to calculate both the amplitude and the multiplicity distribution using eqs.(5 - 7) and therefore it enlarges our possibility to study the Pomeron structure in the perturbative QCD. However the main reason why I gave this somewhat new derivation of a well known solution is to illustrate the new technique of the auxillary function that I am going to use later on to get the solution

to the more complicated problem of Pomeron interaction. To demonstrate how this technique works let us calculate the multiplicity distribution, namely

$$\frac{\sigma_n}{\sigma_t} = \frac{1}{\Psi(y, r, x=1)} \cdot \int \frac{dx}{2\pi i} \frac{1}{x^{(n+1)}} \Psi(y, r, x).$$

where the integration contour over  $x$  is to the right of all singularities of function  $\Psi$ . In the saddle point approximation we can perform the above integral and the answer can be written in the form:

$$\frac{\sigma_n}{\sigma_t} = \sqrt{\frac{n+1}{\omega_0 y}} \cdot \exp\left[-(n+1)(\ln(n+1) - 1) - \omega_0 y - (r-r_0)^2 \left(\frac{\omega_0}{4\delta(n+1)} - \frac{1}{4\delta y}\right)\right]. \quad (14)$$

From eq.(14) we can also calculate the mean  $(r - r_0)^2$  at fixed multiplicity:

$$\langle (r - r_0)^2 \rangle = \frac{\int dr (r - r_0)^2 \sigma_n(y, r)}{\sigma_n(y, r)}.$$

Using eq.(14) we can find that

$$\langle (r - r_0)^2 \rangle = 2 \frac{\delta}{\omega_0} (n+1). \quad (15)$$

Eq. (15) shows a very important property of LLA structure of the Pomeron, namely the fact that the mean log of the transverse momentum increases after the emission of  $n$  gluons.

## 2. $b_t$ dependence of the LLA Pomeron.

The main property of the impact parameter motion of the parton could be understood directly from the uncertainty principle, since

$$\Delta b_t q_t \approx 1. \quad (16)$$

It means that  $\Delta b_t \propto \frac{1}{q_t}$  for each emission where  $q_t$  is the typical transverse momentum of the parton. As we assumed  $q_t > Q_0 \gg 1 \text{ GeV}$  for all produced partons, the displacement of the parton in  $b_t$  we can consider as a small one. Moreover due to the emission of gluons the mean transverse momentum increases at high energy or after  $n \gg 1$  emissions. I hope that this discussion made natural the strict LLA result ( see refs.[6] [10] [11]) that the LLA Pomeron does not depend on the momentum transferred ( $t$ ). Thus, in LLA of perturbative QCD we can consider the Pomeron as frozen in  $b_t$  - space or in other words its exchange is proportional to  $\delta(b_t)$ .

### 3. QCD motivated Pomeron.

Now we can formulate what model for the Pomeron structure we are going to discuss as the first approximation to the “hard ” Pomeron. Namely, we assume that the Pomeron can be reduced to the simple formula:

$$P(y, b_t) = i e^{\omega_0 y} \delta(b_t) . \quad (17)$$

Since we consider the case when initial and final virtuality are equal the oversimplified formula (17) does not take into account the power-like behaviour on  $y$  in eq. (13). Throughout the paper we will use this simplified version of eq. (13) but it should be stressed that it is not hard to incorporate the correct behaviour of eq. (13) in all our calculations, but the simplest expression of eq. (17) makes all our calculations so transparent that we prefer to use this form to clear up the main property of the screening (shadowing) corrections.

## 3 Shadowing corrections.

In this section we are going to discuss how to incorporate the shadowing ( screening ) corrections in the framework of the simplified approach to the Pomeron structure given by eq. (17). There are two origins of the shadowing (screening ) corrections: the interaction between colliding hadrons due to multipomeron exchanges and the interaction between pomerons. The first one is usually taken into account by the so called eikonal approach which is really the only one that is in the market for the description of the shadowing corrections. During the last decade the eikonal approach has squeezed out the more general understanding of the origin and nature of shadowing (screening) corrections and has become a synonym of the shadowing correction in general. This happened partly due to the failed attempts to take into account the pomeron interaction in the framework of so called Reggeon Field Theory ( RFT) [5]. The main goal of this section as well as the whole of this paper is to revive the RFT and to suggest more general approach than the eikonal one to the shadowing correction.

### 3.1 Eikonal Approach.

Let me start with the review of the main ideas and formulas of the eikonal approach that are carried out most compactly in the impact parameter ( $b_t$ ) representation.

Our amplitudes are normalized as follows:

$$\frac{d\sigma}{dt} = \pi |f(s, t)|^2 ; \quad \sigma_{tot} = 4\pi \text{Im} f(s, 0) ,$$

where

$$f(s, t) = \frac{1}{2\pi} \int d\mathbf{b}_t \exp^{i\mathbf{q} \cdot \mathbf{b}_t} a(\mathbf{b}_t, s) \quad (18)$$

and

$$a(s, \mathbf{b}_t) = \frac{1}{2\pi} \int d\mathbf{q} \exp^{-i\mathbf{q} \cdot \mathbf{b}_t} f(s, t) \quad (19)$$

hence we have :  $\sigma_{tot} = 2 \int d\mathbf{b}_t \text{Im} a(s, \mathbf{b}_t)$  and  $\sigma_{el} = \int d\mathbf{b}_t |a(s, \mathbf{b}_t)|^2$

Unitarity requires  $\text{Im} a(s, \mathbf{b}_t) \leq 1$ . In order to satisfy the unitarity constraint it is convenient to express  $a(s, \mathbf{b}_t)$  in terms of the complex eikonal function  $\chi(s, \mathbf{b}_t)$  with ( $\text{Im} \chi \geq 0$ ). i.e.

$$a(s, \mathbf{b}_t) = i[1 - e^{i\chi(s, \mathbf{b}_t)}] \quad (20)$$

which ensures that unitarity is restored on summing up all the eikonal multi-particle exchange amplitudes.

All above formulas are general and the eikonal model starts with two assumptions:

1. At high energies elastic scattering is essentially diffractive and therefore  $\text{Re} \chi$  is small. We assume  $\text{Re} \chi \approx 0$ , then the amplitude  $a(s, \mathbf{b}_t)$  is purely imaginary and determined by the opaqueness  $\Omega(s, \mathbf{b}_t) \equiv \text{Im} \chi$ .

2. The opaqueness

$$\Omega(s, \mathbf{b}_t) = \frac{1}{4\pi} \int d^2 b_t e^{-i\mathbf{q} \cdot \mathbf{b}_t} g^2(t) \text{Im} P(s, t) = s^{\omega_0} \int \frac{d^2 b'_t}{2\pi} g(\mathbf{b}_t - \mathbf{b}'_t) g(\mathbf{b}'_t) , \quad (21)$$

where all notation are obvious from Fig.2 and  $t = -q_t^2$ . Here

$$g(\mathbf{b}_t) = \frac{1}{2\pi} \int d^2 b_t e^{-i\mathbf{q} \cdot \mathbf{b}_t} g(q_t^2) ,$$

where  $g(t)$  is the vertex for the Pomeron - hadron interaction as seen from Fig.2. Eq. (21) establishes the direct relationship between the opaqueness and the Pomeron exchange. Within this assumption the equation (20) sums up the diagrams of Fig.3.

The advantages of the eikonal approach are evident: the exceptional simplicity of the approach and the fact that this approach takes into account the natural scale for the shadowing corrections. It makes this model very attractive and popular. However it should be stressed that there are no theoretical arguments why this approach should

work. The eikonal model looks extraordinarily strange from the point of view of the parton or QCD approach. Indeed, a slight glance at the QCD parton cascade (see Fig. 4) shows us that in spite of very complicated structure of this cascade the number of partons drastically increases mostly due to decay of each particular parton in its own chain of partons. No arguments have been found in QCD why this complicated structure of the parton cascade which could in principle be described as the Pomeron interactions (see Fig. 4) could be reduced to eikonal diagrams. The parton cascade for the eikonal diagrams looks very simple, namely it is only production of the different parton chains by the fast hadron as it is shown in Fig. 5. I would like to draw your attention to the fact that even in the simplest case of the deep inelastic scattering the structure of the parton cascade can be described better by a fan diagram than by an eikonal one (see Fig. 6 and ref. [4] for details).

### 3.2 Pomeron interaction ( " Fan " diagrams ).

In this subsection I am going to discuss "fan" diagrams contribution to hadron - hadron scattering. I consider this problem as the next approximation to reality after the eikonal one. It certainly will be able to teach us how pomeron - pomeron interaction results in the shadowing correction. However I would first like to make some general remarks on the main features of pomeron interactions in QCD.

#### 1. Pomeron interactions in QCD.

The main advantage of QCD in our problem is the fact that we can formulate what we are doing. Our Pomeron in QCD is well established object, namely LLA "ladder" diagrams which lead to eq. (17) in the first rough approximation. So in principle we can calculate in QCD the vertices of interaction between three, four and so on, "ladders". In practice only triple "ladder" interactions have been calculated in specific kinematical regions where virtualities of all interacting partons were large enough ( see refs. [12] [13] [14] ) as well as the amplitude of two "ladder" rescattering (see refs. [15] [16] ). Let me summarize what we have learned from these calculations.

1. In perturbative QCD we can introduce vertices for three and four pomeron interaction (see Fig. 7), which are local in rapidity.

B2. All contributions with integration over small transverse momenta  $k_t$  ( $k_t < Q_0$ ) are cancelled. It means that we can justify calculations in perturbative QCD.

3. The vertices  $\gamma$  and  $\lambda$  in Fig. 7 have different order of magnitude in  $\alpha_s$ . Namely, it turns out that

$$\gamma \propto N_c \alpha_s^2; \quad \lambda \propto \alpha_s.$$

4. The sign of pomeron - pomeron scattering amplitude  $\lambda$  corresponds the attractive forces [15] [16] as was discussed many years ago by B.M. McCoy and T.T. Wu [17].

5. Concerning  $b_t$  - dependence of pomeron - pomeron interaction vertices we can also consider them as a  $\delta$  - function in  $b_t$ .

2. *Strategy of approach.*

Based on this experience with QCD calculation I would like to suggest the following strategy of approach:

1) We start from the simplest formula of eq. (17) for one Pomeron exchange.

2) We introduce the vertices  $g(b_t)$  for the Pomeron interaction with the hadron (see Fig.2).

3) We describe the Pomeron - Pomeron interaction introducing triple Pomeron vertex( $\gamma$ ) and four Pomeron amplitude ( $\lambda$ ) which are local in rapidity and are proportional to  $\delta(b_t)$  with respect to any impact parameter related to the interaction.

3. *Summation of the "fan" diagrams.*

To demonstrate the problems that we face in finding the screening correction contributions let me discuss the simplest nontrivial case: summation of the "fan" diagrams of Fig.6 only, neglecting even the pomeron rescattering (vertex  $\lambda$  in Fig.7).

To solve this problem we develop the same method of auxillary function <sup>1</sup>

$$\Psi(y, x) = \sum_n C_n(y) x^n, \quad (22)$$

in which the coefficients  $C_n(y)$  constitute the probability amplitude for finding  $n$  Pomerons at rapidity  $y$ . For  $\Psi(y, x)$  it is very simple to write down the equation:

$$-\frac{\partial \Psi(y, x)}{\partial y} = \omega_0 x \frac{\partial \Psi(y, x)}{\partial x} - \gamma x^2 \frac{\partial \Psi}{\partial x}. \quad (23)$$

This equation is nothing more than the different form of the equation for  $C_n$  :

$$-\frac{dC_n(y)}{dy} = \omega_0 n C_n - \gamma(n-1)C_{n-1}. \quad (24)$$

The physical meaning of eq. ( 24 ) is clear from Fig.8, the first term describes the propagation of Pomerons which do not interact with each other while the second one annihilates any one of the Pomerons in the interval  $dy$ , replacing it by two others. The

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<sup>1</sup>As far as I knowv this method was firstly applied to the problem of the shadowing correction in ref. [18].

sign minus in front of this term reflects the shadowing (screening) character of the interaction or in other word the fact that our scattering amplitude is pure imaginary at high energy.

Eq. ( 23 ) can be solved and the solution is an arbitrary function of one variable  $\Psi(\kappa)$ , where

$$\kappa = \omega_0(Y - y) + \ln \frac{x}{1 - \frac{\gamma}{\omega_0} x} . \quad (25)$$

The function  $\Psi(\kappa)$  can be found from an initial condition, which for our problem is the following ( see Fig. 6 for notations ):

$$\Psi(\kappa) = x g(b_t - b'_t) \text{ at } y = Y . \quad (26)$$

From eq. (26) we can find that

$$x = \frac{e^\kappa}{1 + \frac{\gamma}{\omega_0} e^\kappa} \text{ and } \Psi = \frac{g(b_t - b'_t) \cdot e^\kappa}{1 + \frac{\gamma}{\omega_0} e^\kappa} . \quad (27)$$

Finally to get the answer for the scattering amplitude at fixed impact parameter  $b_t$  we need to substitute  $y = 0$  and  $x = g(b'_t)$  in the definition of  $\kappa$  and find  $\Psi(\kappa)$  from previous equation. Thus

$$a_{FD}(Y = \ln s, b_t) = \int \frac{d^2 b'_t}{2\pi} \cdot \frac{g(b_t - b'_t)g(b'_t)}{\frac{\gamma}{\omega_0} + e^{-\omega_0 Y} [1 - \frac{\gamma}{\omega_0} g(b'_t)]} . \quad (28)$$

#### 4. Eikonal + " Fan" diagrams.

It is very instructive to get now the formula for the amplitude that takes into account eikonal and "fan" diagrams (see Fig. 9) together. Such a formula can be written in terms of the opaqueness  $\Omega(s, b_t)$  and eq. (20) if

$$\Omega(Y = \ln s, b_t) = e^{\omega_0 Y} \int \frac{d^2 b'_t}{2\pi} g(b_t - b'_t)g(b'_t) + 2[a_{FD}(Y, b_t) - a_{FD}(Y, b_t, \gamma = 0)] . \quad (29)$$

The above expression for  $\Omega$  takes into account in correct way the fact that two sets of the "fan" diagrams with many pomeron interaction coupled to top or bottom part of Fig. 9 - type diagram have the same common part - one Pomeron exchange.

### 3.3 Rescattering of Pomerons.

I have considered a toy - model for the origin of the shadowing correction in the previous subsection, here I would like to discuss a selfconsistent approach in which the

pomeron - pomeron rescattering will be taken into account, since this interaction seems to be the biggest one in QCD ( $\lambda \propto \alpha_s$  while  $\gamma \propto \alpha_s^2$ ). It means that I am going to sum up the diagrams of Fig. 10 - type or in other words we include selfconsistently the interaction of the Pomeron with the hadron which is of the order of  $\alpha_s$  in QCD as well as the Pomeron - Pomeron rescattering ( $\lambda \propto \alpha_s$ ).

The equation for the auxillary function  $\Psi$  for this problem looks as a trivial generalization of eq. (24),namely:

$$-\frac{\partial \Psi(y, x)}{\partial y} = \omega_0 x \frac{\partial \Psi(y, x)}{\partial x} + \lambda x^2 \frac{\partial^2 \Psi(y, x)}{\partial x^2}. \quad (30)$$

It should be stressed that eq. (30) describes the attractive interaction between pomerons ( $\lambda > 0$ ) as was discovered many years ago by Mc Coy and Wu [17] and has been recently rediscovered in QCD ( see refs. [15] [16]).

However this equation cannot be solved in such an easy way as eq. (24). First let us simplify a little bit the equation, introducing new variable  $\eta = \ln x$  and going to  $\omega$  representation:

$$\Psi(y, \eta) = \int \frac{d\omega}{2\pi i} e^{\omega(Y-y)} \psi(\omega, \eta). \quad (31)$$

For  $\psi$  the equation looks as follows:

$$\omega \psi(\omega, \eta) = (\omega_0 - \lambda) \frac{d\psi(\omega, \eta)}{d\eta} + \lambda \frac{d^2 \psi(\omega, \eta)}{d\eta^2}. \quad (32)$$

Eq. (32) can be easily solved by going to a Laplace representation with respect to  $\eta$ :

$$\psi(\omega, \eta) = \int \frac{df}{2\pi i} e^{f\eta} \phi(\omega, f). \quad (33)$$

In  $\omega$  and  $\eta$  representation eq. (32) reduces to:

$$\omega = (\omega_0 - \lambda) f + \lambda f^2. \quad (34)$$

So the solution of eq. (32) finally looks like

$$\Psi(Y - y, \eta) = \int \frac{df}{2\pi i} \phi(f) e^{[(\omega_0 - \lambda)(Y - y) + f\eta]}. \quad (35)$$

The function  $\phi(f)$  should be found from the initial condition

$$\Psi(Y - y, \eta) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{[g(b_t - b'_t)]^n}{n!} e^{n\eta} = 1 - \exp[-g(b_t - b'_t)e^\eta]; \quad (36)$$

at  $y = Y$ .

To satisfy the above condition we need to choose the function  $\phi(f)$  equal to

$$\phi(f) = \Gamma(-f)$$

and the solution can be rewritten in the form:

$$\Psi(Y - y, \eta) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{[g(b_t - b'_t)e^\eta]^n}{n!} e^{[(\omega_0 - \lambda)n + \lambda n^2](Y - y)}. \quad (37)$$

Using the obvious relation:

$$e^{(\omega_0 f + \lambda f^2)(Y - y)} = \int e^{-f\eta'} d\eta' \cdot \frac{1}{2\sqrt{\pi\lambda(Y - y)}} \cdot e^{-\frac{[(\omega_0 - \lambda)(Y - y) + \eta']^2}{4\lambda(Y - y)}}, \quad (38)$$

we can rewrite the answer in a form more convenient for further discussions:

$$\Omega(Y = \ln s, b_t) = \int \frac{d^2 b'_t}{2\pi} \Psi(Y, \eta = \ln g(b'_t)) = \int \frac{d^2 b_t}{2\pi} \int d\eta' \{ 1 - \exp(-g(b_t - b'_t)g(b'_t)e^{-\eta'}) \} \cdot \frac{1}{2\sqrt{\pi\lambda Y}} \cdot e^{-\frac{[(\omega_0 - \lambda)Y + \eta']^2}{4\lambda Y}}. \quad (39)$$

Eq. (39) can be simplified assuming the Gaussian for  $b_t$  - dependence of  $g(b_t)$ , namely

$$g(b_t) = \frac{\sigma_0(s = s_0)}{\pi R_h^2} e^{-\frac{b_t^2}{R_h^2}}, \quad (40)$$

where  $R_h$  is the radius of the hadron while  $\sigma_0$  is the value of the cross section of the hadron - hadron interaction at sufficiently small energy  $s = s_0$ . Performing integration over  $b_t$  we get the result:

$$\Omega(Y, b_t) = \frac{R_h^2}{4\sigma_0} \int d\eta' \{ \ln \left[ \frac{\sigma_0^2}{(\pi R_h^2)^2} e^{-\frac{b_t^2}{2R_h^2} - \eta'} \right] + C - Ei \left( -\frac{\sigma_0^2}{(\pi R_h^2)^2} e^{-\frac{b_t^2}{2R_h^2} - \eta'} \right) \} \cdot \frac{1}{2\sqrt{\pi\lambda Y}} \cdot e^{-\frac{[(\omega_0 - \lambda)Y + \eta']^2}{4\lambda Y}}. \quad (41)$$

At very high energy the dominant contribution in the integral over  $\eta'$  gives the value of  $\eta' \sim -(\omega_0 - \lambda)Y$  and  $\Omega$  turns out to be equal for  $b_t^2 \ll 2r_h^2(\omega_0 - \lambda)Y$ :

$$\Omega(Y, b_t) = \frac{R_h^2}{4\sigma_0} [(\omega_0 - \lambda)Y + \ln \frac{\sigma_0}{R_h^2} + C - \frac{b_t^2}{2R_h^2}]. \quad (42)$$

For  $b_t^2 \gg 2R_h^2(\omega_0 - \lambda)Y$

$$\Omega(Y, b_t) = \frac{\sigma_0}{\pi R_h^2} e^{\omega_0 Y - \frac{b_t^2}{2R_h^2}}. \quad (43)$$

Substituting this expression for  $\Omega$  in eq. (20) we can conclude that the total cross section increases logarithmically with energy:

$$\sigma_t \propto 2\pi R_h^2 \ln s.$$

However this result depends crucially on the Gaussian parametrization of function  $g(b_t)$ . If  $g(b_t) \propto e^{-\frac{b_t^2}{R_h^2}}$   $\sigma_t \rightarrow \ln^2 s$  at high energy.

### 3.4 Pomeron interaction ( General case).

In this subsection I am going to consider the general case and sum up the diagrams of Fig. 11 - type, taking into account both the pomeron - pomeron rescattering ( $\lambda$ ) and the pomeron splitting into two pomerons ( $\gamma^+$ ) as well as the pomeron annihilation ( $\gamma^-$ ) (see Fig.11 for notations). The first question that arises is why we can restrict ourselves to summing only diagrams of the type of Fig.11 or in other words why we neglect the more complicated interactions among pomerons, for example the one pomeron transition to three or even more pomerons. To answer these questions we need to recall that in QCD we have the following order of the magnitudes for our basic interactions:

$$g \propto \alpha_s; \quad \omega_0 \propto N_c \alpha_s; \quad \lambda \propto \alpha_s; \quad \gamma^- \sim \gamma^+ \propto N_c \alpha_s^2. \quad (44)$$

Summing the diagrams of Fig.11 - type we make an attempt to calculate the high energy amplitude within the accuracy of the order of  $O((\alpha_s^3 \ln s)^n)$ . Indeed, if we consider the following set of the small parameters:

$$\alpha_s \ll 1; \quad \alpha_s \ln s \gg 1; \quad \alpha_s^3 \ln s \sim 1, \quad (45)$$

we can reduce our problem to the summation of the Fig. 11 set of the diagrams. The reggeon diagrams give us the possibility to take into account in a constructive way the factorization property of QCD that is very general, at least more general than any leading log approximations.

The equation for auxiliary function  $\Psi$  for the general set of the diagrams of Fig.11 can be written in the form:

$$- \frac{\partial \Psi(Y - y, x)}{\partial y} = \quad (46)$$

$$\omega_0 x \frac{\partial \Psi(Y-y, x)}{\partial x} + \lambda x^2 \frac{\partial^2 \Psi(Y-y, x)}{\partial x^2} - \gamma^+ x^2 \frac{\partial \Psi(Y-y, x)}{\partial x} + \gamma^- x \frac{\partial^2 \Psi(Y-y, x)}{\partial x^2}.$$

This equation can be simplified introducing the new variable  $\eta = \ln x$  and going to  $\omega$  representation (see eq. (31)). For  $\psi(\omega, \eta)$  the equation can be reduced to the form:

$$\omega \psi(\omega, \eta) = (\omega_0 - \lambda - \gamma^+ e^\eta - \gamma^- e^{-\eta}) \frac{d\psi(\omega, \eta)}{d\eta} + (\lambda + \gamma^- e^{-\eta}) \frac{d^2 \psi(\omega, \eta)}{d\eta^2}. \quad (47)$$

Let us find the semiclassical slution of the equation substituting

$$\psi(\omega, \eta) = \exp(\chi(\omega, \eta))$$

and assuming that

$$\frac{d^2 \chi(\omega, \eta)}{d\eta^2} \ll \left[ \frac{d\chi(\omega, \eta)}{d\eta} \right]^2.$$

In this case eq.(47) can be reduced to the algebraic one,namely

$$\omega = [\omega_0 - \lambda - \gamma^+ e^\eta - \gamma^- e^{-\eta}] \frac{d\chi}{d\eta} + [\lambda + \gamma^- e^{-\eta}] \left( \frac{d\chi}{d\eta} \right)^2. \quad (48)$$

Solving this equation we can get the answer:

$$\psi(\omega, \eta) = \phi(\omega) \cdot \exp\left\{ \int_{-\infty}^{\eta} d\eta' \frac{1}{2(\lambda + \gamma^- e^{-\eta'})} \right\}. \quad (49)$$

$$[\lambda + \gamma^+ e^{\eta'} + \gamma^- e^{-\eta'} - \omega_0 - \sqrt{(\lambda + \gamma^+ e^{\eta'} + \gamma^- e^{-\eta'} - \omega_0)^2 + 4\omega(\lambda + \gamma^- e^{-\eta'})}] \}.$$

However solution (34) can be reliable only at sufficiently large  $\omega$  namely at

$$\omega \gg \gamma e^\eta,$$

where  $\chi''_{\eta\eta} \ll \chi'^2_{\eta}$ . At very small  $\omega$  however the last term in the eq. (32) turns out to be negligibly small,so solution (34) is able to describe this region too. The function  $\phi(\omega)$  can be found from the initial condition of eq. (26), namely:

$$\int \frac{d\omega}{2\pi i} \phi(\omega) \exp\left\{ \int_{-\infty}^{\eta} d\eta' \frac{1}{2(\lambda + \gamma^- e^{-\eta'})} \right\}$$

$$[\lambda + \gamma^+ e^{\eta'} + \gamma^- e^{-\eta'} - \omega_0 - \sqrt{(\lambda + \gamma^+ e^{\eta'} + \gamma^- e^{-\eta'} - \omega_0)^2 + 4\omega(\lambda + \gamma^- e^{-\eta'})}] \} =$$

$$= \{ 1 - \exp[-e^n g(b_t - b'_t)] \} . \quad (50)$$

However the solution given by eqs. (49) and (50) cannot be considered as transparent from a physical point of view. It is the reason why I would like to give another solution which has a worse accuracy but it is simple enough to clarify the situation and to demonstrate the main property of the solution. Let us assume that the last term in eq. (47) is small enough to be neglected. In this case the solution looks as the solution of eq. (23), namely that  $\Psi$  is a function of one variable ( $\Psi(\kappa)$ ):

$$\kappa = Y - y - \frac{1}{\Delta} \ln \frac{(x - x_+)x_-}{(x - x_-)x_+} . \quad (51)$$

where

$$x_{\pm} = \frac{-(\omega_0 - \lambda) \pm \sqrt{(\omega_0 - \lambda)^2 - 4\gamma^+ \gamma^-}}{2\gamma^-}$$

and

$$\Delta = 2\gamma^-(x_+ - x_-); \quad x = e^n .$$

The function  $\Psi(\kappa)$  could be found from the initial condition (50) at  $y = Y$ . It is easy to see that  $\Psi(\kappa)$  is equal to:

$$\Psi(\kappa) = 1 - \exp \left\{ -g(b_t - b'_t) \frac{x_+ - x_- \frac{x - x_+}{x - x_-} \cdot e^{-\Delta(Y-y)}}{1 - \frac{x - x_+}{x - x_-} \cdot e^{-\Delta(Y-y)}} \right\} . \quad (52)$$

Using this solution we can calculate the last term of eq.(47). One can see that this term is small enough even at  $y = Y$  and becomes smaller at  $y \rightarrow 0$ .

Thus to get the solution of our problem or in other word to sum up all diagrams of Fig.11 - type we need to:

1. substitute  $x = g(b'_t)$  in eq. (52),
2. integrate over  $b'_t$ ,
3. calculate  $\Omega(s, b_t)$  as

$$\Omega(Y = \ln s, b_t) = \int \frac{d^2 b'_t}{2\pi} \quad (53)$$

$$\{ \Psi(b_t, b'_t, \kappa(\gamma^+ = \gamma^- = 0)) + 2[\Psi(b_t, b'_t, \kappa) - \Psi(b_t, b'_t, \kappa(\gamma^+ = \gamma^- = 0))] \} .$$

4. substitute the opaqueness  $\Omega(s, b_t)$  in eq. (20).

## 4 Large rapidity gaps in hadron - hadron collisions.

Eq. (47) gives us the possibility to discuss the behaviour of the total cross section and elastic cross sections as well as inclusive observables in hadron - hadron collisions. Here I am going to discuss only two problems: the energy behaviour of the inclusive cross section and the probability of a large rapidity gap.

### 4.1 Inclusive cross section.

Accordingly AGK - cutting rules [19] the inclusive cross section can be calculated as sum of the diagrams of Fig. 12 -type. It is obvious that the formula for the inclusive cross section looks as follows:

$$\frac{d\sigma}{dy} = G_{2Ph} \int \frac{d^2 b_t}{2\pi} \Psi_{inc}(Y - y, x = g(b_t)) \cdot \int \frac{d^2 b'_t}{2\pi} \Psi_{inc}(y, x = g(b'_t)) , \quad (54)$$

where all notations are clear from Fig. 12.  $\Psi$  is the solution of eq. (47) but with a different initial condition as compared with eq. (50). Namely,

$$\Psi(Y - y = 0, x) = x . \quad (55)$$

In the approximation that leads to solution (51) one can find the answer for  $\Psi_{inc}$ :

$$\Psi_{inc}(Y - y, x) = \frac{x_+ - x_- \frac{x - x_+}{x - x_-} \cdot e^{-\Delta(Y-y)}}{1 - \frac{x - x_+}{x - x_-} \cdot e^{-\Delta(Y-y)}} . \quad (56)$$

Eq. (56) solves the problem, allowing one to calculate the inclusive cross section.

### 4.2 Large rapidity gap.

Dokshitzer, Khoze and Sjostrand and Bjorken suggested [20] that one considered not the inclusive cross section of hard processes such as Higgs boson production for example (see Fig. 13 ), but the cross section for the event with a very interesting signature, namely, such that no hadrons are produced with rapidities between  $y_1$  and  $y_2$ , except for the Higgs boson and hadrons from it's fragmentation. To obtain the formulae for the cross section of such an event, we need to multiply the expression for the inclusive cross section (see eq.(54) by the probability that the partons with rapidity larger than  $y_1$  do

not interact with partons that have rapidity less than  $y_2$ . Introducing the probability  $P(y_1 - y_2, S, b_t)$  that no inelastic interaction takes place at impact parameter  $b_t$  in the rapidity region  $y_1 - y_2$  at energy  $s$ , one can write the following formula for the survival probability of the rapidity gap

$$\langle |S^2| \rangle = \frac{\int a_{hard} P(y_1 - y_2, s, b_t) d^2 b_t}{\int a_{hard} d^2 b_t}, \quad (57)$$

where  $a_{hard}$  is the inclusive cross section (see eq.(54) and Fig. 12 ). To calculate  $\langle |S^2| \rangle$  one needs to estimate  $P(y_1 - y_2, s, b_t)$ .

1. *Eikonal Approximation.*

Fortunately it is easy to do in the eikonal approach.  $s$  - channel unitarity can be written in the diagonalised form, viz. in  $b_t$  - representation as

$$2 \text{Im} a(s, b_t) = |a(s, b_t)|^2 + G_{in}(s, b_t) \quad (58)$$

where

$$\sigma_{in} = \int d^2 b_t G_{in}(s, b_t). \quad (59)$$

In eikonal approach one can see that from eq.(20) follows that

$$G_{in}(s, b_t) = 1 - e^{-2\Omega(s, b_t)}. \quad (60)$$

From the above equation we can conclude that the factor

$$e^{-2\Omega(s, b_t)} \quad (61)$$

describes the probability  $P(s, b_t)$  which does not depend on  $y_1 - y_2$  within the eikonal approximation. Finally we can obtain the Bjorken formula [20] for  $\langle |S^2| \rangle$  substituting eq.(61) in the expression (57) for  $\langle |S^2| \rangle$ .

2. *General Approach.*

As has been discussed the eikonal approximation oversimplifies the structure of the parton cascade reducing the complicated parton cascade with a rich variety of different parton interactions to simple picture of Fig.5. Using the general equation (47) taking into account the interaction between Pomerons we are able to write down a more general expression for the survival probability of a large rapidity gap than eq.(61). We can also examine how important could be the parton interactions inside of the parton cascade for the value of the survival probability. To find the expression for  $\langle |S^2| \rangle$  we can use eq.(57) and a general expression for the opacity  $\Omega$  (see eq.

(53) ). However we need to take into account that only inelastic interaction due to the Pomeron exchanges and the Pomeron interactions in the rapidity region  $y_1 - y_2$  should be suppressed. The solution of this problem looks very simple and we can summarize the procedure of the solution as follows:

1. One solves eq. (47) with the initial condition of eq.(50). Thus we found the function  $\Psi(Y - y, x, b_t, b'_t)$ .

2. The next step is to find the solution of eq. (47)  $\Psi(y_1 - y)$  with the initial condition

$$\Psi(y_1 - y, x, b_t, b'_t) |_{y=y_1} = \Psi(Y - y_1, x, b_t, b'_t) . \quad (62)$$

3. A solution of eq. (47)  $\Psi(y, x, b'_t)$  should be found which satisfies the initial condition of eq. (50) but with  $g(b'_t)$  instead of  $g(b_t - b'_t)$ .

4. We specify function  $\Psi(y_1 - y, x, b_t, b'_t)$  extracting the value of  $x = x_m$  from the matching condition:

$$\Psi(Y - y_2, x_m, b_t, b'_t) = \Psi(y_2, x_m, b'_t) . \quad (63)$$

5. To calculate the opaqueness  $\Omega(y_1 - y_2, s, b_t)$  we need to substitute in eq. (53)  $\Psi(y_1 - y_2, x_m, b_t, b'_t)$ .

6. The equation (57) with the opaqueness  $\Omega(y_1 - y_2, s, b_t)$  allows us to calculate the survival probability in the general case.

The described general procedure could be specified using one of the previous explicit solutions ( see eqs. (27 ), (39) or (52) ). We do not want to discuss the application of the above solution since it is better to do together with some phenomenological estimates of the value of the vertices for pomeron - pomeron interaction (such as  $\gamma^+$ ,  $\gamma^-$  and  $\lambda$ ). It only should be stressed that the resulting formula turns out to be much more complicated than the eikonal formula. I hope to publish a close investigation of the role of the pomeron interaction elsewhere rather soon.

## 5 Conclusions.

Concluding the paper I would like to repeat once more that an attempt was made in this paper to regenerate the Reggeon Calculus as a way to take into account the pomeron - pomeron interaction to understand the origin and the main property of shadowing (screening ) corrections. The approach is based on two principle assumptions:

1. The only "hard" processes with the typical scale of the transverse momentum of the order of  $Q_0 \gg \mu$  such as  $\alpha_s(Q_0^2) \ll 1$  contribute to the structure of Pomeron.

2. We can introduce vertices for pomeron - pomeron interaction which are local in rapidity and in the impact parameter.

Both assumptions look very natural from experience in perturbative QCD calculations as well as from current experimental information. However we need a much more detailed study of the above assumptions. The main goal in this paper for me was to convince the reader that the calculation of the shadowing corrections could be formulated as a theoretical problem with a restricted number of assumptions that could be checked experimentally at least. I will be happy if somebody will find arguments against my approach since such a discussion will be able to move us toward deeper understanding of the problem.

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## Figure Captions.

- Fig. 1:* The Pomeron as QCD - ladder.
- Fig. 2:* Scattering amplitude with Pomeron exchange.
- Fig. 3:* Eikonal diagrams.
- Fig. 4:* The structure of the parton cascade.
- Fig. 5:* The parton structure of eikonal diagrams.
- Fig. 6:* "Fan" diagrams.
- Fig. 7:* Pomeron interactions.
- Fig. 8:* Graphical representation of the equation for the sum of "fan" diagrams.
- Fig. 9:* "Fan" diagrams in hadron - hadron collisions.
- Fig. 10:* Pomeron - Pomeron rescattering diagrams in hadron - hadron collisions.
- Fig. 11:* Diagrams for the general case of Pomeron interactions.
- Fig. 12:* Inclusive cross section.
- Fig. 13:* Higgs boson production (inclusive cross section).
- Fig. 14:* Higgs boson production (shadowing corrections).

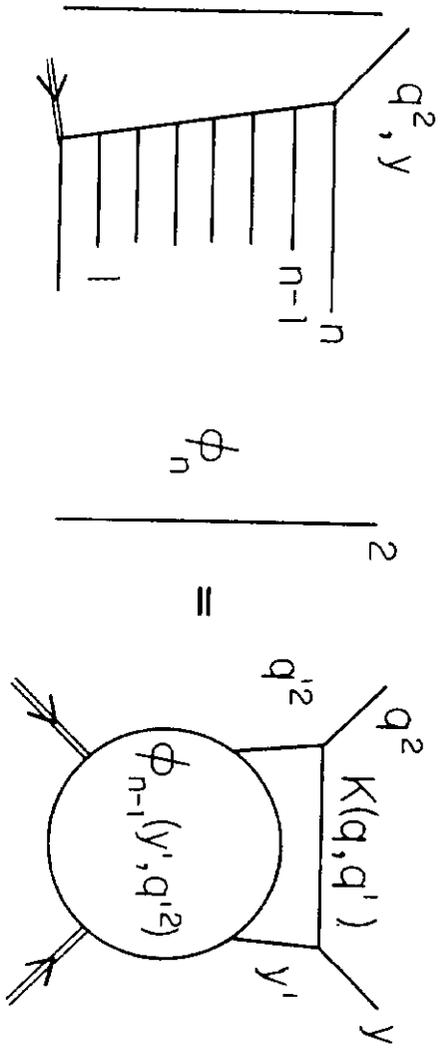


Fig. 1

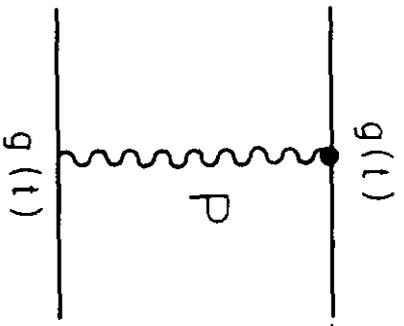


Fig. 2

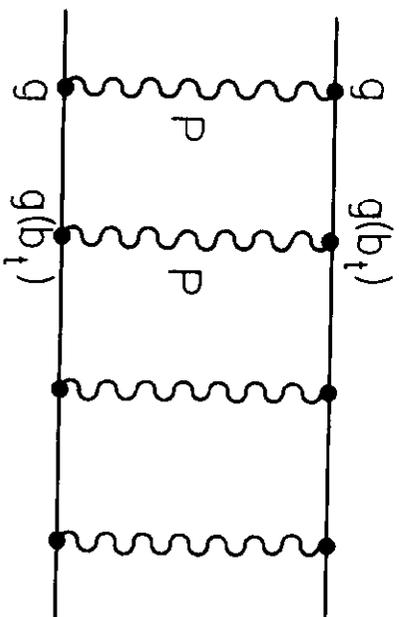
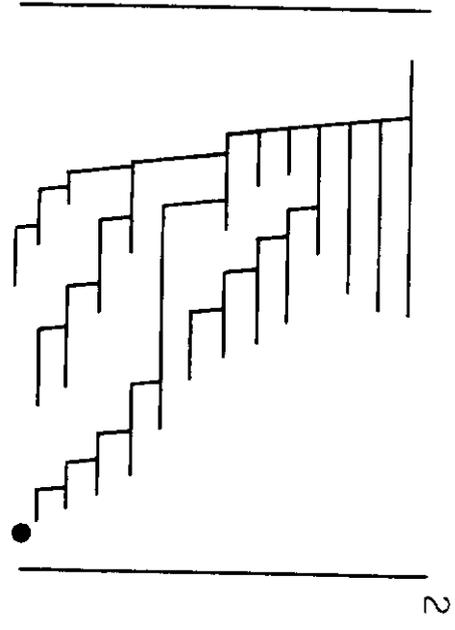


Fig. 3



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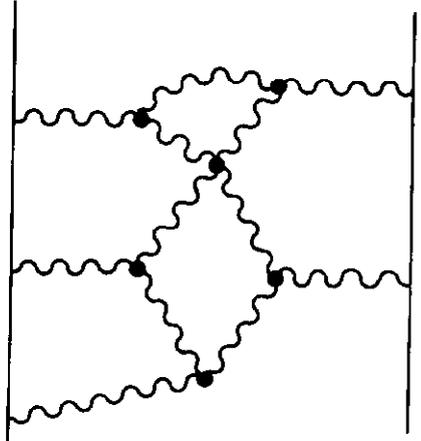


Fig. 4

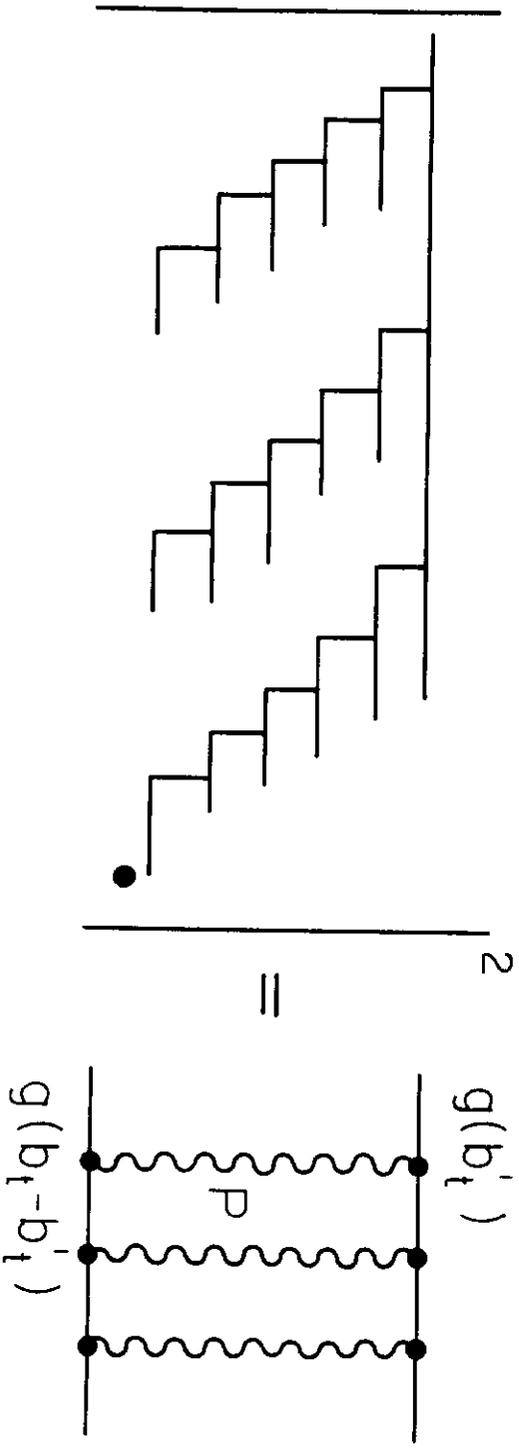


Fig. 5

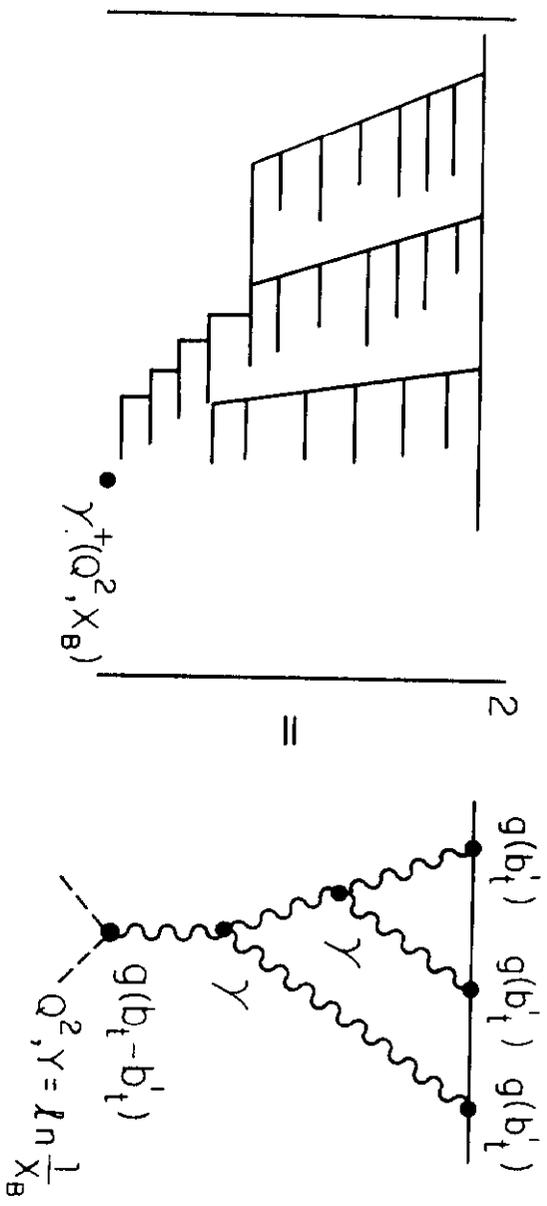


Fig. 6

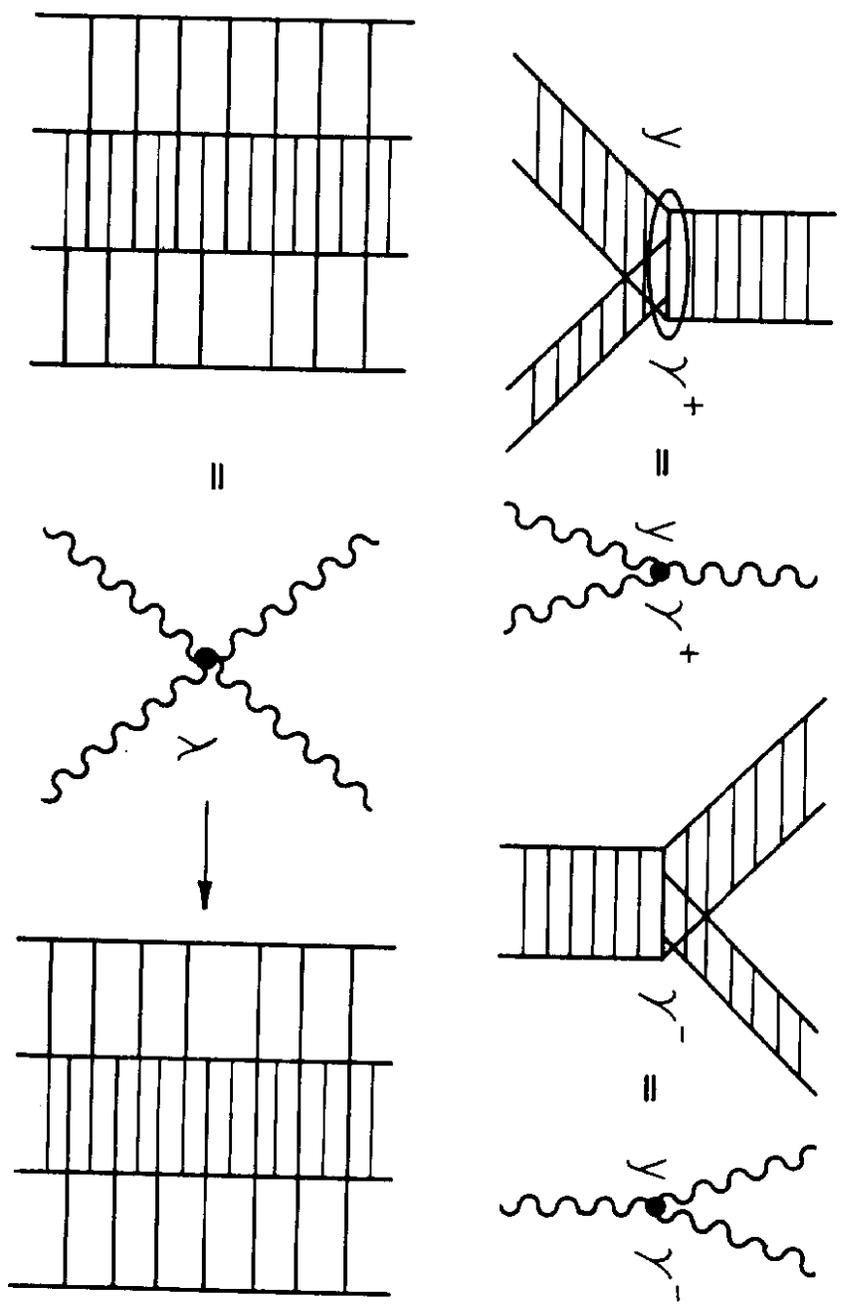


Fig. 7

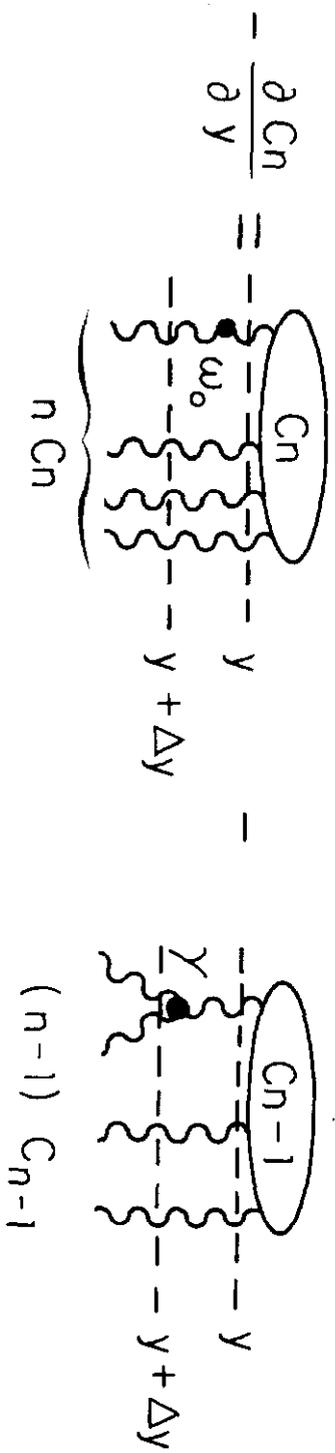


Fig. 8

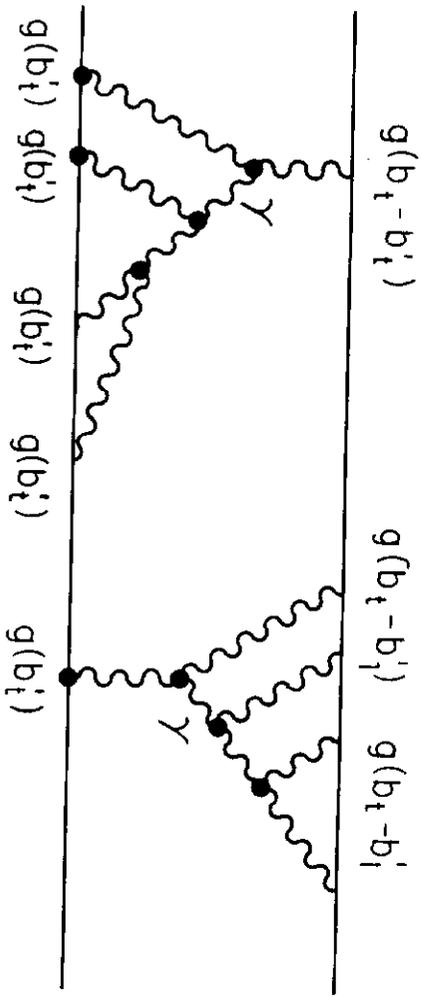


Fig. 9

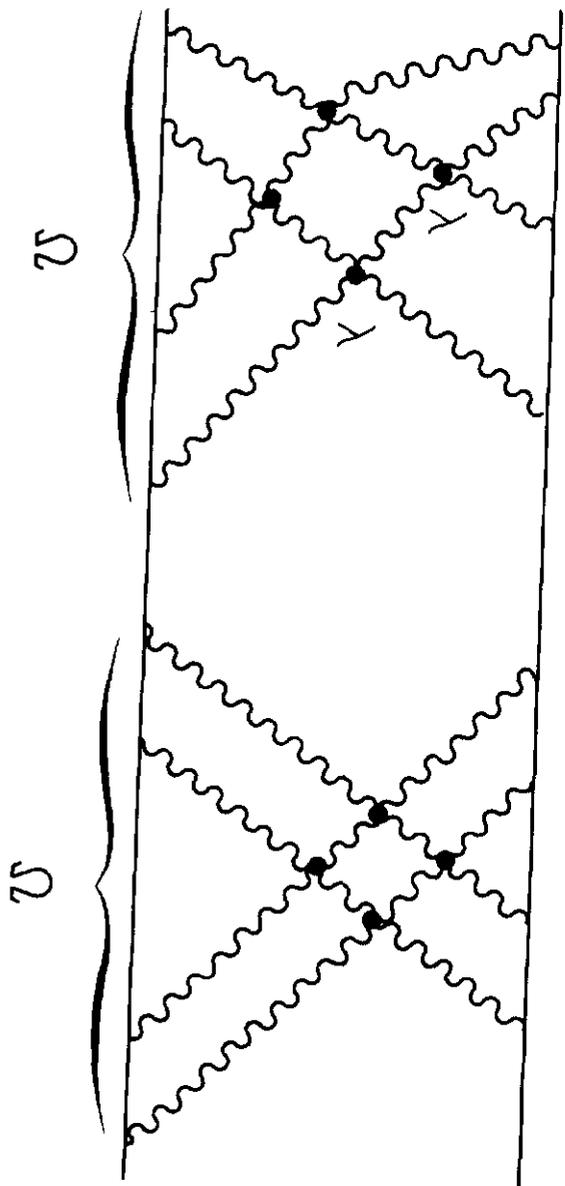


Fig. 10

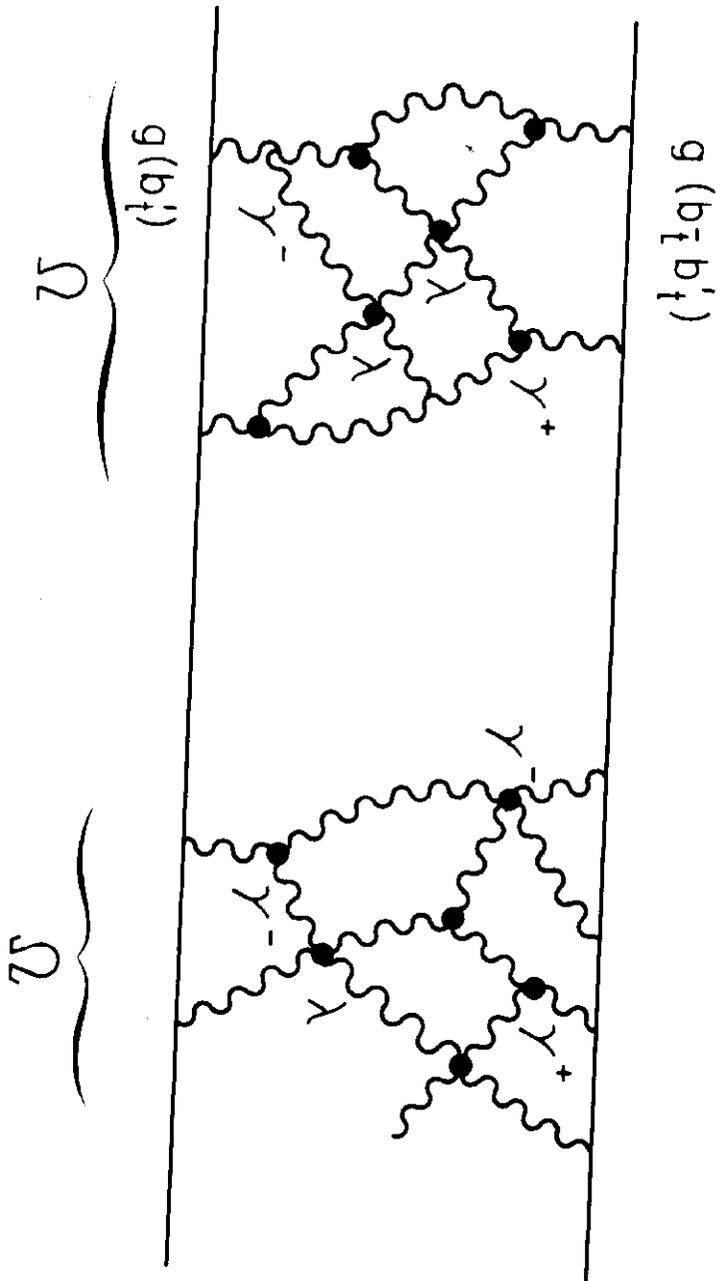


Fig. 11

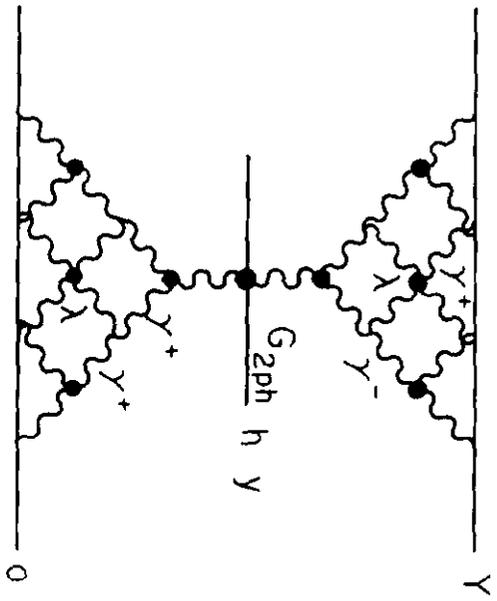


Fig. 12

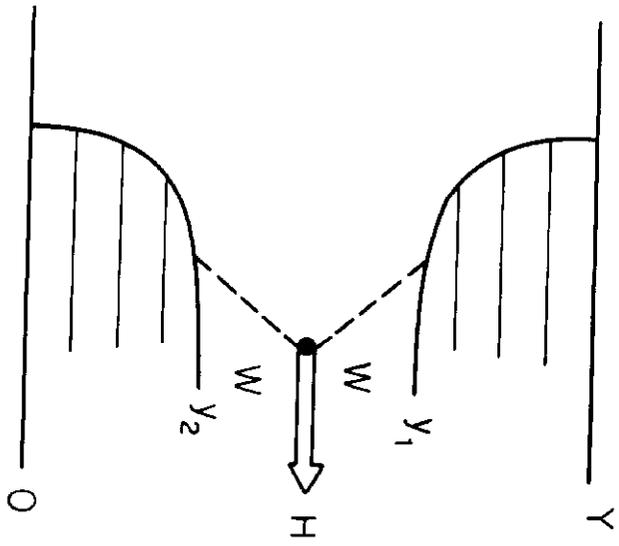


Fig. 13

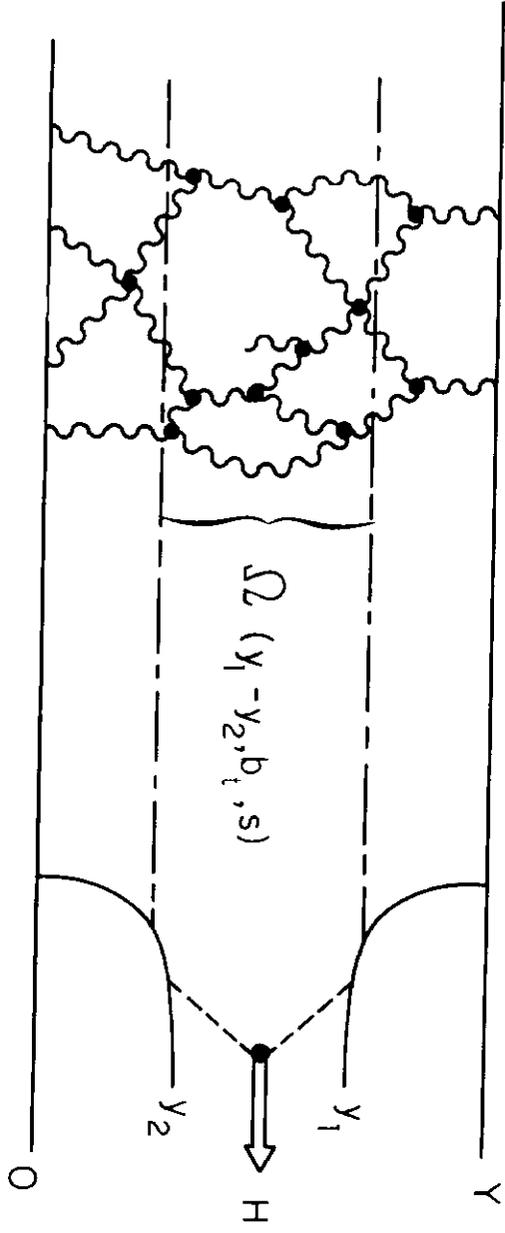


Fig. 14