

## Large Rapidity Gaps in pp Collisions

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### Abstract

The survival probability of large rapidity gaps in pp collisions is calculated for several different eikonal models of the Gaussian form. Results obtained for models based on partonic interactions are quite similar. The Regge-pole model predicts a higher value of  $\langle |S|^2 \rangle$ .

# 1 Introduction

It has recently been suggested [1, 2] that the observation of a large rapidity gap in the  $\eta - \phi$  lego plot, constructed for exceedingly high energy p-p interactions, may serve as a signature for W-W fusion associated with the production of a Higgs boson. The practical utilization of this idea as a useful trigger for rare electroweak processes depends on ones ability to reliably assess the survival probability  $\langle |S|^2 \rangle$ . This is defined [2] as the fraction of events for which spectator events do not fill the rapidity gap of interest.

The survival probability is easily defined in the eikonal model in impact parameter space. We use amplitudes normalised so that

$$\frac{d\sigma}{dt} = \pi |f(s, t)|^2 \quad (1)$$

$$\sigma_{tot} = 4\pi \text{Im}f(s, 0) \quad (2)$$

$$a(s, b) = \frac{1}{2\pi} \int d\mathbf{q} e^{-i\mathbf{q}\cdot\mathbf{b}} f(s, t) \quad (3)$$

From which we derive the b-space formulae [3] :

$$\sigma_{tot} = 2 \int d\mathbf{b} \text{Im}a(s, b) \quad (4)$$

$$\sigma_{el} = \int d\mathbf{b} |a(s, b)|^2 \quad (5)$$

s-channel unitarity implies that  $|a(s, b)| \leq 1$ , and when written in a diagonalized form we have

$$2\text{Im}a(s, b) = |a(s, b)|^2 + G_{in}(s, b) \quad (6)$$

from which we obtain for the inelastic cross section

$$\sigma_{in} = \int d\mathbf{b} G_{in}(s, b) \quad (7)$$

s-channel unitarity is most easily enforced in the eikonal approach where, assuming that  $a(s, b)$  is purely imaginary, we can write

$$a(s, b) = i(1 - e^{-\Omega(s, b)}) \quad (8)$$

where the eikonal  $\Omega(s, b)$  is a real function.

Our assumption that  $a(s, b)$  is purely imaginary is not compatible with analyticity and crossing symmetry. These are, easily restored upon substituting  $s^\alpha \rightarrow s^\alpha e^{-i\frac{\pi\alpha}{2}}$ . From Eqn. (7) we can express  $G_{in}(s, b)$  as a function of  $\Omega(s, b)$

$$G_{in}(s, b) = 1 - e^{-2\Omega(s, b)} \quad (9)$$

Note that  $P(s,b) = e^{-2\Omega(s,b)}$  is the probability that no inelastic interaction takes place at impact parameter  $b$ .

We follow Bjorken [2] and define  $\langle |S|^2 \rangle$  as the normalized multiplication of two quantities. The first is a convolution over the parton densities of the two interacting projectiles presenting the cross section for the hard parton-parton collision under discussion. The second  $P(s,b)$  is the probability that no other interaction takes place in the rapidity interval of interest. In the eikonal formalism we have:

$$\langle |S|^2 \rangle = \frac{\int a_H(s,b)P(s,b)d^2b}{\int a_H(s,b)d^2b} \quad (10)$$

where  $a_H(s,b)$  denotes the amplitude associated with hard collisions that can be expressed through the eikonal  $\Omega_H(s,b)$  using Eqn. (8). Some preliminary calculations of  $\langle |S|^2 \rangle$  have been presented in Ref. [2]. Following this pioneering effort there were also a number of attempts made to estimate  $\langle |S|^2 \rangle$  using Monte Carlo techniques [4, 5]. It is important to note that these assessments of the survival probability are model dependent. A number of models are available [6 - 11] which provide a good reproduction of the data in the ISR - Tevatron range. As we shall show these models differ in their estimates of  $\Omega_H(s,b)$  and  $P(s,b)$  in the high energy limit of LHC and SSC. It is therefore pertinent to carefully check the dependence of  $\langle |S|^2 \rangle$  on the phenomenological input required in Eqn. (10). This is the main aim of this note, where we have attempted a systematic study of  $\langle |S|^2 \rangle$  and its sensitivity to the input parameters.

In the following we assume that both  $\Omega_H(s,b)$  and  $\Omega(s,b)$  in  $a_H(s,b)$  and  $P(s,b)$  are well approximated by Gaussians

$$\Omega_H(s,b) = \nu_H(s)e^{-\frac{b^2}{R_H^2}} \quad (11)$$

$$2\Omega(s,b) = 2\nu(s)e^{-\frac{b^2}{R^2}} \quad (12)$$

This input assumption has been verified by the analysis of Ref.[10, 11], where it has been shown that eikonal models of this form provide an excellent reproduction of the cross section data and particle distributions in the energy range  $5 \leq \sqrt{s} \leq 1800$  GeV. The main advantage of assuming the input in a Gaussian form is that the integration in Eqns. (4) and (10) can be carried out analytically, whence the total cross section

$$\begin{aligned} \sigma_{tot} &= 2 \int db(1 - e^{-\Omega(s,b)}) = 2\pi R^2(s) \sum_{n=1}^{\infty} \frac{(-1)^{n-1} \nu(s)^n}{n!n} \\ &= 2\pi R^2(s)[\ln \nu(s) + C - Ei(-\nu(s))] \end{aligned} \quad (13)$$

$Ei(x)$  denotes the integral exponential function  $Ei(x) = \int_{-\infty}^x \frac{e^t}{t} dt$ , and  $C$  is the Euler constant ( $C = 0.5773$ ). For  $\nu(s) \gg 1$  we have

$$\sigma_{tot} \rightarrow 2\pi R^2(s)[\ln\nu(s) + C] \quad (14)$$

The inelastic cross section is given by

$$\sigma_{in} = 2 \int db(1 - e^{-2\Omega(s,b)}) = \pi R^2(s)[\ln(2\nu(s)) + C - Ei(-2\nu(s))] \quad (15)$$

and therefore  $\sigma_{el} = \sigma_{tot} - \sigma_{in} =$

$$= \pi R^2(s)[\ln(\frac{\nu(s)}{2}) + C + Ei(-2\nu(s)) - 2Ei(-\nu(s))] \quad (16)$$

Again for  $\nu(s) \gg 1$  we have

$$\sigma_{el} \rightarrow \pi R^2(s)[\ln(\frac{\nu(s)}{2}) + C + e^{-2\nu(s)} - 2e^{-\nu(s)}]$$

One can extract the values for  $R^2(s)$  and  $\nu(s)$  using the expressions for  $\sigma_{tot}$  and  $\sigma_{el}$  given in Eqns. (13) and (15). As expected at the Tevatron energy where the cross sections are known, the spread in values of  $R^2(s)$  and  $\nu(s)$  for the different models is rather small. It is only when the model parameters are extrapolated to higher energies does the difference become significant, and allows one to test the theory on which the parametrization is based. We shall estimate the importance of this dependence when making predictions for LHC and SSC energies.

The integration of Eqn. (10) yields:

$$\langle |S|^2 \rangle = a \left(\frac{1}{2\nu}\right)^a \gamma(a, 2\nu) \quad (17)$$

where  $a = \frac{R^2}{R_H^2}$  and  $\gamma(a, x)$  denotes the incomplete gamma function

$$\gamma(a, x) = \int_0^x z^{a-1} e^{-z} dz \quad (18)$$

For  $\nu(s) \gg 1$  Eqn. (17) simplifies to [2]:

$$\langle |S|^2 \rangle = \frac{a\Gamma(a)}{\nu(s)^a} + \frac{a}{\nu(s)} e^{-\nu(s)} \quad (19)$$

A general numerical mapping of  $\langle |S|^2 \rangle$  as a function of  $a$  and  $\nu$  is shown in Figs. 1 and 2.

A realistic assessment of the survival probability is subject to a considerable ambiguity, due mainly to the following reasons:

1) There is no clear definition of the hard component of the b-space scattering amplitude or eikonal. Moreover, it is not clear whether the growth of  $\sigma_{tot}$  with  $s$  is due to the soft or to the hard sector. Bjorken [2] has suggested that one can estimate  $R_H^2$  from the low energy cross section where  $\sigma_{tot}(pp) \approx 40$  mb. Hence  $R_H^2$  is energy independent and  $a > 1$ . Implicit in this estimate is the assumption that  $\sigma_{tot}$  growth is associated with the soft sector. This point of view is not common to all models. In particular, models based on parton interactions [6, 10] associate the growth of  $\sigma_{tot}$  with the hard or semi-hard sector. Hence  $R_H^2$  is energy dependent and  $a \approx 1$ . Bjorken [2] based his numerical estimates on the minijet model [6]. Indeed in this type of model  $R_H^2$  approaches a constant in the high energy limit. Here, the growth in  $\sigma_{tot}$  is due to gluon-gluon interactions which are semi-hard. Having no better insight to this problem, we follow the recipe suggested by Bjorken, and fix  $R_H^2$  from the low energy data at  $\sqrt{s} \approx 10$  GeV.

2) The input information required in Eqn. (17), i.e.  $a$  and  $2\nu$  is obtained from fits to the rising pp and  $\bar{p}p$  cross sections. This rise is a consequence of both a process of blackening, i.e. a rise of  $\nu(s)$ , and expansion, i.e. an increase of  $R^2$ . However, these two processes compensate each other as is readily seen in Eqn. (14). The dependence of  $\langle |S|^2 \rangle$  on these parameters is completely different as is evident from Eqn. (19). As we have noted, there are a number of phenomenological models which reproduce the available data well in the  $10 \leq \sqrt{s} \leq 1800$  GeV energy range. These models differ in their estimates of  $a$  and  $2\nu$ , and we wish to examine the stability of  $\langle |S|^2 \rangle$  with respect to variations of the input parameters  $a$  and  $2\nu$ , as deduced from the phenomenological models.

3) We would like to emphasize that all our estimates of  $\langle |S|^2 \rangle$  are based on the eikonal approach, i.e. on the assumption that the representation for the scattering amplitude given in Eqn. (8), is a valid approximation at high energies. This form allows us to formulate the expression for the survival probability in a simple and transparent manner.

In the following we elaborate on these points by discussing a few input models: Our results are summarized in Table I, where we list the values for  $\nu$ ,  $R_H^2$ ,  $R^2$ ,  $\sigma_{tot}$  and  $\langle |S|^2 \rangle$  for energy values of the Tevatron, LHC and SSC.

## 1.1 Minijet-parton model

In this model [6] the eikonal for the partonic i-j collision is given by

$$\Omega(s, b)(s, b) = W_{ij}(b)\sigma_{ij}^{QCD}(s) \quad (20)$$

where  $\sigma_{ij}^{QCD} \sim s^{J-1}$ . For  $W_{ij}$ , the Chou-Yang formulation is assumed [12]

$$W_{ij}(b) = \frac{\mu_{ij}^2}{96\pi} (\mu_{ij}b)^3 K_3(\mu_{ij}b) \quad (21)$$

Eqn. (21) is reasonably well approximated by a Gaussian. To realize this we have fitted Eqn. (21) numerically, utilizing the parameters for  $\mu_{ij}$  given in Ref. [6]. To estimate the hard component, we follow the method suggested by Bjorken [2] and fix  $R_H^2$  from the model predictions at  $\sqrt{s} = 10$  GeV, where  $\sigma_{tot} \approx 40$  mb. This is a natural choice as in this model, we have relatively small changes of  $R^2(s)$  with energy. Not suprisingly, our estimates of  $\langle |S|^2 \rangle$  are compatible with those of Refs. [2, 3].

## 1.2 Regge-pole model

An impressive reproduction of the experimental total cross section data is obtained [7] by utilizing a simple Regge-pole model. For our high energy analysis we are interested in the super-critical Pomeron where the amplitude is given by:

$$f(s, t) = iC e^{R_0^2 t} s^{\alpha(t)-1} \sin[0.5\pi\alpha(t)] \quad (22)$$

with  $\alpha(t) = 1 + \epsilon + \alpha't$ . Donnachie and Landshoff fit [7] the data with values of the parameters  $C = 21.7$  mb and  $\epsilon = 0.0808$ . In addition we utilize a global fit [8] to B, the nuclear slope, and obtain  $R_0^2 = 5.2$  GeV<sup>-2</sup> and  $\alpha' = 0.2$  GeV<sup>-2</sup>. The b-space transform of Eqn. (20) is

$$a(s, b) = iC \frac{s^\epsilon}{2 |\beta|^2} \exp\left[-\frac{R_1^2 \cdot b^2}{4 |\beta|^2}\right] \cdot [R_1^2 \sin\left(\frac{\pi}{2}\alpha(0) + Z\right) - \frac{\pi}{2}\alpha' \cos\left(\frac{\pi}{2}\alpha(0) + Z\right)] \quad (23)$$

where

$$\begin{aligned} R_1^2 &= R_0^2 + \alpha' \ln s \\ |\beta|^2 &= R_1^4 + \frac{\pi^4 \alpha'^2}{4} \\ Z &= \frac{\pi \alpha' b^2}{8 |\beta|^2} \end{aligned} \quad (24)$$

As both  $\epsilon$  and  $\alpha' \ll 1$ , Eqn. (21) is well approximated by a Gaussian ( see Eqn. (11) ) with

$$\begin{aligned} \nu(s) &= \frac{C R_1^2}{2 |\beta|^2} s^\epsilon \\ R^2(s) &= \frac{4 |\beta|^2}{R_1^2(s)} \end{aligned} \quad (25)$$

The model in the form suggested by Donnachie and Landshoff [7] is not appropriate for fitting data at high energies, as for very high energy it violates unitarity ( $a(s, b=0) > 1$ )

above  $\sqrt{s} \approx 5$  TeV. For our evaluation we use a very similar eikonized version suggested by Cudell and Margolis [9], with  $C = 24$  mb,  $\epsilon = 0.093$  and  $\alpha' = 0.25$  GeV<sup>-2</sup>. For the hard sector our choice is less straightforward than before, as  $R^2(s) \sim \ln s$ , and there is no obvious way of defining the hard component. In Table I we present  $R_H^2$  as the threshold of Eqn. (25). Note that the results obtained from the Regge-pole model are in marked contrast to the other models investigated in this note. The difference will be elaborated upon later.

### 1.3 Lipatov-like Pomeron

A simple parametrization for the Lipatov-like Pomeron [13], has been suggested in Ref. [10]. The eikonal is given by

$$\Omega(s, b) = \frac{a_1 s^{a_2}}{(\ln s)^{a_3}} \cdot e^{-\frac{b^2}{R^2(s)}} \quad (26)$$

where the following two parametrizations of  $R^2(s)$  provide good fits to the data:

$$R_{L1}^2 = a_4 + a_5 (\ln s)^{a_6} \quad (27)$$

$$R_{L2}^2 = a_4 + a_5 \sqrt{\ln s} + a_6 \ln s \quad (28)$$

$a_i$  are fitted parameters. In this model a regular Pomeron with trajectory  $\alpha(0) = 1$ , is appended to the Lipatov-like Pomeron. Again the choice of what to take for  $R_H^2$  is ambiguous, as  $R^2(s)$  is energy dependent and its low energy limit is exceedingly small. We adopt an arbitrary definition as suggested in Ref. [2] and use  $R_H^2$  to be the value at  $\sqrt{s} = 10$  GeV, where  $\sigma_{tot} \approx 40$  mb.

### 1.4 Dual parton model

This is a multi-component model [11] describing soft and semihard multiparticle production. The eikonal is given by

$$\Omega(s, b) = \Omega_S(s, b) + \Omega_H(s, b) - \Omega_{TP}(s, b) - \Omega_L(s, b) \quad (29)$$

where the last two terms correspond to the triple Pomeron and loop contributions. As  $\Omega_{TP}(s, b)$  and  $\Omega_L(s, b)$  are reasonably small, this is effectively a two component model whose parameters are:

$$\Omega_S(s, b) = \frac{\sigma_S}{8B_S} \cdot e^{-\frac{b^2}{4R_S^2}} \quad (30)$$

where

$$\begin{aligned}\sigma_S &= C s^\epsilon \\ R_S^2 &= B + \alpha' \ln s\end{aligned}\tag{31}$$

with  $C = 40.8$  mb,  $\epsilon = 0.076$  and  $\alpha' = 0.24$  GeV<sup>-2</sup>.

$$\Omega_H(s, b) = \frac{\sigma_H}{8B_H} \cdot e^{-\frac{b^2}{4R_H^2}}\tag{32}$$

where  $R_H^2 = B$ . In this type of model  $R_H^2$  is defined as the low energy threshold limit of  $R_S^2(s)$  given by Eqns. (30) and (31). The  $\sigma_H$  is calculated in lowest order QCD and is dependent on the value taken for the  $p_t^{min}$  cutoff. Numerical values for  $\sigma_H$  at different energies are given in [11].

## 2 Conclusions

Our results are summarized in Table I. The results obtained for the various partonic models [6], [10], [11] for  $\langle |S|^2 \rangle$  in the LHC-SSC energy range are remarkably stable. We note that the models of [10] and [11], even though very different in their construction, yield rather similar input parameters, as summarized in Table I. Ref. [6] differs from the above, in that it has the highest values for  $\nu$ . These are compensated for by having the corresponding lowest values for  $a$ , producing final results which are similar to those obtained in Refs. [10] and [11].

The survival probability obtained from the Regge-pole model [8-10] are considerably higher. On examining the input parameters used in our calculation, we find that the difference can be traced to the fact that the  $\nu$  values associated with the Regge-pole model are the smallest. Hence, in order to be compatible with the data the model requires relatively large values of  $R_H^2$  and  $R^2$  which give rise to high  $\langle |S|^2 \rangle$ .

We conclude with a more general comment. Clearly the questionable aspect of a calculation such as presented here, is the fact that the definition of the hard component is not unique. We have followed Bjorken's suggestion [2], and have fixed  $R_H^2$  from the low energy data. It is likely that our estimates for  $R_H^2$  are on the conservative side, so that in reality one could expect even higher values for the survival probability than are given in Table I.

**Table I** Parameters and predictions of different models

Model	$\sqrt{s}$ TeV	$\nu(s)$	$R_H^2$ GeV <sup>-2</sup>	$R^2$ GeV <sup>-2</sup>	$\sigma_{tot}$ mb	$\langle  S ^2 \rangle$ %
Minijet <sup>(6)</sup>	1.8	2.50	14.41	19.74	72	13.2
	16.0	3.90	14.41	22.60	107	5.5
	40.0	4.75	14.41	23.20	121	3.8
Regge <sup>(7-9)</sup>	1.8	1.11	25.41	35.80	76	32.6
	16.0	1.48	25.41	40.16	102	22.1
	40.0	1.68	25.41	41.99	117	18.1
Lipatov 1 <sup>(10)</sup>	1.8	1.60	15.78	25.39	75	19.7
	16.0	2.69	15.78	28.02	113	8.2
	40.0	3.44	15.78	29.37	134	4.9
Lipatov 2 <sup>(10)</sup>	1.8	1.44	16.19	28.64	76	20.6
	16.0	2.24	16.19	32.66	115	9.2
	40.0	2.77	16.19	34.23	137	5.8
Dual parton <sup>(11)</sup> KMRS[B-2]	1.8	1.83	10.56	28.47	75	9.6
	16.0	2.23	10.56	32.67	109	5.3
	40.0	2.43	10.56	34.43	124	4.0

## Figure captions

**Figure 1:** Contours of percentage of the survival probability  $\langle |S|^2 \rangle$  as a function of  $a$  and  $\nu$ .

**Figure 2:** Graph of  $\log$  ( % survival probability ) versus  $a$  for selected values of  $\nu$ .

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# SURVIVAL PROBABILITY

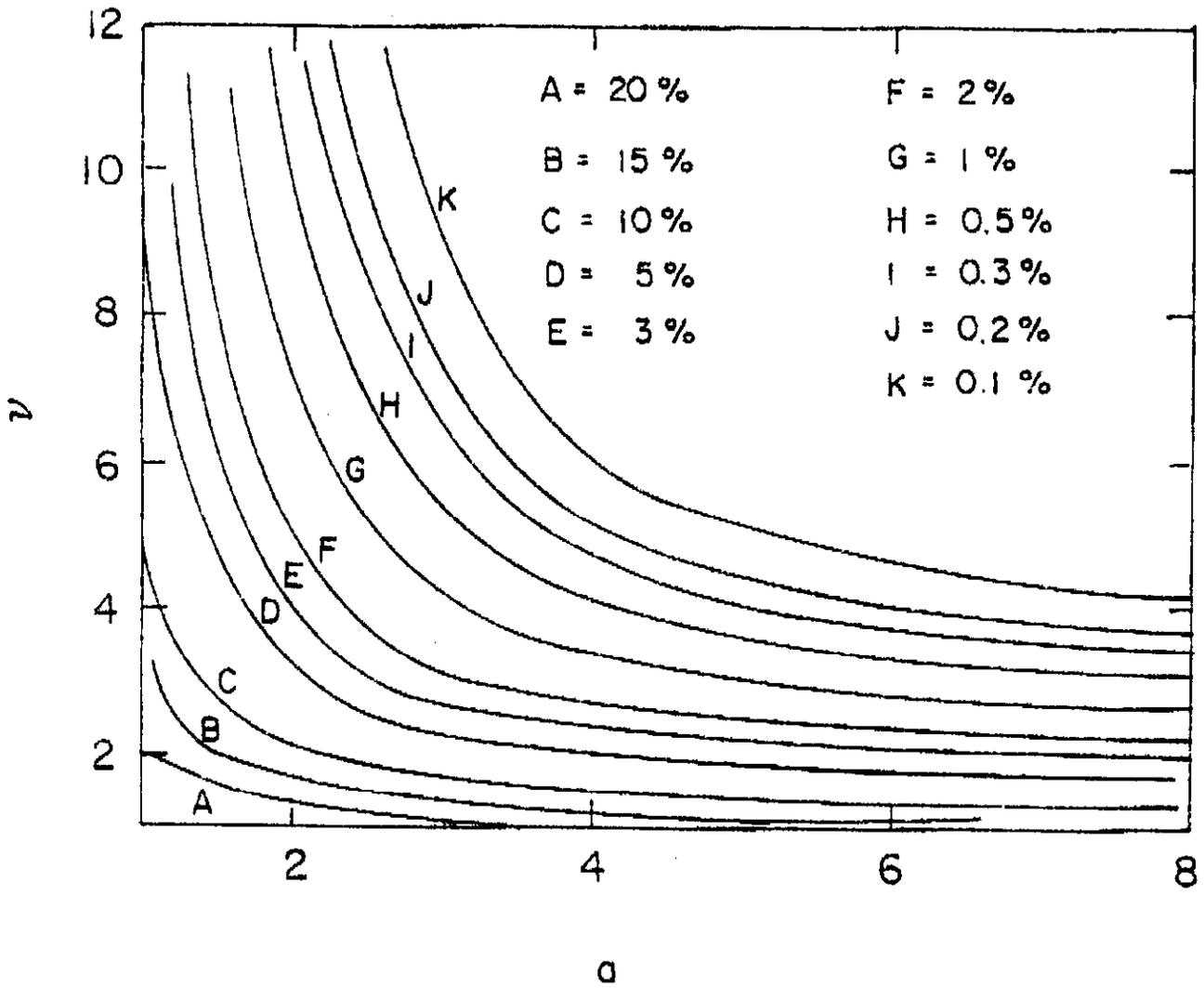


Fig. 1

# GAP SURVIVAL PROBABILITY

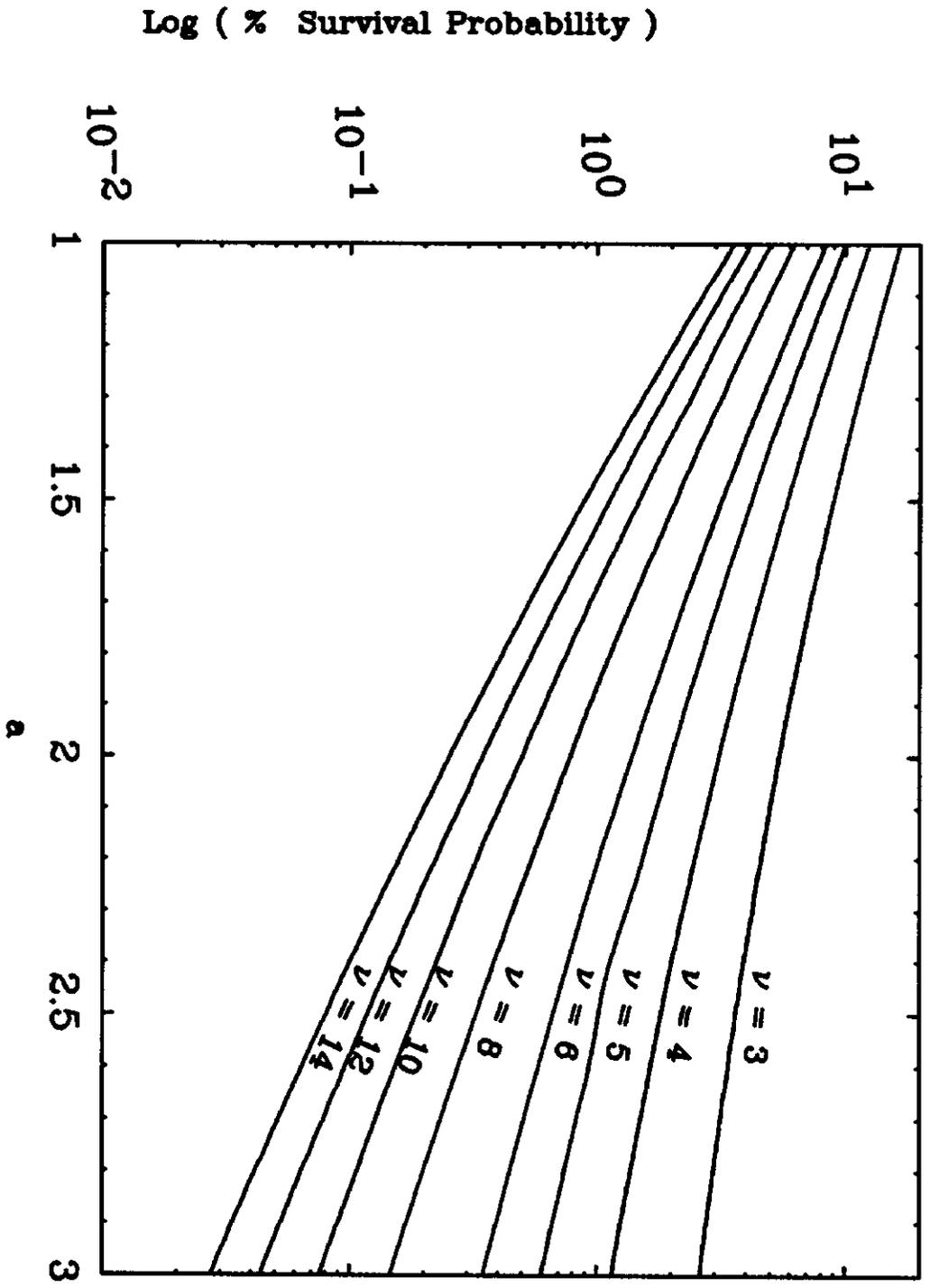


Fig. 2