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**FIRST THEORIST'S GAZE at HERA DATA  
at LOW  $x_B$**

**E. Levin**

*Fermi National Accelerator Laboratory, P.O.Box 500, Batavia, IL 60510, USA  
and*

*St. Petersburg Nuclear Physics Institute, 188350, Gatchina, Russia*

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**Abstract**

**This is a status report on the low  $x_B$  physics in deeply inelastic scattering just after the first experimental data from HERA.**



## I. Introduction

I am viewing this talk as a summary of the recent theoretical and experimental developments in the region of small  $x_B$  just after first experimental data from HERA [1][2][3][4]. In the talk I am going to cover the following topics:

- **Brief theoretical summary** in which I am going to discuss the status of our theoretical understanding of new phenomena that we anticipate in the region of low  $x_B$ . The selling formula of this section is

*“ We do theory right ! ”*

- **25  $nb^{-1}$  HERA physics.** Here I am going to present the first theoretical conclusions from HERA data at low  $x_B$  and convince you that even data from such a small integrated luminosity clarified the situation in region of low  $x_B$  a lot.. Thus

*“ It works ! ”*

- **What is next?** I'll discuss in this section some aspects of new experiments in the region of low  $x_B$ .

*“ Don't miss it!”*

## II. Brief theoretical summary.

1. *Three kinematical regions of QCD.* Let me start with a general description of what we know about QCD, accepting the widespread opinion that QCD is the only candidate for the theory of strong interactions. I hope that all experts will agree with me that we can distinguish three kinematical regions, with a specific approach in QCD to each of them:

A. The region of small distances ( $r_t \ll R_p \sim 1Fm$ ) and low density of partons ( $\rho$ ) where  $\rho$  is equal to

$$\rho = \frac{x_B G(x_B, Q^2)}{\pi R^2} . \quad (2.1)$$

In eq. (2.1)  $x_B G$  is the number of gluons at fixed  $y = \ln(1/x_B)$  and  $Q^2$  while  $\pi R^2$  is the area of the hadron disc in which gluons are distributed. In this region

$$\alpha_s(r_t^2) \ll 1 \text{ and } \rho \ll 1 . \quad (2.2)$$

Here we can use perturbative QCD approach. Frankly speaking only this region has been tested experimentally in the deeply inelastic scattering, in  $e^+e^-$  - annihilation

or in 'hard' production in hadron - hadron collisions. Thus this region is well known and it is only because of the great number of the experiments in this region that we are sure that QCD is really our microscopic basis for the theory of strong interactions.

B.  $r_t \sim R_p \sim 1Fm$ . This is the region where  $\alpha_s \geq 1$ . So in this region we are dealing with nonperturbative QCD and key words here are confinement of quarks and gluons, soft physics, nuclear physics, Regge approach for high energy scattering, quark - gluon plasma formation and so on. We certainly have not solved all problems in this region but I would like to emphasize that we have learned a lot of the main properties of QCD in this region using lattice QCD calculation [5] and QCD sum rules [6]. Unfortunately we still do not know how to use nonperturbative QCD methods for scattering processes and namely for collisions our knowledge of confinement degenerates in such monstres as so called hadronization models.

C. Between the above two regions there is a huge region where the distances are still small ( $r_t \ll R_p \sim 1Fm$ ) but the density of partons  $\rho$  becomes so large that we are not able to apply the ordinary methods of perturbative QCD. Only the edge of this region we can touch in perturbative QCD, namely, the vicinity of the border with region A. The kernel of the problem is a nonperturbative one. However the existence of the transition region where we can use pQCD allow us to make definite predictions for new phenomena that we anticipate in this region of small  $x_B$ . This is the key idea of the strategy for our approach: to move from what we know in pQCD to that region where we have not had the complete theory, but we developed the first approach to the problem and have some hopes to solve the problem theoretically since here still  $\alpha_s \ll 1$ . The most pure theoretical process is deeply inelastic scattering which we are going to consider in the next subsection.

2. *New phenomena at low  $x_B$* . Let me first list new phenomena that we anticipate in the region of small  $x_B$ :

- Increase of gluon ( quark ) density.
- Growth of the mean transverse momentum of gluon (quark ).
- Saturation of gluon density.

Each of these points has different theoretical status. Increase of the gluon density follows just from the solution to the usual (GLAP) [7] evolution equation, namely:

$$x_B G(x_B, Q^2) \propto e^{\sqrt{\frac{4N_c \alpha_s}{\pi} \ln \frac{1}{x_B} \ln \frac{Q^2}{Q_0^2}}}, \quad (2.3)$$

while the growth of the mean gluon transverse momentum can be seen only after resummation of the large contributions of the order of  $(\alpha_s \ln \frac{1}{x_B})^n$  in our perturbation series for  $x_B G(x_B, Q^2)$ . This resummation leads to the BFKL [8] evolution equation

and its solution gives:

$$x_B G(x_B, Q^2) \propto x_B^{-\omega_0}; \quad \omega_0 = \frac{4N_c \ln 2}{\pi} \cdot \alpha_s(Q_0^2); \quad (2.4)$$

$$\ln^2 \frac{p_t^2}{Q_0^2} = \frac{56\zeta(3)N_c}{\pi} \cdot \alpha_s(Q_0^2) \ln \frac{1}{x_B},$$

where  $p_t^2$  is the mean transverse momentum of gluons while  $Q_0^2$  is the initial one in the evolution.

Saturation of the gluon density is the direct consequence of the increase of the gluon density and S-channel unitarity. Since precise this phenomenon is the most important in the region of small  $x_B$  I'll discuss it in the next subsection in detail while the detailed discussion of the first two properties you could find in ref. [9].

3. *Saturation of gluon density.* S - channel unitarity says that the cross section of virtual photon absorption should be smaller than the geometrical size of the hadron (proton), namely

$$\sigma_{\gamma^*p} \ll \pi R^2. \quad (2.5)$$

Since

$$\sigma_{\gamma^*p} = \frac{\alpha_{em} F_2(x_B, Q^2)}{Q^2}$$

we get for

$$\frac{\alpha_{em} e^{\sqrt{\frac{4N_c \alpha_s}{\pi} \ln \frac{1}{x_B} \ln \frac{Q^2}{Q_0^2}}}}{Q^2} \quad \text{or} \quad \frac{\alpha_{em} e^{\frac{24N_c \ln 2}{\pi} \ln \frac{1}{x_B}}}{Q^2} \ll \pi R^2. \quad (2.6)$$

So one can see from eq.(2.6) that at  $x_B < x_{cr}$  where  $\ln \frac{1}{x_{cr}} \propto \ln^2 \frac{Q^2}{Q_0^2}$

$$\sigma_{\gamma^*p} \gg \pi R^2. \quad (2.7)$$

Thus the density of gluons becomes so large that the unitarity constraint is violated, even at a very large value of  $Q^2$ . The last fact is very important since it says that the miraculous confinement forces cannot help us solve this problem.

4. *Nonlinear (GLR) evolution equation.* Thus we should find the solution of the problem in the framework of perturbative QCD. To do this we need to resum the perturbation series in a new small parameter [10], namely

$$W = \frac{\alpha_s}{Q^2} \cdot \rho. \quad (2.8)$$

The first factor in eq.(2.8) is the cross section for gluon absorption by a parton from the hadron. So it is clear that  $W$  has a very simple physical meaning, namely it is the probability of parton (gluon) recombination in the parton cascade. We can rewrite the unitarity constraint (2.5) in the form

$$W \leq 1. \quad (2.9)$$

So  $W$  is the natural small parameter in our problem. It is worthwhile to note that  $W$  can be rewritten through the so called packing factor

$$PF = \langle r_{\text{constituent}}^2 \rangle \cdot \rho. \quad (2.10)$$

Indeed

$$W = \alpha_s \cdot PF. \quad (2.11)$$

The result of the resummation which has been done in ref. [10] can be easily understood considering the structure of the QCD cascade in a fast hadron. Inside of a cascade there are two processes that are responsible for the resulting number of partons:

$$\text{Emission } (1 \rightarrow 2); \text{ Probability } \propto \rho; \quad (2.12)$$

$$\text{Annihilation } (2 \rightarrow 1); \text{ Probability } \propto r^2 \rho^2 \propto \frac{1}{Q^2} \rho^2,$$

where  $r^2$  is the size of produced parton in the annihilation process. For deep inelastic scatterin  $r^2 \propto \frac{1}{Q^2}$ .

It is obvious that at  $x_B \sim 1$  only the production of new partons (emission) is essential since  $\rho \ll 1$ , but at  $x_B \rightarrow 0$  the value of  $\rho$  becomes so large that the annihilation of partons that diminishes the total number of gluons enters to the game.

Finally we can write a simple equation for the density of partons, namely:

$$\frac{\partial^2 \rho}{\partial \ln \frac{1}{x_B} \partial \ln Q^2} = \frac{\alpha_s N_c}{\pi} \rho - \frac{\alpha_s^2 \gamma}{Q^2} \rho^2, \quad (2.13)$$

or in terms of the gluon structure function  $x_B G(x_B, Q^2)$

$$\frac{\partial^2 x_B G(x_B, Q^2)}{\partial \ln \frac{1}{x_B} \partial \ln Q^2} = \frac{\alpha_s N_c}{\pi} x_B G(x_B, Q^2) - \frac{\alpha_s^2 \gamma}{Q^2 R^2} (x_B G(x_B, Q^2))^2. \quad (2.14)$$

Eq. (2.13) is the so called GLR equation [10] and unfortunately even now we need some complicated technique of summation of Feynman diagrams in  $W^n$ 'th order of perturbation theory to calculate the value of  $\gamma$  (see ref. [11] for details) and to understand the kinematical region where we can trust the equation (2.13).

5. *The scale of the shadowing corrections ( SC ).* The second term in eq. (2.14) describes the shadowing corrections and its value crucially depends on the value of  $R^2$ . The physical meaning of  $R^2$  is very simple, namely the fact that in our approach we assume that there are no correlations between gluons except the fact that they are confined in the disc of radius  $R$ . If  $R = R_{proton}$  the value of the SC turns to be negligibly small. If  $R \ll R_{proton}$ , the SC could be large ( see refs. [12][13][14] ). Fortunately during the last year the first theoretical estimates of the value of  $R$  have been done by Braun et al [15] in the framework of QCD sum rules. The result is

$$R = 0.3 - 0.35 \text{ Fm} \sim \frac{1}{3} R_{proton} .$$

This result encourages us to study SC experimentally at the present energy.

6. *Correlations.* As I have discussed we assumed that there are no correlations between gluon cascade except the fact that they are distributed in the disc of radius  $R$ . It means that the probability to find two partons with the same values of  $\ln \frac{1}{x_B}$  and  $\ln Q^2$  ( $P_2$ ) is

$$P_2 \propto \rho^2 . \quad (2.15)$$

However it turns out that this assumption is correct only in the case of the big numbers of colours ( $N_c \rightarrow \infty$ ) [16] [17]. If  $N_c$  is not too big the result of refs. [16] [17] can be rewritten as

$$\frac{P_2}{\rho^2} \rightarrow \exp\left(\frac{2N_c\alpha_s}{\pi} \cdot \frac{1}{(N_c^2 - 1)^2} \ln \frac{1}{x_B} \ln \frac{Q^2}{Q_0^2}\right) . \quad (2.16)$$

We can claim even more, namely that the probability to find  $n$  - gluons is proportional to ( see ref.[18] and ref.[14] for the review):

$$\frac{P_n}{\rho^n} \rightarrow \quad (2.17)$$

$$\rightarrow \exp\left(\frac{2N_c\alpha_s}{\pi} \cdot \frac{1}{(N_c^2 - 1)^2} \cdot \frac{1}{3} n^2(n^2 - 1) \ln \frac{1}{x_B} \ln \frac{Q^2}{Q_0^2}\right) \text{ at } x_B \rightarrow 0 .$$

Eq.(2.16) was obtained as the result of several reduction:

$$\begin{aligned} & [2n \text{ gluon cascades with attractive forces} ] \rightarrow \quad (2.18) \\ & \rightarrow [n \text{ "Pomerons" with attractive forces} ] \rightarrow \\ & \rightarrow [ \text{Nonlinear Schroedinger equation in one dimensional space} + \\ & \quad + \text{ Bethe ansatz for the solution} ] \rightarrow \end{aligned}$$

→ [Bosons (gluons, "Pomerons") behave as fermions].

Experience with the solution to the correlation function tells us that the saturation hypothesis for the gluon density at  $x_B \rightarrow 0$  is quite likely. This hypothesis can be expressed in the form;

$$PF \rightarrow \text{Constant}; \quad x_B G(x_B, Q^2) \propto Q^2; \quad \text{at } x_B < x_\sigma \quad (2.19)$$

where

$$\ln \frac{1}{x_B} = 0.079 \ln^2 \frac{Q^2}{Q_0^2}.$$

I have no time to discuss all theoretical topics in details but you will be able to find them in refs. [9] [10] [11] [14] [19].

### III. 25 nb<sup>-1</sup> HERA physics.

In short, the conclusion of this section could be expressed as

*"What we want (large density of gluons and significant SC at small  $x_B$ )  
is what we get at HERA today!"*

1. *Preliminary experimental data.* Let me first summarize the first experimental result at small  $x_B$  from both collaborations at HERA ( refs. [1][2] [3] [4] ):

1. The increase of the value of the deep inelastic structure function  $F_2$  at  $x_B \rightarrow 10^{-4}$  shows that the gluon density reaches sufficiently big value, namely

$$x_B G(x_B, Q^2) \rightarrow 40 - 50 \text{ at } x_B = 10^{-4} \text{ and } Q^2 = 20 \text{ GeV}^2.$$

The value  $x_B G(x_B, Q^2) = 50$  I took from the *MRS D* parametrization that describes the new data.

2. ZEUS collaboration measured a very important value, namely the cross section of the diffraction dissociation and the result is

$$\frac{\sigma^{DD}}{\sigma_t} = \frac{5.2 \text{ nb}}{80 \text{ nb}} = 6 \cdot 10^{-2} \text{ at } x_B = 10^{-4} \text{ and } Q^2 > 10 \text{ GeV}^2.$$

2. *Proton versus Fe.* Let me interpret a little bit the above experimental data making them closer to our everyday experience. I believe that the best way to do this is to compare the structure of proton at  $x_B = 10^{-4}$  and  $Q^2 = 15 \text{ GeV}^2$  with the nucleus of iron. This comparison is shown in the table below where I calculated the packing factor for the both cases, taking the size of the parton  $R_G^2 = \frac{1}{Q^2}$  and the size of the nucleon  $R_N = 0.86 \text{ fm}$ .

Table 1: Proton versus Fe.

Constituents	Proton ( $x_B = 10^{-4}$ , $Q^2 = 15\text{GeV}^2$ )		Fe
	gluons	gluons	nucleons
Radius of the disc ( R )	$R = R_N$	$R = \frac{1}{3} R_N$	4.2 Fm
Number of constituents	50	50	56
Density of constituents ( $\rho$ )	36.98	332.8	1.01
Packing Factor ( PF )	0.114	1.03	0.75

Inspite of the sufficiently rough estimates we can conclude that:

$$\text{" Proton ( at } x_B = 10^{-4} \text{ and } Q^2 = 15\text{GeV}^2 \text{ ) } = \text{Fe!"}$$

In particular it means that we anticipate sufficiently large SC at HERA.

3. *The value of SC from HERA data.* Let me write the deep inelastic structure function in the form:

$$F_2(x_B, Q^2) = F_2^{GLAP}(x_B, Q^2) - \Delta F_2(x_B, Q^2), \quad (3.1)$$

where  $F_2^{GLAP}$  is the solution of the usual ( GLAP ) evolution equation and  $\Delta F_2$  is the SC. Just from very natural estimates we have

$$\frac{\Delta F_2}{F_2} \propto \alpha_s PF \rightarrow 0.1 \text{ for } Q^2 = 15\text{GeV}^2 \text{ and } x_B = 10^{-4}. \quad (3.2)$$

However the ZEUS data on diffraction dissociation gives us the possibility to estimate the value of SC better. Indeed, it was shown in ref. [20] that we have the relationship between SC and DD cross section directly from AGK cutting rules [21]:

$$|\Delta F_2| = F_2^{DD}, \quad (3.3)$$

where  $F_2^{DD}$  was defined in ref. [20]. In other words

$$\left| \frac{\Delta \sigma_t}{\sigma_t} \right| = \frac{\sigma_{\gamma^*P}^{DD}}{\sigma_t}. \quad (3.4)$$

Therefore directly from ZEUS data we can conclude:

$$\frac{|\Delta F_2|}{F_2} > 6 \cdot 10^{-2}.$$

I would like to emphasize that after such an estimates we can start to discuss not the question whether there is SC or not, but rather the question whether we could describe the value of SC in our theory.

I would like also to note, that the details of the evolution equation for  $F_2^{DD}$  were discussed in ref.[20] and it was suggested to measure the sum  $F_2 + F_2^{DD}$  in which all contributions of SC are cancelled. Thus for this sum we can use the GLAP equation even in the region of very small  $x_B$ .

#### IV. What is ahead?

I have no time to discuss such theoretical problems as the new evolution equation that replaces the GLR one in the case of finite  $N_c$  or the proof of AGK cutting rules that also has an accuracy of the order of  $\frac{1}{N_c^2-1}$ . Here I am going to discuss only one topics, namely the large rapidity gap ( LRG) physics in deep inelastic scattering.

Let us consider the typical process with LRG, namely the production of two jets with large transverse momenta  $p_{1t} \sim -p_{2t}$  and with the rapidity interval between them  $y_1 - y_2 \gg 1$ . The cross section for such process could be written using factorization formula [22] in the form:

$$\frac{d\sigma^{FF}}{d\Delta y} = \int x_1 G(x_1, p_{1t}^2) x_2 G(x_2, p_{2t}^2) \sigma^{hard} , \quad (4.1)$$

where

$$\sigma^{hard} \propto \frac{1}{p_{1t}^2} \cdot \left[ \frac{x_1 x_2 s}{p_{1t}^2} \right]^{\omega_0(p_{1t}^2)} . \quad (4.2)$$

Here  $s = \frac{Q^2}{x_B}$  and  $\omega_0 = C \alpha_s(p_{1t}^2)$  (see refs. [23] [24] [25] for details).

However Bjorken [26] noted that this formula is not correct if you would like to separate the sample without any hadron with rapidity  $y$  between  $y_1$  and  $y_2$  ( $y_1 < y < y_2$ ). It should be stressed that this signature is the very interesting way how to extract experimentally the jet production due to exchange of a colourless "Pomeron". The cross section of the above process should be written in the form [26]:

$$\frac{d\sigma}{d\Delta y} = S^2 \frac{d\sigma^{FF}}{d\Delta y} , \quad (4.3)$$

where  $S^2$  is the survival probability for LRG. In hadron - hadron collisions it has been shown [26] [27] that

$$S^2 = 0.10 - 0.15 \text{ at } \sqrt{s} = 1.8 \text{TeV} ; S^2 \sim 0.05 \text{ at } \sqrt{s} = 40 \text{TeV} .$$

It should be also stressed that all estimates in hadron - hadron collisions for  $S^2$  cannot be done in purely theoretical way. We have to use some model assumptions. In deeply inelastic scattering the situation is quite different: we can develop here a

theoretical approach to calculate  $S^2$  [28] and the value of  $S^2$  turns out to be rather large (  $S^2 \geq 0.7$  [28] ).

So we have a good chance to do LRG physics in deep inelastic scattering even at HERA energies.

## V. Conclusions.

1. In theory we understood that the GLR equation is a good first approximation in the case of  $N_C \rightarrow \infty$  and we have started to develop technique to take into account  $\frac{1}{N_C^2-1}$  corrections (for the review of the status of such attempts see ref. [14]).

2. The preliminary experimental data from both collaborations at HERA show the increase of the gluon density at small value of  $x_B$  and give the reliable estimates for the value of shadowing (screening) corrections from the data on diffraction dissociation in deeply inelastic processes. These data move the central question from whether there is SC or whether they are small to the question whether theory can describe the experimental value of SC if that turns out to be big enough.

3. Deeply inelastic scattering is just the right place for large rapidity gap physics.

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