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$O(\alpha_S)$ Corrections to the Photon Structure Functions $F_2^\gamma(x, Q^2)$ and $F_L^\gamma(x, Q^2)$

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Abstract

We examine the QCD corrections to the structure functions $F_2^\gamma(x, Q^2)$ and $F_L^\gamma(x, Q^2)$ for a real photon target. The pointlike photon contributions from light and heavy quarks are computed through $O(\alpha_S)$. A parameterization of the hadronic (resolved) photon contribution is also included and a comparison is made with the present experimental data. We find that while the pointlike contributions are large for $F_2^\gamma(x, Q^2)$, they are the dominant part of $F_L^\gamma(x, Q^2)$. For $Q^2 > 50$ (GeV/c)² the charm component is as least as large as the light quark component.



Addenda & Errata

- Refs. [7] and [8] should be interchanged. Subsequently, to ref. [8] should be added 'P.Aurenche, P.Chiapetta, M.Fontannaz, J.Ph.Guillet, E.Pilon, preprint LPTHE-Orsay 92/13.'
- Addenda and errata for p.11, just above middle of page: replace '..literature ([5]-[8])' by '..literature ([5]-[7])'. Six lines further below; replace '..which is not available yet.' by '..which are available in [8]'. The next sentence, 'Notice....possible.', should be deleted. Six lines further below, after '..of the scale M^2 .' insert the sentence 'In spite of the fact that a NLL parametrization exists we will restrict our attention to the LL approximation, because it is sufficient for our purpose here and because the relatively poor quality of the data cannot distinguish between the LL and NLL parametrizations.'. In the next sentence, replace 'Because of the missing NLL and NNLL..' by 'Since we omit the NLL and the unknown NNLL.'
- Erratum for p.12, above middle of page: replace '..densities give a...'
by '..densities may give a..'
- Erratum for p.14, end of third paragraph: replace '..with the data.'
by '..with the data [8]'. In the next sentence, replace '..one has to include..' by '..the authors of ref. [8] have included..'
- Erratum for p.15, near bottom of page: replace '..densities, which are not yet available.'
by '..densities [8]'

1 Introduction

In the past two decades there has been considerable interest in the study of photon-photon interactions in electron-positron colliders. When one photon is virtual and the other one is almost real the analogy with deep-inelastic electron-nucleon scattering motivated the introduction of the corresponding structure functions $F_k^\gamma(x, Q^2)$ ($k = 2, L$) for the photon. It was originally observed by Witten [1] that both the x and Q^2 dependence of these structure functions were calculable in perturbative QCD (pQCD) at asymptotically large Q^2 . Thus from a theoretical point of view this process should provide a much more thorough test of pQCD than the corresponding deep-inelastic scattering off a nucleon target, where only the Q^2 evolution of the structure functions is calculable. The original optimism subsided once it was realized that there were complications with experimental confirmation of this prediction at experimental (non-asymptotic) values of Q^2 [2], [3]. For recent reviews see [4]. In particular at small Q^2 there is a contamination of the purely pointlike (unresolved) pQCD contribution by the hadronic (resolved) component of the photon. This latter piece is not calculable in pQCD and must be extracted from experimental data. One of the approaches used is to describe this hadronic piece by parton densities in the photon. For parameterizations see [5], [6], [7] and [8]. For a different approach see [9].

In this paper we will investigate higher-order pQCD corrections in the deep-inelastic structure functions containing both light (massless) quarks and heavy (massive) quarks. We report the results of including these pQCD corrections through order $\alpha\alpha_s$. These corrections have not been considered earlier in the literature. They are the Abelian analogues of the order α_s^2 corrections to light-and heavy-quark production in deep-inelastic lepton hadron scattering contributing to the structure functions $F_k(x, Q^2)$ ($k = 2, L$). For light quarks the non-Abelian results were reported in [10] while for heavy quarks they were reported in [11].

The deep-inelastic structure function $F_2^\gamma(x, Q^2)$ was originally measured by the PLUTO collaboration [12] at PETRA using single tag events in the reaction $e^- + e^+ \rightarrow e^- + e^+ + \text{hadrons}$. In the past few years there have been a series of new measurements at PETRA, PEP and TRISTAN by several groups, including CELLO [13], TPC2 γ [14], TASSO [15], JADE [16], AMY [17], VENUS [18] and TOPAZ [19]. All these groups concentrated on the measurement of the light-quark contribution to $F_2^\gamma(x, Q^2)$. The heavy-quark

component (mainly charm) has been hard to extract due to problems identifying charmed particle decays. In the near future higher-luminosity runs at TRISTAN should yield some information on heavy-quark (mainly charm) production and this is one reason that we study it here. We should mention that there was a previous investigation of pQCD corrections to heavy quark production in [20], where it was assumed that both photons were off-mass-shell and a small value for the photon virtuality was chosen for generating numerical results. Since these authors did not therefore encounter mass singularities they had no need to perform any mass factorization. Hence their method was different from the one we adopt. Finally there exists a possibility that the longitudinal structure function $F_L^\gamma(x, Q^2)$ can be measured at LEP [21]. Therefore we will also present the higher-order pQCD corrections to $F_L^\gamma(x, Q^2)$ which have not been reported previously.

Two-photon reactions are important to understand as background processes to the normal s -channel reactions at present and future e^-e^+ colliders. The latter machines will have a large amount of beamstrahlung [22], [23]. Therefore a basic input is the parton density in a photon, which is one of the topics we discuss.

The paper is organized as follows. In section 2 we present the pQCD corrections up to order α_s , which are used in our calculations. In section 3 we discuss the effects of the higher-order corrections to the structure functions $F_k^\gamma(x, Q^2)$ ($k = 2, L$) including the $O(\alpha_s)$ contributions from the light and heavy quarks.

2 Higher-Order Corrections to the Photon Structure Functions

The deep-inelastic photon structure functions denoted by $F_k^\gamma(x, Q^2)$ ($k = 2, L$) are measured in e^-e^+ collisions via the process (see fig.1)

$$e^-(p_e) + e^+ \rightarrow e^-(p'_e) + e^+ + X, \quad (2.1)$$

where X denotes any hadronic state which is allowed by quantum-number conservation laws. When the outgoing electron is tagged then the above reaction is dominated by the photon-photon collision reaction (see fig.1)

$$\gamma^*(q) + \gamma(k) \rightarrow X, \quad (2.2)$$

where one of the photons is highly virtual and the other one is almost on-mass-shell. The process (2.1) is described by the cross section

$$\frac{d^2\sigma}{dx dy} = \int dz z f_\gamma^e(z, \frac{S}{m_e^2}) \frac{2\pi\alpha^2 S}{Q^4} \left[\{1 + (1-y)^2\} F_2^\gamma(x, Q^2) - y^2 F_L^\gamma(x, Q^2) \right], \quad (2.3)$$

where $F_k^\gamma(x, Q^2)$ ($k = 2, L$) denote the deep-inelastic photon structure functions and $\alpha = e^2/4\pi$ is the fine structure constant. Furthermore the off-mass-shell photon and the on-mass-shell photon are indicated by the four-momenta q and k respectively with $q^2 = -Q^2 < 0$ and $k^2 \approx 0$. Because the photon with momentum k is almost on-mass-shell, expression (2.3) is written in the Weizsäcker-Williams approximation. In this approximation the function $f_\gamma^e(z, S/m_e^2)$ is the probability of finding a photon $\gamma(k)$ in the positron, (see fig.1). The fraction of the energy of the positron carried off by the photon is denoted by z while \sqrt{S} is the c.m. energy of the electron-positron system. The function $f_\gamma^e(z, S/m_e^2)$ is given by (see [24])

$$f_\gamma^e(z, \frac{S}{m_e^2}) = \frac{\alpha}{2\pi} \frac{1 + (1-z)^2}{z} \ln \frac{(1-z)(zS - 4m^2)}{z^2 m_e^2}, \quad (2.4)$$

provided a heavy quark with mass m is produced. The scaling variables x and y are defined by

$$x = \frac{Q^2}{2k \cdot q}, \quad y = \frac{k \cdot q}{k \cdot p_e}, \quad q = p_e - p'_e, \quad (2.5)$$

where p_e, p'_e are the momenta of the incoming and outgoing electron respectively. Following the procedure in [25] the photon structure functions in the QCD-improved parton model have the following form

$$\begin{aligned} \frac{1}{\alpha} F_k^\gamma(x, Q^2) = & x \int_z^1 \frac{dz}{z} \left[\left(\frac{1}{n_f} \sum_{i=1}^{n_f} e_i^2 \right) \left\{ \Sigma^\gamma\left(\frac{x}{z}, M^2\right) C_{k,q}^S\left(z, \frac{Q^2}{M^2}\right) \right. \right. \\ & \left. \left. + g^\gamma\left(\frac{x}{z}, M^2\right) C_{k,g}\left(z, \frac{Q^2}{M^2}\right) \right\} + \Delta^\gamma\left(\frac{x}{z}, M^2\right) C_{k,q}^{NS}\left(z, \frac{Q^2}{M^2}\right) \right] \\ & + \frac{3}{4\pi} x \left[\left(\sum_{i=1}^{n_f} e_i^4 \right) C_{k,\gamma}\left(x, \frac{Q^2}{M^2}\right) + e_H^4 C_{k,\gamma}^H\left(x, Q^2, m^2\right) \right]. \quad (2.6) \end{aligned}$$

Here Σ^γ and Δ^γ represent the singlet and non-singlet combinations of the parton densities in the photon respectively while the gluon density is represented by g^γ . The same notation also holds for the hadronic (Wilson) coefficient functions $C_{k,i}$ ($i = q, g$) where $C_{k,q}^S$ and $C_{k,q}^{NS}$ stand for the singlet and non-singlet coefficient functions respectively, and $C_{k,g}$ denotes the gluonic coefficient function. The photonic coefficient functions for massless and massive quarks are given by $C_{k,\gamma}$ and $C_{k,\gamma}^H$ respectively, where m in (2.6) denotes the heavy-quark mass. The index i in (2.6) runs over all light flavors provided they can be produced in the final state (n_f is the number of light flavors) and e_i, e_H stand for the charges of the light and heavy quarks respectively in units of e . The parton densities as well as the coefficient functions depend on the mass factorization scale M except for the $C_{k,\gamma}^H$ which can be calculated in pQCD without performing mass factorization. Notice that in addition to the mass factorization scale M the quantities in (2.6) also depend on the renormalization scale μ which appears in the pQCD corrections via $\alpha_s(\mu^2)$. However in this paper we will put $\mu = M$. Because of the origin of the photonic parton densities and the two different types of coefficient functions (photonic and hadronic) we will call the first term, represented by the integral, the hadronic (resolved) photon part, and the second term the pointlike (unresolved) photon part. The latter can be split into a light-quark contribution due to $C_{k,\gamma}$ and a heavy-quark contribution due to $C_{k,\gamma}^H$.

In the subsequent discussions in this paper we will neglect all pQCD corrections beyond the first order in α_s , so that we can put $C_{k,q}^S = C_{k,q}^{NS} = C_{k,q}$.

In this case (2.6) can be simplified as follows

$$\begin{aligned} \frac{1}{\alpha} F_k^\gamma(x, Q^2) &= x \int_x^1 \frac{dz}{z} \left[Q^\gamma\left(\frac{x}{z}, M^2\right) C_{k,q}\left(z, \frac{Q^2}{M^2}\right) \right. \\ &\quad \left. + \left(\frac{1}{n_f} \sum_{i=1}^{n_f} e_i^2 \right) g^\gamma\left(\frac{x}{z}, M^2\right) C_{k,g}\left(z, \frac{Q^2}{M^2}\right) \right] \\ &\quad + \frac{3}{4\pi} x \left[\left(\sum_{i=1}^{n_f} e_i^4 \right) C_{k,\gamma}\left(x, \frac{Q^2}{M^2}\right) + e_H^4 C_{k,\gamma}^H\left(x, Q^2, m^2\right) \right], \end{aligned} \quad (2.7)$$

with

$$\begin{aligned} Q^\gamma(z, M^2) &= \left(\frac{1}{n_f} \sum_{i=1}^{n_f} e_i^2 \right) \Sigma^\gamma(z, M^2) + \Delta^\gamma(z, M^2) \\ &= 2 \sum_{i=1}^{n_f} e_i^2 q_i^\gamma(z, M^2), \end{aligned} \quad (2.8)$$

where we have set $q_i^\gamma = \bar{q}_i^\gamma$ in the above equations. From now on we will only use (2.7) for our calculations, the results of which will be presented in the plots in Section 3.

Starting with the parton densities we will follow the prescription in [5] (where Σ^γ has the same meaning and $\Delta^\gamma = q_{NS}^\gamma$) and, in the case where all quarks are light, set

$$q_u^\gamma = q_c^\gamma = q_t^\gamma, \quad (2.9)$$

$$q_d^\gamma = q_s^\gamma = q_b^\gamma. \quad (2.10)$$

Below the charm-quark threshold we have

$$n_f = 3 : \quad Q^\gamma = \frac{8}{9} q_u^\gamma + \frac{4}{9} q_d^\gamma, \quad \sum_{i=1}^3 e_i^2 = \frac{2}{3}, \quad \sum_{i=1}^3 e_i^4 = \frac{2}{9}. \quad (2.11)$$

Above the charm-quark threshold and below the bottom-quark threshold the above quantities are changed into

$$n_f = 4 : \quad Q^\gamma = \frac{16}{9} q_u^\gamma + \frac{4}{9} q_d^\gamma, \quad \sum_{i=1}^4 e_i^2 = \frac{10}{9}, \quad \sum_{i=1}^4 e_i^4 = \frac{34}{81}. \quad (2.12)$$

Finally above the bottom-quark threshold they become

$$n_f = 5 : \quad Q^\gamma = \frac{16}{9} q_u^\gamma + \frac{2}{3} q_d^\gamma, \quad \sum_{i=1}^5 e_i^2 = \frac{11}{9}, \quad \sum_{i=1}^5 e_i^4 = \frac{35}{81}. \quad (2.13)$$

The coefficient functions originate from the following parton subprocesses. In the Born approximation we have the reaction (fig.2)

$$\gamma^*(q) + \gamma(k) \rightarrow q + \bar{q}, \quad (2.14)$$

where q (\bar{q}) stand for light as well as heavy (anti)-quarks. The $O(\alpha_s)$ pQCD corrections are given by the one-loop contributions to process (2.14) (see fig.3) and the gluon bremsstrahlung process (see fig.4)

$$\gamma^*(q) + \gamma(k) \rightarrow q + \bar{q} + g. \quad (2.15)$$

The parton cross section for the Born reaction (2.14) can be found in [3], [26] (light quarks) and [5], [25] (heavy quarks). Notice that the above reactions are very similar to the ones where the on-mass-shell photon $\gamma(k)$ is replaced by a gluon $g(k)$. The cross sections of the photon-induced processes constitute the Abelian parts of the expressions obtained for the gluon-induced processes which are presented up to order α_s^2 for the case of massless quarks in [10] and in the case of massive quarks in [11]. By equating some color factors equal to unity or zero in the latter expressions one automatically obtains the cross sections for the photon-induced processes above. In the case of massless quarks the parton cross sections for (2.14), (2.15) contain collinear divergences which can be attributed to the initial photon being on-mass-shell. These singularities are removed by mass factorization in the following way. We define

$$\hat{\mathcal{F}}_{k,\gamma}(z, Q^2, \epsilon) = \sum_i \int_0^1 dz_1 \int_0^1 dz_2 \delta(z - z_1 z_2) \Gamma_{i\gamma}(z_1, M^2, \epsilon) C_{k,i}(z_2, \frac{Q^2}{M^2}), \quad (2.16)$$

where $\hat{\mathcal{F}}_{k,\gamma}(z, Q^2, \epsilon)$ is the parton structure function, which is related to the parton cross section in the same way as the photon structure function $F_k^\gamma(x, Q^2)$ is related to the cross section $d^2\sigma/dxdy$ in (2.3). It contains the collinear divergences represented by the parameter $\epsilon = n - 4$ (we use dimensional regularization). These divergences are absorbed in the transition functions $\Gamma_{i\gamma}$ ($i = \gamma, q, g$) which depend both on ϵ and on the mass factorization scale M . The coefficient functions $C_{k,i}$ ($i = \gamma, q, g$) are computed in the $\overline{\text{MS}}$ scheme and they appear in the expressions for the photon structure functions in (2.6). In the case $i = \gamma$, where the photon is pointlike, the corresponding transition function is given by

$$\Gamma_{\gamma\gamma}(z, M^2, \epsilon) = \delta(1 - z). \quad (2.17)$$

The other transition functions $\Gamma_{i\gamma}$ ($i = q, g$) can be inferred from the Abelian parts of Γ_{ig} [3], [26], [27]. In the case of massive quarks (heavy-flavor production) the parton structure functions do not have mass singularities. They automatically belong to the pointlike photon contribution and can be identified with the coefficient functions $C_{k,\gamma}^H$.

The coefficient functions can be expanded in α_s as follows

$$C_{k,i} = C_{k,i}^{(0)} + \frac{\alpha_s(M^2)}{4\pi} C_{k,i}^{(1)} + \dots, \quad (2.18)$$

with $i = q, g, \gamma$ and

$$C_{k,\gamma}^H = C_{k,\gamma}^{H,(0)} + \frac{\alpha_s(M^2)}{4\pi} C_{k,\gamma}^{H,(1)} + \dots. \quad (2.19)$$

In zeroth order of α_s , the hadronic coefficient functions are

$$C_{2,q}^{(0)}\left(z, \frac{Q^2}{M^2}\right) = \delta(1-z), \quad (2.20)$$

$$C_{L,q}^{(0)}\left(z, \frac{Q^2}{M^2}\right) = 0, \quad (2.21)$$

$$C_{k,g}^{(0)}\left(z, \frac{Q^2}{M^2}\right) = 0, \quad (k = 2, L). \quad (2.22)$$

In order α_s the hadronic coefficient functions are given by

$$\begin{aligned} C_{2,q}^{(1)}\left(z, \frac{Q^2}{M^2}\right) &= C_F \left\{ \left[\left(\frac{4}{1-z} \right)_+ - 2 - 2z \right] \right. \\ &\quad \times \left\{ \ln \frac{Q^2}{M^2} + \ln(1-z) - \frac{3}{4} \right\} - 2 \frac{1+z^2}{1-z} \ln z + \frac{9}{2} + \frac{5}{2}z \\ &\quad \left. + \delta(1-z) \left\{ 3 \ln \frac{Q^2}{M^2} - 9 - 4\zeta(2) \right\} \right\}, \quad (2.23) \end{aligned}$$

and

$$C_{L,q}^{(1)}\left(z, \frac{Q^2}{M^2}\right) = C_F [4z]. \quad (2.24)$$

The above coefficient functions, which emerge via mass factorization from processes (2.14) and (2.15), (see figs.3 and 4), can be also inferred from the parton subprocesses $\gamma^* + q \rightarrow q$ (with one-loop corrections) and $\gamma^* + q \rightarrow q + g$ in deep-inelastic lepton-hadron scattering. The gluonic coefficient functions $C_{k,g}^{(1)}$ although of order α_s , emerge from the $O(\alpha_s^2)$ process $\gamma^*(q) + \gamma(k) \rightarrow q + \bar{q} + q + \bar{q}$. Nevertheless they contribute in order α_s , after mass factorization has been performed where the corresponding transition function $\Gamma_{g\gamma}$ leads to the scale dependence of the gluon density g^γ . The gluonic coefficient functions are given by

$$C_{2,g}^{(1)}\left(z, \frac{Q^2}{M^2}\right) = n_f T_f \left[4\{z^2 + (1-z)^2\} \left(\ln \frac{Q^2}{M^2} + \ln(1-z) - \ln z \right) + 32z(1-z) - 4 \right], \quad (2.25)$$

$$C_{L,g}^{(1)}\left(z, \frac{Q^2}{M^2}\right) = n_f T_f [16z(1-z)]. \quad (2.26)$$

Notice that the latter coefficient functions can be inferred from the parton subprocess $\gamma^* + g \rightarrow q + \bar{q}$ in deep-inelastic lepton-hadron scattering. The color factors appearing in eqs.(2.23)-(2.26) are given by $C_F = 4/3$ and $T_f = 1/2$ for the case of $SU(3)$.

The photonic coefficient functions in zeroth order of α_s for massless quarks, denoted by $C_{k,\gamma}^{(0)}(z, Q^2/M^2)$, originate from the Born reaction (2.14) (see (fig.2)). They can be derived from (2.25), (2.26) as follows

$$C_{k,\gamma}^{(0)}\left(z, \frac{Q^2}{M^2}\right) = \frac{1}{n_f T_f} C_{k,g}^{(1)}\left(z, \frac{Q^2}{M^2}\right), \quad (k = 2, L). \quad (2.27)$$

From the same reaction we also obtain the heavy-flavor contributions which read

$$C_{2,\gamma}^{H,(0)}(z, Q^2, m^2) = \left\{ \left\{ 4 - 8z(1-z) + \frac{16m^2}{Q^2} z(1-3z) - \frac{32m^4}{Q^4} z^2 \right\} L + \left\{ -4 + 32z(1-z) - 16 \frac{m^2}{Q^2} z(1-z) \right\} \sqrt{1 - \frac{4m^2}{s}} \right\}, \quad (2.28)$$

and

$$C_{L,\gamma}^{H,(0)}(z, Q^2, m^2) = 16z(1-z) \left[\sqrt{1 - \frac{4m^2}{s}} - 2\frac{m^2}{s}L \right], \quad (2.29)$$

where m is the heavy-flavor mass and \sqrt{s} is the c.m. energy of the virtual photon-real photon system. Furthermore we have

$$s = (1-z)\frac{Q^2}{z}, \quad L = \ln \left[\frac{1 + \sqrt{1 - 4m^2/s}}{1 - \sqrt{1 - 4m^2/s}} \right]. \quad (2.30)$$

These formulae were first derived in [28],[29].

In the next order in α_s , process (2.15) (fig.4) and the one-loop corrections to process (2.14) (fig.3) give rise to the coefficient functions $C_{k,\gamma}^{(1)}(z, Q^2/M^2)$ and $C_{k,\gamma}^{H,(1)}(z, Q^2, m^2)$. In $O(\alpha_s)$ the photonic coefficient functions for massless quarks $C_{k,\gamma}^{(1)}(z, Q^2/M^2)$ can be obtained from the Abelian parts of the $O(\alpha_s^2)$ contributions to the coefficient functions $C_{k,g}(z, Q^2/M^2)$ in [10] by applying the same relations as in (2.27). The corresponding heavy-quark coefficient functions $C_{k,\gamma}^{H,(0)}(z, Q^2, m^2)$ can be inferred from the Abelian parts of the $O(\alpha_s^2)$ contribution to $C_{k,g}(z, Q^2, m^2)$ computed for heavy-flavor production in [11]. Both expressions are too long to be put in this paper. ¹ We translate the notation used in this paper into those used in [3], [25] and [26] in Table 1, where we also list the new coefficient functions which were not used in the earlier calculations in the literature.

¹These functions are available from smith@elsebeth.physics.sunysb.edu.

3 Results

In this section we will first discuss the $O(\alpha_s)$ corrections to the hadronic (first) part and the corrections to the pointlike (second) part of the photon structure functions (2.7). In particular we focus our attention on the heavy-flavor contribution (mainly charm) which enters via the photonic part. In the literature attempts have been made to implement the higher-order QCD corrections in the photon structure function [26],[3]. As we have already pointed out above (2.6) we follow the prescription in [25] which is the same as normally given for the hadronic structure functions in deep-inelastic lepton-nucleon scattering [10]. In this case all the nonperturbative effects are hidden in the x -dependence of the parton densities $Q^\gamma(x, M^2)$ and $g^\gamma(x, M^2)$ (2.7). The perturbative parts are given by the splitting functions and the coefficient functions. The former appear in the Altarelli-Parisi (AP) equations which determine the M^2 -dependence of the parton densities. Various parameterizations of the parton densities are given in the literature ([5] - [8]). However they are all of the leading logarithmic (LL) type and a consistent Q^2 evolution of $F_k^\gamma(x, Q^2)$ can only be given when the densities are combined with the lowest order coefficient functions. In our case the latter are given by $C_{2,i}^{(0)}$, $C_{L,i}^{(1)}$, ($i = q, g$) and $C_{L,\gamma}^{(0)}$. Inclusion of the $O(\alpha_s)$ corrections to $C_{2,i}$ ($i = q, g$) and the contributions $C_{2,\gamma}^{(0)}$, $C_{L,\gamma}^{(1)}$ requires a next-to-leading-logarithmic (NLL) parameterization of the parton densities which is not available yet. Notice that the $O(\alpha\alpha_s)$ and $O(\alpha_s^2)$ contributions to the AP splitting functions are known so that the construction of the NLL-parameterization is in principle possible. If we also want to include $C_{2,\gamma}^{(1)}$ one even needs the $O(\alpha\alpha_s^2)$ and $O(\alpha_s^3)$ corrected AP splitting functions which have not been calculated in the literature. These would yield the next-to-next-to-leading-logarithmic approximation (NNLL). Notice that this problem does not exist for the heavy-flavor contributions represented by $C_{k,\gamma}^{H,(l)}$ ($l = 0, 1$) because the latter could be calculated without carrying out mass factorization, demonstrating that they are independent of the scale M^2 . Because of the missing NLL and NNLL parton densities the higher-order pQCD corrections to the coefficient functions have to be considered as an estimate of how the LL approximation to the photon structure function will be altered by including higher-order pQCD effects.

At this moment the LL parton densities give a good description of the data obtained for the structure function $F_2^\gamma(x, Q^2)$ over a wide range of Q^2

values (see below). Inclusion of the higher-order QCD corrections leads to a modification of the nonperturbative parameters describing the x -dependence of the parton densities. Another effect is that the Q^2 -dependence of the photon structure function will be altered when higher-order corrections are included particularly at large Q^2 -values. However the analysis in [25] reveals that the addition of the $O(\alpha_s)$ corrections $C_{2,i}^{(1)}$ ($i = q, g$), the pointlike photon contributions $C_{2,\gamma}^{(0)}$ and the two-loop AP splitting functions to F_2^γ hardly changes the Q^2 -evolution in the region $5.9 (\text{GeV}/c)^2 < Q^2 < 110 (\text{GeV}/c)^2$ accessible to past and present experiments. This led to the conclusion [25] that the LL parameterization for the parton densities is quite adequate to describe the existing data. In our opinion this analysis has two drawbacks, which can be summarized as follows. In spite of the fact that the LL as well as the NLL parton densities give a good description of F_2^γ it does not mean that they will provide us with the same good description for F_L^γ since the coefficient functions for these two structure functions in (2.7) are different. The same conclusion holds for other photon induced processes like e.g. photoproduction of heavy flavors [24]. Another objection is that in the determination of the LL parton densities the heavy-flavor contribution which shows up via $C_{k,\gamma}^H$ in (2.6), is neglected. This might be correct for low Q^2 in view of the limited statistics of the available data but is certainly incorrect for large Q^2 as we will see later on.

Besides the theoretical uncertainties one also has to deal with the quality of the experimental data. At this moment only data for $F_2^\gamma(x, Q^2)$ are known because of the experimental limitation $xy^2 \ll 1$. The available data have been obtained from various experiments where $0.03 < x < 0.8$ and $1.31 < Q^2 < 390 (\text{GeV}/c)^2$ [12] - [19]. However there exists some hope that at LEP $F_L^\gamma(x, Q^2)$ can also be measured [21].

In the subsequent part of this paper we first discuss how the LL description for F_2^γ is modified by the following corrections. They are given by:

- I . The $O(\alpha_s)$ contributions to the hadronic coefficient functions given by $C_{2,i}^{(1)}$ ($i = q, g$).
- II . The photonic coefficient functions due to light quarks $C_{2,\gamma}^{(l)}$ ($l = 0, 1$).
- III . The photonic coefficient functions due to heavy-flavor contributions (mainly charm) represented by $C_{2,\gamma}^{H,(l)}$ ($l = 0, 1$).

In our calculations we will adopt the two-loop corrected running coupling constant as presented in Eq.(10) of [30]. Further we choose $n_f = 4$ in the running coupling constant, which implies $\Lambda_{\overline{MS}} = 0.26 \text{ GeV}/c$. For the LL parton densities we take the DG parameterization given in [5], where $n_f = 3$ (see (2.11)). The latter is also used in the expressions in Section 2. The factorization scale is $M = \sqrt{Q^2}$.

In fig.5 we plot $F_2^\gamma(x, Q^2)/\alpha$ for $Q^2 = 5.9 \text{ (GeV}/c)^2$ as a function of x , where we compare our results with the data of PLUTO [12]. The curves represent the following contributions. The solid line is given by the LL-contribution to F_2^γ which we denote by $F_2^\gamma(LL)$. The dashed line originates from $F_2^\gamma(LL)$ plus the $O(\alpha_s)$ contributions due to I above. The latter constitutes the hadronic part of the structure function, which we will define as $\text{hadronic} = F_2^\gamma(LL) + O(\alpha_s)$. If we also include the pointlike photon part due to II, called light, we get the dotted line indicated by $\text{hadronic} + \text{light}$. Finally we add the pointlike photon part coming from the charmed quark (called heavy) corrected up to $O(\alpha_s)$ due to III so that the total result represented by the dotted-dashed line is given by $\text{hadronic} + \text{light} + \text{heavy}$. The figure reveals an appreciable deviation from the LL prescription when we include higher-order corrections, in particular in the higher x -region. The main changes are introduced by the hadronic contribution due to $C_{2,q}^{(1)}$ and the pointlike photon contribution due to $C_{2,\gamma}^{(0)}$. The steep rise of the dashed line near $x = 1$ can be attributed to soft-gluon radiation which leads to the logarithmic terms $(\ln^l(1-z)/(1-z))_+$ ($l = 0, 1$) in $C_{2,q}^{(1)}$ (2.23). The logarithmic term $\ln(1-z)$ occurring in $C_{2,\gamma}^{(0)}$ (2.27) (see also (2.25)) is responsible for the large negative correction near $x = 1$ and even causes F_2^γ to become negative in that region. The other contributions are unobservable at least within the errors in the present data. The gluonic part due to $C_{2,g}^{(1)}$ (2.25) is negligible and the charm contribution $C_{2,\gamma}^H$ is small even if one includes the $O(\alpha_s)$ corrections. Finally it turns out (see fig.6) that, for $x > 0.5$, the negative contribution due to $C_{2,\gamma}^{(0)}$ is partially compensated by adding the $O(\alpha_s)$ corrections represented by $C_{2,\gamma}^{(1)}$. In fig.7 we have compared the charm contribution to F_2^γ with $F_2^\gamma(LL)$. We see that in the region $0 < x < 0.4$, where charm contributes, it only constitutes about 20% of $F_2^\gamma(LL)$. If one includes the $O(\alpha_s)$ correction this becomes much larger and makes up approximately 30% of $F_2^\gamma(LL)$.

In figs.8-10 we have made the same plots as in figs.5-7 but now for $Q^2 = 51$

$(\text{GeV}/c)^2$, where there is data from the AMY experiment [17]. The figures reveal that the light-quark contribution becomes smaller (fig.9) whereas the charm contribution (fig.10) becomes larger with respect to the hadronic part of F_2^γ when Q^2 increases. At $Q^2 = 51 (\text{GeV}/c)^2$ the charm contribution constitutes 50% of $F_2^\gamma(LL)$ which is hardly altered when one includes the $O(\alpha_s)$ correction. The charm component is also larger than the light-quark part by about a factor of two (compare fig.9 with fig.10). If one could measure the charm component cleanly it would provide a good test of pQCD since one does not need to perform mass factorization to calculate the corrections to this contribution.

We have also computed the bottom-quark contribution but it turns out that this is negligible at this Q^2 so that we did not plot it in the figures. The origin of the suppression of the bottom-quark component can be attributed to the mass as well as the charge of the bottom quark. The former is three times larger than the mass of the charmed quark so there is a phase space suppression. Moreover e_H^4 for the bottom quark contributes a factor of 1/16 when compared with the corresponding e_H^4 factor for the charmed quark. We also studied the plots for $Q^2 = 100 (\text{GeV}/c)^2$, however they did not provide us with additional useful information when they are compared with those obtained for $Q^2 = 51 (\text{GeV}/c)^2$.

Summarizing the above results we conclude that the light-quark and heavy-quark contributions to the pointlike photon part change appreciably the leading logarithmic (LL) description of the photon structure function F_2^γ . This means that the nonperturbative parameters appearing in the existing parton densities like DG in [5] have to be refitted in order to bring the theoretical description of F_2^γ (2.7) in agreement with the data. Notice that besides the coefficient functions one also has to include the higher-order AP splitting functions into this analysis.

Bearing in mind the uncertainties above concerning the leading logarithmic parameterizations of the parton densities we will give a prediction for the longitudinal structure function F_L^γ . For $Q^2 = 5.9 (\text{GeV}/c)^2$ we have plotted the hadronic and pointlike photon parts of F_L^γ in fig.11 where the pointlike photon part is again split up in its light-and heavy-quark (charm) components. Notice that up to order α_s the LL description of F_L^γ coincides with the $O(\alpha_s)$ corrected hadronic part since the longitudinal coefficient functions $C_{L,i}$ ($i = q, g$) are only calculated up to this order. A glance at fig.11 shows that the hadronic part is heavily suppressed with respect to the light-quark

contribution to the pointlike photon part, which is due to $C_{L,\gamma}^{(0)}$ (2.27). The latter will be slightly reduced in the region $x > 0.3$ when one includes the $O(\alpha_s)$ correction $C_{L,\gamma}^{(1)}$ (fig.12). The charm contribution is appreciable in the region $x < 0.3$ in particular when one includes the $O(\alpha_s)$ correction (fig.13). The latter increases the Born approximation for charm production by a factor of two. The reason the hadronic part is suppressed for F_2^γ but not for F_L^γ can be traced back to the differences in the coefficient functions $C_{2,q}$ and $C_{L,q}$. The former starts already in order α_s^0 whereas the latter starts in order α_s . This has to be compared with the photon coefficient functions $C_{k,\gamma}^{(0)}$ ($k = 2, L$), which are both of order α_s^0 , so that $C_{L,q}^{(1)}$ is suppressed by order α_s with respect to the other coefficient functions. The second reason is that $C_{2,q}^{(1)}$ gets large contributions from soft-gluon radiation which are absent in $C_{L,q}^{(1)}$. Notice that the gluon density is less important in the region $0.1 \leq x \leq 1$. Moreover it is convoluted with the coefficient functions $C_{2,g}^{(1)}$ (2.25) and $C_{L,g}^{(1)}$ (2.26) which are both of order α_s and do not contain any soft-gluon enhancements.

If we study F_L^γ at $Q^2 = 51 \text{ (GeV/c)}^2$ (see fig.14) we observe that the ratio between the light-quark contribution and the hadronic component is unaltered with respect to $Q^2 = 5.9 \text{ (GeV/c)}^2$ (see also fig.15). However at $Q^2 = 51 \text{ (GeV/c)}^2$ the charm contribution is of the same size as the light-quark part and becomes much larger than the hadronic component of F_L^γ (fig.16). The reason that the charm and light-quark contributions have the same magnitude can be attributed to the fact that for $Q^2 \gg m_c^2$ the coefficient functions $C_{L,\gamma}^{(0)}$ (2.27) and $C_{L,\gamma}^{H,(0)}$ (2.29) become equal. From fig.16 we also infer that the charm component is increased by about 30% when the $O(\alpha_s)$ contribution $C_{L,\gamma}^{(1)}$ is included. We also computed the bottom-quark contribution to F_L^γ . Like in the case of F_2^γ it turned out that the bottom-quark component is negligible so that it is not shown in the figures. Likewise we did not show the figures for $Q^2 = 100 \text{ (GeV/c)}^2$ since they were not qualitatively different from the ones for $Q^2 = 51 \text{ (GeV/c)}^2$.

A comparison between the plots made for F_2^γ and F_L^γ reveals that the hadronic part of F_L^γ is heavily suppressed with respect to the pointlike photon part contrary to what is observed for F_2^γ . We do not expect that this feature will be altered if we had used the NLL parton densities, which are not yet available. Therefore the longitudinal structure function F_L^γ provides us with a much better test of pQCD than F_2^γ . Originally this feature was expected for F_2^γ [1]. Unfortunately this expectation did not materialize be-

cause of the large hadronic component in F_2^γ . The problem is now left to the experimentalists who have to try to extract the longitudinal structure function from the data via the cross section in (2.3).

Summarizing our findings we have seen that the pointlike photon contribution leads to an appreciable correction to the leading logarithmic description of F_2^γ , in particular in the large x region. Furthermore the pointlike photon component dominates the longitudinal structure function F_L^γ and overwhelms the hadronic part completely. The charm-quark contribution, which is relatively small compared with the light-quark contribution at small Q^2 , becomes of the same magnitude as the latter as Q^2 increases (see the plots for $Q^2 = 51 \text{ (GeV/c)}^2$). This feature is characteristic for both structure functions. Therefore the measurement of the charm contribution alone would provide a clean test of pQCD. As far as the $O(\alpha_s)$ contributions to the pointlike photon part are concerned we observe that they are appreciable at small Q^2 but become less important relative to the $O(\alpha)$ contributions when Q^2 becomes larger. This statement holds for both the light-quark and the heavy-quark contributions.

Acknowledgements

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Table 1.

this paper	[26]	[3], [25]*
$C_{2,g}^{(1)}$ (2.23)	$B_{NS}^{(n)}, B_{\psi}^{(n)}$ (4.10)	B_{NS}, B_q (3.7)
$C_{2,g}^{(1)}$ (2.25)	$B_G^{(n)}$ (4.11)	B_G (3.7)
$C_{2,\gamma}^{(0)}$ (2.27)	$B_{\gamma}^{(n)}$ (4.12)	B_{γ} (3.7)
$\frac{3\alpha_s}{4\pi} \left(\frac{2}{3}\right)^4 C_{2,\gamma}^{H,(0)}$ (2.28)	—	$\frac{1}{x} F_{2,c}^{\gamma}$ (2.13)*
$C_{L,g}^{(1)}$ (2.24)	—	—
$C_{L,g}^{(1)}$ (2.26)	—	—
$C_{L,\gamma}^{(0)}$ (2.27)	—	—
$C_{L,\gamma}^{H,(0)}$ (2.29)	—	—
$C_{k,\gamma}^{(1)}$ [10]	—	—
$C_{k,\gamma}^{H,(1)}$ [11]	—	—

Notations in several papers for the hadronic and photonic coefficient functions. Notice that the expressions in [26] are in Mellin transform space. The blanks mean that these contributions were not considered in the papers quoted.

References

- [1] E. Witten, Nucl. Phys. B120,189 (1977).
- [2] W.A. Bardeen in *Proceedings of the 1981 International Symposium on Lepton and Photon Interactions at High Energies, Bonn*, edited by W. Pfeil (Physikalisches Institut, Universität Bonn, Bonn 1981), p 432.
- [3] M. Glück and E. Reya, Phys. Rev. D28, 2749 (1983).
- [4] J.H. Field in *VIII International Workshop on Photon-Photon Collisions, Jerusalem, April (1988)*, World Scientific, (1989) p.349; S. Kawabata, in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics*, eds S. Hegarty, K. Potter, and E. Quercigh, World Scientific, (1991) p.53.
- [5] M. Drees and K. Grassie, Z. Phys. C28,451 (1985).
- [6] H. Abramowicz, K. Charchula and A. Levy, Phys. Lett. B269, 458 (1991); see also H. Abramowicz et al., DESY 91-057.
- [7] M. Glück, E. Reya and A. Vogt, Phys. Rev. D45, 3986 (1992); *ibid* D46, 1973 (1992).
- [8] L.E. Gordon and J.K. Storrow, M/C.Th.91/29.
- [9] J.H. Field, F. Kapusta and L. Poggioli, Z. Phys. C36, 121 (1987); Phys. Lett. 181, 362 (1986); Z. Phys. C51, 241 (1991).
- [10] E.B. Zijlstra and W.L. van Neerven, Nucl. Phys. B383,525 (1992); Phys. Lett. B273,476 (1991); W.L. van Neerven and E.B. Zijlstra, Phys.Lett. B272, 127 (1991).
- [11] E. Laenen, S. Riemersma, J. Smith and W.L. van Neerven, ITP-SB-92-09; ITP-SB-92-41 (to be published in Nucl. Phys.); Phys. Lett. B291, 325 (1992).
- [12] Ch. Berger *et al.*, (PLUTO Collaboration), Phys. Lett. B107, 168 (1981), *ibid* B142, 111 (1984), *ibid* B149, 421 (1984), Z. Phys C26, 353 (1984), Nucl. Phys. B281, 365 (1987).

- [13] H.J. Behrend *et al.*, (CELLO Collaboration), Phys. Lett. B126, 391 (1983); *XXV International Conf. on High Energy Physics* Singapore, (1990);
- [14] A. Aihara *et al.*, (TPC2 γ Collaboration) Phys. Rev. Lett. 58, 97 (1987); Z. Phys C34, 1 (1987); D. Bintinger *et al.*, Phys. Rev. Lett. 54, 763 (1985).
- [15] M. Althoff *et al.*, (TASSO Collaboration), Z. Phys. C31, 527 (1986);
- [16] W. Bartel *et al.*, (JADE Collaboration) Z. Phys. C24, 231 (1984).
- [17] R. Tanaka (AMY Collaboration) in *IX International Workshop on Photon-Photon Collisions*, March 22-26, 1992, University of California, San Diego, California. KEK preprint 92-37; T. Tanaka *et al.*, Phys.Lett. B277, 215 (1992); T. Sasaki *et al.*, Phys.Lett. B252, 491 (1990); T. Nozaki in *Proceedings of the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics*, eds S. Hegarty, K. Potter, and E. Quercigh, World Scientific, (1991) p.156.
- [18] M. Chiba (VENUS Collaboration) in *Proceedings of the XXVth Rencontres de Moriond (High Energy Hadronic Interactions)* Les Arcs, March, 1991.
- [19] H. Hayashii (TOPAZ collaboration) in *IX International Workshop on Photon-Photon Collisions*, March 22-26, 1992, University of California, San Diego, California.
- [20] C.T. Hill and G.G Ross, Nucl. Phys. B148, 373 (1979).
- [21] A. Ali *et al.*, *Physics at LEP* edited by J. Ellis and R. Peccei, CERN-86-02 Geneva (1986) vol.2, p 81.
- [22] M. Drees and R.M. Godbole, DESY 92-044; Phys. Rev. Lett. 67, 1189 (1991).
- [23] O.J.P. Éboli, M.C. Gonzalez-Garcia, F. Halzen and S.F. Novaes, MAD/PH/701.
- [24] J. Smith and W.L. van Neerven, Nucl.Phys. B374,36 (1992).

- [25] M. Glück, K. Grassie and E. Reya, Phys. Rev. D30, 1447 (1984).
- [26] W.A. Bardeen and A.J. Buras, Phys. Rev. D20, 166 (1979).
- [27] M. Fontannaz and E. Pilon, Phys. Rev. D45, 382 (1992).
- [28] E. Witten, Nucl.Phys. B104,445 (1976); J. Babcock and D. Sivers, Phys.Rev. D18,2301 (1978); M.A. Shifman, A.I. Vainstein and V.J. Zakharov, Nucl.Phys.B136, 157 (1978); M. Glück and E. Reya, Phys.Lett. 83B, 98 (1979); J.V. Leveille and T. Weiler, Nucl.Phys. B147, 147 (1979).
- [29] M. Glück in *Proceedings of the HERA Workshop*, Hamburg. Oct. 1987 ed. R.D. Peccei, vol. 1, p.119; M. Glück, R.M. Godbole and E.Reya, Z.Phys. C38, 441 (1988).
- [30] G. Altarelli, M. Diemoz, G. Martinelli and P. Nason, Nucl. Phys. B308, 724 (1988).

Figure Captions

- Fig.1.** The process $e^-(p_e) + e^+ \rightarrow e^-(p'_e) + e^+ + X$, where X denotes any hadronic state.
- Fig.2.** The lowest order Feynman diagrams contributing to the Born reaction $\gamma^*(q) + \gamma(k) \rightarrow q + \bar{q}$.
- Fig.3.** Feynman diagrams contributing to the one-loop correction to the process $\gamma^*(q) + \gamma(k) \rightarrow q + \bar{q}$. Additional graphs are obtained by reversing the arrows on the quark lines. Graphs containing the external quark self-energies are included in the calculation but not shown in the figure.
- Fig.4.** The order g ($\alpha_s = g^2/4\pi$) Feynman diagrams contributing to the gluon bremsstrahlung process $\gamma^*(q) + \gamma(k) \rightarrow q + \bar{q} + g$. Additional graphs are obtained by reversing the arrows on the quark lines.
- Fig.5.** Corrections to the leading logarithmic (LL) representation of $F_2^\gamma(x, Q^2)$ at $Q^2 = 5.9$ (GeV/c)² (using the DG parameterization), solid line: $F_2^\gamma(LL)$, dashed line: hadronic, dotted line: hadronic + light, dotted-dashed line: hadronic + light + heavy (heavy = charm). For an explanation of hadronic, light, and heavy see the text. The data are from PLUTO [12].
- Fig.6.** The light-quark contributions to $F_2^\gamma(x, Q^2)$ compared with $F_2^\gamma(LL)$ at $Q^2 = 5.9$ (GeV/c)², solid line: $F_2^\gamma(LL)$, long-dashed line: Born contribution ($C_{2,\gamma}^{(0)}$), short-dashed line: $O(\alpha_S)$ corrected contribution ($C_{2,\gamma}^{(0)} + C_{2,\gamma}^{(1)}$). The data are from PLUTO [12].
- Fig.7.** The charm-quark contributions to $F_2^\gamma(x, Q^2)$ compared with $F_2^\gamma(LL)$ at $Q^2 = 5.9$ (GeV/c)², solid line: $F_2^\gamma(LL)$, long-dashed line: Born contribution ($C_{2,\gamma}^{H,(0)}$), short-dashed line: $O(\alpha_s)$ corrected contribution ($C_{2,\gamma}^{H,(0)} + C_{2,\gamma}^{H,(1)}$). The data are from PLUTO [12].
- Fig.8.** Corrections to the leading logarithmic (LL) representation of $F_2^\gamma(x, Q^2)$ at $Q^2 = 51$ (GeV/c)² (using the DG parameterization). The notation is as in Fig.5. The data are from AMY [17].

- Fig.9.** The light-quark contributions to $F_2^\gamma(x, Q^2)$ compared with $F_2^\gamma(LL)$ at $Q^2 = 51$ (GeV/c)². The notation is as in Fig.6. The data are from AMY [17].
- Fig.10.** The charm-quark contributions to $F_2^\gamma(x, Q^2)$ compared with $F_2^\gamma(LL)$ at $Q^2 = 51$ (GeV/c)². solid line: $F_2^\gamma(LL)$, The notation is as in Fig.8. The data are from AMY [17].
- Fig.11.** Corrections to the hadronic part of $F_L^\gamma(x, Q^2)$ (hadronic) at $Q^2 = 5.9$ (GeV/c)² (using the DG parameterization), solid line: hadronic, dotted line: hadronic + light, dotted-dashed line: hadronic + light + heavy (heavy = charm). For an explanation of hadronic, light and heavy see the text.
- Fig.12.** The light-quark contributions to $F_L^\gamma(x, Q^2)$ compared with the hadronic part of $F_L^\gamma(x, Q^2)$ (hadronic) at $Q^2 = 5.9$ (GeV/c)², solid line: hadronic, long-dashed line: Born contribution ($C_{2,\gamma}^{(0)}$), short-dashed line: $O(\alpha_s)$ corrected contribution ($C_{2,\gamma}^{(0)} + C_{2,\gamma}^{(1)}$).
- Fig.13** The charm-quark contribution to $F_L^\gamma(x, Q^2)$ compared with the hadronic part of $F_L^\gamma(x, Q^2)$ (hadronic) at $Q^2 = 5.9$ (GeV/c)², solid line: hadronic, dashed line: Born contribution ($C_{2,\gamma}^{H,(0)}$), short-dashed line: $O(\alpha_s)$ corrected contribution ($C_{2,\gamma}^{H,(0)} + C_{2,\gamma}^{H,(1)}$).
- Fig.14.** Corrections to the hadronic part of $F_L^\gamma(x, Q^2)$ (hadronic) at $Q^2 = 51$ (GeV/c)² (using the DG parameterization). The notation is the same as in Fig.11.
- Fig.15.** The light-quark contributions to $F_L^\gamma(x, Q^2)$ compared with the hadronic part of $F_L^\gamma(x, Q^2)$ (hadronic) at $Q^2 = 51$ (GeV/c)². The notation is the same as in Fig.12.
- Fig.16** The charm-quark contribution to $F_L^\gamma(x, Q^2)$ compared with the hadronic part of $F_L^\gamma(x, Q^2)$ (hadronic) at $Q^2 = 51$ (GeV/c)². The notation is the same as in Fig.13.

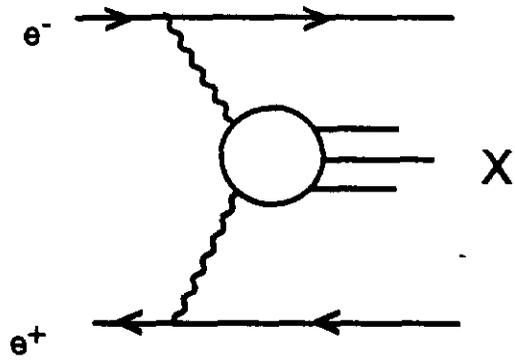


Figure 1.



Figure 2.

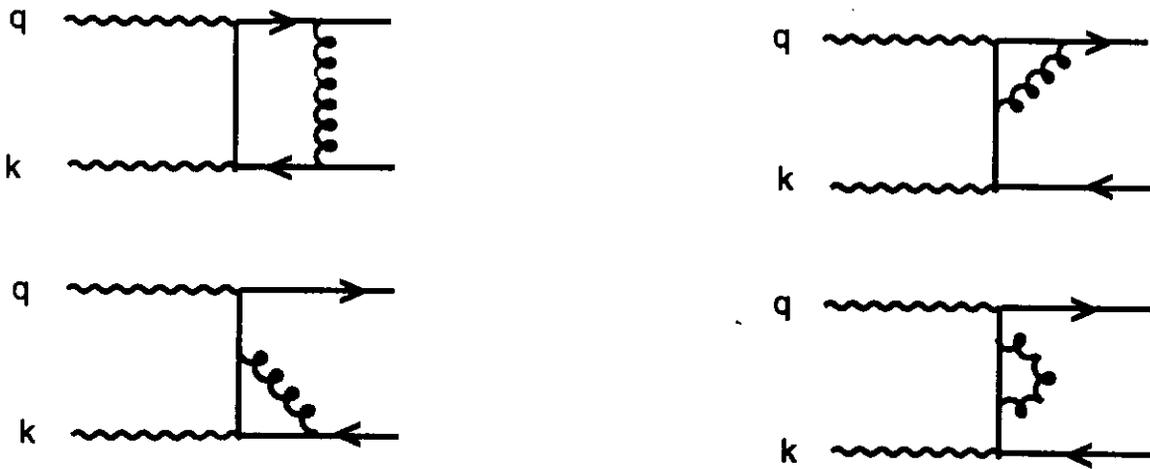


Figure 3.

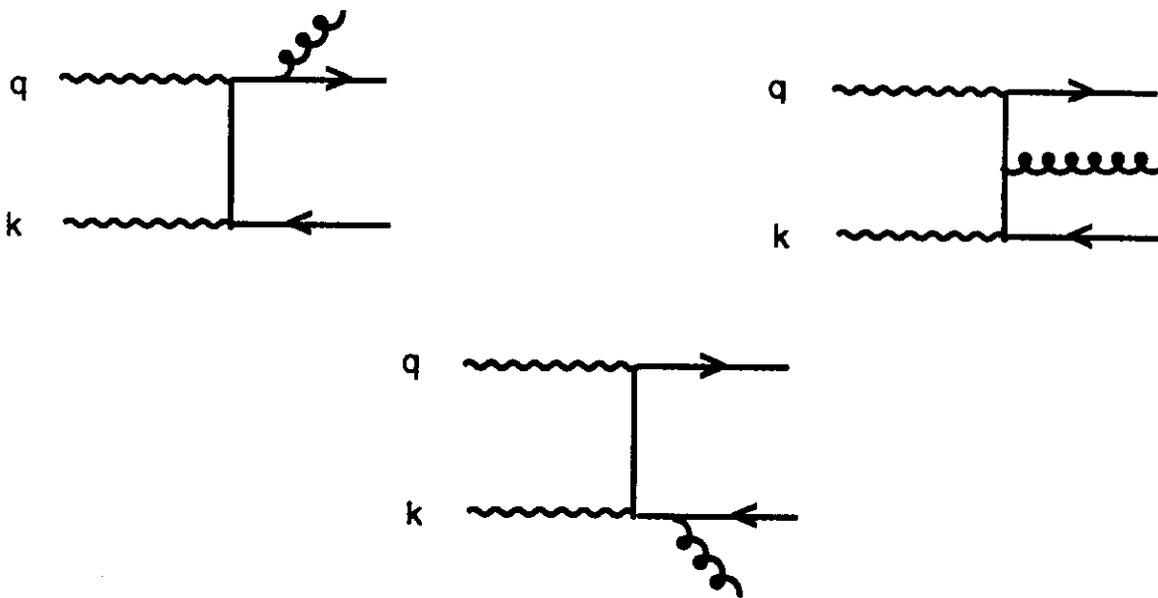


Figure 4.

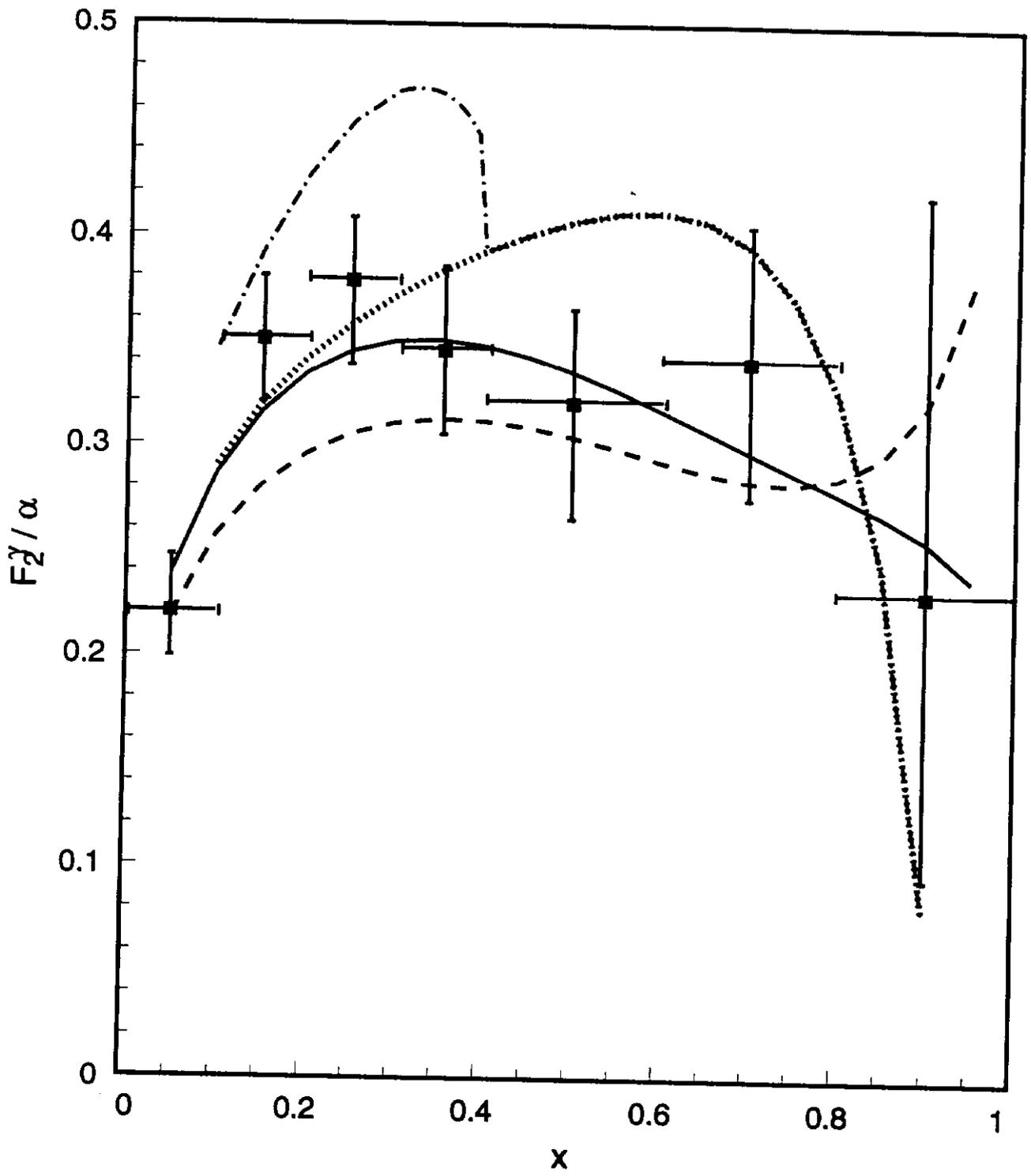


Figure 5.

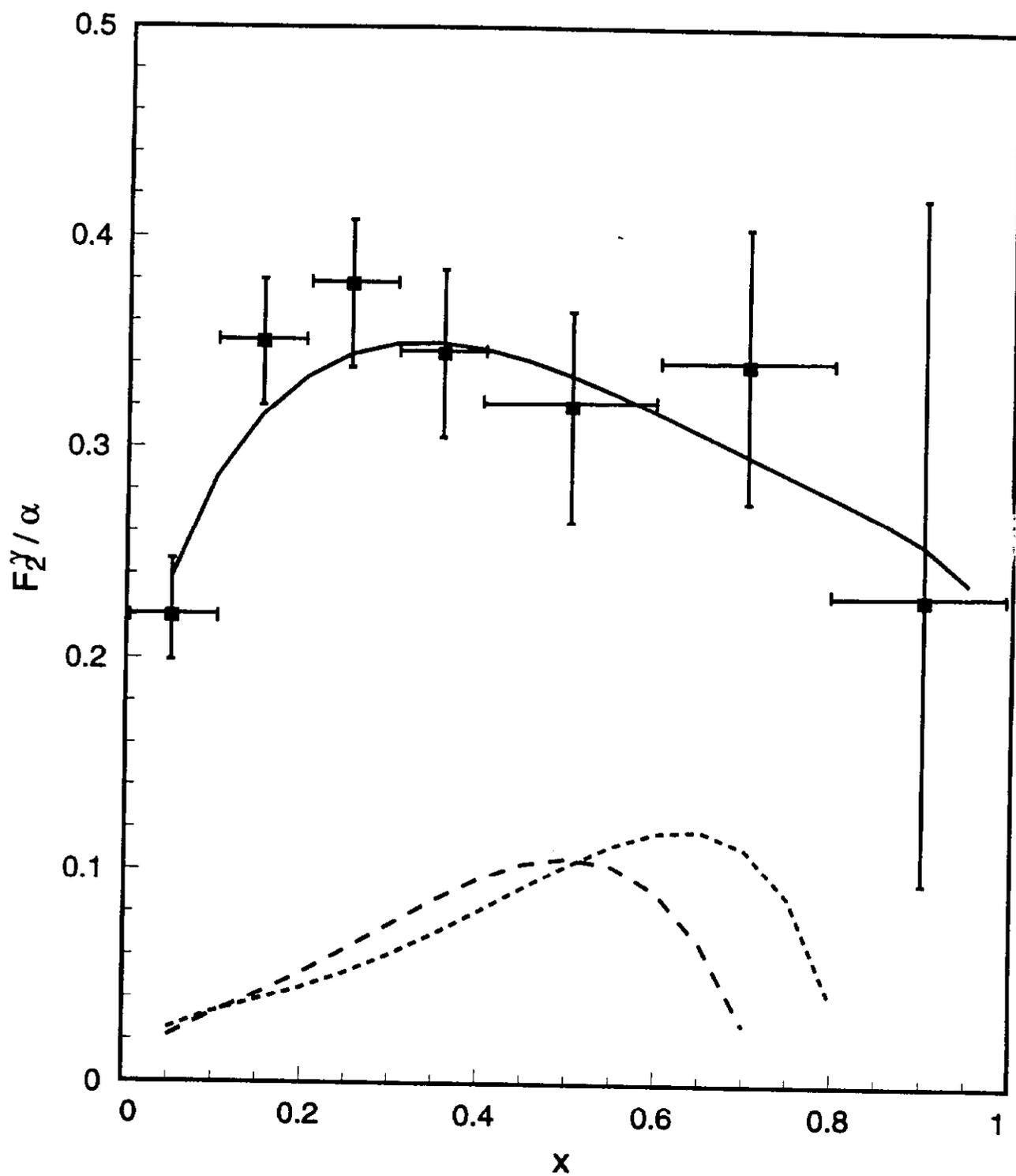


Figure 6.

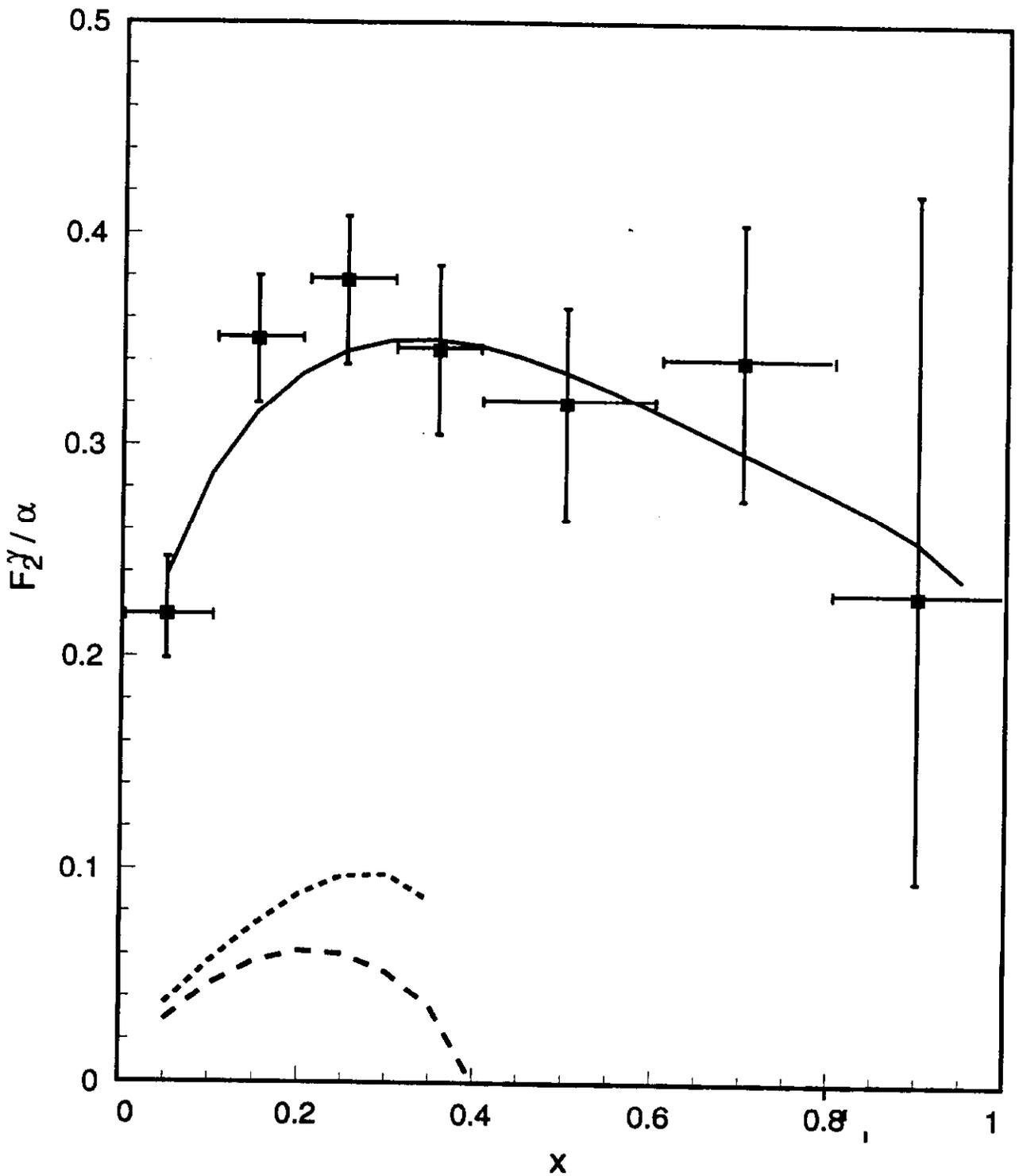


Figure 7.

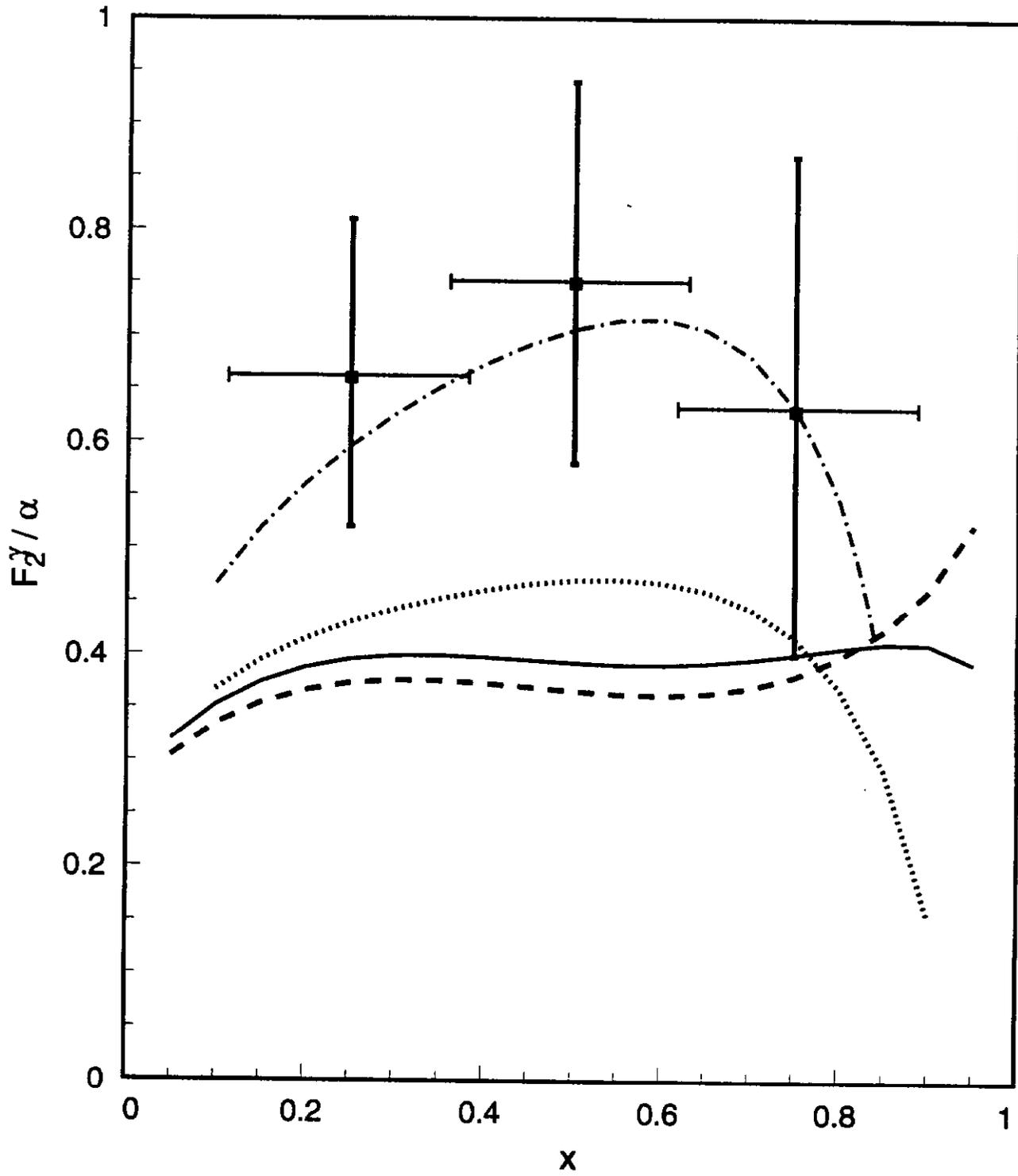


Figure 8.

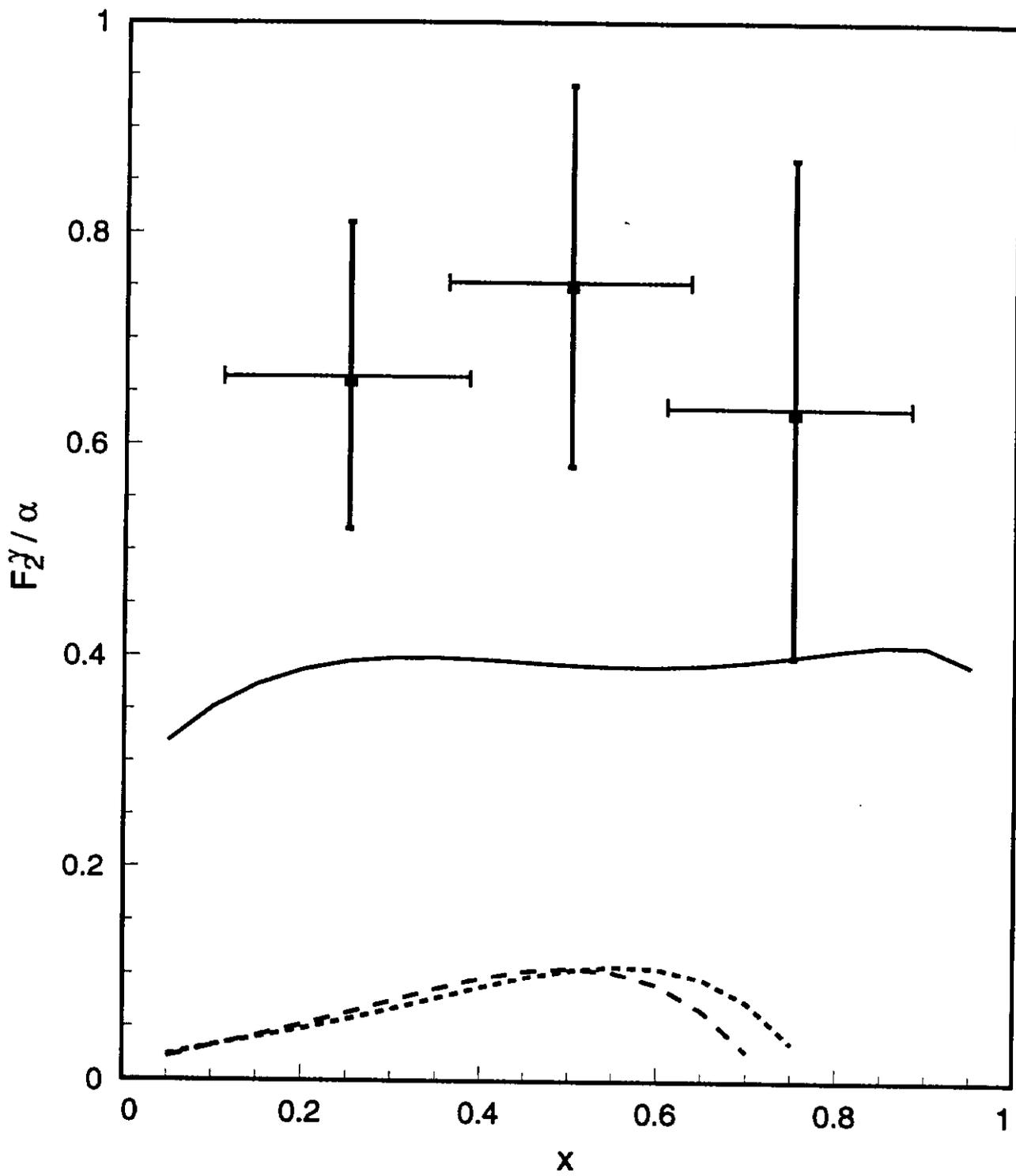


Figure 9.

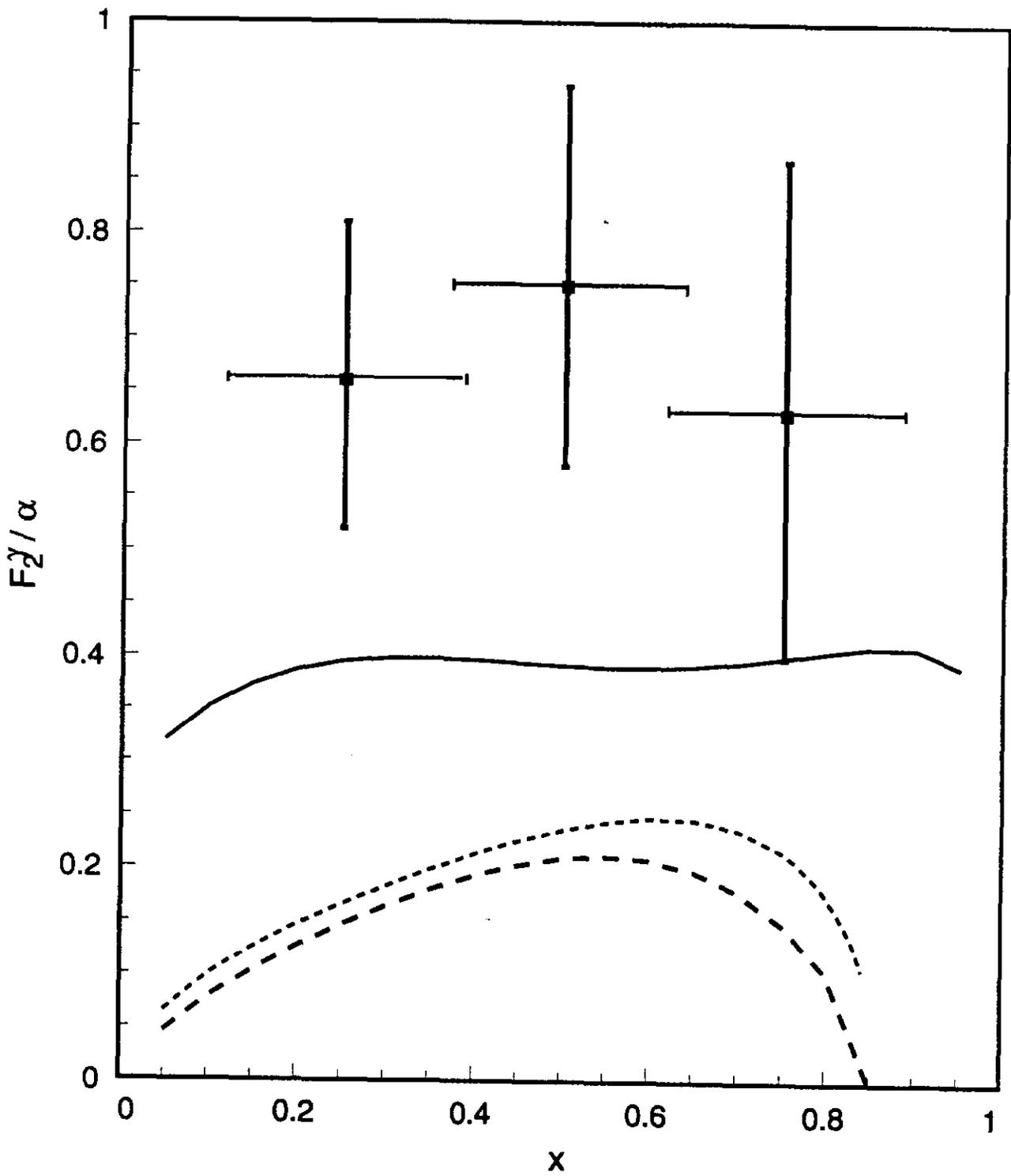


Figure 10.

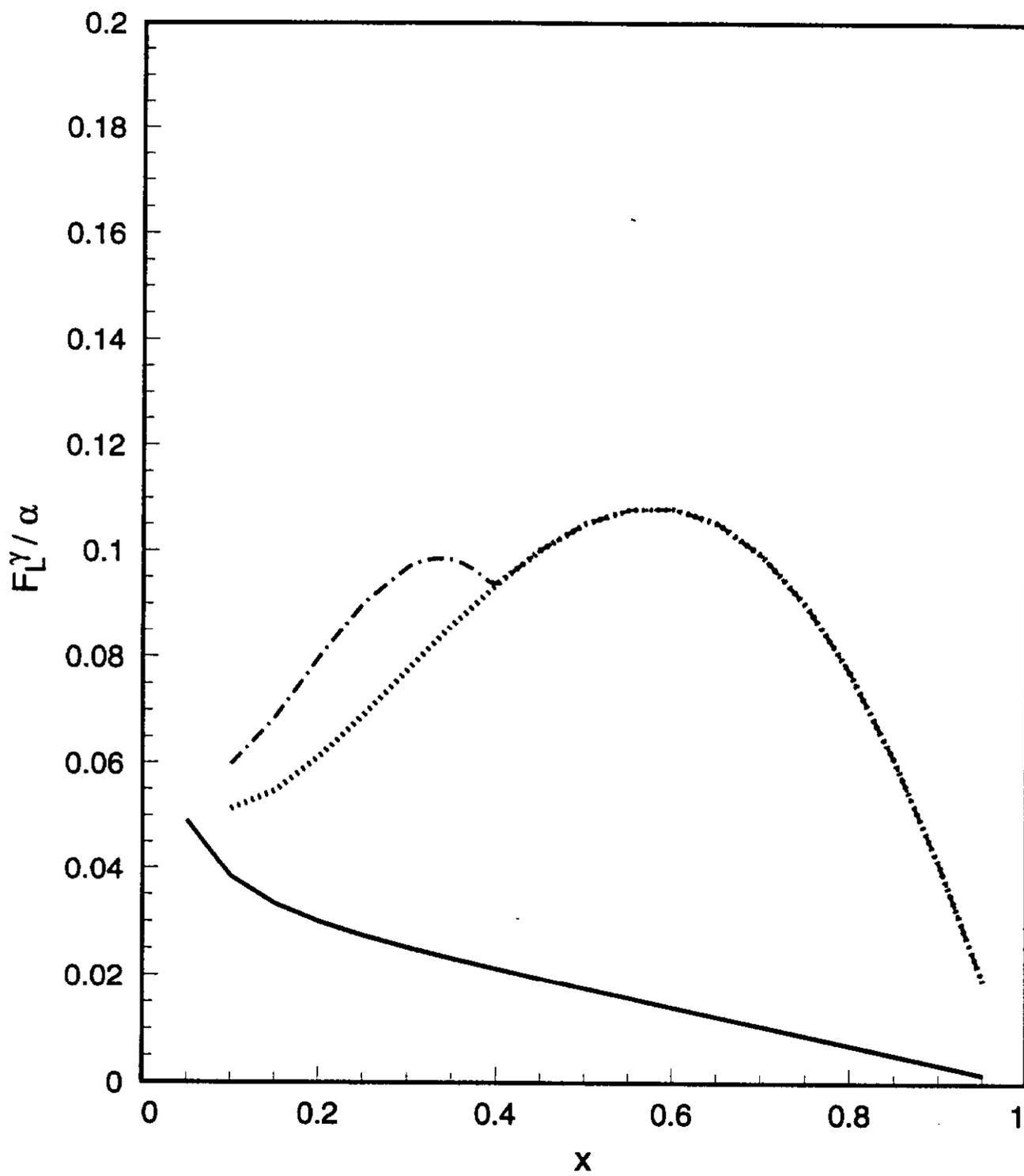


Figure 11.

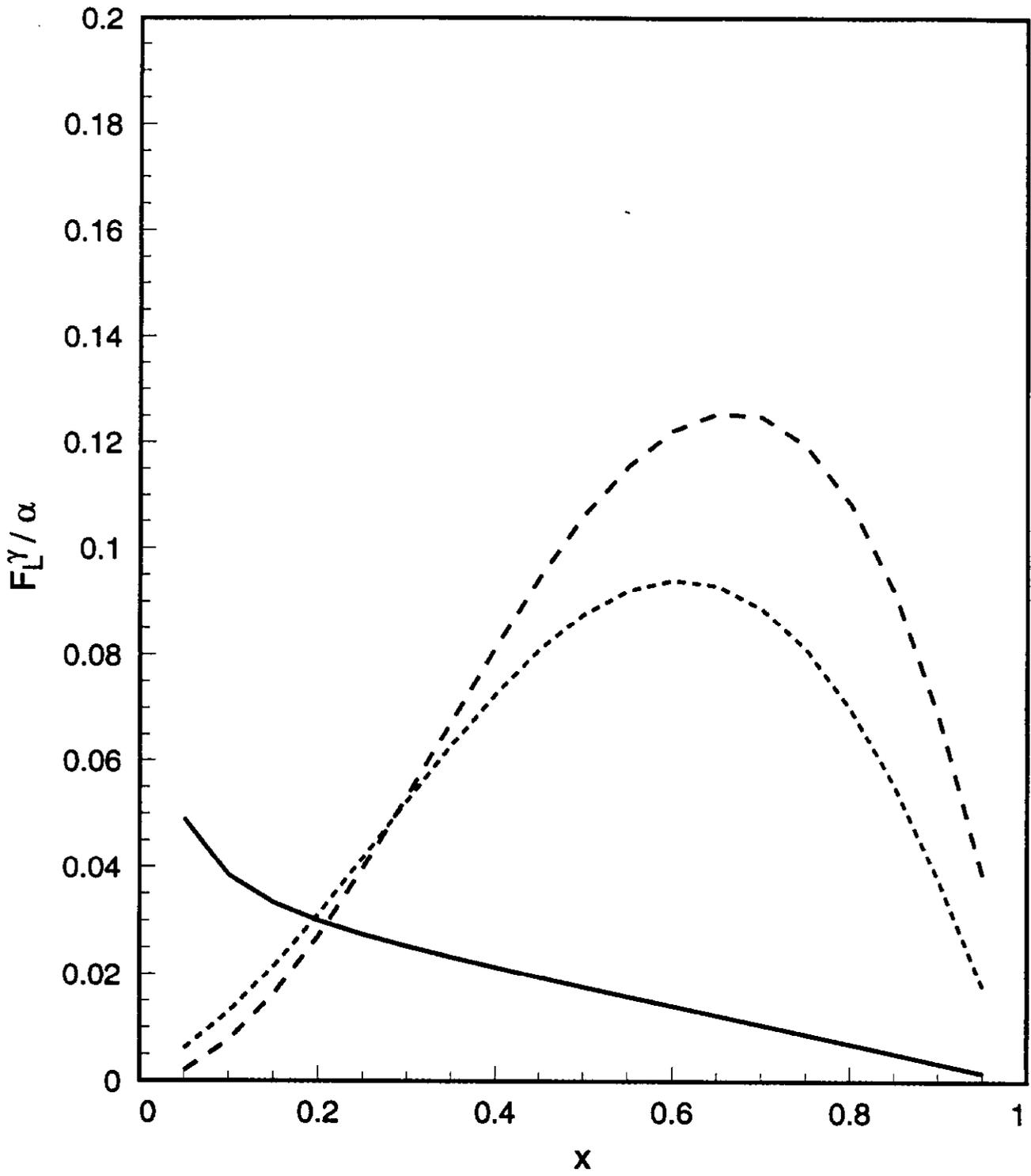


Figure 12.

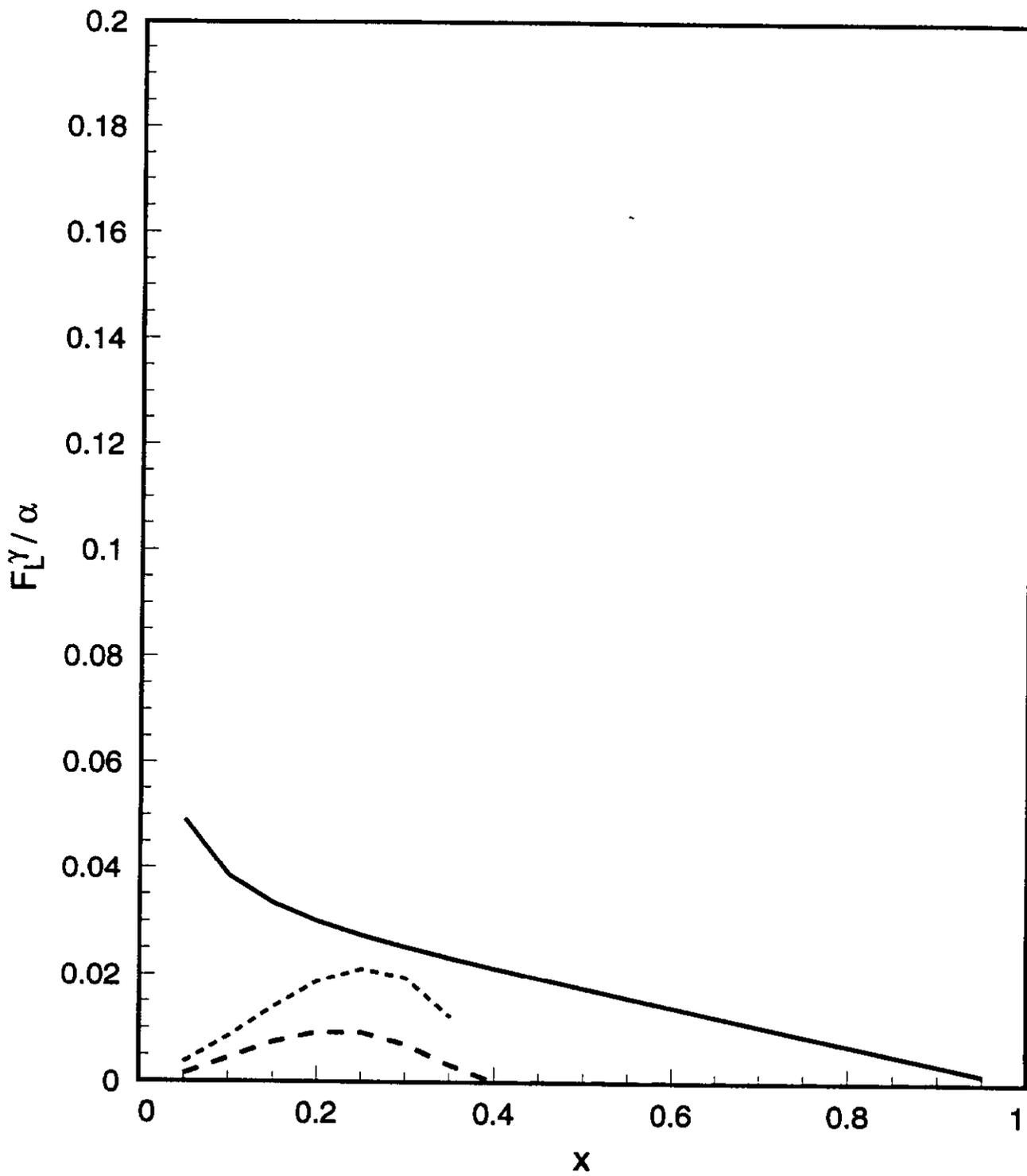


Figure 13.

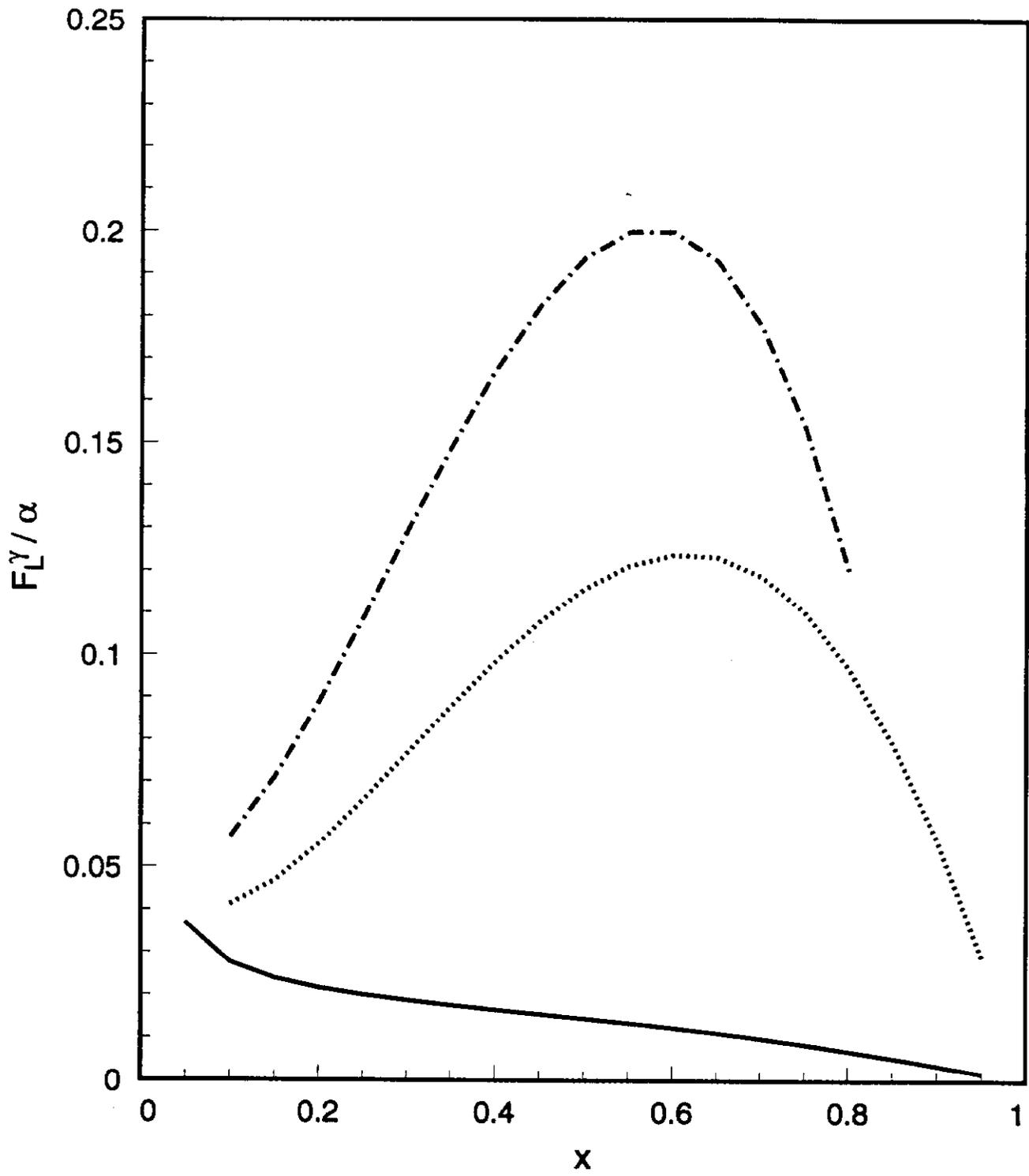


Figure 14.

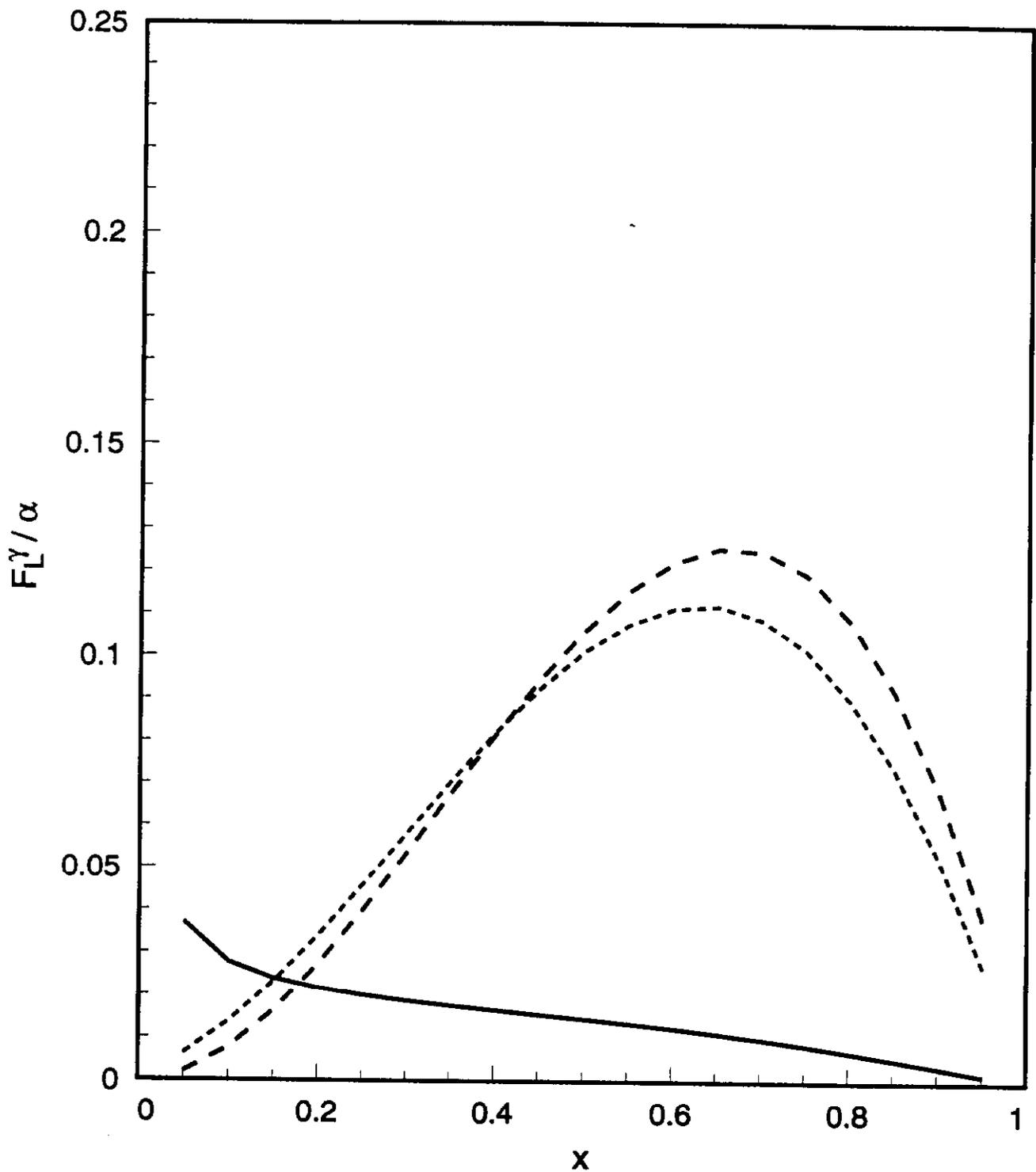


Figure 15.

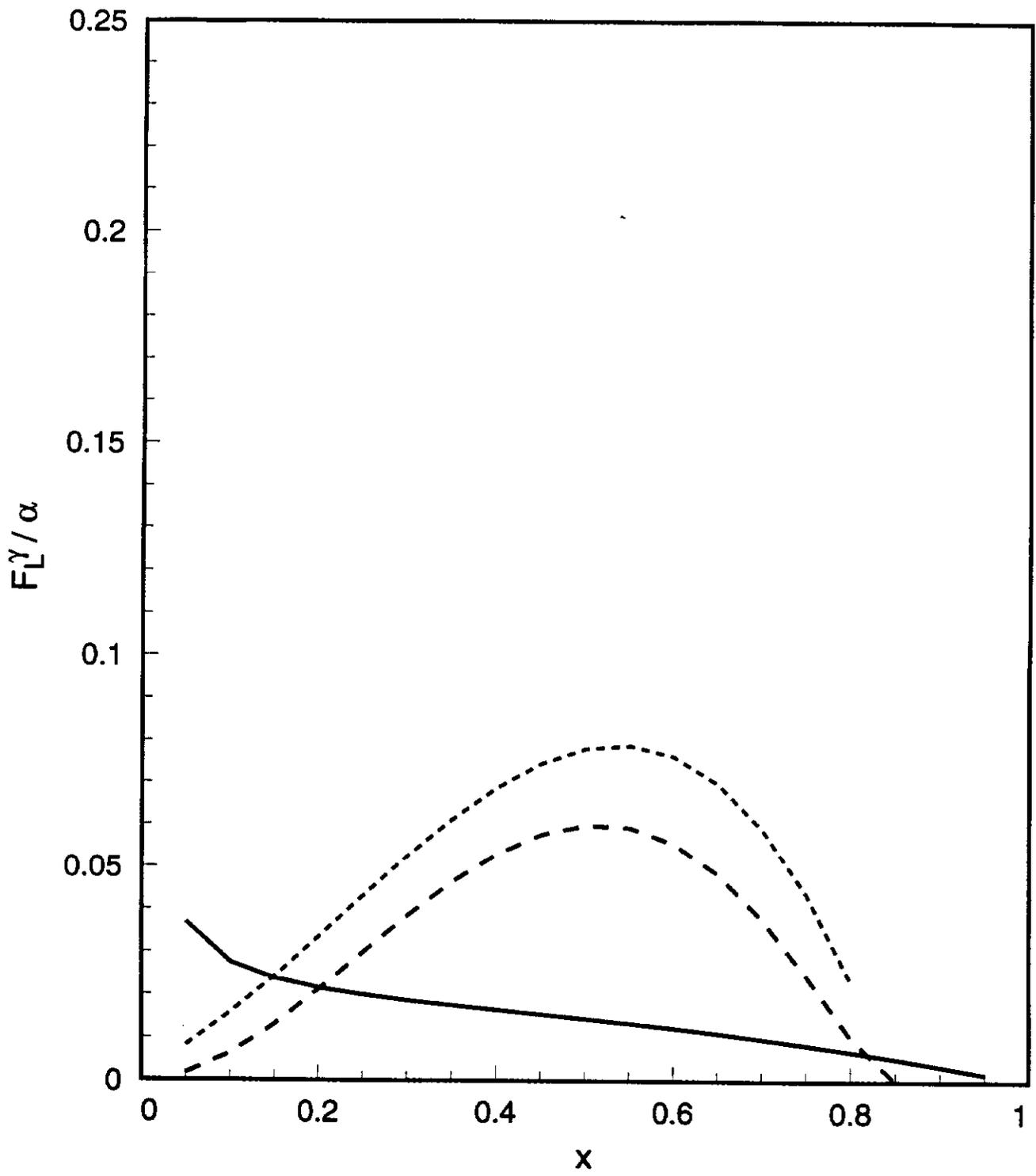


Figure 16.