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# Is the Momentum Space Optimally Used with the FODO Lattices?

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ABSTRACT

The available momentum space of a FODO lattice is determined by the maximum value of the dispersion function ($\delta x = D_X \,\partial p/p$). In a regular FODO lattice the dispersion function oscillates between its maximum and minimum values, which are always positive. The maximum value of the dispersion function in a FODO cell of a fixed length depends on the cell phase difference. An example of a new lattice, in which the dispersion function is lowered to half its value in the same FODO cell, is presented. The available momentum space in the new lattice is raised to twice that in the FODO lattice by allowing the dispersion function to oscillate between the same positive and negative values. The maxima of the dispersion function in the new lattice have half the value of those within the regular 90° cells.

## 1 INTRODUCTION

The optimum value of the dispersion function, in a lattice that consists of a row of FODO cells, depends primarily on the phase difference within the cell. A phase difference of $\Phi$=90° for a fixed-length cell represents an optimum [1] value with respect to the dispersion function as well as the betatron functions. An example of the FODO cell used in the present Relativistic Heavy Ion Collider (RHIC) lattice at Brookhaven is used for comparison. A new lattice with better momentum dependence is designed by using a combination of the same FODO cells and additional insertions [2] with a $\pi$-phase difference.

## 2 TWOFOLD IMPROVEMENT IN MOMENTUM DEPENDENCE

The available momentum space of the lattice depends on the maximum value of the dispersion function. For the same value of the admittance-available beam offset, the maximum momentum ($\sigma_{p\,max}$) is defined from the definition of the beam size $\sigma_{tot}$ :

$$\sigma^2_{tot} = \sigma^2_{Twiss} + (D_x\,\sigma_p)^2, \quad \text{with} \quad \sigma^2_{Twiss} = \varepsilon_N\,\beta_{Twiss} / 6\pi\,\gamma\beta, \tag{1}$$

where $\gamma\beta$ are the relativistic factors, $\varepsilon_N$ is the normalized emittance in mm mrad, $D_x$ is the horizontal dispersion function, and $\beta_{Twiss}$ is the betatron amplitude. Thus the available momentum aperture is

$$\sigma_p = [\sqrt{(\sigma^2_{tot} - \sigma^2_{Twiss})}] / D_x\,. \tag{2}$$

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Equation (2) shows that the momentum aperture is inversely proportional to the value of the dispersion function.

The dispersion function could oscillate between positive and negative values instead of always having positive values as in the lattice made of FODO cells. This was accomplished [3-6] in previous designs of the imaginary $\gamma_t$ lattices. A lattice in which the dispersion function is both positive and negative could avoid the transition energy, which is an additional advantage over regular FODO-cell-based lattices. A method of designing lattices without transition was reported earlier.[4, 7]

The FODO cell used for comparison is like a cell from the present RHIC lattice design. The RHIC lattice within the arcs consists of a row of FODO cells. The magnets are 9.45 meters long, and the focussing and defocussing quadrupoles are both 1.11 meters long. If the phase difference in the FODO cell is 90°,the maxima of the beta functions are $\beta_{x\ max}$ = 49.98 m, $\beta_{y\ max}$ = 50.02 m; and $D_{x\ max}$ =1.554 m, $D_{x\ min}$ =0.751 m. The phase difference of the FODO cells in the two rings of the present RHIC lattice is slightly different from 90° ($\phi_x$=80.43°, $\phi_y$=85.30° in the inner arc cells). The maximum of the dispersion function is $D_{x\ max}$ =1.841 m.

Figure 1 shows the betatron functions within a lattice made of three FODO cell in a row, with a total length L= 88.761 m. The betatron functions are also presented in Table 1. The emittance is assumed to be $\varepsilon$=20 $\pi$ mm mrad, and the acceptance depends on the available aperture.

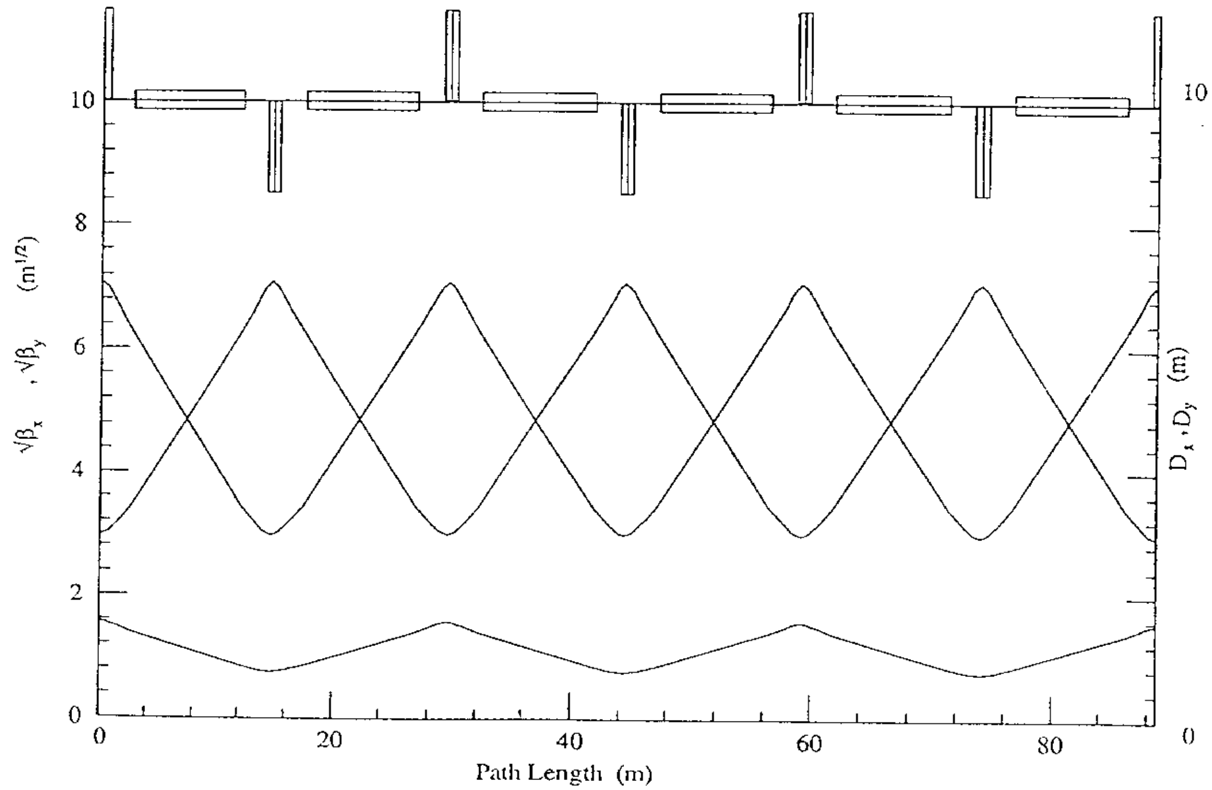


Figure 1 Betatron functions within three FODO cells in a row.

TABLE 1

| | $\beta_{x\,max}$ | $\beta_{x\,min}$ | $\beta_{y\,max}$ | $D_{x\,max}$ | $D_{x\,min}$ | $\nu_x$ | $\nu_y$ | $\xi_x$ | $\xi_x$ | $\gamma_t$ |
|---|---|---|---|---|---|---|---|---|---|---|
| FODO(90°) | 49.98 | 8.75 | 50.02 | 1.554 | 0.751 | .750 | .750 | –.897 | –.900 | 18.79 |
| RHIC | 49.71 | 10.55 | 48.55 | 1.841 | 0.939 | .670 | .711 | –.777 | –.809 | 17.01 |
| New | 55.17 | 1.35 | 49.49 | 0.887 | –.880 | .727 | .411 | –1.69 | –.589 | i176.0 |

A new block of three cells is constructed by using two of the FODO cells described above plus a $\pi$-insertion constructed from two doublet quadrupoles with a 9.45-meter-long dipole between. The first quadrupole of the doublet replaces the focussing quadrupole of the FODO cell in front of the insertion. The total length of this block is L= 73.451 m. Figure 2 shows the elements and the betatron functions within the new block. Analytical formulae and other details of the construction of such blocks are presented elsewhere.[7] The excellent properties and good stability of this kind of lattice were also studied earlier.[7]

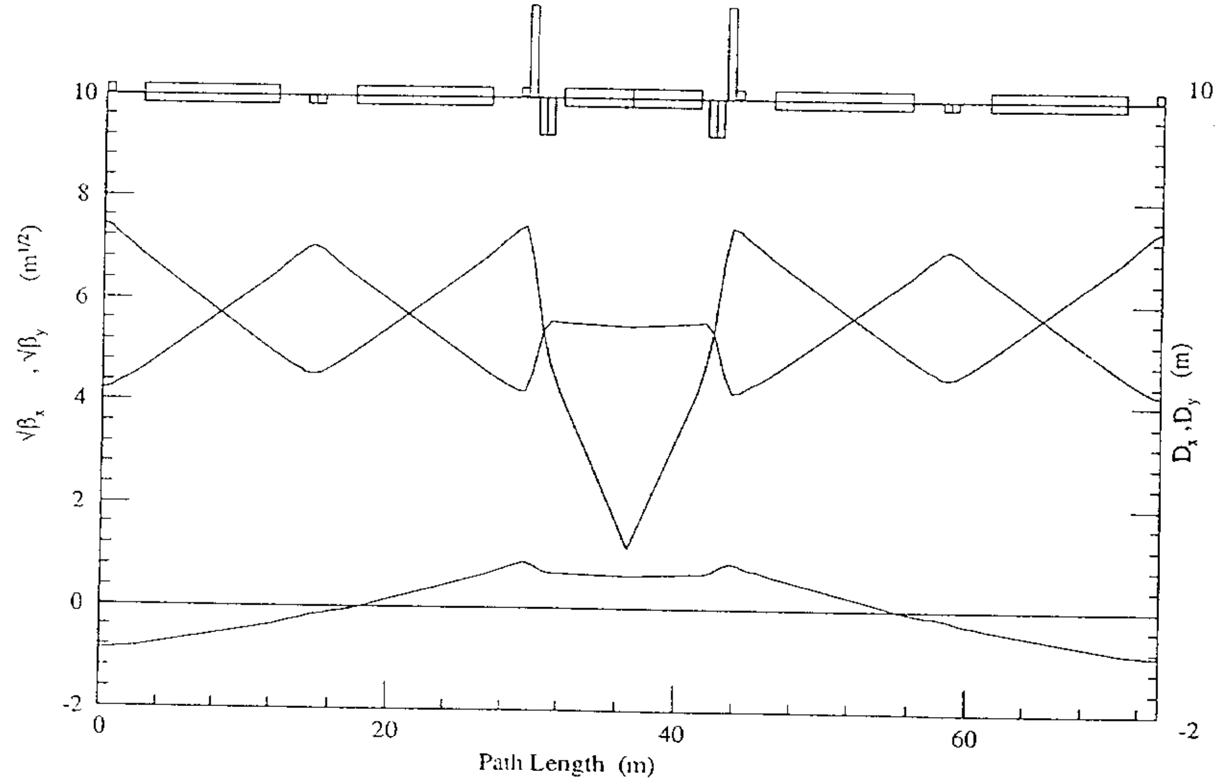


Figure 2 Betatron function within the basic block of the new lattice. The block consists of two FODO cells and a $\pi$-insertion.

## 3 CONCLUSION

A new type of lattice could be constructed, by using a combination of FODO cells and $\pi$-insertions, which provides double the momentum space of a regular FODO cell without losing much in the geometrical compaction factor. The new lattice not only improves the momentum dependence over that of the FODO design but could even be optically very stable, with smaller sextupole-induced second-order tune shifts.

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