Fermi National Accelerator Laboratory

FERMILAB - Conf - 93/014-A
February 1993

On the Meaning of $\Delta T/T$ [a]

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One of the most interesting aspects of the discovery of Microwave Background Radiation (MBR) anisotropy by the COBE satellite[1] is the ability to compare these anisotropies with the amplitude of density inhomogeneities we measure. Combining these two, we can get a "unified" view of the inhomogeneities present in our universe on a broad range of scales. To make this comparison we must be able to translate $\Delta T/T$ into $\delta\rho/\bar{\rho}$, the mass overdensity. This latter quantity we may try to determine from the distribution of galaxies and their velocities. The standard translation, which is often referred to as "the Sachs-Wolfe formula", is given by

$$\frac{\Delta T}{T} = \frac{1}{3c^2}\Phi_{\rm ls} \qquad \frac{1}{a^2}\nabla^2\Phi = 4\pi G\delta\rho. \tag{1}$$

Here Φ is the gravitational potential perturbation at the last scattering surface, in the direction one is measuring T, a is the cosmological scale factor, and ∇^2 is Laplacian w.r.t. comoving coordinates. The formulae in Eq. 1 are appropriate for adiabatic perturbations grown by gravitational instability in a matter-dominated, flat ($\Omega_0 = 1$), FRW cosmology on scales much larger than the horizon at last scattering. Most discussions of the implications of the COBE results have assumed this formula. It is important to realize that the conditions for the validity of Eq. 1 may not be satisfied, and that the $\delta\rho/\bar{\rho}$ implied by the COBE detection will be different in different scenarios.

The results of Sachs and Wolfe[2] are much more general than Eq. 1, and can easily be derived from the geodesic equations for photons. Gravitationally induced MBR anisotropies can be decomposed[3] according to which type of metric fluctuations formed them: one of two different gravitational potentials, vorticity perturbations, or gravity waves. The cogent features to be discussed can be illustrated by consideration of just the potential fluctuations in the case where there are no large anisotropic stresses. Then there is just one potential, Φ, which is the same as the Newtonian potential. In terms of Φ the Sachs-Wolfe integral is written

$$\frac{\Delta T}{T} = \frac{1}{4}\frac{\delta\rho_{\rm r}}{\bar{\rho}_{\rm r}}\bigg|_{\rm ls} + \frac{1}{c^2}\Phi_{\rm ls} + \frac{2}{c^2}\int\frac{\partial}{\partial t}\Phi(\mathbf{x}_\gamma(t),t)\,dt \tag{2}$$

[a] *to appear in the proceedings of* Texas/PASCOS '92

where the first term is the radiation density fluctuation at the last scattering surface (w.r.t. the Newtonian time slicing), the second term is the usual gravitational redshift, and the last term is an integral along the photon path from last-scattering to the observer. When the conditions for Eq. 1 are satisfied, $\delta\rho/\bar{\rho} \propto a$, and hence Φ is constant in time, and therefore the integral in Eq. 2 does not contribute. For adiabatic perturbations $\delta\rho_r/\bar{\rho}_r = -\frac{8}{3}\Phi/c^2$ on super-horizon scales and one obtains Eq. 1.

A point which should be stressed is that the standard result, given in Eq. 1, represents a high degree of cancellation of the different terms in Eq. 2. The first term nearly cancels the second, and the third term is zero. Most deviations from the requirements for Eq. 1 will not lead to such a fortuitous cancellation, and one will obtain a larger $\Delta T/T$ for a fixed $\delta\rho$ today. Note that the gravitational waves and vortical perturbations we have neglected can only increase $\Delta T/T$ even more. Hence in alternative scenarios, anisotropy measurements on scales larger than 1° (e.g. COBE) will probably indicate a smaller $\delta\rho$ than one obtains using Eq. 1. Given the result that the COBE measurements have forced the "CDM model" into having a $\delta\rho$ which is uncomfortably large when compared with small-scale galaxy clustering[4], one might find the lowering of the ratio $\delta\rho/(\Delta T/T)$ desirable.

To make these ideas more concrete, consider the case where there is no perturbation at the time of last scattering, after which some of the matter is moved around by non-gravitational forces, say in a "late-time phase transition"[5], and inhomogeneities in Φ result. Thus the first two terms of Eq. 2 are zero, but the integral will not be. Since the integral has a factor of 2 multiplying the potential, which is 6 times greater than the prefactor in Eq. 1, one can see that it is not difficult to find larger $\Delta T/T$'s. Jaffe, Frieman, and the author[6] have considered this problem in some detail. We have found that on scales much less than the present horizon, a practical lower limit on $\Delta T/T$ comes from growing the potential $\propto \int^t a^{-1} dt'$, in which case

$$\frac{\Delta T}{T} = \frac{1}{c^2}\Phi_f \sqrt{\frac{2\lambda}{\Delta\eta}} \leq \frac{1}{3c^2}\Phi_f \sqrt{\frac{\lambda}{333\, h^{-1}\mathrm{Mpc}}} \tag{3}$$

where λ is the wavelength of the perturbation, $\Delta\eta$ is the comoving distance light has traveled during the time the potential grows to it final value, Φ_f. The last inequality comes from the requirement that $\Delta\eta$ can be no larger than the present horizon ($6000\, h^{-1}$Mpc). Thus production of perturbations at late times can only suppress anisotropies on small scales ($\lesssim 3°$) and will increase them on larger scales. Since it is unlikely that one will produce the anisotropies in the optimal way, it seems likely that one will never in practice suppress the $\Delta T/T$ by producing the inhomogeneities after last scattering. Note that this is exactly the opposite of what was expected[5].

A class of models which is similar to the "late-time" models are models involving topological defects, such as cosmic strings, cosmic textures, and global monopoles. Unlike the late-time models, the seeding of inhomogeneities on a given scales occurs roughly when that scale enters the horizon, i.e. $\Delta\eta \approx \lambda$. Hence for the large scales we are considering, those which enter the horizon after last-scattering, we may use Eq. 3 to obtain

$$\frac{\Delta T}{T} \approx \frac{\sqrt{2}}{c^2}\Phi_f. \tag{4}$$

This estimation is roughly consistent with more detailed calculations[7,8,9]. (See Ref. 8 for a more thorough discussion.) All estimates imply that $\delta\rho$ is smaller than would be predicted by Eq. 1, and this result is largely due to the aforementioned factor of six.

So far it has been argued that $\delta\rho$ might be smaller than is implied by Eq 1. This does not necessarily imply that $\delta\rho/\bar{\rho}$ is also smaller. One can make $\delta\rho/\bar{\rho}$ larger by decreasing $\bar{\rho}$. This is achieved by changing the cosmology. For example in an open, Lemaître, or "loitering universe"[10] $\Delta T/T$ is small for a fixed $\delta\rho/\bar{\rho}$ largely because $\bar{\rho}$ is small today.

In summary, this note was meant to clarify the relationship between $\delta\rho/\bar{\rho}$ and $\Delta T/T$ by stressing the following points: 1) Determining $\delta\rho$ from measurements of MBR anisotropy is *model dependent*, 2) Eq. 1, the standard model, is for most practical purposes an *upper limit* on the $\delta\rho$ one would obtain in other models, 3) for seed models $\delta\rho$ is a few times smaller than the standard result, which can be understood in terms of the 2 prefactor in Eq. 2 versus the $\frac{1}{3}$ prefactor in Eq. 1.

ACKNOWLEDGEMENTS

Special thanks to Shoba Veeraraghavan, Andrew Jaffe, and Josh Frieman for their insights on these matters. This work was supported in part by the DOE and by grant NAGW-2381 of the NASA.

REFERENCES

1. Smoot G. et al. 1992. Astrophys. J. Lett. **396**: L1.

2. Sachs, R. & Wolfe, A. 1967. Astrophys. J. **147**: 73.

3. Bardeen, J. 1980. Phys. Rev. D **22**: 1882.

4. Efstathiou, G., Bond, J.R., & White, S.D.M. 1992. Mon. Not. R.A.S. **258**: 1p.

5. Hill, C., Schramm, D., & Fry, J. 1989. Comm. Nucl. Part. Phys. **19**: 25.

6. Jaffe, A., Stebbins, A. & Frieman, J. 1992. Fermilab-Pub-92/362-A.

7. Bennett, D. & Rhie, S. 1992. LLNL preprint.

8. Stebbins, A. & Veeraraghavan, S. 1992. Fermilab-Pub-92/188-A.

9. Pen, U., Spergel, D., & Turok, N. 1992. Imperial/TP/92-93/04.

10. Sahni, V., Feldman, H., and Stebbins. A. 1992. Astrophys. J. **385**: 1.