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Rapidity gaps in deep inelastic scattering

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Abstract

A theoretical approach is developed to large rapidity gap physics in deeply inelastic scattering. The formula for the survival probability of the large rapidity gap is obtained, which turns out to be quite different from Bjorken's formula for hadron-hadron collisions.

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I. Introduction

The main goal of this paper is to develop Bjorken's ideas [1] on large rapidity gap physics for deeply inelastic processes at low x_B.

We have sufficiently good theoretical understanding of deeply inelastic scattering at low x_B (see ref. [2] and recent reviews [3]). At least we can write the new (GLR) nonlinear evolution equation for the deep inelastic structure function that takes into account the screening (shadowing) corrections. Nowthat we have a more solid theoretical background in deeply inelastic scattering we can start to study the behaviour of more complicated processes such as processes with a large rapidity gap between produced particles.

The result of this paper is a formula for the survival probability of the large rapidity gap in the deeply inelastic processes that is obtained within the theoretical accuracy. I hope that this formula could be very instructive for the case of hadron-hadron collision where we are able to get only intuitive if correct description of the survival probability.

To understand the problems that we face discussing the large rapidity gap physics let me consider the production of a Higgs particle via WW fusion in pp collision as was suggested by Bjorken [1] (see Fig. 1). The cross section of this reaction can be described by simple factorized formula due to AGK cutting rules [4] and / or the factorization theorem [5].

$$f(y-\Delta y, y_H, p_{Ht}) \;=\; \frac{d\sigma}{dy_H dp^2_{Ht}} \;= \tag{1.1}$$

$$\int \phi\left(q^2_{1t}, x_1\right)\phi\left(q^2_{2t}, x_2\right)\cdot d^2q_{1t}d^2q_{2t}\cdot\sigma_{hard}\left(q^2_1, q^2_2, x_1x_2S\right)(q_1+q_2\rightarrow q_1q_2H)\;,$$

where

$$y=\ln S,; \quad \Delta y \;=\; y_1 \;-\; y_2\;, \tag{1.2}$$

while y_1 (y_2) is rapidity of produced quark. In eq. (1.1) we use so called the transverse momentum factorization approach [6]. Furthermore σ_{hard} is the cross section for the hard subprocess:

$$\bar{q}_1(x_1, q_{1t}) + q_2(x_2, q_{2t}) \rightarrow \bar{q}_1(x_1, p_{1t}) + q_2(x_2, p_{2t}) + H \tag{1.3}$$

which is marked in Fig. 1 by a dotted line, and function ϕ is closely related to the deep inelastic structure function namely

$$\alpha_s(\, x\, F(x, q^2)\,) \;=\; \int^{q^2} \alpha_s(q'^2)\; \phi(q'^2, x) dq'^2 \tag{1.4}$$

We do not need to know the exact formula for the hard cross section in eq. (1.1), for us it is only important that the rapidity gap between two produced quarks with transverse momenta $p_{1t} =\sim p_{2t}$ is large enough.

At first sight this mechanism of Higgs production has an excellent signature for the experimental detection. The event topology is very remarkable: two collimated $(p_{1t} \approx -p_{2t})$ jets with rapidities y_1 and y_2 and no hadrons between them except the Higgs boson and the secondary particles from its decay. (see Fig. 2 where this event is drawn in a lego–plot). However, the diagram of Fig. 1 can not give the value of the cross section for the event with such a striking signature. Indeed each parton with $x > x_1$ can interact with a parton with $x < x_2$. (in lab frame, see Fig. 3) and such an interaction generally speaking produces a lot of partons (hadrons) with rapidities between y_2 and y_1

Thus to calculate the cross section of Higgs production with the rapidity gap we need to multiply the value calculated by eq. (1) by factor $< S^2 >$ which gives the survival probability of the rapidity gap. Bjorken suggested a formula [1] that allows one to calculate $< S^2 >$.

In the next section I am going to discuss this formula trying mainly to clear up its physical meaning and to point out some shortcoming of the Bjorken approach.

In section 3 I'll consider the survival probability of the large rapidity gaps in the deep inelastic scattering. The equation for $< S^2 >$ will be derived and a solution will be found that gives the possibility to discuss the large rapidity gap processes on a solid theoretical ground.

Section 4 is devoted to concluding comments and suggestions for further theoretical work.

II. Bjorken's formula

To start with let me remind youof several general properties of hadron-hadron collisions originating from analiticity, crossing symmetry and unitarity on the amplitudes of elastic scattering.

1.) S-channel unitarity looks simple in the impact parameter representation in which the elastic amplitudes $a(s, b_t)$ are related to the amplitude in s, t representation $f(s, t)$ in the following way

$$a(s, b_t) \; = \; \frac{1}{2\pi} \int d^2 q_t \, e^{i(\mathbf{b_t}, \mathbf{q_t})} \; f(s, t = -q_t^2) \; ; \tag{2.1}$$

$$f(s, t = -q_t^2) \; = \; \frac{1}{2\pi} \int d^2 b_t \, e^{-i(\mathbf{b_t}, \mathbf{q_t})} \; a(s, b_t) \; .$$

2.) S-channel unitarity gives the relation between $a(s, b_t)$ and $G_{in}(s, b_t)$, where $G_{in}(s, b_t)$ is the contribution of all inelastic interactions to the imaginary part of the elastic amplitude. Thus, unitarity says

$$2 \, Im \, a(s, b_t) \; = \mid a(s, b_t) \mid^2 \; + \; G_{in}(s, b_t) \; . \tag{2.2}$$

3.) All observables can be rewritten through $a(s, b_t)$, $f(s, b_t)$ and $G_{in}(s, b_t)$ in the following form.

$$\begin{aligned} \sigma_{tot} \; &= \; 2\pi \int d^2 b_t \; Im \, a(s, b_t); \\ \sigma_{in} \; &= \; \pi \int d^2 b_t \, G_{in}(s, b_t) \, ; \\ \sigma_{el} \; &= \; \pi \int d^2 b_t \mid a(s, b_t) \mid^2 ; \\ \sigma_{tot} \; &= \; \sigma_{el} \; + \; \sigma_{in} \; ; \\ \sigma_{tot} \; &= \; 4\pi f(s, t = 0) \; (\text{optical} \quad \text{theorem}) \; . \end{aligned} \tag{2.3}$$

4.) At high energy assuming that $a(s, b_t)$ is mostly imaginary we can introduce function $g(s, b_t)$ as

$$a(s, b_t) = i \, (1 - g(s, b_t)) \tag{2.4}$$

In eq. (2.4) $g(s, b_t)$ is real and has a very simple physical meaning, namely $g^2(s, b_t)$ is the probability that the hadron does not interact inelastically. Indeed, directly from unitarity constraint of eq.(2.2) we can see that

$$G_{in}(s,b_t) = 1 - g^2(s,b_t) \tag{2.5}$$

so $G_{in}(sb_t)$ is the probability of an inelastic interaction while g^2 is just the probability that the hadron penetrates through the target without inelastic interaction.

5.) In the eikonal model

$$g^2(s,b_t) \;=\; e^{-\Omega(s,b_t)} \tag{2.6}$$

and Bjorken in his calculation used

$$\Omega(s,b_t) \;=\; \nu\, e^{-\frac{b_t^2}{R^2(s)}} \tag{2.7}$$

It is easy to check that eq.(2.6) gives usual formula for the eikonal description of the rescattering processes in hadron–hadron interaction (for a couple of recent papers see ref.[7]).

We have prepared all ingredients to get Bjorken's formula for $< S^2 >$ but we need to rewrite the formula of eq. (1.1) for our inclusive cross section in b_t representation. For this purpose let introduce the function $\Psi(b_t, q^2, x)$ such that

$$\phi(x,q^2) \;=\; \int d^2 b_t \Psi(b_t, q^2, x) \tag{2.8}$$

We also can consider our hard Higgs production process to be located at very small value of b_t Indeed, just from uncertainty principle $b_t \sim 1/p_{1t}$ in our process, but for Higgs production $p_{1t} \sim p_{2t} \sim M_W$. So, the typical value of b_t for our hard process turns to be very small. Finally

$$f(b_t, y, \Delta y) \;= \tag{2.9}$$

$$= \int \Psi(x_1, q_{1t}^2, b_t')\, d^2 q_{1t} \sigma_{hard}(b_t' - b_t'', \Delta y)\, \Psi(x_2, q_{2t}^2, b_t - b_t'')\, d^2 q_{2t}\, d^2 b_t'\, d^2 b_t'' \;=$$

$$= \int \Psi(x_1, q_{1t}^2, b_t'^2)\, d^2 q_{1t}\, \Psi(x_2, q_{2t}, (b_t - b_t')^2)\, d^2 q_{2t} \sigma_{hard}(0, \Delta y)\, d^2 b_t'$$

Using eq. (2.9), we can write down the expression for $< S^2 >$, that is equal to

$$< S^2 > = \frac{\int g^2(s,b_t)\, f(b_t, Y, \Delta y\,)\, d^2 b_t}{\int f(b_t, Y, \Delta y)\, d^2 b_t} \tag{2.10}$$

Using eikonal approximation for $g^2(s, b_t^2)$ (see eqs. (2.6) and (2.7)) and assuming that

$$f(b_t, Y, \Delta y)) = \frac{1}{R_0^2} e^{-\frac{b_t^2}{R_0^2}} , \qquad (2.11)$$

the formula of eq. (2.10) can be reduced to Bjorken formula (see ref. [1]). It should be stressed that R_0^2 in eq. (2,10) is responsible for t dependence of the deep inelastic structure functions (ϕ) in eq. (1.1) and it does not depend on energy in the Leading Log Approximation (LLA) of perturbative QCD.

The simplest way to understand the physical meaning of eq. (2.5) (or eq. (2.6)) is to consider the hadron-hadron interaction in the parton model in lab frame where one ofthe colliding hadrons is at rest. (see Fig. 4.).

$\Omega(s, b_t)$ describes the probability of interaction of one wee parton coming from one " ladder" (Fig. 4) with the target. Thus eq. (2.6) gives the probability that the incoming hadron does not interact with the target via the exchange of one "ladder" (or in other words via the exchange of one Pomeron). Now it is clear that eq.(2.5) and (2.6) cannot be justified since we have no arguments why more complicated diagrams (see Fig. 5 for example) do not contribute to the expression for $g^2(s, b_t)$. However a hadron-hadron collision is not a good laboratory to study true structure of the parton cascade and the generalization of formulas (2.5) and (2.6). Unfortunately we have no theory to describe hadron-hadron interaction with guaranteed theoretical accuracy and we use a model approaches to discuss so called "soft" hadron physics. That is the reason why I prefer to switch to the deeply inelastic process for a generalization of Bjorken's formula (see eqs. (2.5) (2.6)(2.10) and (2.11); here we can use QCD as the theory of the parton cascade and we can arrive at a definite conclusion with respect to the survival probability of the large rapidity gap.

III. The survival probability in deep inelastic scattering

1.) *The structure of the parton cascade in QCD.*

In deeply inelastic scattering the structure of the parton cascade is described by function $\phi(y = \ln 1/x_B, q_t^2)$ (see eq. (1.2)) that gives the number of partons with $y = \ln 1/x_B$ and $r = \ln q^2/\Lambda^2$, in the area of the order of $1/q^2$ in the transverse phase. In other words ϕ is just the number of partons with fixed y and r that can interact with the virtual probe.

The evolution equation for ϕ in the region of small x_B looks as follows [2,8]:

$$\frac{\partial\phi(y,q_t^2)}{\partial y} = \frac{N_c\alpha_s}{\pi}\int K(q^2,q'^2)\,\phi(y,q_t'^2)\,\frac{d^2q_t'}{2\pi}\cdot\{1-\frac{\alpha_s\phi(y,q_t^2)}{\phi_0}\} \qquad (3.1)$$

where $\phi_0 = \frac{16}{27\pi}R^2$ (see ref. [8] for details). and $K(q^2,q'^2)$ is the kernel of the $BFLK$ - equation [9]:

$$K(q^2,q'^2)\phi(y,q^2) = \frac{\phi(y,q'^2)}{(q-q')_t^2} - \frac{q_t'^2\phi(y,q^2)}{(q-q')_t^2(q_t'^2+(q-q')_t^2)}. \qquad (3.2)$$

The GLR equation (3.1) sums up so called "fan" diagrams (see Fig. 6). and takes into account the recombination of gluon as well as their emission. (see refs.[2,3] for details).

For our purposes, to get the survival probability of the large rapidity gap we have to rewrite GLR equation (3.1) in the b_t - representation as has been discussed above. It was shown in refs.[2,3] that in the leading log $\frac{1}{x_B}$ approximation of perturbative QCD the kernel of the evolution equation and the splitting functions that describe the triple ladder vextex γ (see Fig. 6) do not depend on the momentum transferred along the ladders, or in other words can be considered as δ - function in b_t-representation. It means that function $\Psi(b_t,x_B,q^2)$ could be rewritten as

$$\Psi(b_t,x_B,q^2) = \phi(x_B,q^2)F(b_t) \qquad (3.3)$$

with the normalization constraint

$$\int d^2b_tF(b_t) = 1 \qquad (3.4)$$

In the LLA we can even claim that

$$\int d^2b_t\, e^{i(\mathbf{Q_t},\mathbf{b_t})}\ F(b_t) = G(Q_t^2) \qquad (3.5)$$

where $G(Q_t^2)$ is the electromagnetic form factor of a hadron and Q_t is the momentum transferred along the single "ladder".

2.) *The evolution equation in b_t*

Now we are easily able to obtain the evolution equation in b_t which sums the "fan" diagrams if we look at Fig. 7. Fig. 7 shows the main properties of the "fan" diagrams with respect to dependence on b_t in the LLA of perturbative QCD. Taking into account the factorization of function ϕ in b_t and (y,r) we can easily write down the equation:

$$\begin{aligned}\frac{\partial\Psi(b_t;y,q^2)}{\partial y} &= \frac{\alpha_s N_c}{\pi}\int K(q'^2,q^2)\cdot\Psi(b_t,y,q'^2)\\ &\quad \{1-\frac{\alpha s}{\Psi_0}M(b_t)\Psi(b_t,y,q^2)\}\end{aligned} \tag{3.6}$$

where $\Psi_0 = \frac{16}{27}$ and function $M(b_t)$ is normalized in such a way that

$$\int d^2b_t F^2(b_t)M(b_t) = \frac{1}{\pi R^2}, \tag{3.7}$$

where R^2 is the same radius as in eq. (1.1) (see also ref. [8]). It is more convenient to rewrite eq. (3.6) introducing a slightly different function

$$\bar{\Psi}(b_t;y,q^2) = \frac{\alpha s}{\Psi_0}M(b_t)\Psi(b_t^2;y,q^2) \tag{3.8}$$

so that for $\bar{\Psi}$ the equation has the form:

$$\frac{\partial\bar{\Psi}(b_t;y,q^2)}{\partial y} = \frac{\alpha_s N_c}{\pi}\int K(q^2,q'^2)\,\bar{\Psi}(b_t;y,q'^2)\cdot\{1-\bar{\Psi}(b_t;y,q^2)\} \tag{3.9}$$

This form of the evolution equation looks much more transparent because it is clear that 1 is the unitarity limit for $\bar{\Psi}$. Namely, $\bar{\Psi}$ has the simple physical interpretation of a probability, that could be smaller or equal to 1. The differential form of eq. (3.9) also has a very simple physical meaning and reflects the fact that each parton can interact with the other emitting it's own parton branch as it is shown on Fig. 6.

It means that s-channel unitarity constraint of eq. (2.2) could be rewritten in terms of $\tilde{\Psi}$ in the following way.

$$2\,\tilde{\Psi}(b_t;y.q^2) \;=\; |\tilde{\Psi}(b_t;y,q^2)|^2 \;+\; G_{in}(b_t;y,q^2)\;. \tag{3.10}$$

If we introduce the probability $(g^2_{in}(b_t,y,q^2)$ that the parton with fixed y and q^2 does not interact with the target as

$$G_{in}(b_t;y,q^2) \;=\; 1 \;-\; g^2_{in}(b_t;y,q^2) \tag{3.11}$$

we can see from eq.(3.11) that

$$g^2_{in} \;=\; \{\;1 \;-\; \tilde{\Psi}(b_t;y,q^2)\;\}^2\;. \tag{3.12}$$

3.) The equation for the survival of large rapidity gap.

Due to simple structure of the parton cascade that is reduced to the "fan " diagrams of Fig. 6 we can easily find the probability for the large rapidity gap in deeply inelastic scattering. For this purpose we need an equation such that it takes into account the fact that the parton with $y > y_1$ do not interact inelastically with the parton with $y < y_2$ (see fig.8). Using eq. (3.12) we can write the following equation for the function (see eq. (2.10))

$$< S^2 > \int d^2b_t f(b_t,Y,\Delta y) \;=\; F(b_t;Y,\Delta y,q^2)\;:$$

$$F(b_t;Y;\Delta y,q^2) \;= \tag{3.13}$$

$$=\; F(b_t;y_1,\Delta y,q^2) \;+\; \frac{\alpha_s N_c}{\pi}\int_{y_1}^{Y} dy'dq'^2 K(q,q')F(b_t;y',\Delta y,q'^2\cdot g^2_{in}(q^2,y')$$

Substituting eq. (3.12) in eq. (3.13) we get finally

$$F(b_t;Y,\Delta y,q^2) \;=\; F(b_t,y_1,\Delta y,q^2) + \frac{\alpha_s N_c}{\pi}\int dq'^2\int_{y_1}^{Y} dy' K(q^2,q'^2)$$

$$F(b_t,y',\Delta y,q'^2)\{1-\tilde{\Psi}(b_t^2,y',q^2)\}^2 \tag{3.14}$$

where the nonintegral term is equal to

$$F(b_t,Y,\Delta y,q^2) \;=\; f(b_t^2;y_1,\Delta y,q^2)\cdot g^2_{in}(b_t^2,y_1)$$

$$= f(b_t^2,y_1,\Delta y,q^2)[\,1-\tilde{\Psi}(b_t^2,y_1)\,]^2 \tag{3.15}$$

and it plays a role of the initial condition for the differential equation

$$\frac{dF(b_t,y,\Delta y;q^2)}{dy} = \frac{\alpha_s N_c}{\pi}\int K(q^2,q'^2)F(b_t,y,\Delta y,q'^2\cdot\{1-\bar{\Psi}(b_t,y,q^2)\}^2 . \quad (3.16)$$

4). The solution.

Eq. (3.16) can be solved using semi-classical approach (see refs.[2,10]) in which we are trying to find a solution in the form

$$F(b_t,y,\Delta y,q^2) = f(b_t^2)\cdot e^{\omega y - k r} \quad (3.17)$$

where $\omega'_y, \omega'_r(k'_y,k'_r) \ll \omega(or k)$. Whithin the semi-classical approximation we can use the property of the kernel $K(q^2,q'^2)$, namely

$$\int K(q^2,q'^2)(q'^2)^{-k} = \kappa(k)(q^2)^{-k} \quad (3.18)$$

where$\kappa(k) = 2\Psi(1) - \Psi(k) - \Psi(1-k)$ and $\Psi(k) = \frac{d\ln\Gamma(k)}{dk}$(($\Gamma(k)$ is the Euler gamma function) substituting eq. (3.17) and taking into account eq. (3.18) we reduce eq. (3.16) to the form

$$\frac{dF(b_t,y,\Delta y,q^2)}{F\,dy} = \frac{\alpha_s N_c}{\pi}\kappa(k)\{1-\bar{\Psi}(b_t,y,q^2)\}^2 \quad (3.19)$$

while the evolution equation in the semiclassical approach looks like

$$\frac{d\bar{\Psi}}{\bar{\Psi}\,dy} = \frac{\alpha_s N_c}{\pi}\kappa(k)\{1-\bar{\Psi}(b_t,y,q^2)\} \quad (3.20)$$

Comparing eqs.(3.19) and (3.20) we see that

$$\frac{1}{F}\frac{dF}{dy} = \frac{d\bar{\Psi}}{\bar{\Psi}dy} - \frac{d\bar{\Psi}}{dy} . \quad (3.21)$$

From eq. (3.21) we can easily reconstruct the solution of eq. (3.14) with the initial conditions (3.15). The answer is

$$F(b_t^2,y,\Delta y,q^2) = F(b_t^2,y_1,\Delta y,q^2)\cdot\frac{\bar{\Psi}(y,b_t)}{\bar{\Psi}(y_1,b_t)}\,e^{-\bar{\Psi}(y,b_t)+\bar{\Psi}(y_1,b_t)} . \quad (3.22)$$

From eq. (3.22) we can get the formula for $< S^2 >$ dividing eq. (3.22) by the expression for the inclusive production of Higgs boson which is shown in Fig. 9.

Finally

$$< S^2 > \; = \; \frac{1}{\tilde{\phi}_{LLA}(y - y_1, q^2)} \cdot$$
$$\cdot \int d^2 b_t \cdot T(b_t) \frac{\bar{\Psi}(y, b_t, q^2)}{\bar{\Psi}(y_1, b_t, q^2)} e^{-\bar{\Psi}(y, b_t, q^2) + \bar{\Psi}(y_1, b_t, q^2)} , \qquad (3.23)$$

where

$$T(b_t) \; = \; \int d^2 b'_t F(b_t - b'_t) F(b'_t) \; .$$

$F(b_t)$ was introduced in eq. (3.3). From the dipole formula for electromagnetic form factor of the proton we obtain that

$$T(b_t) \; = \; \frac{1}{48} m^5 b_t^3 K_3(m b_t) \; .$$

if

$$G(t) \; = \; \frac{1}{[1 \; + \; \frac{|t|}{m^2}]^2} \; .$$

$\tilde{\phi}_{LLA}$ is the solution of the linear evolution equation,namely eq.(3.9) ($\bar{\Psi} = \tilde{\phi} F(b_t)$) without the nonlinear term. This contribution appears in eq. (3.23) since in the expression for the inclusive cross section all shadowing corrections between partons with $y > y_1$ and with $y < y_2$ cancel due to AGK - cutting rules.

5.) b_t - dependence of $\bar{\Psi}$.

To complete the discussion of the rapidity gap in the deep inelastic scattering we need to know the b_t - dependence of the structure function $\bar{\Psi}$. Eq.(3.9) contains no explicit dependence on b_t. It means that the whole dependence on b_t originates from the initial condition or, better said, from the matching procedure with the solution of the linear evolution equation. The solution of such a kind of nonlinear equation as eq.(3.9) has been discussed in ref.[11] and the method developed there can be easily generalized for eq. (3.9). However we prefer to use here the semi-classical approach,developed for this equation in refs. [2,10] to illustrate all properties of the solution and the matching procedure in an explicit way.

To simplify the algebra we also prefer to solve the equation in Double Log Approximation (DLA) in which the kernel $\kappa(k)$ has a very simple form

$$\kappa(k) \; = \; \frac{1}{1 \; - \; k}$$

(see ref.[11] for details). Using the general approach of ref.[11] we can rewrite eq.(3.9) in the form:

$$\frac{d^2\bar{\Psi}(b_t,t)}{dt^2} = \frac{d\bar{\Psi}(b_t,t)}{dt} - \frac{1}{4}\bar{\Psi}[1 - \bar{\Psi}], \tag{3.24}$$

introducing new variable

$$t = \frac{4N_c\alpha_s}{\pi}(y - y_0) - (r - r_0), \tag{3.25}$$

where $y = \ln(1/x_B)$ and $r = \ln(q^2/\Lambda^2)$. We also neglected the dependence of α_s on r for simplicity.

For the solution of eq. (3.24) in the semi-classical approach we make the ansatz:

$$\bar{\Psi} = e^{\Omega(b_t,t)}. \tag{3.26}$$

Assuming that $\Omega(b_t,t)$ is a smooth function of t ($\frac{d^2\bar{\Psi}}{dt^2} \ll (\frac{d\bar{\Psi}}{dt})^2$) we can calculate

$$\frac{d^2\bar{\Psi}}{dt^2} = [\frac{d\Omega}{dt}]^2 e^{\Omega}$$

and obtain a simple equation for Ω:

$$\{\frac{d\Omega}{dt}\}^2 - \frac{d\Omega}{dt} + \frac{1}{4} = \frac{1}{4}e^{\Omega}. \tag{3.27}$$

This equation can easily be solved:

$$\bar{\Psi} = \frac{1}{[1 + C(b_t)e^{-\frac{t}{4}}]^2}. \tag{3.28}$$

The matching procedure goes as follows:

1. At $t \to 0$ $\bar{\Psi}$ should be equal to

$$\bar{\Psi}(b_t,q^2,x_B) = \frac{\alpha_s}{\Psi_0}M(b_t)F(b_t)\cdot\phi_{LLA}(x_B,q^2), \tag{3.29}$$

where ϕ_{LLA} is the solution of the linear GLAP evolution equation [12].

2.At $t = 0$ $\phi_{LLA} = 1$. So one can find $C(b_t)$ from eq.(3.19).

$$C(b_t) = \frac{\sqrt{\Psi_0}}{\sqrt{\alpha_s}}[M(b_t)F(b_t)]^{-\frac{1}{2}} - 1. \tag{3.30}$$

Unfortunately we do not know anything about $M(b_t)$ except eq.(3.8). However we can consider two limiting cases:

1. R^2 in eq. (3.8) is the radius of a hadron. It means that gluons are uniformly distributed in the whole hadron disc. It seems natural to assume that $M(b_t)$ is $Const(b_t)$. Thus

$$C(b_t) \;=\; \sqrt{\frac{\Psi_0}{\alpha_s}}\{F(b_t)\}^{-\frac{1}{2}}\;. \tag{3.31}$$

For the nucleon $F(b_t)$ is given by

$$F(b_t) \;=\; \frac{m^2 b_t}{2}K_1(mb_t)\;,$$

from the dipole formula for the electromagnetic form factor of the proton. At large b_t $F(b_t) \rightarrow exp(-mb_t)$ so $C(b_t) \propto exp[(mb_t)/2]$.

From eq.(3.18) we see that $\bar{\Psi}$ is of $O(1)$ in this case only if

$$mb_t < \frac{1}{2}t = \frac{2N_c\alpha_s}{\pi}(y - y_0) - \frac{1}{2}(r - r_0)\;. \tag{3.32}$$

Thus the eq. (3.18) gives us the typical picture of the black disc for a hadron when $y \rightarrow \infty$. It should be stressed that eq. (3.18) with $C(b_t)$ from eq. (3.21) leads to a distribution over b_t which is quite different from the eikonal formula (see eqs. (2.10) and (2.11)).

2. R^2 in eq.(3.7) is small($R \sim 0.1Fm$). It means that we assume the picture of a hadron with two radii: radius of the hadron ($R_h \sim 1Fm$) and radius of the constituent quark ($R_Q \sim 0.1Fm$). The gluons are confined in the disc with the smallest radius. In this case

$$M(b_t) \;=\; F_Q(\frac{b_t}{R_Q})F^{-2}(\frac{b_t}{R_h})\;. \tag{3.33}$$

We do not know the exact form of F_Q but assuming the same dipole formula as for proton form factor but with typical mass M larger that m ($m \propto \frac{1}{R_Q} \gg m \propto \frac{1}{R_h}$ we get

$$C(b_t) \;=\; \sqrt{\frac{\Psi_0}{\alpha_s}}\{F_Q(\frac{b_t}{R_Q})F(\frac{b_t}{R_h})\}^{-\frac{1}{2}} \tag{3.34}$$

and for large b_t we have

$$C(b_t) \;\propto\; e^{\frac{M-m}{2}b_t}\;, \tag{3.35}$$

which gives us the same asymptotic behaviuor of the interaction radius as in the previous case.

Looking at eq. (3.18) we can even conclude that our matching procedure can be reduced to the statement, that our solution is the solution for the function ϕ (see eq. (3.1)) which depends on new variable

$$t(b_t) \;=\; t \;-\; 4\ln[\sqrt{\frac{\alpha_s}{\Psi_0}}C(b_t)]\;. \tag{3.36}$$

In particular, the correct solution of the problem is

$$\tilde{\Psi} \;=\; \frac{1}{\pi R^2}x_{sat}(t(b_t))\;, \tag{3.37}$$

where x_{sat} is described in ref.[11].

Eq. (3.28) gives the full answer to the question what is the b_t - dependence of the deep inelastic structure function assuming:

1. the GLR evolution equation (eqs. (3.1) and (3.8) in the whole kinematic region of y and r.

2. the δ - function in b_t for the kernel and vertices in the LLA of perturbative QCD.

More detail analysis of possible improvements of the answer should include the random walk of the partons in b_t. The first attempt was made in ref. [13] but much more work is needed to obtain deeper understanding of this problem.

IV. Conclusions.

The main result of the paper is formula (3.23) for the survival probability of the large rapidity gap in deeply inelastic scattering. Let me stress several points that could be instructive for further development of large rapidity gap physics in hadron - hadron collisions.

1. Eq. (3.23) is quite different from the Bjorken formula obtained in the eikonal approximation. It is very important for the origin of this difference to be absolutely transparent,namely: the correct formula takes into account the absence of the inelastic interaction between partons with $y > y_1$ and $y < y_2$, while Bjorken formula simplifies

the situation and replaces such a probability by the probability that a wee parton does not interact with the target inelastically.

2.Eq. (3.23) was obtained assuming that the GLR evolution equation (eqs. (3.1) and (3.9)) is valid in the whole kinematic region of deeply inelastic scattering. IN fact the GLR is only proven in the restricted kinematic region where $\tilde{\Psi}$ is still small ($\tilde{\Psi} \propto \alpha_s$). So we can trust the result only for sufficiently small values of $\tilde{\Psi}$.

3. Eq. (3.23) could be considered as the simplest case, since the real structure of the parton cascade looks much more complicated.

V. Acknowledgements

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Figure Captions.

Fig. 1: The process of Higgs boson production in hadron - hadron collisions.

Fig. 2: Lego -plot for the structure of the inelastic event for Higgs boson production in hadron - hadron collisions.

Fig. 3: The parton - parton interaction inside the parton cascade for Higgs boson production in hadron - hadron collisions.

Fig. 4: The structure of the parton cascade related to the eikonal approximation.

Fig. 5: The space - time picture of the parton cascade in hadron - hadron collisions in lab. frame.

Fig. 6: The structure of the parton cascade in deeply inelastic scattering.

Fig. 7: The equation for $\Psi(b_t, y, q^2)$.

Fig. 8: Graphical representation of the equation for the sum of "fan" diagrams for Higgs boson production in deep inelastic scattering.

Fig. 9: The inclusive cross section for Higgs boson production in deeply inelastic scattering.

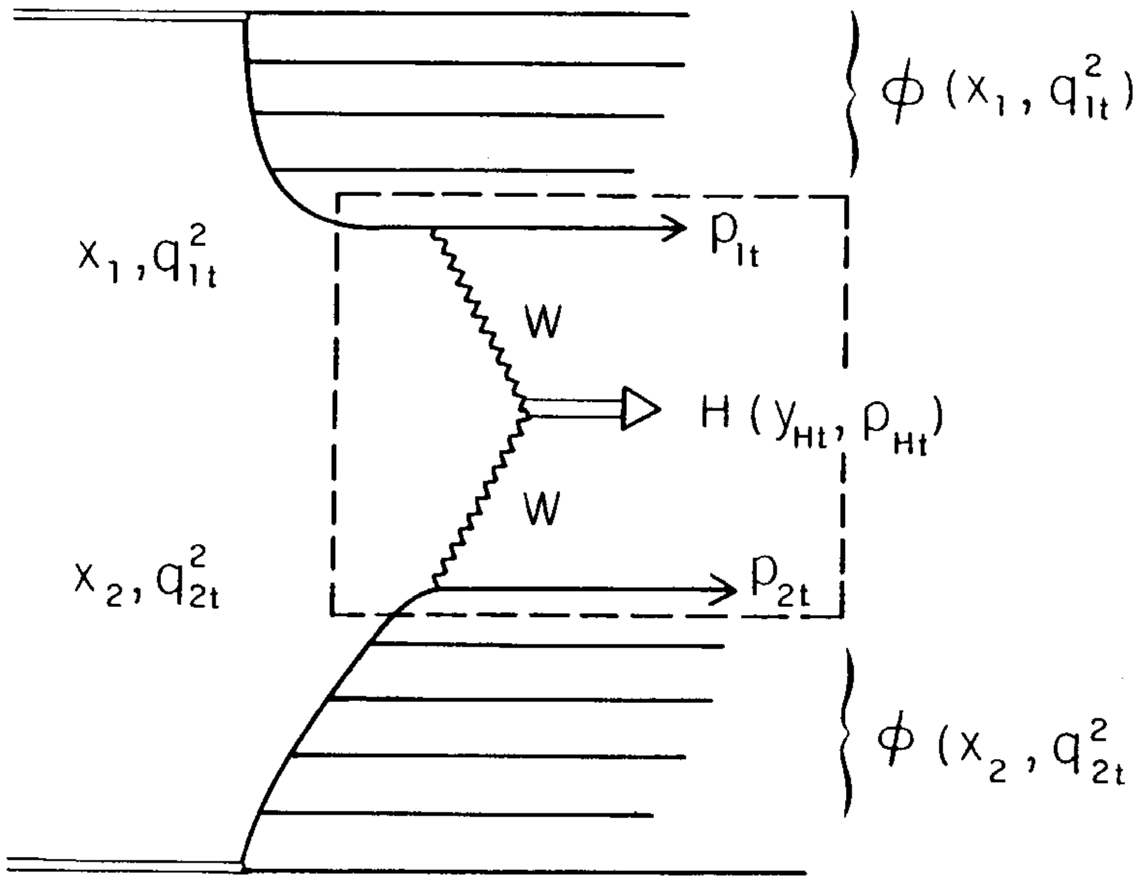


Fig. 1

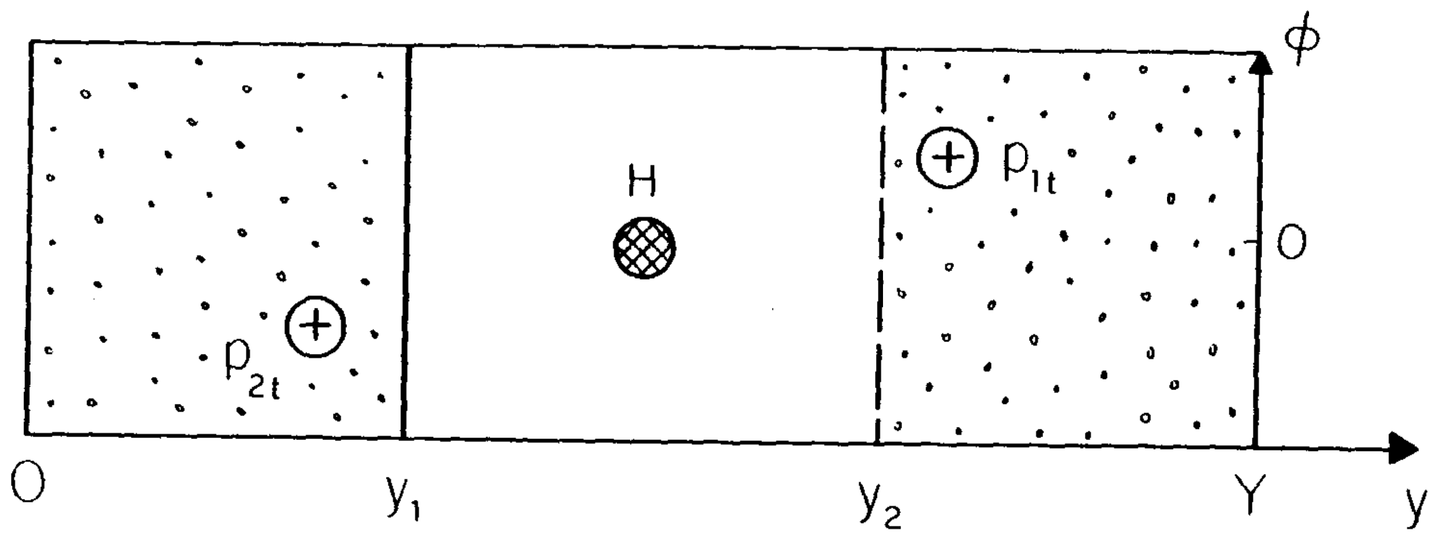


Fig. 2

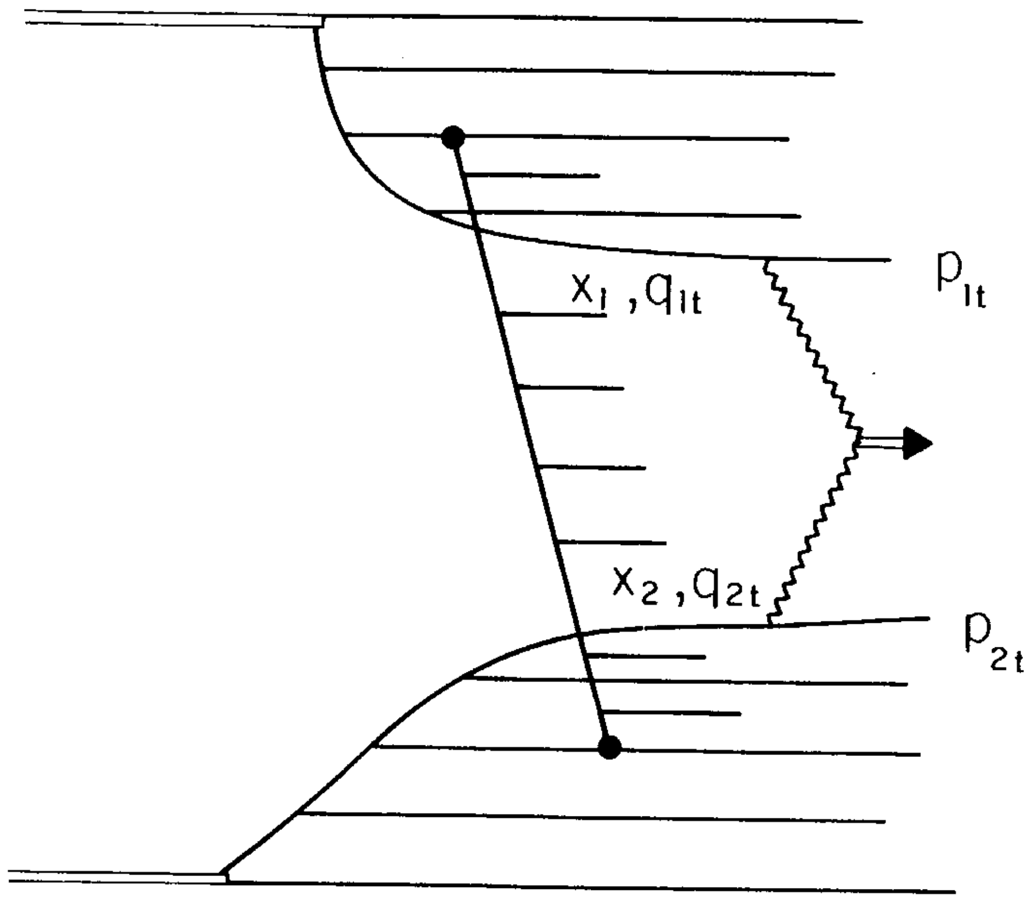
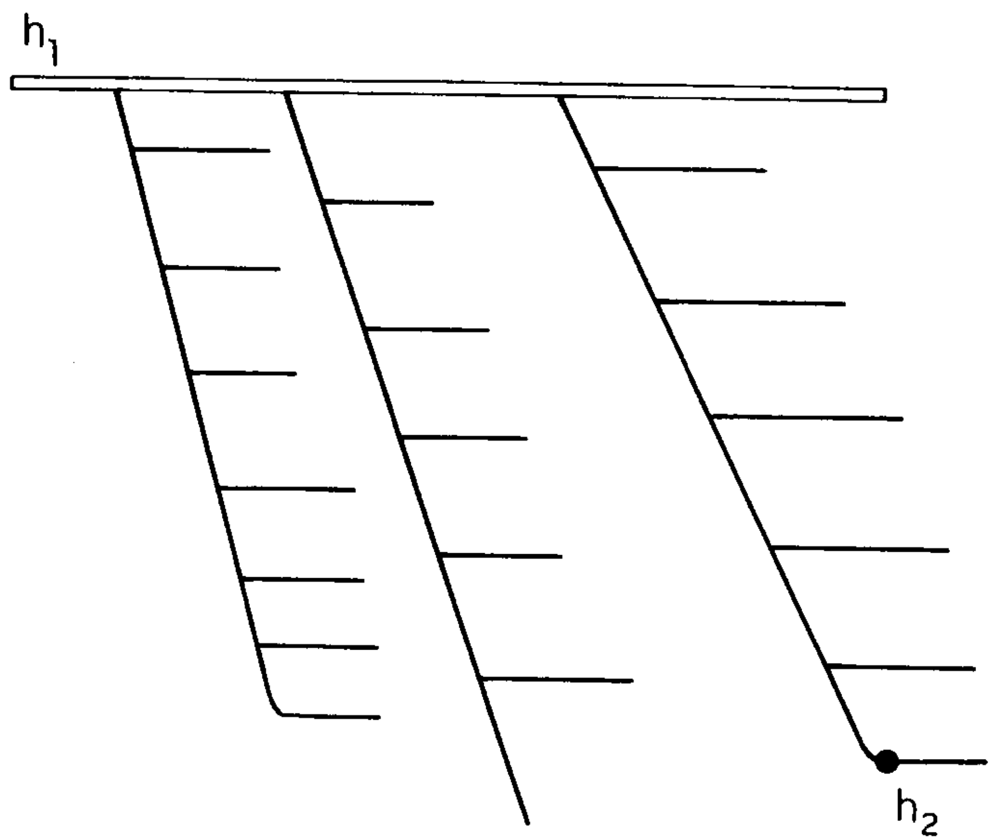


Fig. 3



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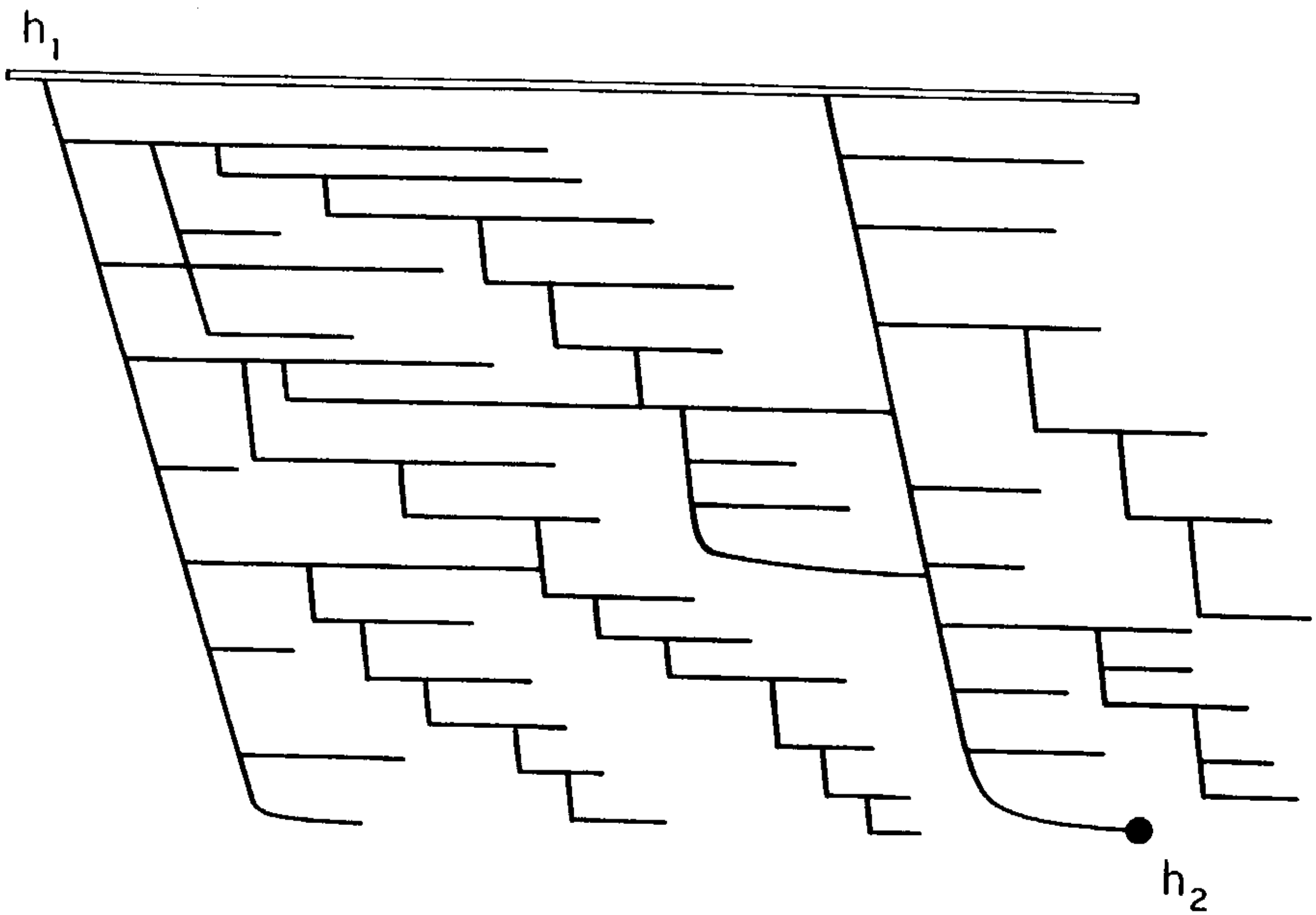

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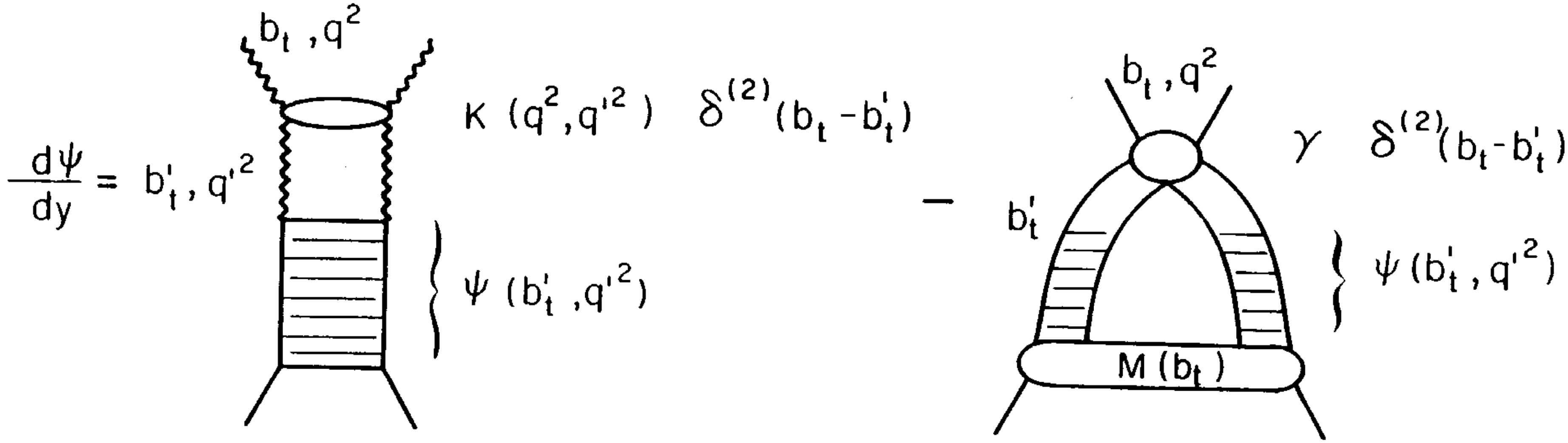

Fig. 6

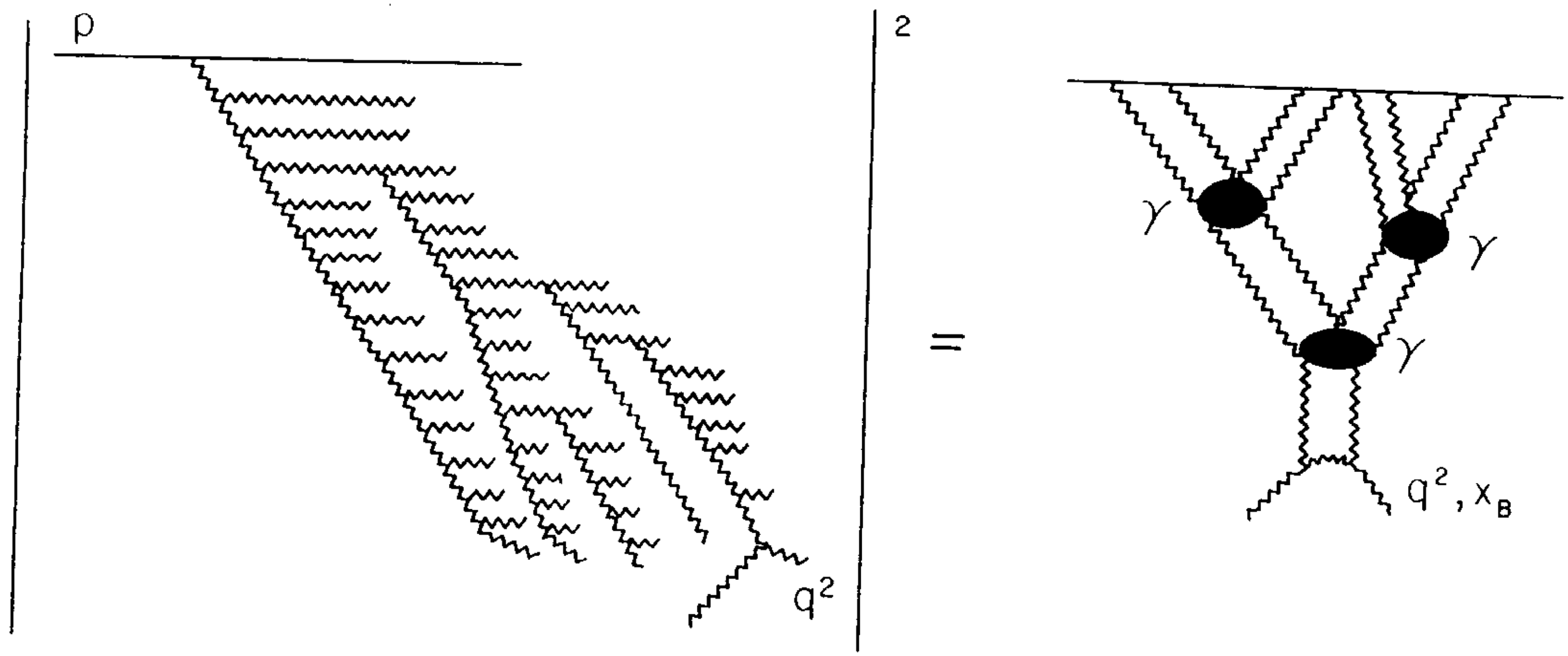

Fig. 7

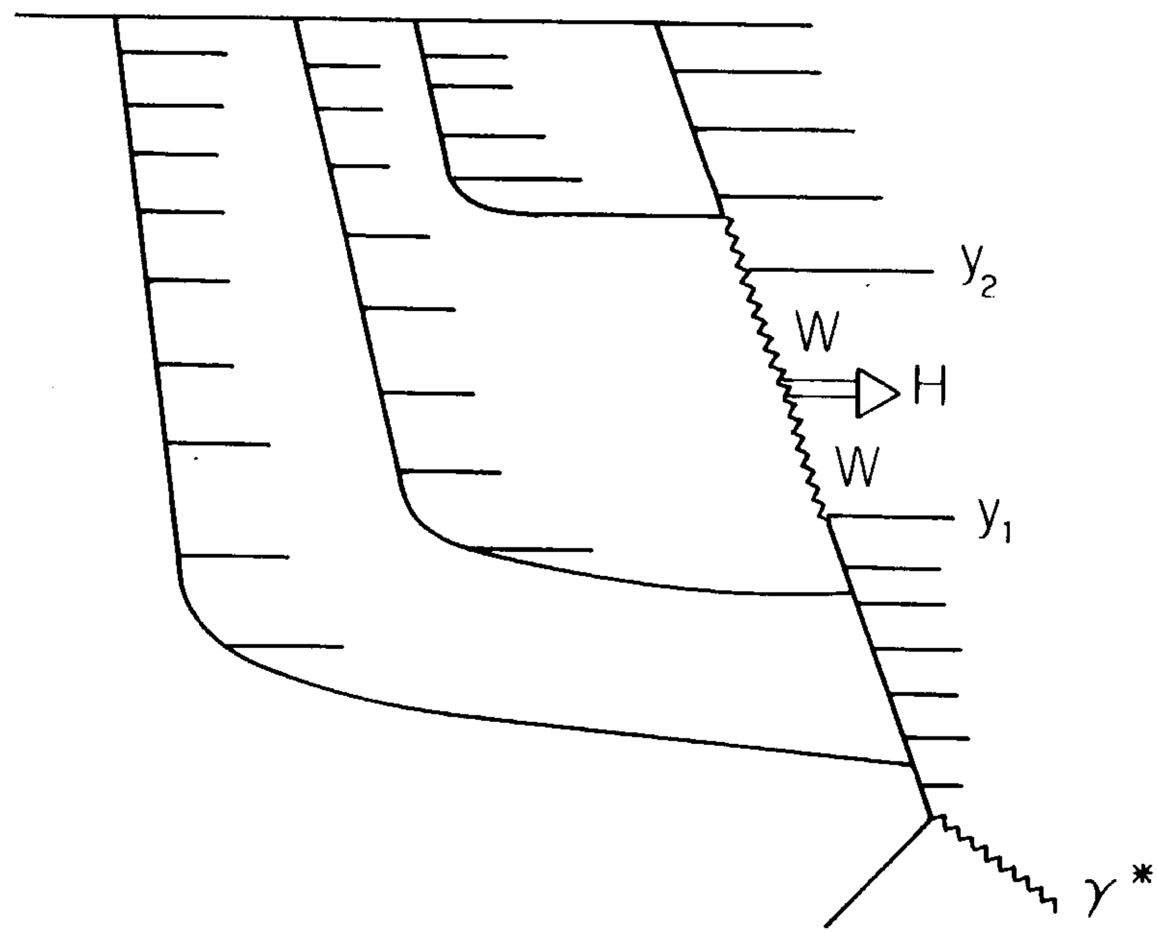

Fig. 8

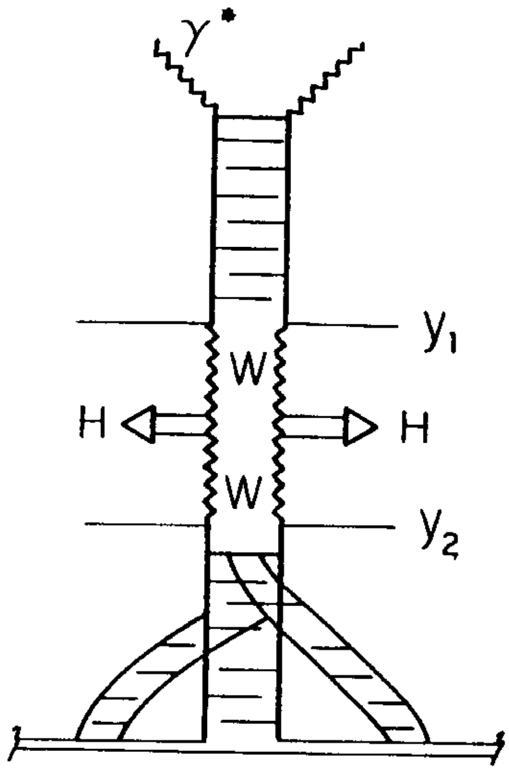

Fig. 9