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Extracting CKM parameters from B decays*

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Abstract

This note extracts CKM (Cabibbo-Kobayashi-Maskawa) parameters from currently triggerable B-decay modes. The classic $B_d \to J/\psi K_S$ asymmetry measures the angle β, one of the angles of the CKM unitarity triangle. The other angles of that triangle are more difficult to extract. A tagged, time-dependent study of $B_s \to J/\psi\phi$ extracts the angle γ. Such a study of $B_d \to J/\psi\rho^0$ independently determines γ, where $B_d \to J/\psi K^*$ needs to be studied for normalization purposes. A tagged study of the classic $B_d \to \pi^+\pi^-$ extracts α if the penguin amplitude is negligible. The penguin may be sizeable, however. An involved isospin analysis is then required. It measures α by disentangling the penguin from the tree amplitude. At hadron accelerators, this isospin analysis would require a tagged, time-dependent study of $B_d \to \pi^0\pi^0$, which is currently impossible. This note presents alternatives for measuring α. The angle could be obtained from studies of exclusive modes that are governed by $b \to d\ \ell^+\ell^-$, such as $B \to \rho\ell^+\ell^-$. The branching ratio for such an exclusive mode is tiny, at the few 10^{-8} level. Another method for measuring this angle requires the study of both $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$. Many more modes could be used to extract CKM parameters, if triggering on secondary vertices becomes feasible. The methods discussed here require high precision. They require tremendous effort experimentally and theoretically. Experiment will guide us toward the feasible modes and theory must accurately estimate ratios of related strong matrix elements.

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1. Introduction

Not all the three angles α, β, and γ of the CKM (Cabibbo-Kobayashi-Maskawa) [1] unitarity triangle [2] can be straightforwardly extracted; see Figure 1. The angle β is the easiest to extract and can be obtained from the $B_d \to J/\psi\ K_S$ asymmetry with negligible hadronic uncertainty [3]. The uncertainty of strong matrix-elements cancels in a ratio which determines the $B_d \to J/\psi K_S$ asymmetry.

The determination of α and γ is more difficult on two fronts. First, the straightforward asymmetries, such as for $B_d \to \pi^-\pi^+$ and $B_s \to \rho^0 K_S$, may not suffice to determine the angles of the CKM triangle, because of penguin diagrams. Elaborate methods have been proposed to overcome this problem [4] - [11]. Second, all existing methods employ modes that cannot currently be triggered on at hadron accelerators.

This note shows how triggerable modes, those with di-leptons in the final state, can extract all the angles of the unitarity triangle: α, β and γ. Further, all the angles can be obtained from triggerable B_d modes alone. Additional triggerable B_s modes are available for the extraction, if the resolution is good enough to observe time-dependent effects of the B_s. Time-integrated rate-asymmetries are not as useful, because they are badly diluted by the large mixing parameter x_s,

$$\frac{\int_0^\infty dt \left[\Gamma\left(B_s(t) \to f\right) - \Gamma\left(\bar{B}_s(t) \to \bar{f}\right)\right]}{\int_0^\infty dt \left[\Gamma\left(B_s(t) \to f\right) + \Gamma\left(\bar{B}_s(t) \to \bar{f}\right)\right]} \sim \frac{\mathrm{Im}\lambda}{x_s} \quad \text{for large } x_s\ . \tag{1}$$

Before turning to the triggerable modes, we briefly review the existing methods. The angle α can be determined from the $B_d \to \pi^+\pi^-$, $\rho^\pm\pi^\mp, a_1^\pm\pi^\mp$ asymmetries [4], when penguin contributions are negligible. For sizeable penguins, elaborate isospin analyses extract α by disentangling the penguin from the tree [5]. Since the $\pi\pi$ isospin analysis requires time-dependent studies of $B_d \to \pi^0\pi^0$ decays at hadron machines, it will not work in the foreseeable future [6]. A combined Dalitz plot and isospin analysis of $B \to \rho\pi$ may be able to determine α [7]. The angle α can also be extracted from six decay modes related to $B_d \to D^0 K_S$ [8], or variants thereof [9].

Most methods extract γ from tagged, time-dependent studies of specific B_s-decays. Experimentalists will have to learn how to observe the rapid B_s-oscillations for very large B_s mixing,

$$x_s \gtrsim 20\ . \tag{2}$$

The $B_s \to \rho^0 K_S$ asymmetry would measure γ, if penguins could be neglected. But penguins may not be negligible, and γ cannot be cleanly extracted from the asymmetry. Anyway, the branching ratio of this color-suppressed mode is expected to be tiny, at the 10^{-7} level [10].

The angle γ can be extracted from tagged, time-dependent studies of [10] $B_s \to D_s^{\pm} K^{\mp}$ or $B_s \to D^0\phi$ [8]. Penguins cannot contribute and the branching ratio for the color-favored $B_s \to D_s^{\pm} K^{\mp}$ mode is expected to be large,

$$B(B_s \to D_s^{\pm} K^{\mp}) \sim 2 \times 10^{-4} \,. \tag{3}$$

Modes of beautiful hadrons with neutral D's can be used to extract γ. Neither tagging nor time-dependence is required for this extraction. The angle γ is obtained by measuring the rates of six processes, $B \to D^0K, \bar{D}^0K, D^0_{CP}K$ and their CP-conjugated partners [8], [11]. Here D^0_{CP} denotes that the neutral D is seen in modes with definite CP parity.

We designate by K^r those resonances of K^0 which can appreciably be seen both in modes that determine their kaon flavor and in modes where the kaon flavor is lost. The K^r_{CP} denotes CP-eigenmodes of the K^r resonance, which are modes with undetermined kaon flavor. Two such resonances are K^{*0} and $K_1(1270)$. The K^{*0} is seen in its $K^+\pi^-$ mode two-thirds of the time and in its $K_S\pi^0$ mode one-sixth of the time. Because the π^0 may be difficult to detect in a hadronic environment, we consider the $K_1(1270)$. It is not too broad, $\Gamma \simeq 90$ MeV, and is seen appreciably in the $K_0^{*+}(1430)\pi^-$ and $K^{*+}\pi^-$ modes that tag the original kaon-flavor. A mode where the original kaon-flavor is lost is

$$B(K_1(1270) \to K_S\rho^0) = 0.07 \,. \tag{4}$$

A Dalitz plot analysis distinguishes among the various modes of $K_1(1270)$.

Time-dependent studies are reviewed in Section 2. Section 3 shows that a tagged, time-dependent study of $B_s \to J/\psi\phi$ measures γ or alternatively α. It notes, in passing, that a similar study of color-allowed modes, $B_s \to D_s^+D_s^-, D_s^{*+}D_s^{*-}, D_s^+D_s^{*-}, D_s^{*+}D_s^-$, extracts the same CKM angle. Section 4 mentions that the $b \to d + c\bar{c}$ transition involves non-spectator amplitudes at the few percent level compared to the dominant spectator one. The interference between the non-spectator with the spectator amplitude could result in direct CP violation [12]. Section 5 exploits this interference to measure γ from a tagged, time-dependent study of any of the following processes of $B_d \to J/\psi\ \rho^0, J/\psi\omega, J/\psi\pi^0, B_s \to J/\psi\ K_S, B_s \to J/\psi\ \bar{K}^{*0}, B_s \to J/\psi\ \bar{K}^r$. The measurement of γ can be performed regardless of whether direct CP violation occurs. Section 6 sketches the extraction of the angle α from CKM suppressed exclusive rare decays governed by $b \to d + \ell^+\ell^-$, such as $B \to \rho\ \ell^+\ell^-, B \to \pi\ \ell^+\ell^-, B \to \omega\ \ell^+\ell^-$. If we could succeed in triggering on secondary vertices, many more modes could be used to measure CKM parameters. Although some of those modes are briefly discussed in Sections 3 and 5, Sections 7 and 8 are devoted entirely to them. Section 7 determines α from the $B_d \to \pi^+\pi^-$ mode when the penguin graph contributes sizeably, without recourse to an isospin analysis. The determination of CKM parameters from many additional modes, governed by $b \to d +$ charmless, is covered in Section 8. The short hand $b \to d +$ charmless denotes any of the $b \to du\bar{u}, b \to dd\bar{d}$ and $b \to ds\bar{s}$ quark transitions. Sections 5 - 8 extract the CKM angles up to discrete ambiguities. We chose not to discuss them because the treatment would become more cumbersome. The time to analyze all the possible ambiguities is when data has been accumulated. Unfortunately this lies many years into the future. Conclusions can be found in Section 9.

The CKM angles can be extracted only when ratios of related strong matrix elements are accurately known, except in the method described in Section 3. Those ratios will have to be analyzed carefully.

We have organized this report in terms of measuring the CKM angles α, β, and γ, because this has become the popular description of the CKM model. However, on a more fundamental level, the CKM matrix can be parametrized by a single CP-violating parameter and three magnitudes. Within that context, the report extracts various CKM combinations. The extractions allow the overdetermination of the CKM model.

2. Time-Dependence

This section reviews time-dependent amplitudes for the decay of a neutral B to a final state f [13, 14, 4]. This intriguing phenomenon occurs because of $B^0 - \bar{B}^0$ mixing. The time-evolutions of initially unmixed B^0 and $\bar{B}^0$ are

$$|B^0(t)\rangle = c(t)\,|B^0\rangle + i\,\frac{q}{p}\,s(t)\,|\bar{B}^0\rangle\ , \tag{5}$$

$$|\bar{B}^0(t)\rangle = c(t)\,|\bar{B}^0\rangle + i\,\frac{p}{q}\,s(t)\,|B^0\rangle\ , \tag{6}$$

where

$$c(t) = e^{-i\,\frac{m_L+m_H}{2}t}\,e^{-\frac{\Gamma}{2}t}\,\cos\frac{\Delta m t}{2}\ , \tag{7}$$

$$s(t) = e^{-i\,\frac{m_L+m_H}{2}t}\,e^{-\frac{\Gamma t}{2}}\,\sin\frac{\Delta m t}{2}\ . \tag{8}$$

The parameters q and p are the coefficients which relate the B^0 and $\bar{B}^0$ to the mass-eigenstates. The CKM model predicts

$$|q/p| \approx 1, \tag{9}$$

to an accuracy of 10^{-3} for the B_d system, and to 10^{-4} for the B_s system. The ratio q/p is essentially a phase given by

$$\frac{q}{p} = \frac{V_{tb}^* \, V_{tx}}{V_{tb} \, V_{tx}^*} \tag{10}$$

where $x = d$ or s for the B_d or B_s system. Define the CP-conjugated final-state as

$$|\bar{f}\rangle = CP|f\rangle \, . \tag{11}$$

Consider the four time-dependent rates of an initially unmixed neutral B to f and $\bar{f}$.

$$\begin{aligned} \Gamma\left(B^0(t) \to f\right) &= e^{-\Gamma t}\Big\{|\langle\, f|B^0\rangle|^2 \cos^2\frac{\Delta m t}{2} + |\langle\, f|\bar{B}^0\rangle|^2 \sin^2\frac{\Delta m t}{2} \\ &- |\langle\, f|B^0\rangle|^2 \mathrm{Im}\,\lambda \sin\Delta m t\Big\} \, , \end{aligned} \tag{12}$$

$$\begin{aligned} \Gamma\left(\bar{B}^0(t) \to f\right) &= e^{-\Gamma t}\Big\{ |\langle\, f|\bar{B}^0\rangle|^2 \cos^2\frac{\Delta m t}{2} + |\langle\, f|B^0\rangle|^2 \sin^2\frac{\Delta m t}{2} \\ &+ |\langle\, f|B^0\rangle|^2 \mathrm{Im}\,\lambda \sin\Delta m t\Big\} \, , \end{aligned} \tag{13}$$

$$\begin{aligned} \Gamma\left(\bar{B}^0(t) \to \bar{f}\right) &= e^{-\Gamma t}\Big\{|\langle\, \bar{f}|\bar{B}^0\rangle|^2 \cos^2\frac{\Delta m t}{2} + |\langle\, \bar{f}|B^0\rangle|^2 \sin^2\frac{\Delta m t}{2} \\ &+ |\langle\, \bar{f}|B^0\rangle|^2 \mathrm{Im}\,\lambda' \sin\Delta m t\Big\} \, , \end{aligned} \tag{14}$$

$$\begin{aligned} \Gamma\left(B^0(t) \to \bar{f}\right) &= e^{-\Gamma t}\Big\{|\langle\, \bar{f}|B^0\rangle|^2 \cos^2\frac{\Delta m t}{2} + |\langle\, \bar{f}|\bar{B}^0\rangle|^2 \sin^2\frac{\Delta m t}{2} \\ &- |\langle\, \bar{f}|B^0\rangle|^2 \mathrm{Im}\,\lambda' \sin\Delta m t\Big\} \, , \end{aligned} \tag{15}$$

where

$$\lambda \equiv \lambda\left(B^0 \to f\right) \equiv \frac{q}{p}\,\frac{\langle\, f|\bar{B}^0\rangle}{\langle\, f|B^0\rangle} \, , \tag{16}$$

$$\lambda' \equiv \lambda(B^0 \to \bar{f}) \equiv \frac{q}{p}\,\frac{\langle\, \bar{f}|\bar{B}^0\rangle}{\langle\, \bar{f}|B^0\rangle} \, . \tag{17}$$

The distinction between an initial unmixed B^0 and $\bar{B}^0$ is called tagging. Whenever we speak about a tagged, time-dependent study of $B^0 \to f$, we mean the study of all four time-dependent rates, Eqs. (12)-(15). For final states which are CP eigenstates, only two distinct time-dependent rates exist.

A tagged, time-dependent study measures the magnitudes of the unmixed amplitudes,

$$|\langle f|B^0\rangle|,\ \ |\langle f|\bar{B}^0\rangle|,\ \ |\langle \bar{f}|\bar{B}^0\rangle|,\ \ |\langle \bar{f}|B^0\rangle| \tag{18}$$

and

$$\mathrm{Im}\,\lambda(B^0 \to f),\ \ \mathrm{Im}\,\lambda(B^0 \to \bar{f}). \tag{19}$$

Strictly speaking the four time-dependent rates of Eqs. (12)-(15) were derived under the assumption that there is no lifetime difference ($\Delta\Gamma$) between the heavy and light B^0. While $\Delta\Gamma$ may be observable in the B_s system [15, 16],

$$\Delta\Gamma/\Gamma \sim 10\%, \tag{20}$$

it is negligible for the B_d system. Tagged, time-dependent fits with non-zero $\Delta\Gamma$ extract, in addition to the above observables, Eqs. (18)-(19), the quantities [13]

$$\mathrm{Re}\,\lambda(B^0 \to f)\ \text{ and }\ \mathrm{Re}\,\lambda(B^0 \to \bar{f}). \tag{21}$$

The λ's will be known without any ambiguity.

3. γ (or α) from $B_s \to J/\psi\phi$

The angle γ can be extracted from tagged, time-dependent measurements of $\bar{B}_s \to J/\psi\ \phi$ [13, 17]. The unmixed $\bar{B}_s \to J/\psi\ \phi$ amplitude is dominated by the CKM combination $V_{cb}V_{cs}^*$, and $B(\bar{B}_s \to J/\psi\ \phi) \approx 10^{-3}$ can be inferred from the measured $B(B_d \to J/\psi K^{*0})$ [18]. The amplitude with the different CKM combination $V_{ub}V_{us}^*$ is negligible, because it is suppressed by three orders of magnitude [12]. A tagged, time-dependent study of $B_s \to J/\psi\ \phi$ measures

$$\lambda = \frac{V_{tb}^*\,V_{ts}}{V_{tb}\,V_{ts}^*}\quad\frac{V_{cb}\,V_{cs}^*}{V_{cb}^*\,V_{cs}}\,. \tag{22}$$

The final state has both CP-even and CP-odd components, diluting the CP asymmetry. An angular analysis can disentangle the CP-odd from the CP-even contributions [19, 20]. It extracts λ without loss in statistical accuracy when one CP-parity dominates. But even when no CP-parity dominates, the full angular distribution measures Imλ with a statistical accuracy that at worst would require no more than four times the statistics compared to a definite CP eigenstate [20]. CLEO and ARGUS results indicate that the helicity-zero $B_d \to J/\psi\ K^{*0}$ amplitude is dominant [21]. This result suggests that the final state of $B_s \to J/\psi\ \phi$ is mainly CP-even.

The tagged, time-dependent distribution (for the CP-even part) is

$$\Gamma\left(\overset{(-)}{B_s^0}(t) \to J/\psi\ \phi\right) \sim e^{-\Gamma t}\left\{1 \overset{(+)}{-} \mathrm{Im}\ \lambda\ \sin\ \Delta m t\right\}, \tag{23}$$

where Γ denotes the B^o lifetime, Δm the positive $B^o - \bar{B}^o$ mass difference, and

$$\mathrm{Im}\lambda = 2|V_{cd}|\left|\frac{V_{ub}}{V_{cb}}\right|\sin\gamma\left(1 + \mathcal{O}\left(\theta^2\right)\right) \tag{24}$$

$$= 2|V_{cd}|\left|\frac{V_{td}}{V_{cb}}\right|\sin\beta\left(1 + \mathcal{O}\left(\theta^2\right)\right) \tag{25}$$

$$= 2\left|\frac{V_{td}\ V_{ub}}{V_{cb}^2}\right|\sin\alpha\left(1 + \mathcal{O}\left(\theta^2\right)\right) \tag{26}$$

The sine of the Cabibbo angle is denoted by $\theta \equiv \sin\ \theta_c = 0.22$.

Measuring Imλ requires the tagged, time-dependent study of $B_s \to J/\psi\ \phi$. Once Imλ is measured, we can either choose to determine $\sin\gamma$ by using Eq. (24) with the by then accurate measurement of $|V_{ub}/V_{cb}|$. Or $\sin\alpha$ can be extracted from Eq. (26) with the by then well known quantities $|V_{td}V_{ub}/V_{cb}^2|$.

The CKM model predicts large values for γ,

$$0.3 \lesssim \sin\gamma \leq 1. \tag{27}$$

Equation (27) and present measurements of $|V_{ub}/V_{cb}|$ guarantee that the interference term never vanishes,

$$0.01 \lesssim \mathrm{Im}\lambda \lesssim 0.05. \tag{28}$$

Thus the CKM model predicts *nonvanishing* CP violation at the θ^2 level. Measuring Imλ = 0.05 (to 3σ) requires the observation of 3600 tagged $B_s \to J/\psi\ \phi$ decays, assuming perfect tagging and time-resolution.

The same CP violating interference term occurs for the modes governed by $b \to c\bar{c}s$, such as $B_s \to D_s^+ D_s^-, D_s^{*+} D_s^{*-}, D_s^+ D_s^{*-}, D_s^{*+} D_s^-$. Those modes are dominantly CP-even [16]. To increase the data sample, they could be added and measure Imλ up to a correction that depends upon the dilution coming from the CP-odd parity.

4. Direct CP Violation with $b \to d(c\bar{c})$

The branching ratio for the color and CKM suppressed exclusive decay mode $H_b \to H_d J/\psi$ is,

$$B(H_b \to H_d J/\psi) \sim 5 \times 10^{-5} \ . \tag{29}$$

Here H_b (H_d) denotes a bottom (down)-quark flavored hadron. Rate asymmetries at the few percent level are possible [12]. They require neither tagging nor time-dependences, except for modes of neutral B mesons where H_d decays into a CP eigenstate.

The comparison between the $H_b \to H_d J/\psi$ process with its CP-transformed partner may exhibit CP violation not only in a rate comparison, but in other decay parameters as well. For instance, compare

$$\begin{array}{rcl}
B^- \to J/\psi\pi^- & \text{versus} & B^+ \to J/\psi\pi^+, \\
B^- \to J/\psi a_1^- & \text{versus} & B^+ \to J/\psi a_1^+, \\
B^- \to J/\psi\rho^- & \text{versus} & B^+ \to J/\psi\rho^+, \\
\bar{B}_s \to J/\psi K^{*0} & \text{versus} & B_s \to J/\psi \bar{K}^{*0}, \\
\bar{B}_s \to J/\psi K^+\pi^- & \text{versus} & B_s \to J/\psi K^-\pi^+, \\
\Xi_b^0 \to J/\psi\Lambda & \text{versus} & \bar{\Xi}_b^0 \to J/\psi\bar{\Lambda}, \\
\Omega_b^- \to J/\psi\Xi^- & \text{versus} & \bar{\Omega}_b^+ \to J/\psi\bar{\Xi}^+ \ .
\end{array}$$

The amplitude for the process is

$$A \equiv A(H_b \to H_d J/\psi) = \xi_c a_2 + \xi_u a_u \ , \tag{30}$$

while that for the CP-conjugated process is

$$\bar{A} \equiv A(\bar{H}_b \to \bar{H}_d J/\psi) = \xi_c^* a_2 + \xi_u^* a_u . \tag{31}$$

Here

$$\xi_q = V_{qb} V_{qd}^*, \quad q = u, c, t, \tag{32}$$

are CKM combinations, and a_2 and a_u strong matrix elements which probably differ in their final-state phases. Unitarity of the CKM matrix eliminated the ξ_t contribution to the amplitude. One CP violating observable is the rate asymmetry,

$$\begin{aligned}
\text{Asym} &\equiv \frac{|A|^2 - |\bar{A}|^2}{|A|^2 + |\bar{A}|^2} \simeq \\
&\simeq -2\sin\gamma \ \operatorname{Im}\left(\frac{a_u}{a_2}\right) \left|\frac{V_{ub}V_{ud}}{V_{cb}V_{cd}}\right|
\end{aligned} \tag{33}$$

The CKM model predicts $\sin\gamma$ to be large, Eq. (27). A recent report has shown that [12]

$$|a_u/a_2| \simeq 0.05 \ , \tag{34}$$

where a_u is estimated from the one-loop electroweak contributions. The conventional non-leptonic penguin amplitude requires at least three gluons to create the J/ψ. The final-state phase difference is currently being investigated [22].

There is no need to limit ourselves to the $H_b \to J/\psi H_d$ modes. CP asymmetries at the 1% level occur for the truly semi-inclusive $b \to c\bar{c}d$ mode. For instance, the asymmetry would show up when all the B^- modes governed by $b \to dc\bar{c}$ are summed over, such as $B^- \to D^- D^0$, $D^{*-}D^0, D^-D^{*0}, D^{*-}D^{*0}, J/\psi\pi^-, J/\psi\rho^-, J/\psi a_1^-$, etc. Neither tagging nor time-dependences are required to observe this CP violating effect. The inclusive $b \to dc\bar{c}$ asymmetry can be more reliably calculated than the exclusive ones. It is trivial to obtain the asymmetry from the existing literature which considered $b \to d$ + charmless. Some choice exclusive modes have been studied [23, 24]. The CPT theorem requires that [25]

$$\Gamma\left(b \to dc\bar{c}\right) - \Gamma\left(\bar{b} \to \bar{d}c\bar{c}\right) \;=\; -\left(\Gamma\left(b \to d + \text{charmless}\right) - \Gamma\left(\bar{b} \to \bar{d} + \text{charmless}\right)\right) \tag{35}$$

Thus the inclusive $b \to dc\bar{c}$ asymmetry can be estimated from the published calculations of $b \to d$ + charmless [26]. But let us review what is involved in calculating the inclusive $b \to dc\bar{c}$ asymmetry. First, the q^2 dependence of the virtual gluon of the penguin graph is tightly constrained, $4m_c^2 < q^2 < m_b^2$. The gluon is hard and can be treated perturbatively. Further, the absorptive part relevant for CP violation emerging from the u-quark loop is not kinematically suppressed. Asymmetries at the percent level result. This perturbative treatment is more justifiable for the inclusive $b \to dc\bar{c}$ process than for the exclusive modes, such as $B^- \to D^0 D^-$, $D^0 D^{*-}, D^{*0}D^-, D^{*0}D^{*-}$, because of rescattering among them. It is possible that some of the exclusive modes will show larger CP-violating effects, which will be compensated by smaller effects with other modes.

In conclusion, the CKM-suppressed modes governed by the $b \to d\ c\bar{c}$ transition may show CP violating effects at best at the few percent level. Although $\sin\gamma$ is proportional to the rate asymmetry, we cannot extract it, due to our lack of understanding about final-state interactions. The effects of final state interactions largely cancel in ratios of related processes. Such ratios may then allow the extraction of CKM parameters, which will be the topic of the next sections.

5. γ from $B_d \to J/\psi\rho^0$

Section 4 focussed on exclusive modes governed by $b \to dc\bar{c}$. Those modes involve two interfering amplitudes with the relative CKM angle γ. If the strong matrix elements could be calculated from first principles, the CKM parameters could be extracted from CP-violating effects and other observables. However, our ability to estimate strong matrix elements is meager. Although we are currently not able to estimate strong matrix elements, we may be able to estimate their ratios more reliably. This note extracts CKM parameters by using such ratios. The extraction of γ will be illustrated with the $B_d \to J/\psi\rho^0$ mode, where $B_d \to J/\psi K^{*0}$ serves as normalization. However, each of the modes, $B_d \to J/\psi\pi^0, J/\psi\omega, D^+D^-, D^{*+}D^-, D^+D^{*-}, D^{*+}D^{*-}$, etc., extracts γ, with the normalization coming from $B_d \to J/\psi K_S, J/\psi K^{*0}, D_s^+D^-, D_s^{*+}D^-, D_s^+D^{*-}, D_s^{*+}D^{*-}$, etc., respectively.

For the exclusive modes governed by $b \to dc\bar{c}$, factorization applied to the effective Hamiltonian predicts a penguin to tree amplitude-ratio of a few percent [12, 22], $|a_u/a_2| \sim 0.05$. This section demonstrates that γ can be extracted regardless of whether or not the final-state phase difference—that is, the phase of a_u/a_2—vanishes.

Consider then the $B_d \to J/\psi\rho^0$ mode and use $B_d \to J/\psi K^{*0}$ as normalization. The amplitude of the unmixed $\bar{B}_d \to J/\psi\rho^0$ is

$$\bar{A} \equiv \langle\, J/\psi\rho^0|\bar{B}_d\rangle = \xi_c a_2 + \xi_u a_u = \xi_c a_2[1 + ze^{-i\gamma}] = \xi_c a_2 \bar{b}\,, \tag{36}$$

while that for the unmixed $B_d \to J/\psi\rho^0$ is

$$A \equiv \langle\, J/\psi\rho^0|B_d\rangle = \xi_c^* a_2[1 + ze^{i\gamma}] = \xi_c^* a_2 b\,. \tag{37}$$

The normalization comes from the CKM favored mode $B_d \to J/\psi K^{*0}$,

$$\langle\, J/\psi K^{*0}|B_d\rangle = v_c^* a_2 + v_u^* a_u = v_c^* a_2[1 + \mathcal{O}(10^{-3})]\,, \tag{38}$$

where $v_q = V_{qb}V_{qs}^*$. Since the final state consists of two spin one particles, three helicity amplitudes contribute. A full angular analysis disentangles them, as shown in Appendix A. The helicity zero amplitude probably dominates the decay, as in $B_d \to J/\psi K^{*0}$ [21]. We assume that to be the case so as to illustrate the point most simply. Otherwise a full angular analysis will obtain the CKM-parameter; see Appendix A. A tagged, time-dependent $B_d \to J/\psi\rho^0$ study determines

$$|A|^2,\ |\bar{A}|^2\,,\ \text{ and }\ \mathrm{Im}\lambda(B_d \to J/\psi\rho^0)\,. \tag{39}$$

The last observable combined with the observation of CP-violation in $B_d \to J/\psi K_S$ yields $\arg(\bar{b}/b)$, because

$$\lambda(B_d \to J/\psi\rho^0) = -\lambda(B_d \to J/\psi K_S)\frac{\bar{b}}{b}\,. \tag{40}$$

In fact, the isospin related processes, $B^{\mp} \to J/\psi\rho^{\mp}$, measure $|\bar{A}|$ and $|A|$ without tagging and without time-dependence. Then the determination of $\mathrm{Im}\lambda(B_d \to J/\psi\rho^0)$ might not require time-dependence. It does require tagging, however. By normalizing with $B_d \to J/\psi K^{*0}$, we measure $|b|^2, |\bar{b}|^2$, since

$$\begin{aligned}\left|\frac{\langle J/\psi\rho^0|B_d\rangle}{\langle J/\psi K^{*0}|B_d\rangle}\right|^2 &= \left|\frac{V_{cd}}{V_{cs}}\right|^2 r\left|1+ze^{+i\gamma}\right|^2 = \\ &= \left|\frac{V_{cd}}{V_{cs}}\right|^2 r\left|b\right|^2.\end{aligned} \tag{41}$$

Here r is the ratio of the strong matrix elements a_2 and will come from theory [27],

$$r \equiv \left|\frac{a_2(B_d \to J/\psi\rho^0)}{a_2(B_d \to J/\psi K^{*0})}\right|^2. \tag{42}$$

It must be accurately calculated and need not be close to 1. Eq. (41) determines $|b|$, since $|V_{cd}/V_{cs}| \approx \theta$, r will be given from theory, and the left-hand side of Eq. (41) is a ratio of rates. Fig. 2 shows the two amplitude triangles,

$$\bar{b} = 1 + ze^{-i\gamma}, b = 1 + ze^{i\gamma}\,. \tag{43}$$

The points B and $\bar{B}$ are equidistant from O, and the angle between $O\bar{B}$ and OB is 2γ. Let us extract γ from the observables, which are $|b|, |\bar{b}|$ and $\arg(\bar{b}/b)$.

We measure the lengths of b and $\bar{b}$ and the angle between them. Let us draw them. The point O is not yet fixed. It is equidistant from points B and $\bar{B}$, that is–point O is somewhere on line d, see Fig. 2. However, normalization demands that EO is of unit length. The location of point O is thus determined, and the angle γ can be obtained.

Studies of $B_d \to J/\psi\rho^0$ and $B_d \to J/\psi K^{*0}$ extract the angle γ. The extraction is accomplished by observing the interference between the spectator and non-spectator diagrams. The angle γ can be extracted whether or not direct CP violation occurs—that is, whether z has a phase or is real—as long as $|z|$ does not vanish. The extraction requires the knowledge of the ratio of matrix elements, r.

The specific mode $B_d \to J/\psi\rho^0$ suffers from drawbacks. The small phase of $\bar{b}/b$ must be disentangled from the large CP violating interference terms, $\lambda(B_d \to J/\psi K_S)$ and $\lambda(B_d \to J/\psi\rho^0)$, see Eq. (40). The final-state interactions may differ for the $J/\psi K^{*0}$ and $J/\psi\rho^0$ modes, and thus r may not be able to be calculated accurately. Furthermore, even within the $SU(3)$ limit, the ratio r is not exactly 1, because the W-exchange diagram contributes to $B_d \to J/\psi\rho^0$ but does not to $B_d \to J/\psi K^{*0}$. This is an academic problem, because the W-exchange diagram is expected to be highly suppressed compared to the spectator one. Those drawbacks can be overcome by tagged, time-dependent studies of specific B_s modes as shown in Appendix B.

What makes this method difficult is that the interfering amplitudes, which are governed by different CKM combinations, are so unequal, $|z| \ll 1$. Tagged, time-dependent studies of $B_d \to D_s^{\pm}K^{\mp}$ and $B_s \to J/\psi\phi$ are most likely superior in extracting γ. Our motivation to present the $B_d \to J/\psi\rho^0$ method is two-fold. It may not be possible to study time-dependences of B_s accurately, if x_s is large. Secondly, we wanted to point out that, in principle, triggerable B^0-modes allow the extraction of CKM parameters in addition to β.

An analogous method extracts α from exclusive modes governed by the $b \to d\ell^+\ell^-$ transition. This is the topic of the next section. The interfering amplitudes are of similar strength, and the interference term yields the phase between the two unmixed amplitudes without any disentangling. Those are two important advantages over the modes discussed in this section. The branching ratio is miniscule, however.

6. α from $B \to \rho\, \ell^+\ell^-$

The angle α can be extracted from the once CKM-suppressed exclusive rare processes, governed by $b \to d\ell^+\ell^-$. The branching ratio is tiny, at the few $\times 10^{-8}$ level.

$$B(B \to \rho\, \ell^+\ell^-) \sim 10^{-7} - 10^{-8}\,. \tag{44}$$

The modes of interest are three body decays, such as $B \to \rho\ell^+\ell^-, \pi\ell^+\ell^-, a_1\ell^+\ell^-, B_s \to \bar{K}^*\ell^+\ell^-$, etc. The amplitude varies around the Dalitz plot. The variation [28] and the short- and long-distance [29] contributions have been theoretically analyzed for the CKM-favored $b \to s\ell^+\ell^-$ modes. The analyses can be modified to apply to the modes of interest here $b \to d\ell^+\ell^-$ [30]. Our aim is only to sketch several ways to extract α. We thus simplify and treat the amplitude as a complex number. We are currently investigating how to optimize the extraction of α. The unmixed amplitudes are

$$\begin{aligned} \langle\, \rho^0\ell^+\ell^-|B_d\rangle &= \xi_t^* z_t[1 + z\, e^{-i\alpha}] = \xi_t^* z_t E, \\ \langle\, \rho^0\ell^+\ell^-|\bar{B}_d\rangle &= \xi_t z_t[1 + z\, e^{i\alpha}] = \xi_t z_t \bar{E}, \end{aligned} \tag{45}$$

where z_t is a strong matrix element, and z depends on ratios of strong matrix elements and on the magnitude of the CKM combination $|V_{ub}V_{ud}|/|V_{tb}V_{td}|$. This parameter z differs from the one in Section 5 and varies around the Dalitz plot. An honest determination of α must take the variation into account, which however is ignored here as stated above.

Large direct CP violation occurs with exclusive rare modes governed by $b \to d\ell^+\ell^-$ [30]. The parameter z is of order unity with a large final state phase difference [30]. A few options exist to determine α. The $B_d \to K^{*0}\ell^+\ell^-$ mode can be used as normalization,

$$\langle K^{*0}\ell^+\ell^-|B_d\rangle = v_t^* z_t[1 + \mathcal{O}(10^{-2})] \,. \tag{46}$$

Then we get

$$\left|\frac{\langle \rho^0\ell^+\ell^-|B_d\rangle}{\langle K^{*0}\ell^+\ell^-|B_d\rangle}\right|^2 = \left|\frac{V_{td}}{V_{ts}}\right|^2 |1 + ze^{-i\alpha}|^2 = \left|\frac{V_{td}}{V_{ts}}\right|^2 |E|^2 \,, \tag{47}$$

where we omit an $SU(3)$ breaking ratio of order unity, for simplicity. Clearly, by the time experiments capable of measuring this ratio will be feasible, the CKM ratio $|V_{td}/V_{ts}|$ will be well-known. A tagged, time-dependent study of $B_d \to \rho^0\ell^+\ell^-$ yields

$$|E|,\ |\bar{E}| \text{ and} \tag{48}$$

$$\mathrm{Im}\lambda(B_d \to \rho^0\ell^+\ell^-) = \mathrm{Im}\frac{\bar{E}}{E} \,. \tag{49}$$

The moduli of the unmixed amplitudes can also be obtained from the isospin related charged B decays, $B^\pm \to \rho^\pm\, \ell^+\ell^-$. Neither tagging nor time-dependence is required. Note that the interference term informs us directly about the relative phase between E and $\bar{E}$, without having to involve another CP-violating measurement, in contrast to Eq. (40). The angle α is extracted "in analogy" to the extraction of γ from $B_d \to \rho^0 J/\psi$ [31]. If time-dependent B_s-measurements are feasible, α could also be determined from B_s modes; see Appendix B.

A second variant could be to measure only the moduli of the two "unmixed" amplitudes and use the calculated z. This suffices to extract α. The moduli could be obtained from the charged B-decays, $B^\pm \to \pi^\pm\ell^+\ell^-, a_1^\pm\ell^+\ell^-$, etc. The mode $B \to K^{(*)}\ell^+\ell^-$ would provide the normalization. The two moduli could alternatively come from $B_s \to \bar{K}^{*0}\ell^+\ell^-$ and $\bar{B}_s \to K^{*0}\ell^+\ell^-$, which are self-tagging since K^{*0} is seen in its $K^+\pi^-$ mode. Theoretical uncertainties are probably reduced since the B_d-mode with identical particle content $B_d \to K^{*0}\ell^+\ell^-$ could be used for normalization. Neither tagging nor time-dependence would ever be necessary. A third variant eliminates normalization. The two moduli of the unmixed amplitudes, the interference term $\arg(\bar{E}/E)$, and the calculated z suffice to determine α.

The amplitude ratio z is of order unity for the exclusive $b \to d\ \ell^+\ell^-$ processes, in contrast to the exclusive $b \to dJ/\psi$ modes where it is tiny at the few percent level. Thus, the angle α may be more readily extracted than the angle γ by the method discussed here.

The extraction of α is also possible from modes with a photon, by using variant 2. The angle α cannot be extracted from methods that involve an interference term λ, for modes with a photon. The interference term vanishes because only one helicity occurs from the B^0-decay and the other from the $\bar{B}^0$-decay [32]. Variant 2 measures the moduli of the two amplitudes of $B^+ \to a_1^+\gamma$ and $B^- \to a_1^-\gamma$, or $B^\pm \to \rho^\pm\gamma$. It normalizes from $B \to K^*\gamma$. The two moduli of $B_s \to \bar{K}^{*0}\gamma$ and $\bar{B}_s \to K^{*0}\gamma$ also extract α, where the normalization comes from $B_d \to K^*\gamma$ or alternatively from $B_s \to \phi\gamma$. The parameter z here is different from that of $B_d \to \rho^0\ell^+\ell^-$ or $B \to \rho^0 J/\psi$, and is in principle calculable. A first step was taken by Soares, who calculated [32]

$$z \approx (0.09 + i\,0.13)|V_{ub}/V_{td}|\,. \tag{50}$$

More theoretical work is required in calculating this z reliably. Because the final state is simpler than non-leptonic modes, there is more hope that theory will estimate z reliably.

In summary, exclusive modes governed by the $b \to d$ transition extract the angle α. In addition to the information coming from the relevant modes, the extraction requires experimental and theoretical input. Experiments must inform us about $|V_{td}/V_{ts}|$, and theory about z and about ratios of strong matrix elements.

7. α from $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$

The tagged $B_d \to \pi^+\pi^-$ mode extracts α, for negligible penguin amplitudes. It may occur, however, that penguin contributions are significant compared to the tree one. An elaborate isospin analysis could determine α by disentangling the tree from the penguin [5]. At hadron accelerators, it requires a tagged, time-dependent study of the $B_d \to \pi^0\pi^0$ mode, which at present cannot be achieved [6].

This section presents alternative measurements of α in the case of large penguins. We assume that flavor $SU(3)$ for B-decays [33] and its breaking terms will be well understood. One way could be to have a tagged, time-dependent $B_d \to \pi^+\pi^-$ study with the normalization coming from $B_d \to K^+\pi^-$. This method is analogous to the extraction of γ from the tagged, time-dependent $B_d \to J/\psi\rho^0$ study, where the normalization comes from $B_d \to J/\psi K^*$, see Section 5.

The accuracy on α depends on how dominant the penguin is over the tree amplitude in $B_d \to K^+\pi^-$. The more dominant the penguin compared to the tree, the more accurately α could, in principle, be extracted. Information as to the strength of the penguin amplitudes could be obtained by comparing the branching ratio of the $\pi^+\pi^-$ mode to the $K^+\pi^-$ one and to those of pure penguin modes, such as $B^- \to K^-\phi, K_S\pi^-, B_d \to \phi K_S$.

Another alternative uses tagged, time-dependent studies of the charged two-body modes, $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$. This method will be the focus of the section. The unmixed

B_d-amplitudes are

$$\langle\, \pi^+\pi^-|\bar{B}_d\rangle = \xi_t a_t + \xi_u a_u = \xi_t a_t(1 + z\ e^{i\alpha}) = \xi_t a_t \bar{a}\ , \tag{51}$$

$$\langle\, \pi^+\pi^-|B_d\rangle = \xi_t^* a_t(1 + z\ e^{-i\alpha}) = \xi_t^* a_t a\ . \tag{52}$$

The interference term is given by

$$\lambda_d = \frac{\bar{a}}{a} = \frac{1 + ze^{i\alpha}}{1 + ze^{-i\alpha}}\ . \tag{53}$$

Here ξ_q are the relevant CKM combinations, a_t, a_u are the two strong matrix elements, and z depends on their ratio and on the ratio $|\xi_u/\xi_t|$. Note that a_t, a_u, and z denote different quantities from the ones of previous sections. The two unmixed B_s-amplitudes and the interference term are

$$\begin{aligned}
\langle\, K^+K^-|\bar{B}_s\rangle &= v_t a_t + v_u a_u = v_t a_t[1 + rze^{-i\gamma}] = v_t a_t \bar{b}\ ,\\
\langle\, K^+K^-|B_s\rangle &= v_t^* a_t[1 + rze^{i\gamma}] = v_t^* a_t b\ ,\\
\lambda_s &= \frac{\bar{b}}{b} = \frac{1 + rze^{-i\gamma}}{1 + rze^{i\gamma}}\ .
\end{aligned} \tag{54}$$

The b, $\bar{b}$ and r differ from the ones defined in Section 5. For simplicity, $SU(3)$ flavor symmetry is assumed, although much effort will have to be directed toward estimating corrections to it. The parameter r is a ratio of CKM elements and will be well known:

$$r = \left|\frac{V_{us}V_{td}}{V_{ud}V_{ts}}\right|, \tag{55}$$

and so will the relative normalization of the unmixed B_d- and B_s-amplitudes,

$$\left|\frac{\xi_t}{v_t}\right| = \left|\frac{V_{td}}{V_{ts}}\right|. \tag{56}$$

The tagged, time-dependent study of $B_d \to \pi^+\pi^-$ informs about $|a|, |\bar{a}|$, and arg $(\bar{a}/a)$, while that of $B_s \to K^+K^-$ measures $|b|, |\bar{b}|$, and arg $(\bar{b}/b)$. Figure 3 shows the two B_d-amplitude triangles,

$$a = 1 + ze^{-i\alpha}\ ,\ \bar{a} = 1 + ze^{+i\alpha}\ , \tag{57}$$

and the two B_s ones,

$$b = 1 + rze^{+i\gamma}\ ,\ \bar{b} = 1 + rze^{-i\gamma}\ . \tag{58}$$

The angle $\angle AC\bar{A}$ is 2α, while $\angle BC\bar{B}$ is 2γ.

We wish now to demonstrate the extraction of the CKM parameters α and γ. The tagged, time-dependent $B_d \to \pi^+\pi^-$ study obtains the phase, arg$(\bar{a}/a)$, and the moduli of the unmixed amplitudes, $|a|$ and $|\bar{a}|$. The two moduli are determined up to an overall constant that can be chosen ***arbitrarily***. The choice of the overall constant fixes the magnitudes of $|b|$ and $|\bar{b}|$. The two magnitudes, $|b|$ and $|\bar{b}|$, as well as $\arg(\bar{b}/b)$ are obtained from the tagged, time-dependent B_s-study. Draw the triangle $AO\bar{A}$. The lengths of its two sides and its angle $\angle AO\bar{A}$ are known. Point C lies on the straight line d that bisects $A\bar{A}$. The shape of the $BO\bar{B}$ triangle is known. Its orientation relative to $AO\bar{A}$ is fixed because points B and $\bar{B}$ must be equidistant from d. Point C is found since the ratio of lengths is known,

$$CB/CA = r\ , \tag{59}$$

thus determining the angles α and γ. If the penguin of $B_d \to \pi^+\pi^-$ is sizeable, b and $\bar{b}$ will be indistinguishable and γ will not be determined, because $CB \ll CA$. For a negligible penguin of $B_d \to \pi^+\pi^-$—that is, $z \gg 1$—the interference term λ_d determines α, and γ could be obtained from the method outlined here.

Instead of extracting the CKM angles α and γ when $|V_{td}/V_{ts}|$ is used as input, the procedure could be inverted by supplying a CKM angle and determining $|V_{td}/V_{ts}|$ and the other CKM angle. The key point is that a simultaneous study of $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ can be used to extract CKM parameters.

8. α from $b \to d +$ charmless

The method used in the previous section can be extended to many additional modes, where the B_d and B_s modes are related by flavor $SU(3)$. There may be doubts as to the validity of flavor $SU(3)$ for the final state. For instance, the symmetry is badly broken for D^0 modes [18],

$$\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^0 \to \pi^+\pi^-)} \simeq 2\ . \tag{60}$$

Perhaps the invariant mass of D^0 still lies within the resonance region, and the breakdown is due to different resonance structures. If so, flavor $SU(3)$ would be a rather good symmetry for B-decays, because the B-mass is much above the resonance region. We do not really understand the symmetry breakdown for the final states of D^0 that are related by flavor $SU(3)$. We could, however, consider final states that are identical in particle content and differ only in their invariant mass, one coming from decays of the heavy hadron and the other coming from the $SU(3)$-related heavy hadron. For the B-mass region, the final state interactions are expected to be similar for the modes with identical particle content coming from decays of the B_d and $\bar{B}_s$. *Only* the $SU(3)$ relation between the initial states (B_d and $\bar{B}_s$) and between the transition currents must be investigated. Table 1 lists examples of such modes. In analogy to the last section, the angles α (and perhaps γ too) can be extracted. For classes 1-2, the phase between the two unmixed amplitudes can be disentangled; for a similar discussion, see Section 5. For modes that involve a single K^0 resonance, K^r, the moduli of the two unmixed amplitudes can be obtained from untagged and time-integrated data samples. Determining the interference term requires a tagged, time-dependent study with K^r_{CP}, however. Theory and experiment will guide us to those modes that have small theoretical uncertainties and that are experimentally feasible.

Many neutral B modes may be used in the future to extract CKM parameters. The extraction is done by disentangling the CKM parameters from strong matrix elements. The disentangling is accomplished by simultaneously studying related modes, where most theoretical uncertainties cancel. We could either study $SU(3)$ related modes, or B_d and $\bar{B}_s$ modes with identical particle content.

Table 1: $\bar{B}_s$ and B_d modes with identical particle content.

Class	$\bar{B}_s$ transition	B_d transition	Examples
1	$b \to d\, u\bar{u}$	$\bar{b} \to \bar{s}\, u\bar{u}$	$\rho^0 K_S, \omega K_S, \eta K_S, \eta K^r, \rho^0 K^r, \omega K^r$
2	$b \to d\, s\bar{s}$	$\bar{b} \to \bar{s}\, s\bar{s}$	$\phi K_S, \phi K^r$
3	$b \to s\, d\bar{d}$	$\bar{b} \to \bar{d}\, s\bar{s}$	$K^0\bar{K}^0, K^r\bar{K}^{r\prime}, K^0\bar{K}^r, K^r\bar{K}^0$

9. Conclusion

Triggerable B_d modes can extract each of the three angles of the unitarity triangle. It is well known that a tagged study of $B_d \to J/\psi K_S$ measures β. It is, however, not as well known that a tagged, time-dependent study of $B_s \to J/\psi\phi, D_s^+ D_s^-$ determines γ [13, 17]. The determination requires the value of $|V_{ub}/V_{cb}|$ as input. The angle γ can still be measured, even if accurate time-dependent B_s-studies are not feasible, perhaps because x_s is too large.

The measurement could come from a tagged, time-dependent study of $B_d \to J/\psi\rho^0$, where $B_d \to J/\psi K^{*0}$ would serve as normalization. Although it could be done, it is a challenge in both experimental and theoretical aspects. The small phase between the two unmixed amplitudes must be disentangled from two large interference terms; see Eq. (40). Further, the magnitudes of those two unmixed amplitudes must be measured very well. On the theoretical front, much effort must be expended to accurately calculate a ratio of strong matrix elements, $a_2(B_d \to J/\psi\rho^0)/a_2(B_d \to J/\psi K^{*0})$. The exclusive modes governed by $b \to d\ell^+\ell^-$ extract the angle α. Here the phase between the two unmixed amplitudes is generally large. It is measured from an interference term without any disentangling. The rate is tiny, however.

If triggering on secondary vertices becomes feasible, many more modes could be used to measure CKM parameters. For instance, a combined $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ analysis can measure CKM parameters. Extractions with other such modes are discussed throughout the note.

In conclusion, precision measurements with beautiful hadrons allow the extraction of various CKM parameters. The CKM model will thus be tested by overconstraining it. The extractions require copious amounts of beautiful hadrons and additional theoretical input as to ratios of strong matrix elements. The first requirement can be fulfilled at hadron accelerators. The second one requires much additional study. We look forward to stimulating interactions between experimentalists and theorists as to what modes are feasible and as to what methods have the least theoretical uncertainties.

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Appendix A: Helicity Amplitudes and the Extraction of CKM Angles

The extraction of the relevant CKM angle is possible even when several helicity amplitudes exist. We prefer to explain the main idea by an example instead of keeping the discussion abstract. The generalization to other cases is straightforward. Although Section 5 focuses on $B_d \to J/\psi \rho^0$, this appendix discusses the $\bar{B}_s \to J/\psi K^{*0}$ mode, because additional subtleties occur. Consider the mode

$$\begin{array}{ll} \bar{B}_s \to J/\psi & K^{*0} \\ & \hookrightarrow K^+\pi^- . \end{array} \tag{61}$$

Three helicity amplitudes contribute,

$$A_{\lambda\lambda} = \langle\, J/\psi(\lambda)\; K^{*0}(\lambda)|\bar{B}_s\rangle, \;\; \lambda = \pm 1,\, 0 \,. \tag{62}$$

The CP-conjugated partner,

$$\begin{array}{ll} B_s \to J/\psi & \bar{K}^{*0} \\ & \hookrightarrow K^-\pi^+ , \end{array} \tag{63}$$

involves

$$\bar{A}_{\lambda\lambda} = \langle\, J/\psi(\lambda)\; \bar{K}^{*0}(\lambda)|B_s\rangle, \; \lambda = \pm 1,\, 0. \tag{64}$$

We find it useful to define

$$H_+ \equiv A_{++} + A_{--} \,, \tag{65}$$

$$H_- \equiv A_{++} - A_{--} \,, \tag{66}$$

$$H_0 \equiv 2A_{00} \,. \tag{67}$$

A full angular analysis can determine the following observables [34, 20],

$$|H_+|^2,\; |H_-|^2,\; |H_0|^2 \,, \tag{68}$$

$$\mathrm{Re}\;\; H_+\; H_0^* \,, \tag{69}$$

$$\mathrm{Im}\, H_+\; H_-^*, \;\; \mathrm{Im}\, H_-\; H_0^* \,. \tag{70}$$

The magnitudes and relative phases of H_+, H_-, H_0 are observables [35].

The CP-conjugated process involves

$$\bar{H}_+ \equiv \bar{A}_{++} + \bar{A}_{--} \ , \tag{71}$$

$$\bar{H}_- \equiv \bar{A}_{++} - \bar{A}_{--} \ , \tag{72}$$

$$\bar{H}_0 \equiv 2\bar{A}_{00} \ . \tag{73}$$

CP invariance requires

$$|\bar{H}_+| = |H_+|, \ |\bar{H}_-| = |H_-|, \ |\bar{H}_0| = |H_0| \ , \tag{74}$$

$$\mathrm{Re} \ \bar{H}_+ \ \bar{H}_0^* = \mathrm{Re} \ H_+ \ H_0^* \ , \tag{75}$$

$$\mathrm{Im} \ \bar{H}_+ \ \bar{H}_-^* = -\mathrm{Im} \ H_+ \ H_-^*, \ \mathrm{Im} \ \bar{H}_- \ \bar{H}_0^* = -\mathrm{Im} \ H_- \ H_0^* \ . \tag{76}$$

In terms of strong matrix elements (denoted by $p_c, p_u, m_c, m_u, z_c, z_u$) and CKM combinations ξ_q,

$$H_+ = \xi_c \ p_c + \xi_u \ p_u, \tag{77}$$

$$H_- = \xi_c \ m_c + \xi_u \ m_u, \tag{78}$$

$$H_0 = \xi_c \ z_c + \xi_u \ z_u \ . \tag{79}$$

The CP-conjugated process involves

$$\bar{H}_+ = \eta(\xi_c^* p_c + \xi_u^* p_u) \ , \tag{80}$$

$$\bar{H}_- = -\eta(\xi_c^* m_c + \xi_u^* m_u) \ , \tag{81}$$

$$\bar{H}_0 = \eta(\xi_c^* z_c + \xi_u^* z_u) \ . \tag{82}$$

The phase η is arbitrary when the $\bar{K}^{*0}$ is seen in its $K^-\pi^+$ mode. It is $+1(-1)$ when $\overset{(-)}{K}{}^{*0}$ is seen in its $K_S\pi^0(K_L\pi^0)$ mode. Were the $B_d \to J/\psi\rho^0$ mode considered, η would be $+1$.

The $K_S\pi^0$ mode of the $\overset{(-)}{K}{}^{*0}$ allows the $B_s \to J/\psi\ \bar{K}^{*0}$ and $\bar{B}_s \to J/\psi\ K^{*0}$ amplitudes to interfere. The time-dependences of the helicity amplitudes are

$$H_k(t) = H_k\left[c(t) + i\ s(t)/\lambda_k\right], \tag{83}$$

$$\bar{H}_k(t) = \bar{H}_k\left[c(t) + i\ \lambda_k\ s(t)\right], \tag{84}$$

where

$$\lambda_k \equiv \frac{q}{p}\frac{H_k}{\overline{H}_k}, \tag{85}$$

for $k = +, -, 0$.

As discussed after Eq. (70), the observables are the magnitudes and relative phases of the three $H_k(t)$. Obviously that is true also for the three $\bar{H}_k(t)$. A tagged time-dependent study of $\bar{B}_s \to J/\psi(K_S\pi^0)_{K^*}$ determines the relevant interference terms. By disentangling them, we measure

$$\arg\left(H_k/\bar{H}_k\right) \text{ for } k = +, -, 0\ . \tag{86}$$

The much more copious self-tagged data sample of $\bar{B}_s \to J/\psi(K^+\pi^-)_{K^*}$ obtains

$$|H_k| \text{ and } |\bar{H}_k|,\ \ k = +, -, 0\ . \tag{87}$$

In the $m_d = m_s$ limit, a full angular analysis of the process

$$B_d \to J/\psi \quad K^{*0} \hookrightarrow K^+\pi^-\ , \tag{88}$$

provides the magnitudes and relative phases of

$$p_c,\ m_c,\ z_c\ . \tag{89}$$

For instance,

$$H_+\left(B_d \to J/\psi\ K^{*0}\right) \cong \frac{V_{cs}}{V_{cd}}\ \xi_c^*\ p_c \tag{90}$$

and similarly for H_0 and H_-. Statistics are doubled when the CP-conjugated mode $\bar{B}_d \to J/\psi\ \bar{K}^{*0}$ is considered as well. The angle γ can now be extracted in several independent ways. Alternatively the magnitudes and relative phases of p_c, m_c, z_c could be obtained from an untagged, time-integrated study of $B_s \to J/\psi\phi$.

Appendix B: CKM Extraction With B_s Modes

The angles γ and α can be extracted from time-dependent B_s-studies. Specific B_s-modes probably reduce theoretical uncertainties, because the $\bar{B}_d$-mode with identical particle content could be used as normalization. The uncalculable final state interactions mostly cancel in ratios of amplitudes. The latter part of Section 5 discussed drawbacks of the extraction of γ from the $B_d \to J/\psi\rho^0$ mode. Those drawbacks can be overcome by studies of specific B_s modes governed by $b \to d{+}J/\psi$, such as $\bar{B}_s \to J/\psi K_S$ or $\bar{B}_s \to J/\psi K^r$. A tagged, time-dependent study of such B_s-modes extracts γ. The normalization could come from the untagged, time-integrated, CKM-favored mode of the other neutral B-species, $B_d \to J/\psi K_S$ or $B_d \to J/\psi K^r$, respectively [36]. The final states have identical particle content and differ only in their invariant mass by about 100 MeV [37]. The uncalculable final-state interactions cancel to a large extent in the ratio of strong matrix elements,

$$r' = \left|\frac{a_2(B_s \to J/\psi K_S)}{a_2(B_d \to J/\psi K_S)}\right|^2 . \tag{91}$$

Perhaps this ratio will be estimated more accurately than r; see Eq. (42). The phase between the two unmixed amplitudes of $\bar{B}_s \to J/\psi K_s(J/\psi K^r)$ and $B_s \to J/\psi K_s(J/\psi \bar{K}^r)$ is determined by disentangling it from two interference terms,

$$\lambda(B_s \to J/\psi K_s(J/\psi K^r_{CP})) = -\lambda(B_s \to J/\psi\phi|_{CP=+})\bar{b}'/b' . \tag{92}$$

The b' and $\bar{b}'$ describe the unmixed amplitudes of $B_s \to J/\psi K_s(J/\psi\bar{K}^r)$ and $\bar{B}_s \to J/\psi K_s$ $(J/\psi K^r)$, in analogy to b and $\bar{b}$ of Eq. (43). And $\lambda(B_s \to J/\psi\phi|_{CP=+})$ denotes the right-hand side of Eq. (22), and is obtained from a CP study of $B_s \to J/\psi\phi$; see Section 3.

Whereas the imaginary parts of the two interference terms of Eq. (92) are small at order θ^2, the ones of $\lambda(B_d \to J/\psi\rho^0)$ and $\lambda(B_d \to J/\psi K_s)$ are much larger. It thus may be easier to disentangle the phase between the two unmixed amplitudes for $B_s \to J/\psi K_s(J/\psi\bar{K}^r)$ than for $B_d \to J/\psi\rho^0$. Finally, there are no W-exchange diagrams for $\bar{B}_d$ and B_s decay modes to $J/\psi K_s(J/\psi\bar{K}^r)$.

What is the merit of extracting γ from $B_s \to J/\psi\bar{K}^r$ compared to $B_s \to J/\psi K_S$? A tagged, time-dependent study of $B_s \to J/\psi K_S$ has to determine simultaneously three observables, the moduli of the two unmixed amplitudes and the interference term $\lambda(B_s \to J/\psi K_s)$.

In contrast, a tagged, time-dependent study of $B_s \to J/\psi K^r_{CP}$ needs to measure only the interference term $\lambda(B_s \to J/\psi K^r_{CP})$. The moduli of the two unmixed amplitudes $B_s \to J/\psi \bar{K}^r$ and $\bar{B}_s \to J/\psi K^r$ are provided from the *much larger* untagged and time-integrated data sample. On the other hand, the $J/\psi K^r$ mode involves several helicity amplitudes. Angular correlations need to disentangle them, unless $J/\psi K^r$ is dominated by a single helicity amplitude. In contrast, the $J/\psi K_s$ mode involves only one helicity amplitude.

The modes $B_s \to K_S \ell^+\ell^-(\bar{K}^r \ell^+\ell^-)$ extract α. A tagged time-dependent study of $B_s \to K_S \ell^+\ell^-(\bar{K}^r \ell^+\ell^-)$ determines the moduli of the two unmixed amplitudes and the interference term. The moduli of the two unmixed amplitudes with a $\overset{(-)}{K^r}$ can be determined without tagging and without time-dependence, as explained above. The normalization could use the same final state from $\bar{B}_d, \bar{B}_d \to K_S \ell^+\ell^-(\bar{K}^r \ell^+\ell^-)$. The phase between the unmixed amplitudes requires some disentangling, however,

$$\lambda(B_s \to K_S \ell^+\ell^-(K^r_{CP} \ell^+\ell^-)) = -\lambda(B_s \to J/\psi\phi|_{CP=+})\lambda^*(B_d \to J/\psi K_S)\bar{E}/E \,. \tag{93}$$

$\overset{(-)}{E}$ denotes the relevant unmixed amplitude of $B_s \to K_S \ell^+\ell^-(\bar{K}^r \ell^+\ell^-)$, in analogy to Eq. (45). The observables are the two magnitudes of the unmixed amplitudes and their relative phase. They allow the determination of α .

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Figure Captions

Figure 1: The CKM unitarity triangle.

Figure 2: The two amplitude triangles b and $\bar{b}$, see Eq. (43). The angle between $O\bar{B}$ and OB is 2γ.

Figure 3: The four amplitude triangles $a, \bar{a}, b$ and $\bar{b}$. The angle between $\bar{A}C$ and AC is 2α, and the one between BC and $\bar{B}C$ is 2γ.

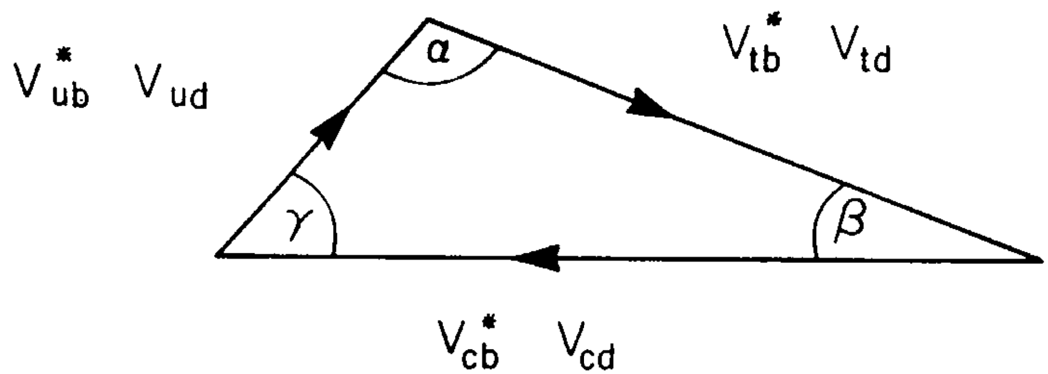

Figure 1

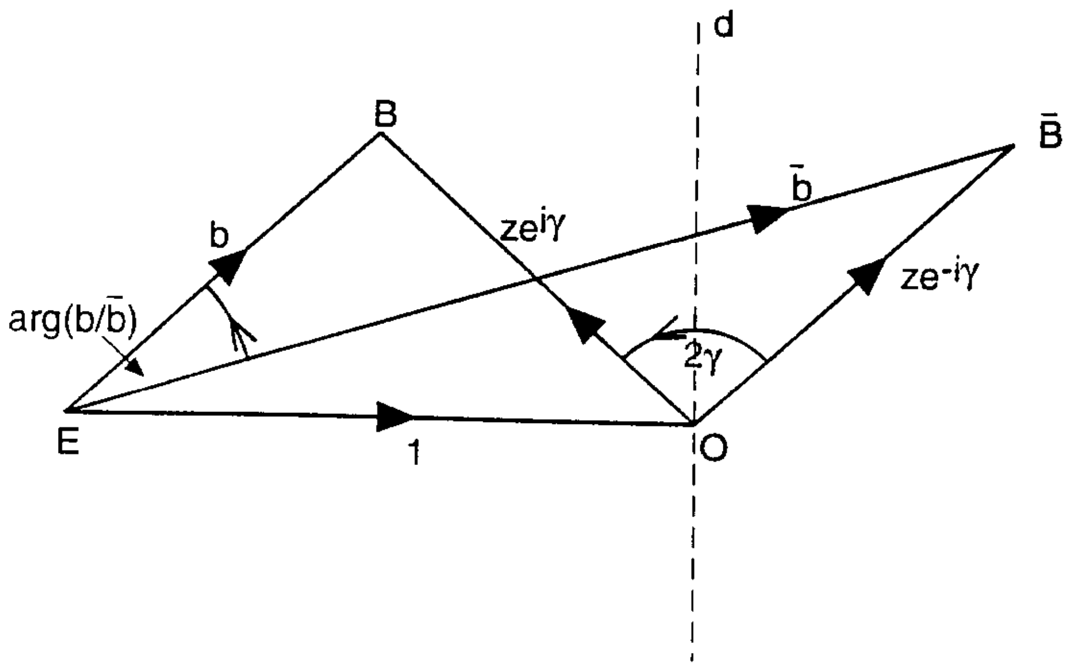

Figure 2

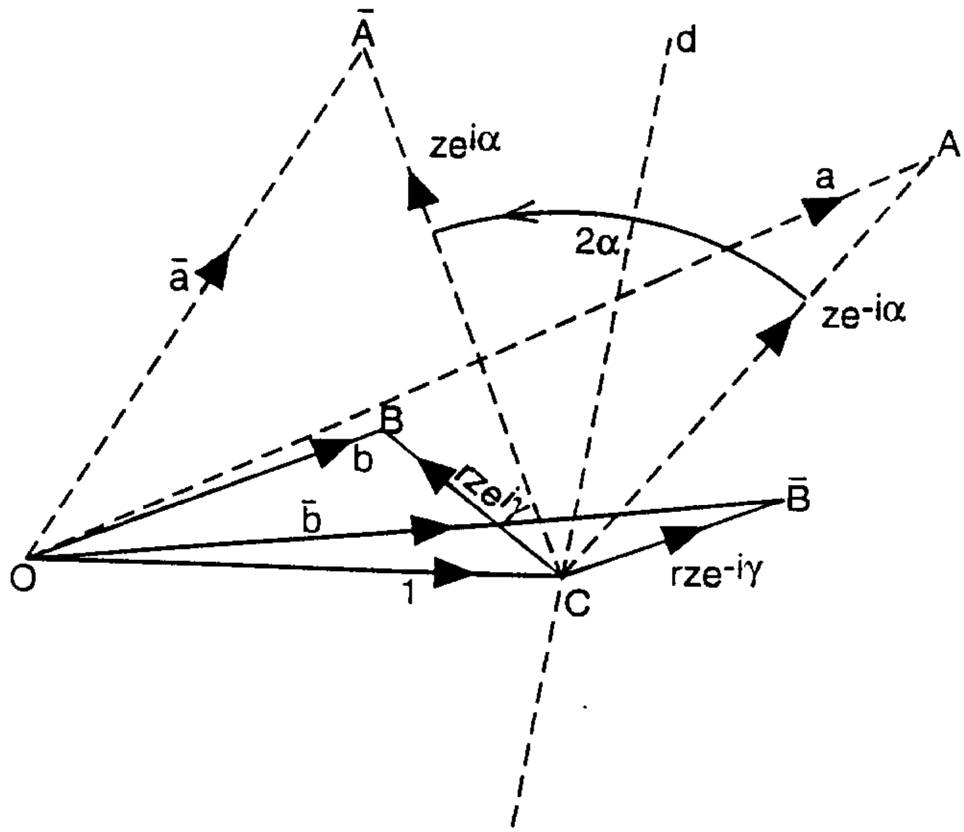

Figure 3