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A Potential Window to the GUT*

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Abstract

The possibility that the form of the inflationary potential can be reconstructed from a knowledge of the primordial power spectra of scalar (density) and tensor (gravity wave) perturbations is discussed and reviewed. It is suggested that measurements of the scalar spectral index and the amplitude of the tensor spectrum at one scale should be possible in the near future. This would be sufficient to determine whether the potential is convex or concave over the limited region relevant to the formation of large-scale structure in the universe. Such information would provide an 'observation' of physics at energy scales above 10^{14}GeV.

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1 Introduction

Einstein described the introduction of a cosmological constant into the field equations of general relativity as 'the biggest blunder of his life.' In view of this, the idea that such a term might play an influential role in the history of the universe has proved remarkably popular. Theoretical cosmologists have traditionally looked to such a term when attempting to reconcile theory with observation and indeed Einstein's original motivation for considering a Λ-term was to cancel the theoretically predicted expansion of the universe [1]. Some time later the steady-state scenario was developed to account for the apparently low age of the universe deduced from observations of the expansion rate [2]. In recent years the inflationary scenario has been developed in an attempt to resolve some of the puzzles of the hot big bang model [3]. Inflation explains why the universe appears remarkably homogeneous and spatially flat on large scales and why monopoles formed during phase transitions are not observed at the present epoch.

During inflation the universe is dominated by the self-interaction potential energy $V(\phi)$ of a quantum scalar field ϕ and this false vacuum energy plays the role of an effective cosmological constant for a finite time interval. This vacuum energy leads to a negative pressure and hence gravitational repulsion. In this scenario the matter content of the observed universe is formed when the false vacuum decays at the end of inflation. Historically, Gliner [4] first realized that a positive vacuum energy density is equivalent to a cosmological constant and this idea was extended by Zel'dovich [5]. However, the idea that matter came from the expansion of a universe dominated by a cosmological constant can be traced to a paper by McVittie [6]. He found that a negative pressure in general relativity can be converted into matter due to the expansion of the universe if the quantity $\rho + 3p$ is constant, where ρ and p are the energy density and pressure of the matter sector. This quantity is approximately constant during inflation.

Current estimates suggest that over 1600 papers have been published on various aspects of the inflationary scenario since its proposal by Guth and others in 1981 [3]. However, despite such extraordinary effort there remain some unresolved problems with the scenario. From a particle physics viewpoint one of the most pressing problems with inflation is the identity of the scalar field. Essentially the problem is that there are too many candidates. These include the Higgs bosons of grand unified theories, the extra degrees of freedom associated with higher metric derivatives in extensions to general relativity, a time-varying cosmological constant and the radius of the internal space in Kaluza-Klein theories (for a recent review see [7]).

In general this scalar field is loosely referred to as the *inflaton*. Traditionally one chooses a specific model of particle physics and then compares the theoretical predictions of the model with observation. This enables the region of parameter space consistent with observation to be determined. However, in view of the large number of plausible models currently on the market, and motivated by recent advances and results in observational cosmology, one might ask whether observations of large-scale structure in the universe can be employed to 'reconstruct' the particle physics and in particular the functional form of the inflationary potential [8-10]. We know from the observed quadrupole anisotropy of the cosmic microwave background radiation (CMBR) [11] that the final stages of the inflationary epoch occurred at or below the grand unified (GUT) scale and this method therefore provides a potential window on the physics of the early universe at scales of order 10^{16}GeV. This corresponds to 10^{-35}s after the moment of creation. The purpose of this talk then is to review the reconstruction procedure and discuss whether such a suggestion is feasable.

The different inflationary models may be classified into two main groups. Those in the first end via completion of a strong first-order phase transition, whilst those in the second end when the scalar field moves from a relatively flat region to a steeper region of its potential (i.e. the slow-roll to waterfall transition). In many cases first-order inflation models can be expressed as slow-roll models after an appropriate conformal transformation on the metric tensor and in this talk we shall therefore adopt the working hypothesis that inflation was driven by a single, self-interacting scalar field minimally coupled to Einstein gravity. We further assume that the initial conditions were specified by some appropriate quantum theory of gravity at the Planck scale. In general this is the simplest scenario and we shall refer to it as *basic inflation*. It is certainly possible that a more complicated theory might be required to accurately describe the physics of the very early universe, but at this stage in the game we should at least attempt to rule out the simplest models first before considering other possibilities further.

2 The Formation of a Primordial Power Spectrum

In reconstruction we do not choose a particle physics model as an initial condition. Consequently we may assume *nothing* about the functional form of the potential *except that it leads to an epoch of inflationary expansion.* To proceed, therefore, we must first consider how the dynamics of the scalar field can be determined in full generality. Only the last 60 e-foldings or so of inflationary expansion have significant observational consequences and in a very real sense this represents the final stages of the inflationary expansion. It is therefore reasonable to assume that any initial anisotropies and inhomogeneities in the topology of the space-time have been smoothed out by this stage. For the purposes of reconstruction, therefore, one may assume the Friedmann-Robertson-Walker (FRW) metric. It follows that the energy and momentum equations for a universe dominated by the inflaton field are

$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right) - \frac{k}{a^2} \tag{2.1}$$

and

$$2\dot{H} = -\kappa^2\dot{\phi}^2 + \frac{k}{a^2}, \tag{2.2}$$

where $a(t)$ is the scale factor of the universe, $H \equiv \dot{a}/a$ is the expansion rate, a dot denotes differentiation with respect to cosmic time t and $\kappa^2 \equiv 8\pi m_P^{-2}$, where m_P is the Planck mass. The spatial sections of the space-time are open, flat or closed for $k = \{-1, 0, +1\}$ respectively and we choose units such that $\hbar = c = 1$.

The field equation for the scalar field represents the local conservation of energy and momentum. It follows directly from the geometrical property that the boundary of a boundary is identically zero:

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \tag{2.3}$$

where a prime denotes differentiation with respect to ϕ. The time dependence in the energy equation may be eliminated by rewriting the scalar field equation in terms of the energy density $\rho \equiv \frac{1}{2}\dot{\phi}^2 + V$ [12]. If $\dot{\phi}$ does not pass through zero during the interval of interest (i.e. the field does not oscillate), Eq. (2.3) simplifies to

$$\rho' = -3H\dot{\phi}, \qquad \dot{\phi} \neq 0. \tag{2.4}$$

The general solution to this equation is

$$t = -3\int^{\phi} d\phi' H(\phi') \left(\frac{d\rho}{d\phi'}\right)^{-1} \tag{2.5}$$

and implies that the inflaton field may be employed as an effective time coordinate. It follows that $6H^2 = -\rho' X'/X$, where $X(\phi) \equiv a^2(\phi)$, and the energy equation reads

$$\rho' X' + 2\kappa^2 \rho X = 6k. \tag{2.6}$$

The field equations reduce to the remarkably simple form

$$2H'a' = -\kappa^2 H a, \qquad 2H' = -\kappa^2 \dot{\phi}, \tag{2.7}$$

for $k = 0$ and substitution of Eq. (2.7) into Eq. (2.1) determines the potential via the Hamilton-Jacobi differential equation [13]:

$$(H')^2 - \frac{3}{2}\kappa^2 H^2 = -\frac{1}{2}\kappa^4 V(\phi). \tag{2.8}$$

For reasons of technical simplicity, inflation is often discussed within the context of the slow-roll approximation, but the Hamilton-Jacobi formalism allows the dynamics of the inflaton field to be investigated in full generality. Indeed, this framework has further applications within the context of higher-order and scalar-tensor gravity theories which are conformally equivalent to Einstein gravity minimally coupled to a self-interacting scalar field [14].

One may define the two parameters [15]:

$$\epsilon \equiv 3\frac{\dot{\phi}^2/2}{V + \dot{\phi}^2/2} = \frac{2}{\kappa^2}\left(\frac{H'}{H}\right)^2 \tag{2.9}$$

$$\eta \equiv -\frac{\ddot{\phi}}{H\dot{\phi}} = \frac{2}{\kappa^2}\frac{H''}{H} = \epsilon - \frac{\epsilon'}{\sqrt{2\kappa^2\epsilon}}. \tag{2.10}$$

Modulo constants of proportionality ϵ is a measure of inflaton's kinetic energy relative to its total energy density and η measures the ratio of the field's acceleration relative to the friction acting on it due to the expansion of the universe. We shall refer to them as the *energy* and *friction* parameters respectively. The slow-roll approximation applies when the magnitudes of these parameters are small in comparison to unity, i.e. $\{\epsilon, |\eta|\} \ll 1$. Inflation proceeds when $\ddot{a} > 0$ and this is equivalent to the condition $\epsilon < 1$. It is interesting that only $H(\phi)$ and its first derivative determine whether the strong energy condition is violated and in principle inflation can proceed if $|\eta|$ is very large. This is the case if the field is located within the vicinity of a local maximum in the potential.

This provides us with sufficient background to determine the perturbation spectra. Let us briefly review how primordial fluctuations are generated during inflation [16]. Density perturbations, $\delta\rho$, arise after the field has rolled down the potential well. Quantum fluctuations in the field during inflation produce a time shift in how quickly the rollover occurs, thus producing a $t \neq$ constant hypersurface for $\delta\rho =$ constant. In other words, for a $t =$ constant 3-surface, there is a density distribution produced by the kinetic energy of the inflaton field.

In a universe with density field $\rho(\mathbf{x})$ and mean density ρ_0, the density contrast is defined as $\delta(\mathbf{x}) = \delta\rho(\mathbf{x})/\rho_0 = (\rho(\mathbf{x}) - \rho_0)/\rho_0$. This contrast is most conveniently expressed as a Fourier expansion $\delta(\mathbf{x}) \propto \int d^3k\delta_k \exp(-i\mathbf{k}.\mathbf{x})$, where we ignore the constant of proportionality. The density perturbation on a scale λ is then given by

$$\left(\frac{\delta\rho}{\rho}\right)^2_\lambda \propto k^3|\delta_k|^2\Big|_{\lambda=k^{-1}} . \tag{2.11}$$

If the strong energy condition is violated, physical length scales grow more rapidly than the Hubble radius H^{-1}, so a given scale that starts sub-Hubble radius can pass outside the Hubble radius during inflation and reenter after inflation during the radiation- or matter-dominated epochs. This implies that quantum fluctuations associated with a given length scale will be present when that length scale reenters the Hubble radius. The amplitude of this fluctuation when it reenters after inflation is given by

$$\left(\frac{\delta\rho}{\rho}\right)^{\mathrm{hor}}_\lambda \equiv \frac{m}{\sqrt{2}}A_S(\phi) = \frac{m\kappa^2}{8\pi^{3/2}}\frac{H^2(\phi)}{|H'(\phi)|}, \tag{2.12}$$

where the quantities on the right-hand-side are to be evaluated when the scale λ crossed the Hubble radius during inflation [16]. In the uniform Hubble constant gauge the constant m equals 4 or 0.4 if the fluctuation reenters during the radiation- or matter-dominated era respectively.

In a similar fashion quantum fluctuations in the graviton field are also redshifted beyond the Hubble radius during inflation. The gravitational wave (tensor) spectrum is calculated in the transverse-traceless gauge, where $+$ and $\times$ denote the two independent polarization states of the metric perturbation. The classical amplitude of the fluctuation satisfies the massless Klein-Gordon equation, so the graviton behaves as a massless, minimally coupled scalar field with two degrees of freedom $\psi_{+,\times}$. The spectrum of tensor fluctuations is then given by

$$A_G(\phi) = \frac{\kappa}{4\pi^{3/2}}H(\phi), \tag{2.13}$$

where once again the quantities on the right-hand-side are evaluated when the scale first crosses the Hubble radius during inflation [17].

The amplitude of the scalar fluctuations at horizon crossing is related to the primordial power spectrum $P(k)$ via the relationship $P(k) \propto A_S^2(k)k \propto k^{n(k)}$, where the function $n(k)$ defines the *spectral index*. The spectral indices of both the scalar and tensor fluctuations may be expressed in terms of the energy and friction parameters:

$$1 - n \equiv d\ln[A_S^2(\lambda)]/d\ln\lambda = 2\left(\frac{2\epsilon_* - \eta_*}{\epsilon_* - 1}\right) \tag{2.14}$$

$$n_G \equiv d\ln[A_G^2(\lambda)]/d\ln\lambda = \frac{2\epsilon_*}{1-\epsilon_*}, \tag{2.15}$$

where $*$ indicates that the energy and friction parameters should be evaluated when a particular scale first crosses the Hubble radius [9,15,18].

This concludes our review of scalar field dynamics and the formation of primordial power spectra in the very early universe. We shall now summarize the observational predictions generic to basic inflation and will then discuss the formalism that enables a reconstruction of the inflationary potential to be made, at least in principle!

3 Observational Predictions of Basic Inflation

Since the discovery of anisotropic structure on the CMBR there has been renewed interest in the possibility that observational tests of inflation might be possible within the near future. It is therefore constructive to summarize the most generic predictions of basic inflation.

Prediction I: Inflation puts the 'bang' into the big bang. The standard hot big bang scenario assumes as an initial condition that the universe is expanding at some arbitrarily early epoch and then traces the history of the universe after that epoch. In the inflationary scenario, however, the gravitational repulsion of the false vacuum leads naturally to a quasi-exponential expansion of the scale factor and the observed Hubble expansion at the present epoch is therefore a prediction of all inflationary models.

Prediction II: The density parameter $\Omega \equiv \rho/\rho_c = 8\pi G/3H^2$ measures the ratio of the energy density of the universe to the critical energy density at a given epoch. When the strong energy condition of general relativity is satisfied the energy equation (2.1) implies that $\Omega = 1$ is an unstable equilibrium point. However during inflation the strong energy condition is violated and Ω approaches unity exponentially fast. Although Ω has been evolving away from unity since inflation ended, it follows immediately that the more inflation there is, the closer the density parameter is to unity at the present epoch. If the onset of inflation is determined by the initial conditions at the Planck epoch, the scenario therefore predicts that the current value of the density parameter measured on the horizon scale should be

$$\Omega_0 = 1 \pm 10^{-5}, \tag{3.1}$$

where the error of 10^{-5} arises from the quantum effects that are responsible for the generation of the primordial power spectrum and is estimated from the observed quadrupole anisotropy in the CMBR. (Models where Ω_0 differs significantly from unity can be constructed, but they require very special, and rather unnatural, initial conditions [19]. Consequently they do not satisfy our rather subjective definition of basic inflation).

This prediction provides one of the main observational tests of the inflationary scenario. When combined with the constraints from nucleosynthesis, which imply that the baryonic matter can contribute at most 10% of the critical density, it leads to the prediction that at least 90% of the observable universe must consist of non-baryonic dark matter. The discovery of dark matter would provide strong support for inflation, although of course failure to detect such particles would not disprove the scenario!

Prediction III: For completeness we mention that there should not be a significant abundance of monopoles left over from some early phase transition, because their energy density is exponentially redshifted by the inflationary expansion. The question of whether cosmic strings can form after inflation is as yet unresolved, although they can do so if the reheating temperature is sufficiently high. However we shall not pursue these questions further here.

Predition IV: It is clear that the energy parameter determines the ratio of the amplitudes of the scalar and tensor modes. For scales that reenter after decoupling this ratio is given by

$$\frac{A_G}{A_S} = \frac{\sqrt{2}}{\kappa}\frac{|H'|}{H} = \sqrt{\epsilon}. \tag{3.2}$$

We see immediately that the requirement that inflation occurs, i.e. $\epsilon < 1$, implies that

$$A_G < A_S. \tag{3.3}$$

An observation violating this condition at any scale would immediately rule out the general class of models we are considering.

Prediction V: Substitution of Eq. (3.2) into the definition of the tensor spectral index leads to the relationship

$$\frac{1}{2}n_G(\lambda) = \frac{A_G^2(\lambda)}{A_S^2(\lambda) - A_G^2(\lambda)}. \tag{3.4}$$

This expression is valid for an arbitrary inflationary potential and illustrates a fundamental connection between the forms of the scalar and tensor fluctuation spectra. At each scale λ it relates the amplitudes of the two spectra to the tilt of the tensor spectrum. In principle these three quantities can be determined observationally and potentially this equation provides a powerful discriminator of the inflationary hypothesis. We shall therefore refer to it as the *consistency* equation [9].

We see from Eq. (3.4) that $\epsilon < 1$ also leads to the prediction that

$$n_G \geq 0. \tag{3.5}$$

This implies that the amplitude of the primordial gravitational wave spectrum must always *increase* with increasing wavelength. This follows because the amplitude of the tensor fluctuations when they reenter varies in direct proportion to the expansion rate of the universe when that scale first crossed the Hubble radius during inflation. However, if $k = 0$, Eq. (2.2) implies that H is always decreasing and, since the first scales to go superhorizon are the last to reenter, this implies $n_G > 0$.

Some comments are in order at this stage. The expressions (2.12) and (2.13) for the scalar and tensor fluctuations are only strictly valid to lowest-order in the energy and friction parameters. The same is true for the consistency equation. In other words we have assumed that the slow-roll condition is valid in deriving these equations and this goes against the spirit of reconstruction. This assumption is necessary because the full analytical expressions for the perturbation spectra are unknown at present. Improved expressions valid to first-order in ϵ and η are

$$A_S \simeq -\frac{\sqrt{2}\kappa^2}{8\pi^{3/2}}\frac{H^2}{H'}\left[1 - (2C+1)\epsilon + C\eta\right] \tag{3.6}$$

$$A_G \simeq \frac{\kappa}{4\pi^{3/2}} H \left[1 - (C+1)\epsilon\right], \tag{3.7}$$

where $C = -2 + \ln 2 + \gamma \simeq -0.73$ is a numerical constant and $\gamma \approx 0.577$ is the Euler constant [20]. However, for reasons of technical simplicity, we shall employ the lowest-order expressions in the remainder of this talk.

We now proceed to develop the formalism that allows the functional form of the inflationary potential to be reconstructed from a knowledge of the primordial power spectra of scalar and tensor fluctuations.

4 Recontruction of the Inflationary Potential from the Primordial Power Spectra

At this stage it is worth remarking that only a very small region of the inflationary potential is available for reconstruction. Scales of cosmological interest span the narrow range $1h^{-1}$Mpc to $6000h^{-1}$Mpc, where $H_0 = 100h$ km s^{-1} Mpc^{-1} is the current expansion rate of the universe. These scales correspond to galaxies through to the size of the current horizon. Such a range of scales first crossed the Hubble radius during $\ln 6000 \approx 9$ e-foldings of accelerated expansion and this represents a very small part of the total inflationary era.

We shall assume that the spectra $A_S(\lambda)$ and $A_G(\lambda)$ have been determined observationally, at least over a small range of scales, by combining large-scale structure and CMBR experiments. The reconstruction of the inflationary potential now follows by parametrizing the full set of solutions in terms of the functional $H[\lambda(\phi)]$. The expressions for the amplitudes of the scalar and tensor fluctuations become

$$\begin{aligned} A_S(\lambda) &= \frac{\sqrt{2}\kappa^2}{8\pi^{3/2}} H^2(\lambda) \left| \frac{d\lambda}{dH} \frac{d\phi}{d\lambda} \right| \\ A_G(\lambda) &= \frac{\kappa}{4\pi^{3/2}} H(\lambda), \end{aligned} \tag{4.8}$$

respectively.

There exists a one-to-one correspondence between a given length scale λ and the value of ϕ when that scale crossed the Hubble radius during inflation. The physical size of a scale at the present epoch is simply $\lambda(\phi) = H^{-1}(\phi)a_0/a(\phi)$, where $H(\phi)$ and $a(\phi)$ are determined at the epoch when the scale had physical size $H^{-1}(\phi)$ (i.e. when it crossed the Hubble radius during inflation). The value of $a(\phi)$ is related to the size of the scale factor at the end of inflation, a_e, via the expression

$$a(\phi) = a_e \exp[-N(\phi)], \tag{4.9}$$

where

$$N(\phi) \equiv \int_{t_e}^{t} H(t')dt' = -\frac{\kappa^2}{2} \int_{\phi}^{\phi_e} d\phi' H(\phi') \left(\frac{dH(\phi')}{d\phi'} \right)^{-1} \tag{4.10}$$

is the number of e-foldings of growth from a particular value of ϕ to the end of inflation at ϕ_e (defined in general as the point where ϵ reaches unity). This implies that

$$\lambda(\phi) = \frac{\exp[N(\phi)]}{H(\phi)} \frac{a_0}{a_e}, \tag{4.11}$$

and differentiation of this equation with respect to the inflaton field yields

$$\frac{d\lambda(\phi)}{d\phi} = \pm\frac{\kappa}{\sqrt{2}}\left(\frac{A_S}{A_G} - \frac{A_G}{A_S}\right)\lambda. \tag{4.12}$$

An expression for the potential, as parametrized by the scale λ, follows by substituting Eq. (4.8) directly into the Hamilton-Jacobi equation (2.8):

$$V[\phi(\lambda)] = \frac{16\pi^3 A_G^2(\lambda)}{\kappa^4}\left[3 - \frac{A_G^2(\lambda)}{A_S^2(\lambda)}\right]. \tag{4.13}$$

Finally, we need to determine how λ varies with ϕ. Integration of Eq. (4.12) yields the function $\phi = \phi(\lambda)$ given by

$$\phi(\lambda) = \pm\frac{\sqrt{2}}{\kappa}\int^{\lambda}\frac{d\lambda'}{\lambda'}\frac{A_S(\lambda')A_G(\lambda')}{A_S^2(\lambda') - A_G^2(\lambda')}. \tag{4.14}$$

An alternative form for $\phi(\lambda)$ may be derived by substituting the consistency equation (3.4) into Eq. (4.14):

$$\phi = \pm\frac{\sqrt{2}}{\kappa}\int^{A_G} dA'_G\,\frac{A_S[A'_G]}{A'^2_G}. \tag{4.15}$$

Without loss of generality the arbitrary integration constant has been eliminated by performing a linear translation on the value of ϕ. This second expression proves particularly useful in the reconstruction process if the functional form of A_S as a function of A_G is known.

The functional form $V(\phi)$ of the potential is deduced by inverting Eq. (4.14) and substituting into Eq. (4.13). The reconstruction equations are Eq. (4.13) and (4.14) along with the consistency equation (3.4). We conclude that *full reconstruction requires a knowledge of both the scalar and tensor perturbation spectra.* However, recall that the tensor spectral index is just $n_G = 2d\ln A_G/d\ln\lambda$. If the tensor spectrum is known, this implies that one may employ the consistency equation to derive the scalars. Unfortunately the consistency equation is a first-order, ordinary differential equation, so the reverse is not true! If one only has a knowledge of the scalar spectrum (and from an observational point of view this is the most likely scenario), one must integrate the consistency equation to determine the tensors and this necessarily introduces an arbitrary constant into the tensor spectrum. The non-linear nature of the consistency equation implies that the scalar spectrum alone does not uniquely determine the tensor spectrum, and hence the functional form of the inflationary potential.

As a corollary, an arbitrarily accurate determination of the scalar spectrum is insufficient to reconstruct the potential. *A minimal knowledge of the primordial gravitational wave spectrum is required.* Indeed the amplitude of the tensor spectrum at one scale is sufficient to determine the integration constant and lift the degeneracy. We shall discuss in Section 6 some of the possible observational routes whereby the necessary information may be gathered.

5 Reconstruction from a Constant Scalar Spectral Index

In this section we shall derive the general class of inflationary potentials that leads to a spectrum of scalar fluctuations with a *constant* spectral index. We assume a power law of the form

$$A_S(\lambda) \propto \lambda^{(1-n)/2} \propto k^{(n-1)/2}. \tag{5.1}$$

This class of spectra is the simplest extension to the scale-invariant Harrison-Zel'dovich spectrum ($n = 1$) and at any rate it is likely that more complicated features in the spectrum will not be measureable observationally in the near future.[1] However, recent results already constrain power spectra of the form (5.1). The data from COBE suggests that (5.1) is consistent at the 1-σ level if n lies in the range 0.6 to 1.6. These limits are independent of the dark matter components. If one includes clustering data and assumes a cold dark matter (CDM) model, the lower limit becomes $n > 0.7$ at 95% confidence if gravitational waves do not contribute significantly to the CMBR temperature anisotropy and $n > 0.84$ if they do [15].

The functional forms of $H(\phi)$ that lead to such spectra may be derived by equating Eqs. (2.12) and (5.1), taking logarithms, and differentiating with respect to the scalar field to remove all undetermined constants [21]. The $(\ln \lambda)'$ term may be rewritten after substitution of Eq. (4.12) and this leads to a second-order differential equation in $H(\phi)$ [21]:

$$2(5-n)\frac{(H')^2}{H} - 2H'' = -(n-1)\kappa^2 H. \tag{5.2}$$

The order of this equation may be reduced by using the identity

$$2H'' \equiv \frac{d(H')^2}{dH} \tag{5.3}$$

and we arrive at the non-linear, first-order differential equation

$$\frac{d(H')^2}{dH} - (5-n)\frac{H'^2}{H} = \frac{n-1}{2}\kappa^2 H. \tag{5.4}$$

This admits the exact integral

$$(H')^2 = \frac{n-1}{n-3}\frac{\kappa^2}{2}H^2 + CH^{5-n}, \qquad n \neq 3, \tag{5.5}$$

where C is an arbitrary integration constant. Eq. (5.5) has a number of solutions depending on the signs of C and n. We have summarized them in Table 1 and the second integration constant has been absorbed into the value of the scalar field [22].

The inflationary potential follows by substituting these solutions into the Hamilton-Jacobi equation (2.8). Its form can not be determined uniquely unless the sign of the integration constant is specified. This constant determines the energy scale at which the fluctuations are first formed during inflation and therefore such a specification requires knowledge of the gravitational wave spectrum on at least one scale. To investigate this further let us consider the $n < 1$ solutions in more detail when C is negative semi-definite.

[1] Throughout this talk the term 'near future' refers to any timescale contained within the next decade.

C/n	$n<1$	$n=1$	$n>1$
$C<0$	$\Lambda\mathrm{sech}^m(\omega\phi)$	NS	NS
$C=0$	$\exp\left(\pm\sqrt{\frac{1}{2}\frac{n-1}{n-3}}\kappa\phi\right)$	Const.	NS
$C>0$	$\Lambda\mathrm{cosech}^m(\omega\phi)$	$\frac{1}{\sqrt{C}\phi}$	$\Lambda\sec^m(\omega\phi)$

Table 1: The functional forms for $H(\phi)$ that lead to a primordial power spectrum of scalar fluctuations with constant spectral index are tabulated for positive, vanishing or negative C and $n-1$. The parameter Λ is a positive–definite constant determined by C and 'NS' implies no real solution for $H(\phi)$ exists in the given region. $m = 2/(3-n)$ and $\omega^2 = |(n-1)(n-3)|\kappa^2/8$ if $n<3$. The form of the potential follows from the Hamilton-Jacobi equation (2.8). We see that the integration constant C must be specified for the potential to be uniquely determined. This can only be done at an observational level if one has knowledge of the primordial gravitational wave spectrum on at least one scale of interest.

Both the hyperbolic and exponential potentials lead to a constant spectral index n. What distinguishes the two solutions is the relative amplitude of the gravitational wave spectrum. This is determined by the ratio (3.2). The energy parameter in the two cases is given by

$$\epsilon = \left(\frac{n-1}{n-3}\right) \tag{5.6}$$

and

$$\epsilon = \left(\frac{n-1}{n-3}\right)\left[\tanh\left(\sqrt{\frac{(n-1)(n-3)}{8}}\kappa\phi\right)\right]^2 \tag{5.7}$$

for $C=0$ and $C<0$ respectively. In the latter case it can be shown that cosmological scales crossed the Hubble radius during inflation when $|\kappa\phi|$ was very small, thereby implying that the contribution of the gravitational waves is exponentially suppressed in this model [18].

It is worth remarking that the solutions presented in Table 1 are general and therefore valid for all values of the scalar field. This implies that one can always expand the potentials as a Taylor series about some specific value ϕ_0 and then perform a linear translation on the field [23]. It is interesting to note that the expansions for the $\{n<1, C<0\}$ and $\{n>1, C>0\}$ are both given by

$$H(\phi) = \Lambda\left[1+\left(\frac{n-1}{8}\right)\kappa^2\phi^2 + \mathcal{O}(\phi^4)\right]. \tag{5.8}$$

This implies that an almost constant spectral index will always be obtained if $H(\phi)$ has an expansion of the form (5.8) to lowest order in ϕ. Such an expansion is valid because all large-scale structure observations correspond to a very narrow region of the inflationary potential and since the field must be rolling down its potential at a sufficiently slow rate for inflation to proceed in the first place, the relative change in the position of the field over the 9 e-foldings associated with large-scale structure observations is expected to be tiny.

Scalar Spectrum	Gravitational Waves Important	Gravitational Waves Negligible
Small Tilt	ϵ large $2\epsilon \approx \eta$	ϵ small $\lvert\eta\rvert$ small
Significant Tilt	ϵ large $\lvert\eta\rvert$ large	ϵ small $\lvert\eta\rvert$ large

Table 2 - Illustrating the connection between tilt and the gravitational wave production in terms of the magnitudes of the energy and friction parameters.

Potentials of this form arise in a number of particle physics models. For $n < 1$ the potential can be interpreted physically as a bulk viscous stress acting on the energy-momentum tensor of a perfect barotropic fluid [18]. The self-interaction potentials of the pseudo-Nambu-Goldstone bosons also have the form (5.8) in the small angle approximation [24]. Furthermore, the recently proposed hybrid inflationary scenario is driven by potentials with an expansion given by (5.8) and predicts $n > 1$ [25].

Finally we summarize the relationship between the tilt of the scalar power spectrum and the gravitational wave contribution in Table 2 [26]. The important quantities are the energy and friction parameters. These are described as 'large' when they are significantly larger than zero but still less than unity and 'small' when they are very close to zero.

6 Can the Primordial Gravitational Waves be Observed?

In this section we shall assess the current possibilities for reconstructing a part of the inflationary potential. It is clear that at some level one requires knowledge of the gravitational wave spectrum and there are three possible methods by which such a spectrum may be determined.

1. Direct Detection: The most obvious method is via direct detection. For inflation driven by exponential potentials the spectral indices satisfy $n_G = 1 - n$ and are uniquely determined by the energy parameter via Eq. (5.6). Hence, increasing the tilt increases the contribution of the gravitational waves to the CMBR anisotropy on large angular scales. However, tilting the spectrum reduces the amplitude on the smaller scales relevant for direct detection. The predicted amplitude of the waves on scales relevant to the most sensitive direct probes of gravity waves, such as the Laser Interferometer Gravity-wave Observatories (LIGO), are too low to be detected [43]. Consequently this route does not appear promising.

2. Polarization of the CMBR: Polarization of the CMBR can arise if the microwave radiation is scattered by free electrons due to Thomson scattering in the presence of a gravitational wave. In principle measurements of the CMBR polarization can provide valuable information regarding the contribution of gravitational waves to large-scale temperature anisotropies [27]. Recent numerical calculations suggest, however, that such an effect is very difficult to detect in practice, since the combined polarization of the tensor plus scalar is typically less than 1%. Therefore this approach does not seem particularly promising either.

3. Comparison of small and large scale CMBR anisotropies: The most promising method for determining the gravitational waves at a particular scale is by comparing the large and small angle anisotropies (for a discussion of these issues see [28]). The surface of last scattering is located at a redshift $z_{\rm LSS} \approx 1100$ and the angle subtended by the horizon scale at that redshift is approximately $\theta_{\rm LSS} \approx (1+z_{\rm LSS})^{-1/2} \approx 2^o$. This implies that anisotropy measurements on angular scales above $\theta \approx 2^o$ directly determine the primeval form of the fluctuations. On smaller angular scales the fluctuations have reentered the horizon. After reentry the gravitational waves behave as relativistic matter and their energy density redshifts as the fourth power of the scale factor. This implies that only scalar modes affect the CMBR anisotropy below $\approx 2^o$. If the fluctuations are statistically independent and Gaussian-distributed, the scalar and tensor fluctuations on scales above 2^o add in quadrature. Consequently *a comparison of large and small scale CMBR experiments may yield information regarding the influence of the primordial gravitational waves.*

Let us investigate this further. The CMBR anisotropies can be expanded into spherical harmonics [29]

$$\frac{\Delta T}{T}(\mathbf{x},\theta,\phi) = \sum_{l=2}^{\infty}\sum_{m=-l}^{l} a_{lm}(\mathbf{x})\, Y^l_m(\theta,\phi), \tag{6.1}$$

where $\mathbf{x}$ denotes the observer's position and the coefficients $a_{lm}(\mathbf{x})$ are Gaussian random variables with mean and variance determined uniquely by the harmonic l:

$$\langle a_{lm}(\mathbf{x})\rangle = 0 \quad ; \quad \langle |a_{lm}(\mathbf{x})|^2\rangle \equiv \Sigma_l^2. \tag{6.2}$$

A given inflationary model predicts values for the averaged quantities

$$\langle Q_l^2\rangle = \frac{1}{4\pi}(2l+1)\Sigma_l^2, \tag{6.3}$$

where the average is taken over *all* observer positions, but the observed multipoles measured from a single point in space are given by

$$Q_l^2 = \frac{1}{4\pi}\sum_{m=-l}^{l}|a_{lm}|^2. \tag{6.4}$$

For sufficiently small angles on the CMBR sky the lth harmonic of the expansion is given approximately by $l \approx (200^o/\theta)$. Hence for harmonics smaller than $l \approx 20$ the dominant contribution to the CMBR arises solely from the Sachs-Wolfe effect when photons are either red or blue shifted as they climb out of, or fall into, gravitational potential wells [30]. If one assumes a constant spectral index $n < 3$, the variances of the coefficients in the harmonic expansion of the scalar fluctuations are related by

$$\Sigma_l[S] = \Sigma_2[S]\frac{\Gamma[l+(n-1)/2]}{\Gamma[l+(5-n)/2]}\frac{\Gamma[(9-n)/2]}{\Gamma[(3+n)/2]}. \tag{6.5}$$

Hence, in models where gravitational waves are not significant, measurements of the lower-order harmonics provide a fit to the spectral index.

For models where the tensor fluctuations are important, however, the anisotropies from such modes are

$$\Sigma_l^2[T] = 144\pi^5 G(2l+1)\frac{(l+2)!}{(l-2)!}\left(\frac{H_0}{2}\right)^{n-1} C(n)\int_0^\infty dk k^{n-2} I_l^2(k), \tag{6.6}$$

where

$$I_l(k) \equiv \int_{k\eta_E/\eta_0}^{k} dy \frac{J_{l+1/2}(k-y)}{(k-y)^{5/2}}\frac{J_{5/2}(y)}{y^{3/2}}, \tag{6.7}$$

η_E and η_0 denote conformal time at recombination and at the present era and $C(n) = P(k)k^{1-n}/4\pi$, where $P(k)$ is the power spectrum [31].

A very useful relationship follows after numerical integration of this expression [32]. It can be shown that the ratio

$$\frac{\Sigma_l^2[S]}{\Sigma_l^2[T]} = \frac{2}{m^2}\frac{A_S^2}{A_G^2} = \frac{25}{2}\epsilon \tag{6.8}$$

is independent of l if n is approximately constant and close to unity. For potentials driven by an exponential potential comparison with Eq. (5.6) therefore implies that

$$\frac{\Sigma_l^2[S]}{\Sigma_l^2[T]} = \frac{\Sigma_l^2[S]}{\Sigma_l^2 - \Sigma_l^2[S]} = \frac{25}{2}\left(\frac{n-1}{n-3}\right), \tag{6.9}$$

where we assume that the expectations add in quadrature, i.e. $\Sigma_l^2 = \Sigma_l^2[S] + \Sigma_l^2[T]$ [32,33]. This is a remarkable result and suggests that inflation driven by exponential potentials can be observationally tested if the spectral index and ratio of the scalars to tensors on large angular scales can be determined. Moreover, deviations from this relationship will tell us how far away we are from the exponential potential model.

Since the large-scale Sachs-Wolfe effect is potentially due to both scalar and tensor modes, we must consider smaller scales ($\theta < 2^o$), where the gravitational waves are unimportant, to determine separate normalizations for these two components. However, because these scales were sub-horizon at the surface of last scattering, complex physical processes do not allow simple expressions for the anisotropies to be written down. On the other hand a 'Doppler peak' is expected to occur at $l \equiv l_{DP} \approx 120 - 200$ due to effects such as Thomsom scattering off moving electrons [34]. In principle, therefore, a measurement of the height of this peak gives the scalars at l_{DS} and comparison with the low multipole anisotropies may allow the tensors to be separated out. If this can be done, it would imply that the COBE satellite is indeed the first experiment to make a direct measurement of the primordial spectrum of gravity waves!

The dark matter in the universe affects the small-scale CMBR anisotropy via a transfer function, although it seems that this is not so important for degree-scale experiments. However, the possible effects of reionization must also be considered. To make progress it seems that one must consider a hypersurface in the full parameter space of observational cosmology and reconstruct within the context of this plane. We shall therefore conclude this section by summarizing what we feel to be the most important parameters that need to be determined, along with some 'favorite' values.

1. Ω_0: Inflation predicts that $\Omega_0 = 1$ and observational support for this prediction comes from the QDOT redshift survey of the Infrared Astronomy Satellite (IRAS) [35]. Their results suggest that $\Omega_0^{0.6} b_I^{-1} = 0.86 \pm 0.15$, where b_I is the IRAS bias parameter.

2. Age of the universe: If there is no cosmological constant, the age of the $\Omega_0 = 1$ universe is $t_0 = 6.52h^{-1}$Gyr. Redshift surveys suggest that the current value of the expansion rate is $0.4 \leq h \leq 1.0$, but if $h > 0.6$, the age of the universe is smaller than the oldest globular clusters in the galaxy, $t_{\rm star} = (13-15) \pm 3$Gyr. Therefore low values of h are favored. The Sunyaev-Zel'dovich effect distorts the CMBR when the electromagnetic waves interact with the hot gas in galaxy clusters and a value of the Hubble constant can then be determined by constraining the size of the cluster along the line of sight. Birkinshaw et al. [36] find $h = (0.4 - 0.5) \pm 0.12$ whilst Jones et al. [37] find $0.2 < h < 0.75$. These observations favour the lower region.

3. Baryonic dark matter: Limits on the percentage of baryonic dark matter in the universe deduced from observing the primordial abundances of the light elements are [38]

$$\Omega_B h^2 = 0.0125 \pm 0.0025. \tag{6.10}$$

Saturating the lower bound on h yields the upper limit $\Omega_B \leq 0.09$ and one must therefore choose some form of non-baryonic dark matter if $\Omega_0 = 1$.

4. Cosmological Constant: A number of observations suggest that the cosmological constant may be a candidate for this dark matter [39,40]. We note however that gravitational lensing effects imply that $\Omega_\Lambda \leq 0.6$ and a best fit to clustering results implies $\Omega_\Lambda \leq 1 - (0.2 \pm 0.1)h^{-1}$, at least for $n < 1$ [41].

5. Reionization history: In the standard picture recombination occurs at a redshift $z \approx 1300$ and the ionization fraction X_e is given by $1 + z_{LSS} \approx (0.03 X_e \Omega_B h)^{-2/3}$ if $\Omega_0 = 1$. Thus the surface of last scattering could have formed as late as $z_{\rm LSS} \approx 76$ if $X_e = 1$, $h = 0.4$ and we employ the upper limit for Ω_B. Such effects alter the position of the Doppler peak and this implies that some assumptions about the ionization history of the universe must be made.

It seems reasonable to assume that no reionization occurred and to then specify $\Omega_0 = 1$, $\Omega_B = 0.1$, $\Omega_\Lambda = 0$ and $h = 0.5$ as a first approximation. The free parameters are then the spectral indices of the scalar and tensor fluctuations, the relative amplitude of the fluctuations on the quadrupole scale and the form of the dark matter (or equivalently the transfer function).

The current state of play with the observations suggests that the most realistic way to reconstruct is within the context of a specific dark matter model and a constant spectral index. There are a number of active and proposed dark matter searches in operation and one might hope to gain some insight into the dark matter in the near future. The most promising method of determining the primordial power spectrum is through measurements of the peculiar velocity field. The development of the POTENT method, in particular, is potentially very useful because it assumes that the velocity is simply given by the divergence of a scalar [42]. All matter interacts via gravity, so peculiar velocities measure the mass spectrum and *not* the galaxy spectrum. In the linear regime

$$\mathcal{P}_v(k) = \frac{1}{25\pi}\left(\frac{aH}{k}\right)^2 \frac{2}{m^2} A_S^2(k) T^2(k), \tag{6.11}$$

where the spectrum of the modulus of the velocity v is defined as $\mathcal{P}_v = V(k^3/2\pi^2)\langle|\delta_v|^2\rangle$, the Fourier components $\delta v(k)$ are defined over the physical volume V and $T(k)$ represents the transfer function. This equation shows how the spectrum is related *directly* to the amplitude of the scalar fluctuations when they reenter the Hubble radius. In principle, therefore, the scalar spectral index can be reconstructed.

The idea then is that the height of the Doppler peak at $l \approx l_{DP}$ can be determined once the spectral index and transfer function of the dark matter are known and this leads to a predicted value for what should be observed. Eq. (6.9) implies that

$$\frac{\Sigma_2^2[S]}{\Sigma_2^2[T]} = \frac{\Sigma_{l_{DS}}^2[S]}{\Sigma_{l_{DS}}^2[T]} \tag{6.12}$$

and this expression is independent of the spectral index. In principle this gives us a measurement of the amplitude of the gravitational waves at the scale l_{DP} and should be sufficient to lift the degeneracy intrinsic to the reconstruction procedure. Once the degeneracy has been lifted this will allow the integration constant to be determined.

7 Conclusion

In this talk we have summarized the general observational predictions of the simplest class of basic inflationary models. The formalism that allows one to reconstruct the functional form of the inflationary potential from the scalar and tensor perturbation spectra was then discussed. Full reconstruction of the potential does not appear viable within the foreseeable future. However, it is quite likely that an accurate determination of the spectral index will be made within the next few years. We presented the full class of general solutions that lead to scalar fluctuations with a constant spectral index and it was found that the precise form of the potential is rather sensitive to the gravitational wave contribution to large angle CMBR anisotropies. If $n > 1$, the potential is convex and, if its functional form over the large-scale structure range is extrapolated to the origin, it has a global minimum located at $V \neq 0$. If $n < 1$ and the gravitational wave contribution is negligible, the field is located within the vicinity of a local maximum instead. On the other hand, if $n < 1$ and the tensor spectrum is important, the potential takes an exponential form.

These general solutions can be expanded as a Taylor series about some value of ϕ, which typically is taken to be the value of the inflaton field when the scale corresponding to the quadrupole first crossed the Hubble radius during inflation. This is consistent because all large-scale structure observations correspond to only a few e-foldings of inflationary expansion and, since the scalar field is moving slowly during inflation, the relative change in the value of ϕ in this range is expected to be very small. This implies that any potential that has a Taylor expansion equivalent to these general solutions to lowest order in ϕ will also produce a scalar spectrum with constant n.

We therefore conclude that partial reconstruction of the inflationary potential, within the approximation that the spectral index is constant, is a realistic possibility. We have shown that a determination of the spectral index, together with knowledge of the gravitational wave amplitude on one scale, will be sufficient to determine whether the potential is convex or concave over the narrow range of scales associated with large-scale structure. Although this is a rather limited piece of information, it would nevertheless correspond to an observation of physics at the GUT scale.

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