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THE CURRENT STATUS OF OBSERVATIONAL CONSTRAINTS ON COSMIC STRINGS

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Abstract

The observational restrictions on the cosmic string scenario for the formation of large scale structure are evaluated. These restrictions are due to the spectrum of gravitational radiation emitted by oscillating string loops, anisotropies in the cosmic microwave background caused by the strings, and evaporating black holes formed from collapsed cosmic string loops. It is shown that the only free parameter of the scenario, the cosmic string mass-per-unit-length, μ, is severely restricted.

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Observational constraints on cosmic strings

Cosmic strings are tubes of topologically bound quanta, which may have formed during a phase transition during the early universe, and which may contribute to the formation of large scale structure. The properties of cosmic strings have been well investigated, both analytically and numerically, over the past 15 years. Roughly, the following understanding of how cosmic strings behave on the large and small scales has developed. On large scales, long, wiggly strings sweep out wakes in the cosmological fluid, leaving overdense regions which will grow to form galaxies and clusters. On small scales, small loops are chopped off and ejected from the long strings. These loops may then contract and expand under their own tension, radiating gravitational waves. Processes on both the large and small scales contribute to the gross features of the cosmic string scenario, and the observational properties of cosmic strings.

The major, cosmological role of cosmic strings is to produce the perturbations in the cosmological fluid which lead to the formation of large-scale structure. Numerical and analytic estimates find that

$$\mu \sim 1 - 4 \times 10^{-6}$$

is required for a successful scenario [1]. (I have set $G = c = 1$). That is, for this range of values of μ, cosmic strings produce a spectrum of density fluctuations in a fluid composed primarily of dark matter which are expected to lead to the formation of the structures observed today. It will be important to compare this value with the limits imposed on μ through observation.

In this article, I will review the major observational constraints on cosmic strings. It will be seen that the spectrum of stochastic gravitational radiation is strongly restricted by the measurement of residuals in pulsar timing. The anisotropy in the microwave background produced by cosmic strings is strongly restricted by the COBE observations. As well, should cosmic string loops collapse to form black holes, the evaporating black holes, and therefore the cosmic strings, may be restricted by measurements of the diffuse photon background. These three observational limits will be summarized in this article.

Spectrum of gravitational waves emitted by oscillating string loops

The emission of gravitational radiation by a network of cosmic strings has been studied analytically through the application of the "one-scale" model of cosmic strings and the results of numerical simulations of cosmic strings. The spectrum of gravitational waves emitted by cosmic strings [2] is constrained by the nucleosynthesis limit on relativistic particle species, and the pulsar timing limit on a stochastic gravitational wave background.

The nucleosynthesis limit requires that the energy density in neutrinos plus any relativistic particle species beyond the photons and electrons present at nucleosynthesis contain no more than the equivalent energy density of N_ν neutrino species. This may be translated into a restriction on the energy density in gravitational radiation at nucleosynthesis

$$\Omega_{gr} \leq 0.163 \times (N_\nu - 3)\Omega_{rad},$$

where 3 light neutrino species have been assumed. The current observational limit of $N_\nu \leq 3.6$ [3] based on observations of element abundances yields

$$\mu \leq 10^{-5}.$$

Thus, the nucleosynthesis limit on cosmic string gravitational radiation, a limit on the mass-per-unit-length μ, is not very restrictive.

The pulsar timing limit requires that the energy density in gravitational radiation, in a logarithmic frequency interval at $\omega = (8.2\text{years})^{-1}$ satisfy the inequality [4]

$$\frac{d\Omega_{gr}}{d\ln\omega}\Big|_{\omega=(8.2\mathbf{years})^{-1}} \leq 1.0 \times 10^{-7} h^{-2}.$$

In this expression, h is the Hubble parameter such that $h = 1$ for $H_0 = 100 km/s/Mpc$. The value quoted here is the 95% confidence level result.

There is a problem, however, concerning the shape of the spectrum of gravitational radiation emitted by cosmic string loops. Numerical simulations suggest that the power emitted in radiation is distributed among modes of oscillation as $P \propto n^{-q}$ where $q = 4/3$ [5] and the mode n is proportional to the frequency. Analytic work suggests that this value of q is characteristic of very cuspy loops [6]. Rather, the power in radiation emitted by non-cuspy but kinky loops, expected to be the dominant population of loops, is distributed as $q = 2$ [7]. The difference is very important, as the observational limit in the case of cuspy loops is much stronger than for kinky loops.

$$\mu \leq \begin{cases} 3 \times 10^{-7} h^{-8/3} & \text{q=4/3} \\ 5 \times 10^{-6} h^{-2} & \text{q=2} \end{cases}$$

Thus, for $q = 2$ the limit is not too restrictive, although for $q = 4/3$ it would appear that the cosmic string scenario is ruled out for $h \geq 0.65$. Under a certain set of assumptions it would appear that the cosmic string scenario is ruled out, although no definite, confident claim may be stated.

The resolution of the problem of the frequency distribution of power may be a combination of both the cuspy and kinky loop cases. The numerical simulations have limited resolution of the small-scale structure on loops. It may be that the loops initially chopped off the long strings are very cuspy, with a power spectrum described by $q = 4/3$. However, the parent loops may then self-intersect and fragment to form more kinky, but less cuspy, $q = 2$ loops. I hope to examine the plausibility and effects of such behavior on the pulsar timing limits in the future.

Anisotropies in the cosmic microwave background radiation

The anisotropies observed in the cosmic microwave background by COBE [8] may serve to constrain μ by restricting the magnitude and spectrum of fluctuations produced by cosmic strings. An effort to numerically model the anisotropies produced by cosmic strings is currently underway [9]. This project will produce the most detailed study of cosmic string induced CMBR perturbations carried out to date. I shall briefly describe the basic aspects of this collaboration.

The numerical simulation of Allen and Shellard [10] has been adapted to evolve a network of cosmic strings lying within the past horizon volume from the time of last scattering to the present. This numerical simulation models the evolution of a large cosmic string network, delivering the string network stress-energy tensor. That is, it computes the string position, velocity, and energy. It is then possible to calculate the metric perturbations induced by the string as

$$h_{\mu\nu} = \int \sqrt{-g'} d^4x' G(x,x')_{\mu\nu}{}^{\alpha\beta} T_{\alpha\beta}(x').$$

In this equation, $T_{\alpha\beta}$ is the stress-energy tensor and $G(x,x')_{\mu\nu}{}^{\alpha\beta}$ is the Green's function for the perturbations induced by the cosmic strings. Next, the shift in temperature induced in a photon as it travels from source to observer may be calculated by applying the Sachs-Wolfe [11] formula:

$$\frac{\delta T}{T} = -\frac{1}{2} \int_\lambda d\lambda \partial_t h_{ij} e^i(\lambda) e^j(\lambda).$$

Here, λ parametrizes the geodesic path of the photon, the e^k are orthonormal basis vectors, and h_{ij} gives the spatial part of the metric fluctuation produced by the cosmic strings. It is then a well-posed, although laborious problem to calculate the temperature fluctuations.

Ultimately, this program yields a map of the temperature on the celestial sphere. Qualitatively, when projected in false colors, this map is rather pleasing to the eye. Analysis of the properties of the map, however, may be carried out by decomposing the temperature distribution into spherical harmonics.

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta, \phi)$$

The statistical properties of the simulated CMBR map are uniquely determined by the a_{lm} coefficients. Thus, observation of a signal or an upper limit on the strength of anisotropies in the CMBR may be expressed in terms of an upper bound on the a_{lm} coefficients.

The COBE group has reported an observed quadrupole anisotropy [8]

$$Q^2 = \frac{5}{4\pi} \sum_{m=-2}^{2} |a_{2m}|^2 = 13 \pm 4\mu K.$$

Recent work by [12] simulating CMBR perturbations produced by cosmic strings on a small patch of the sky indicate that $\mu \leq 2 \times 10^{-6}$ is required for compatibility with this observation. Comparing preliminary results of our work, computation of the a_{lm} coefficients, with the COBE observation, we find that $\mu \sim 10^{-6}$ is marginally compatible with observation. This might suggest that cosmic strings are either incompatible with, or responsible for the observed anisotropy. We expect that a full analysis should be complete well before these conference proceedings appear.

Evaporating black holes formed from collapsed cosmic string loops

A cosmic string loop which, contracting under tension, falls within its Schwarzschild radius will form a black hole. A population of primordial black holes may form in this manner. In turn, the evaporation of black holes formed from collapsed cosmic string loops may be used to limit μ. In a recent work [13], we have examined the cosmological implications of a population of primordial black holes formed from collapsed cosmic string loops.

This work made use of the cosmic string one-scale model, the model of the quantum-mechanical decay of black holes through Hawking radiation, and the assumption that a fraction f of all newly formed loops collapse to form black holes. Ultimately, for $\mu = 10^{-6}$ we arrive at the expression

$$\frac{d\Omega_\gamma(t_0)}{d\ln\omega}\Big|_{\omega=100MeV} = 10^{-9} f$$

for the fraction of critical energy density in photons radiated by black holes formed from collapsed cosmic string loops, in a logarithmic frequency interval at $\omega = 100 MeV$, as observed today.

Observations of the diffuse photon background [14] find

$$\frac{d\Omega_\gamma(t_0)}{d\ln\omega}|_{\omega=100MeV} \leq 10^8 h^{-2}.$$

This yields the limit

$$\mu \leq 10^{-6} \quad \text{for} \quad f = 10^{-17}.$$

For increasing f, the value of μ compatible with observation decreases. It does not appear very simple, however, to make a reasonable estimate of this fraction f. One might naively estimate that every perturbation on a string loop must be fine tuned to an accuracy of one part in $\mu^{-1} \sim 10^6$. Then, the n parameters describing a cosmic string loop will be finely tuned, so that it may collapse to form a black hole, for one in μ^{-n} loops. Such a guess, however, assumes that all perturbations on string loops occur with equal frequency, and that all loop parameters contribute equally in determining whether a black hole may form. Probably, the loops produced by a cosmic string network may be described by a some set of common properties. We hope in the near future to carry out a careful numerical analysis to determine the properties of realistic loops, and then the fraction of those loops collapsing to form black holes.

Summary

The strongest current limit on the cosmic string mass-per-unit-length μ, due to CMBR anisotropies, requires $\mu \leq 2\times10^{-6}$. Under certain assumptions regarding the properties and behavior of cosmic strings, pulsar timing measurements may restrict $\mu \leq 3 \times 10^{-7} h^{-8/3}$. If the model of cosmic strings and cosmic string gravitational radiation used to arrive at this limit are well founded, it would appear that cosmic strings are too light to provide the necessary perturbations for the formation of large scale structure. Otherwise, cosmic strings remain marginally compatible with current observations as a viable method of generating large scale structure. As discussed in this review, several projects are currently underway to analyze and test in further detail the viability of the cosmic string scenario.

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