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On The Interaction Between A Cosmic String And A Primordial Magnetic Field

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ABSTRACT

I study the interaction of a neutral, non-superconducting cosmic string with an external magnetic field. Two observations result from this study. First, a magnetic field of sufficient strength may enhance the size of wakes behind supersonic cosmic strings. Second, as an extension of the electrostatic "image charge" effect to magnetostatics, a body of constant magnetization near a cosmic string experiences a self-force due to the conical spacetime.

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I. Introduction

The role played by magnetic fields in cosmology is poorly understood. Furthermore, there exists no satisfactory explanation for the generation of the primordial magnetic field, which may be necessary to explain the the presence of intercluster and galactic magnetic fields today. (For a sample of the mechanisms proposed to generate a primordial field, see the recent review by Coles [1].) It would only be necessary to find a mechanism to generate a microscopic seed field [2], as there are several astrophysical processes by which such a microscopic field may be amplified to such magnitudes as are observed today in galaxies and the intercluster medium. It is generally agreed, however, that such a field could only be generated by some sort of exotic process in the early universe. Only exotic phenomena could provide the turbulent motion necessary to generate the vorticity which would lead to the development of a magnetic field in a conducting plasma (see for example the mechanisms described in [1,3,4]). It has been proposed that cosmic strings may be responsible for the magnetic fields [5,6]; turbulent motion in the wake behind a cosmic string moving through the conducting plasma may produce and sustain a magnetic field. However, to date only crude estimates of the process by which strings generate magnetic fields have been made.

In this paper, I shall examine the interaction of a large-scale magnetic field with a neutral, non-conducting cosmic string. The motivation for this investigation is that the behavior of a large-scale magnetic field in the conical spacetime about a cosmic string may have some significant observational consequences. The first observation made, in section II, will be that magnetic fields may enhance the size of wakes, which serve as the density fluctuations which evolve to form clusters and galaxies (see for example [7]). I will describe a naive analysis which would suggest that these wakes are easily enhanced by a weak, primordial field. I will then turn to a more rigorous analysis to show that the wakes are not easily enhanced by realistic, cosmological magnetic fields. The second observation made, in section III, will be that there is an "image" effect for bodies of constant magnetization near a cosmic string, and hence a magnetic self-interaction. It will be demonstrated that both effects, however, are of negligible cosmological consequence.

II. Magnetic Fields and Wakes

A magnetic field which is sustained by a quasi-neutral, conducting plasma may play a role in the behavior of the fluid near a neutral, non-conducting cosmic string. I will attempt to elucidate this behavior in this section.

It is instructive to first present a naive analysis which results in an *incorrect* description of the behavior of the magnetic field near a supersonic cosmic string. I would claim naively that when the tension in the magnetic field lines, $p_{\rm mag} = \frac{1}{8\pi}B^2 = \frac{1}{2}\rho_{\rm fluid}v^2_{\rm Alfven}$, is comparable to the inwards-directed pressure caused by the conical geometry of the string spacetime, $p_{\rm wake} = \frac{1}{2}\rho_{\rm fluid}v^2_{\rm wake}$, where $v_{\rm wake} \sim G\mu$, the magnetic field will prevent the infall of matter into the wake.

That is, when $p_{\rm mag} > p_{\rm wake}$, matter will not fall into the wake behind a cosmic string. If I only need to satisfy $v_{\rm Alfven} > v_{\rm wake}$, it would appear that a weak, primordial magnetic field may resist the collapsing matter. For the maximum strength primordial field consistent with observations, $v_{\rm Alfven} > v_{\rm wake}$ from the time of recombination until a redshift as late as $z \approx 10$ in the matter era. The notion of $p_{\rm wake}$, however, is unphysical. Perturbations propagate at the speed of sound $v_{\rm sound}$ in the fluid, rather than at the speed $v_{\rm wake}$. Therefore, it will not be until $v_{\rm Alfven} > v_{\rm sound}$ that the tension in the magnetic field will prevent the flow lines to converge.

The case in which a string passes through the plasma supersonically may be interesting when a magnetic field of sufficient strength is present, as I will show in this more rigorous, *correct* analysis.

It is a well-known result that a magnetic field may serve to increase the angular size of a shock [8,9,10], although the application of this effect to the wakes produced by a cosmic string is new. From the magnetohydrodynamic equations [11,12,13] it follows that the physical velocity of propagation of a disturbance in the plasma by the magnetic field is given by the Alfven speed

$$\vec{v}_{\rm Alfven} = \frac{\partial \omega}{\partial \vec{k}} = \frac{\vec{B}}{\sqrt{4\pi\rho}}. \qquad (II.1)$$

Thus, the presence of the magnetic field serves to increase the effective speed of sound along the direction of the magnetic field. A perturbation will propagate along a direction $\hat{n}$ with velocity

$$v^2_{\rm perturbation} = v^2_{\rm sound} + (\hat{n} \cdot \vec{v}_{\rm Alfven})^2. \qquad (II.2)$$

The characteristic surface, or Mach surface, on which the perturbation travels outward from its source is no longer a sphere, but an ellipsoid, prolate along the direction of the magnetic field. The Mach angle, the angular size of the shock or wake produced by a supersonic source, will then depend on the relative directions of $\hat{n}$ and $\vec{v}_{\rm Alfven}$.

I shall consider the scenario in which a cosmic string travels supersonically through a magnetic field $\vec{B}$, ordered on length scales $\lambda_{\rm mag}$, which is sustained by the cosmological fluid, an excellent conductor. For the case in which $\vec{v}_{\rm string} \parallel \vec{B}$, the wake angle $\theta_{\rm wake}$ is given by [8,9,10]

$$\sin^2 \theta_{\rm wake} = \frac{M^2 + A^2 - 1}{M^2 A^2} \qquad (II.3)$$

where $M = v_{\rm string}/v_{\rm sound}$ is the Mach number and $A = v_{\rm string}/v_{\rm Alfven}$ is the Alfven number. The result of the presence of the magnetic field, as can be seen from equation (II.3), is that for a fixed value of M, the angular size of the shock increases with increasing $v_{\rm Alfven}$, until the shock disappears. Alternatively, the effect is to lower the effective Mach number of the string in the fluid, the ratio between the string speed and the effective speed of sound, and thus widen the angular size of the shock. This is the first result of this paper.

Thinner, unenhanced wakes will form on scales larger than λ_{mag}, the characteristic length scale on which the magnetic field is oriented. It is possible to imagine that if a cosmic string were to travel on a path roughly parallel to the random walk magnetic field lines, a series of wide wakes would develop.

It will be useful to estimate the magnitude of this effect. Based on measurements of Faraday rotation of extragalactic sources, the strongest present-day magnetic field which is consistent with observations is $B \approx 10^{-10}G$ on length scales $\lambda_{mag} \approx 100kpc$ [14]. However, at no time is the corresponding Alfven speed comparable to the speed of sound or the string. Therefore, a realistic cosmological magnetic field is too weak to amplify the size of cosmic string wakes. Should a very strong magnetic field persist for a very short period before dissipating, this amplification of the wake size may be cosmologically interesting. It may serve to segregate the plasma from the dark matter, which does not experience the effect of the magnetic field. Still, there is no evidence to suggest that such a strong magnetic field on cosmological length scales may have endured for any amount of time during the era of galaxy formation.

III. Image Magnetic Fields

A body of uniform magnetization near a cosmic string will experience an attractive force toward the string due to the conical spacetime geometry about the string. This attractive force may be understood as the interaction of the body with a second, "image" body of uniform magnetization in the same way a charged particle experiences a repulsive force near a cosmic string [15,16,17,18]. I will give a more detailed study of this effect in this section.

The magnetostatics of a magnetic body may be constructed in analogy to the electrostatics of a charge distribution. A magnetic body contributes an effective current density to the medium [19] $\vec{J}_M = \vec{\nabla} \times \vec{M}$. Then, defining $\vec{H} = \vec{B} - 4\pi\vec{M} = -\vec{\nabla}\Phi_M$, Maxwell's equations yield

$$\nabla^2 \Phi_M = 4\pi \vec{\nabla} \cdot \vec{M} = -4\pi\rho_M. \qquad (III.1)$$

So, the electrostatic charge density has been replaced by the divergence of the magnetization. Then, given the Greens function for the Poisson equation, the magnetostatic potential is

$$\Phi(x) = -\int_V d^3x' \vec{\nabla}' \cdot \vec{M}' G(x;x') + \int_{\partial V} da' \hat{n}' \cdot \vec{M}' G(x;x'). \qquad (III.2)$$

The novel effects due to the cosmic string are to be included in the Greens function.

The Greens function for the Poisson equation in a cosmic string spacetime has been well studied by Linet and Smith [17,18] for the cases of electrostatics and Newtonian gravity. They found that the Greens function may be written as

$$G(x;x') = \frac{p}{\pi\sqrt{2r_< r_>}} \int_{u_o}^{\infty} du (\cosh u - \cosh u_o)^{-1/2} \frac{\sinh pu}{\cosh pu - \cos(\phi - \phi')}. \qquad (III.3)$$

Here, $\cosh u_o = (r_<^2 + r_>^2 + (z-z')^2)/2r_< r_>$ and $p = 1/(1 - 4G\mu/c^2)$. The effect of the angle deficit is to affect an "image" particle on the other side of the string. This Greens function may be applied to the magetostatic equation (III.2).

I will now examine the case of a body of uniform magnetization near a cosmic string. Consider a sphere of constant magnetization $\vec{M} = M_o \hat{z}$ a distance L from a cosmic string which lies along the $\hat{z}$ axis. To the interior of this sphere of radius a, $\vec{\nabla} \cdot \vec{M} = 0$, so the only contribution to the magnetostatic potential comes from

$$\Phi_M(x) = \int_{\partial V} da' \hat{n}' \cdot \vec{M} G(x;x'). \qquad (III.4)$$

While evaluation of this expression is very complicated, ultimately, the net result is that there appears a magnetic dipole $\vec{m} = \frac{4\pi}{3} a^3 \vec{M}$ at coordinates $(\rho, \theta, z) = (L, 0, 0)$ as well as an image dipole $\vec{m}_{image} = k(p)\vec{m}$ at $(L, \pi, 0)$. Here, the coordinate θ may take the values $\theta \epsilon [0, 2\pi]$, and $k(p)$ is the function given by Linet and Smith for the strength of the image as a function of the string's mass-per-unit-length. This is the second result of this paper.

It will be interesting to study the properties of this system of magnetic dipoles. For $G\mu \ll 1$, $k(p) \approx 4G\mu$. The potential energy of such a system of dipoles is

$$\begin{aligned} U &= \frac{\vec{m}_1 \cdot \vec{m}_2 - 3(\hat{n} \cdot \vec{m}_1)(\hat{n} \cdot \vec{m}_2)}{|\vec{x}_1 - \vec{x}_2|^3} \\ &= \frac{8\pi^2 G\mu}{9} {M_o}^2 \frac{a^6}{L^3} \end{aligned} \qquad (III.5)$$

where $\hat{n}$ is the unit vector pointing along the direction $\vec{x}_1 - \vec{x}_2$. The energy density of this system is approximately $\rho_{dipole} \approx G\mu {M_o}^2 (a/L)^6$. Due to the large exponent in the ratio of the two distance scales in the last expression, the magnitude of the energy density in the dipole system is suppressed for magnetic fields which do not cross the cosmic string, $a \ll L$. For realistic values of the magnetization, this energy density is negligible compared to the field and string energy densities.

Another case to consider is that of a large scale magnetic field generated in the wake of a cosmic string passing through a plasma. I will assume, for this simple example only, that the fluid motion behind the string is sufficiently turbulent to generate a magnetic field aligned parallel to the string (although the orientation doesn't really matter here), and which fills the entire wedge region behind the string. This can be simply understood as a thin wedge of uniform magnetization behind the cosmic string. There will be a dipole interation between the newly generated field and the image field on the other side of the string. The effect, as was pointed out in [20], of this interaction is to exert a drag force on the cosmic string. That is, the string does work by pulling or dragging along the plasma fluid through the magnetic interaction. The energy in such a system is approximately $U \approx (\alpha l)^2(v_s t)M_o{}^2 G\mu$ where αl is the size of the wiggle on the string, expressed as a fraction of the horizon radius, $\alpha \leq 10^{-3}$, and v_s is the velocity of the string. Comparing this string energy-loss mechanism with the power radiated in gravitational waves,

$$\frac{P_{\rm mag}}{P_{\rm grav}} \approx \frac{(\alpha l)^2(v_{\rm string}t)M_o{}^2 G\mu/t}{\gamma_{\rm grav}G\mu^2} \approx \alpha^2 v_{\rm string}\Omega_{\rm mag}/\gamma_{\rm grav}G\mu \ll 1. \qquad (III.6)$$

Here, $\gamma_{\rm grav} \approx 10^2$ is a dimensionless coefficient for the power radiated in gravitational waves. For realistic values of the magnetization, the energy loss for a cosmic string through the magnetic drag is much less efficient than through the emission of gravitational waves, which is similar to the results of [20] for the drag by a gravitational image.

IV. Conclusion

In this brief study of the novel interaction of a magnetic field with a neutral, non-conducting cosmic string, I have made several observations. First, a plasma which supports a magnetic field of sufficient strength may enhance the size of a wake behind a cosmic string. However, the required magnetic field strength is unrealistically large to be cosmologically interesting. My second observation, that a body of uniform magnetization near a cosmic string observes a second, image body, is also of negligible cosmological consequence. The cosmic string energy dissipated by a primordial magnetic field and its image is much less than the energy lost by the string through other mechanisms.

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