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THE EFFECT OF CHROMATIC DECOHERENCE ON TRANSVERSE INJECTION OSCILLATION DAMPING

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In order to eliminate or reduce transverse emittance growth during transfers between accelerators, transverse damper systems are used to eliminate residual dipole oscillations before phase space dilution takes place. In transfers where the target accelerator has high chromaticity or the beam has a large momentum spread, phase space dilution due to chromatic decoherence can take place on a scale short compared to the damping time of the transverse injection oscillation damper. The effect of the damper on the beam phase space is not clear while the coherent oscillation is suppressed by this decoherence. The purpose of this paper is to quantify the effectiveness of dampers at eliminating emittance blowup at transfers in the presence of chromatic decoherence.

1. Chromatic Decoherence

When a beam is injected with a transverse error, the resultant coherent betatron oscillation can be written as

$$x(n) = x_0 \cos(2\pi \nu_x n) \quad , \qquad (1)$$

where n is the turn number after injection, ν_x is the betatron tune of the target accelerator, and for clarity sake the injection error is assumed to be purely positional of amplitude x_0. Since every particle is assumed to have the same betatron tune, this coherent oscillation would continue forever with unchanged amplitude if no damper acted on the beam.

Assuming that the phase space distribution of the beam is Gaussian in all dimensions, and assuming that all particles have the same synchrotron tune ν_s, then in the case of nonzero chromaticity $\xi_x = d\nu_x/(dE/E)$ the betatron motion is described by the equation[1]

$$x(n) = x_0 \cos(2\pi \nu_x n) \exp\left[-\frac{2\xi_x^2 \sigma_\delta^2}{\nu_s^2} \sin^2(\pi \nu_s n) \right] \quad , \qquad (2)$$

where σ_δ is the fractional momentum spread of the beam. Of course, due to the sinusoidal nature of the RF voltage, synchrotron tune is a function of longitudinal oscillation amplitude. Therefore, the complete recoherence predicted by this equation each time n is a multiple of $1/\nu_s$ is actually reduced each synchrotron period until the coherent betatron oscillation has completely and permanently decohered into transverse emittance growth.

2. Simulation Results

It is possible to simulate chromatic decoherence and the effect of dampers by using a multiparticle simulation computer program. By tracking a large number of test particles through a theoretical accelerator containing the appropriate physics and compiling the average position and rms width of each phase space plane each turn, one can predict the output of actual accelerator diagnostics. The number of test particles used was 1000.

In this paper a consistent set of conditions has been simulated. The only parameters of interest which are modified are the bunch length (to simulate either a long or short bunch), the chromaticity, and the feedback gain. Unless otherwise stated, the conditions simulated are those listed in table 1.

Table 1: Standard parameters used in the computer simulation runs reported in this paper. The values are chosen to approximate injection into the Tevatron Collider.

Parameter	Value	Units
Initial Geometric Emittance	6×10^{-9}	m-rad
Initial Beam Size	0.34	mm
Fractional RMS Energy Width	0.27×10^{-3}	
Synchrotron Tune	3.28×10^{-3}	
Chromaticity	10	
$\sigma_\delta \xi_x / \nu_s$	0.82	
Initial Injection Error	1	mm
Feedback Gain	0.01	
RF Period	18.81	nsec
RMS Bunch Length (Short)	0.78	nsec
RMS Bunch Length (Long)	3.13	nsec

In figures 1 and 2, the evolution of the injection betatron oscillation and the rms beam size is simulated assuming a short bunch. In cases such as this, where the bunch length is much smaller that the bucket length, the beam particles predominantly sample the linear part of the RF sine wave and the recoherence phenomenon repeats for many synchrotron periods. Particles with larger synchrotron oscillation amplitudes have a synchrotron tune lower than small amplitude particles. If the rms bunch length becomes comparable to the RF period the nonlinearity of the buckets prevents all of the particles from rephasing after a small-amplitude synchrotron oscillation has taken place. Figures 3 and 4 show the evolution of the coherent betatron oscillation and beam size in this situation. Because a large percentage of particles are at larger amplitude, the bulk modulation frequency is also reduced.

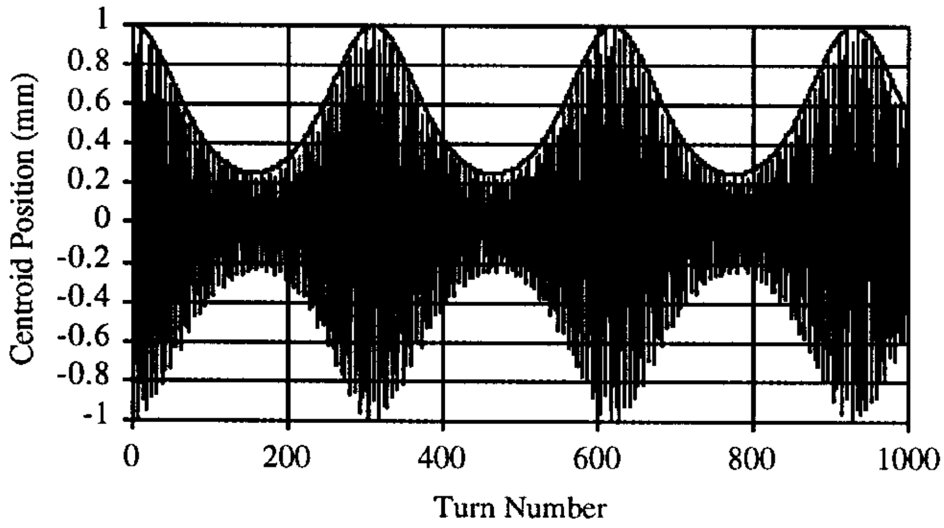

Figure 1: Example of the comparison between the output of a multiparticle simulation program and equation 2 (upper smooth curve) for a short bunch.

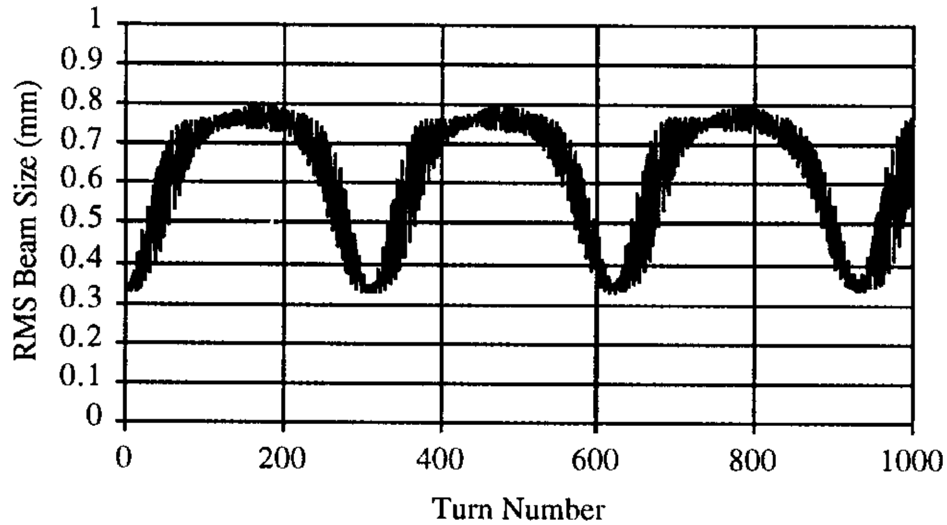

Figure 2: Same run as shown in figure 1, computer program simulation of the rms beam size just after injection in the presence of chromatic decoherence.

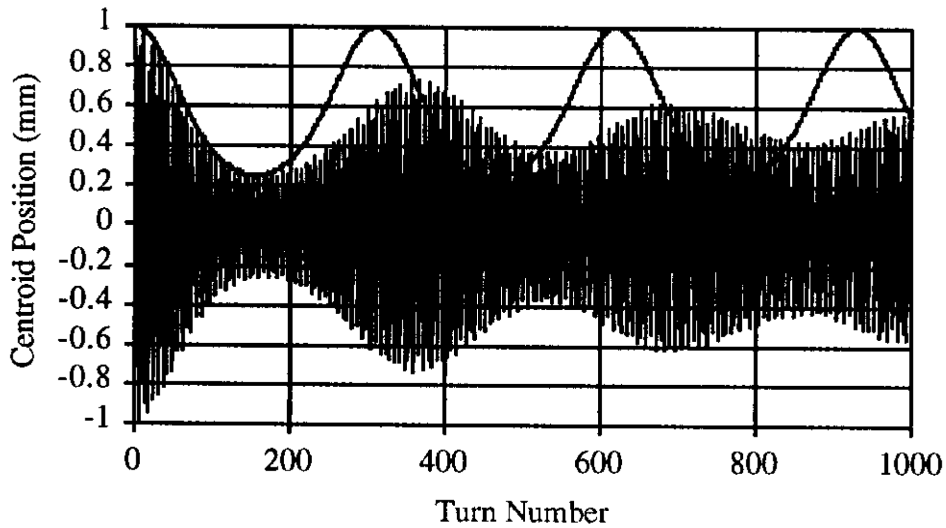

Figure 3: Same conditions as simulated in figure 1, but with a long bunch. Note that the bulk synchrotron frequency is slower due to the dependence of synchrotron tune on amplitude.

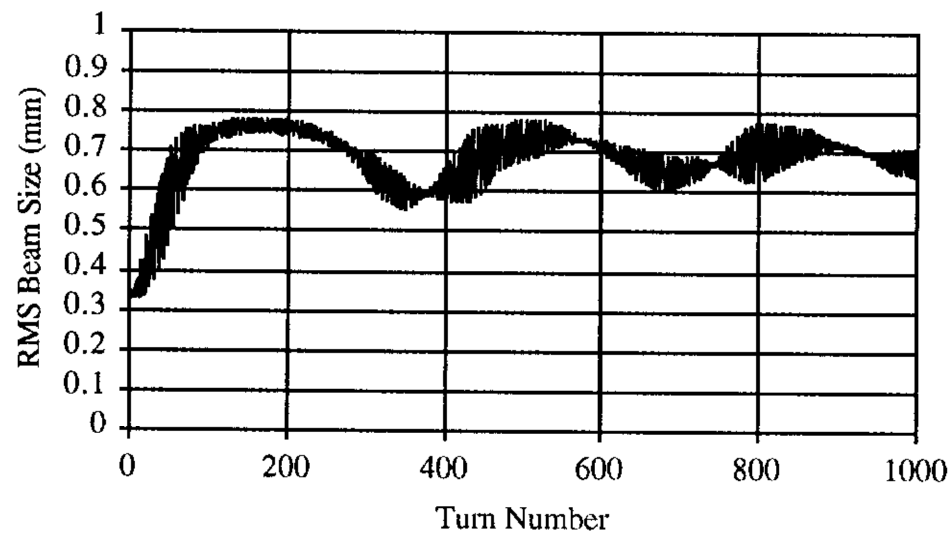

Figure 4: Beam size evolution from the same run displayed in figure 3. Note that with the larger tune spread of the longer bunch, the permanent conversion of coherent amplitude to emittance growth occurs more rapidly.

The dependence of equilibrium emittance as a function of injection error (normalized to the beam size) can be calculated theoretically[2]. Figure 5 is the result of this calculation, where D is the dilution factor defined as the equilibrium emittance divided by the initial emittance and δ is the injection error divided by the beam size. This curve was calculated using a 95% criterion for the emittance and dividing by 6 to generate a rms value (assuming that the distribution was Gaussian). According to the curve, for the standard parameters listed in table 1 the value of δ is 2.9. Looking at figure 5, this corresponds to an emittance dilution of a factor of 4, or beam size step increase of a factor of 2. The data in figure 4 confirm this prediction.

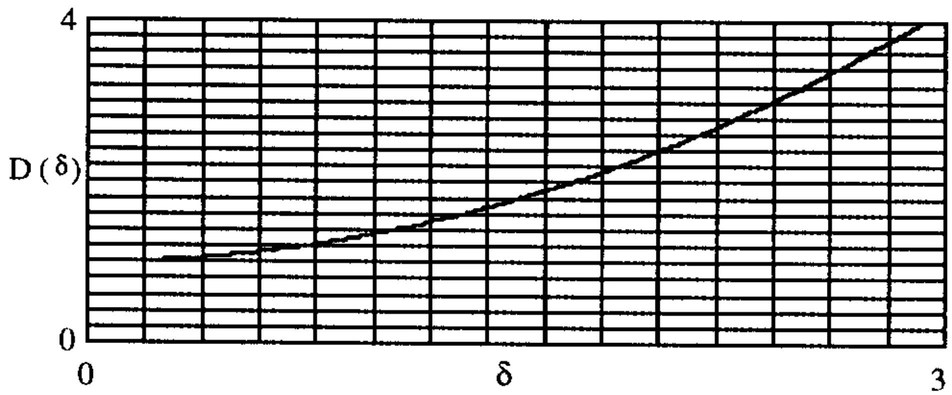

Figure 5: Emittance dilution factor as a function of the transverse injection oscillation amplitude in units of beam size at injection. The horizontal axis is 0.2 per division, the vertical axis is 20% emittance growth per division.

3. Dampers

The effect of a damper is to reduce the oscillation amplitude of an initial betatron oscillation as an exponential function of turn number. A damper is typically composed of a pickup, a preamplifier, a delay, a power amplifier, and a kicker where the distance between the pickup and kicker is an odd multiple of 90° of betatron phase advance. Figure 6 contains a sketch of such a system.

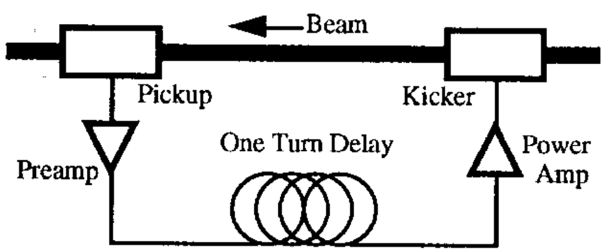

Figure 6: Sketch of a typical transverse injection oscillation damper system.

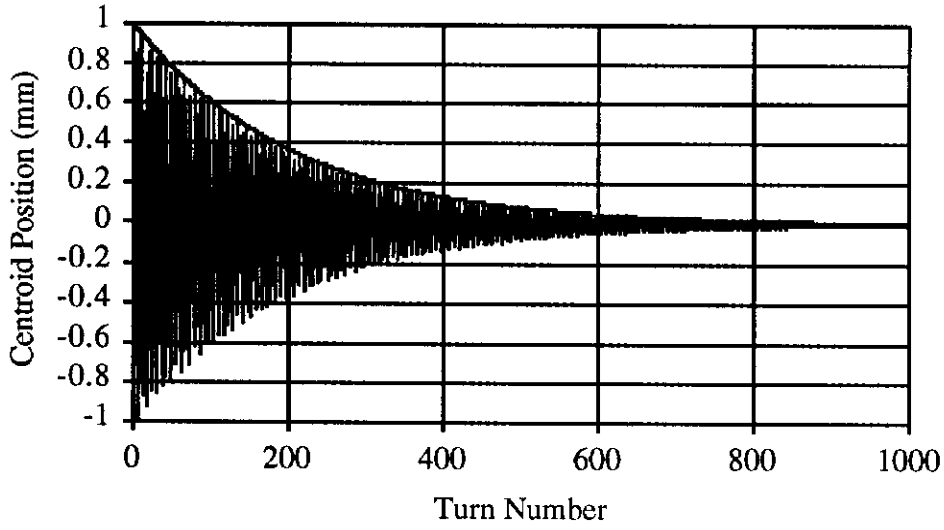

Figure 7: Multiparticle computer simulation result and analytical prediction (upper smooth curve) for damping of coherent motion due to a transverse injection oscillation damper system. For this run the chromaticity was set to zero, eliminating the chromatic decoherence shown earlier.

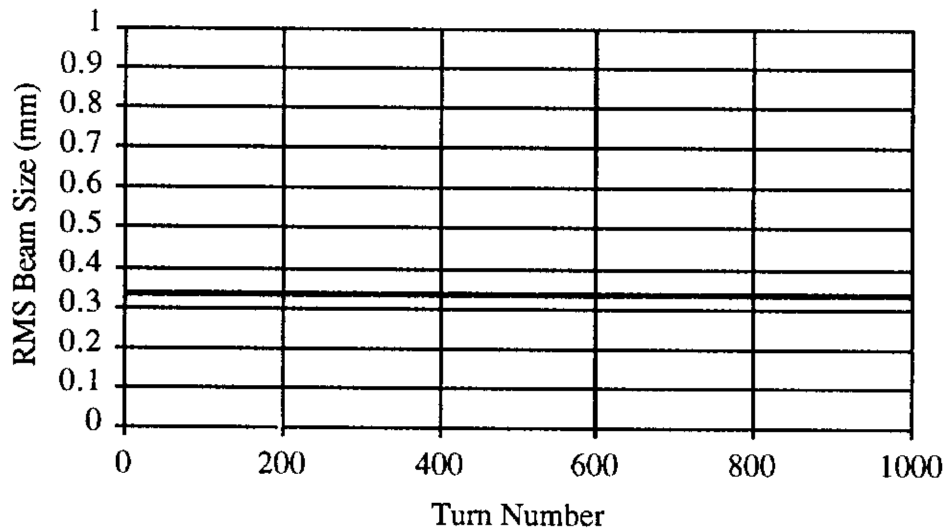

Figure 8: Same run as shown in figure 7, but showing the evolution of the rms beam size when no decoherence mechanism is competing with the damper system.

The gain g of such a system is defined to be the fractional correction the damper makes each turn. A unity gain system therefore completely removes the beam angle at the kicker which corresponded to the measured beam offset at the pickup. For cases where g << 1, the equation describing the motion of the beam centroid with turn number is an exponential with a characteristic decay time of 2/g

$$x(n) = x_0 \cos(2\pi \nu_x n) \exp\left[-\frac{n}{\left(2/g\right)}\right] \quad . \tag{3}$$

Figures 7 and 8 show the evolution of the centroid oscillation and the beam size of a bunch injected with an initial position error, zero chromaticity, and a damper system active with a gain of 0.01. Note that as in the previous section, the results of multiparticle simulations agree very well with theoretical predictions.

4. Damping During Chromatic Decoherence

The point of this paper is to combine the above two phenomena, chromatic decoherence and transverse damping, to assess the utility of injection oscillation dampers as a tool to prevent emittance growth at transfers. Given the fact that a theoretical calculation has not yet been performed, this analysis is limited to multiparticle simulation program results. As a result, in order to make the results presented here applicable to transfers in other accelerators, the worst case scenario of an injection error much larger than the beam size is simulated.

There are only 4 parameters which specify the dynamics of injection damping in the presence of chromatic decoherence. First, ν_s determines the time scale of the problem. Second, $2\nu_s/g$ determines the damping time (in units of synchrotron periods) in the absence of decoherence. Third, the parameter $\sigma_\delta \xi_x/\nu_s$ determines the depth of the chromatic modulation of the coherent betatron oscillation amplitude. Fourth, bunch length determines the speed (again, in units of synchrotron periods) at which a coherent betatron oscillation is permanently converted into emittance growth.

Since the time scale set by the synchrotron tune is only relevant for extrapolation of results to other accelerators, it can be considered to be an arbitrary parameter in the simulation results presented below. The other three parameters actually determine the effectiveness of the injection damping system. Therefore, though it can be accomplished by another choice of variable, in this paper chromaticity and feedback gain are varied in order to understand the effect of each parameter on the asymptotic emittance after the initial oscillation has damped down. In order to demonstrate the effect of the rate of permanent emittance growth (determined by a finite bunch length), all simulations are performed for both long and short bunch conditions.

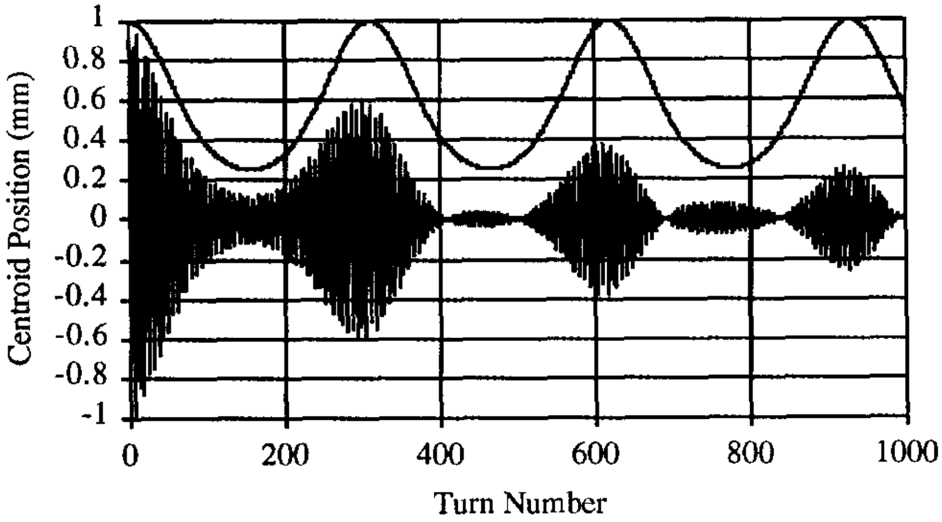

Figure 9: Damping and decoherence with a short bunch. The upper curve is simply the prediction from equation (2) not including the damping.

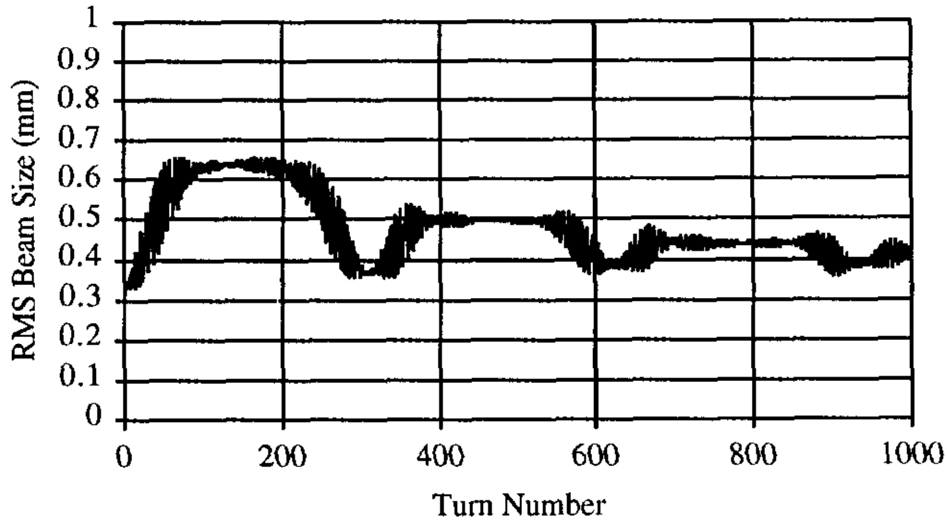

Figure 10: Damping and decoherence with a short bunch for the same simulation run displayed in figure 9. Note that for the standard conditions in table 1, some permanent emittance growth occurs before the coherent oscillation is completely damped.

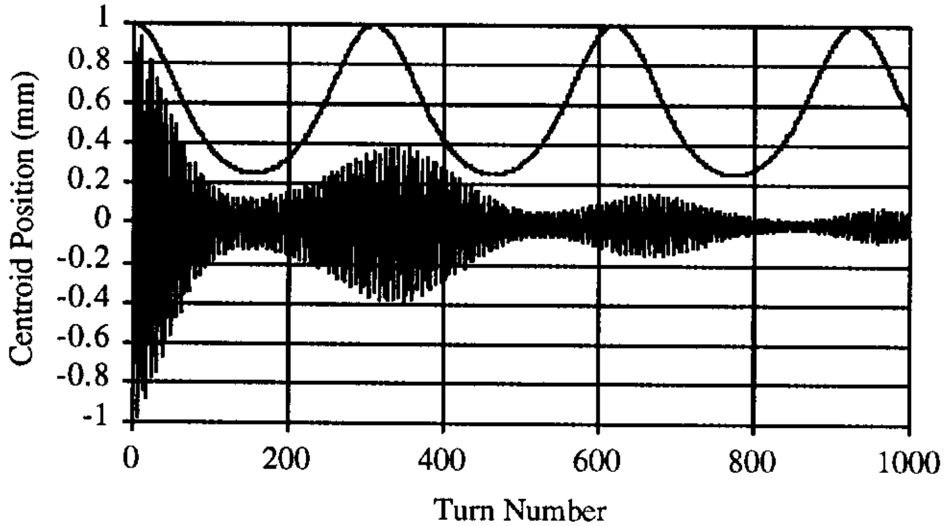

Figure 11: Damping and decoherence with a long bunch. Again, the bulk synchrotron frequency is reduced due to the large synchrotron amplitude particles sampling the nonlinear restoring force of the sinusoidal RF voltage.

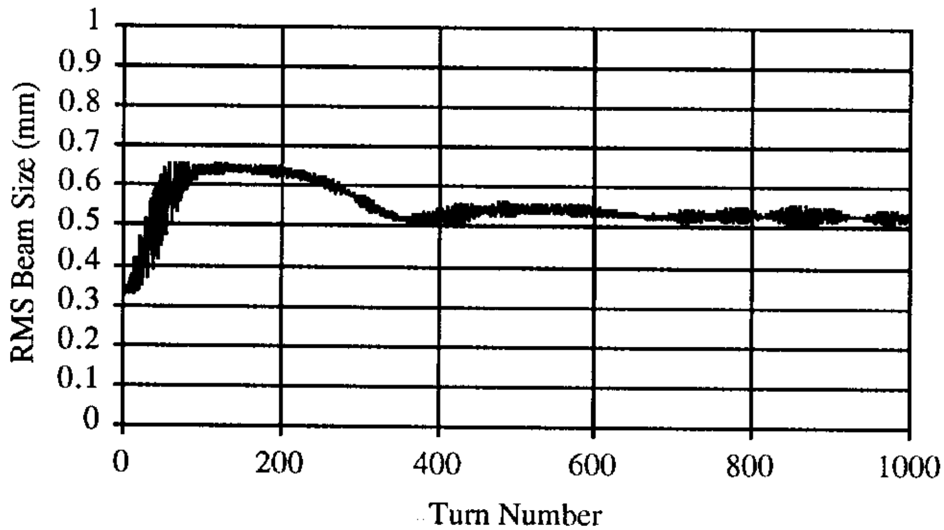

Figure 12: Damping and decoherence with a long bunch. For long bunches the equilibrium emittance is asymptotically approached much more quickly than in the short bunch case.

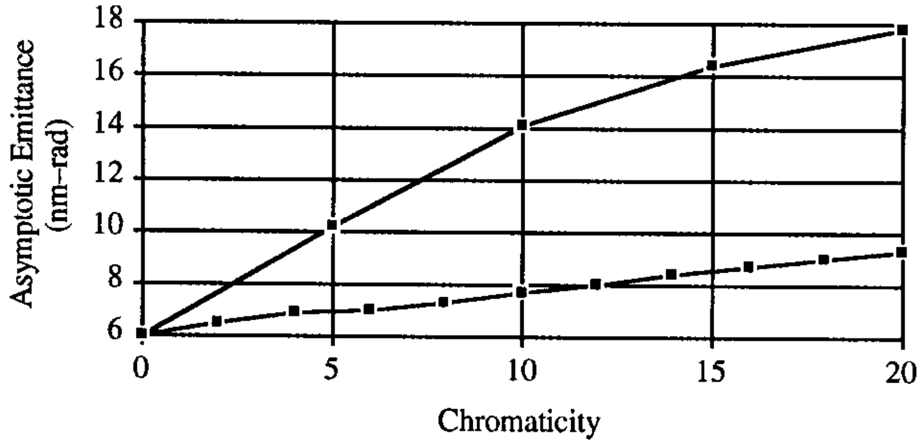

Figure 13: The dependence of the asymptotically attained emittance as a function of chromaticity for a short (bottom) and long (top) bunch.

The effectiveness of injection betatron oscillation damping vs. chromaticity is the first study presented. Since the figure of merit is the increase in emittance due to the injection error during transfer, a plot of equilibrium emittance vs. chromaticity is displayed in figure 13. With both long and short bunch results plotted, the first feature to notice in figure 13 is the fact that the faster the permanent decoherence rate, the larger the emittance growth with increasing chromaticity. In order to keep the emittance growth below 10%, a long bunch must have a chromaticity of less than 1 unit, while for a long bunch a chromaticity of approximately 2 units is tolerable. This is in the case of a damping time approximately two thirds of a synchrotron period (2/g=200 turns).

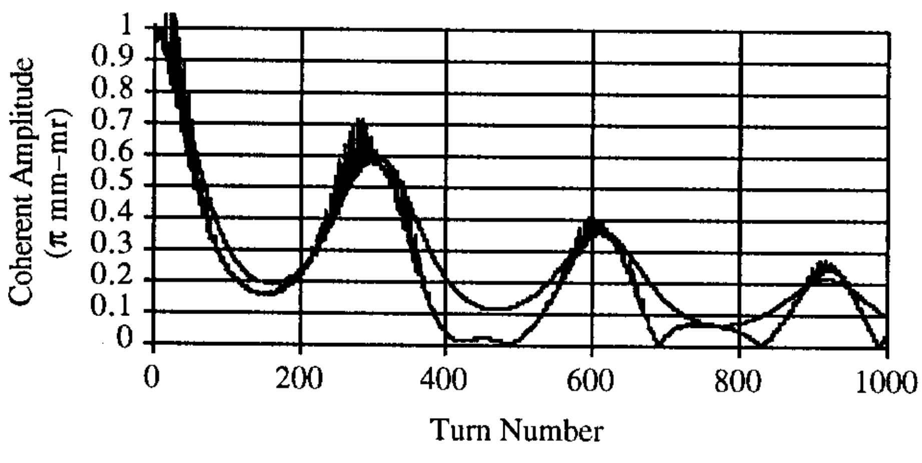

Figure 14: Amplitude of a coherent betatron oscillation vs. turn number for a short bunch. The smooth curve superimposed on the data, and in most cases above the simulated data, is just equation (2) multiplied by an exponential decaying factor.

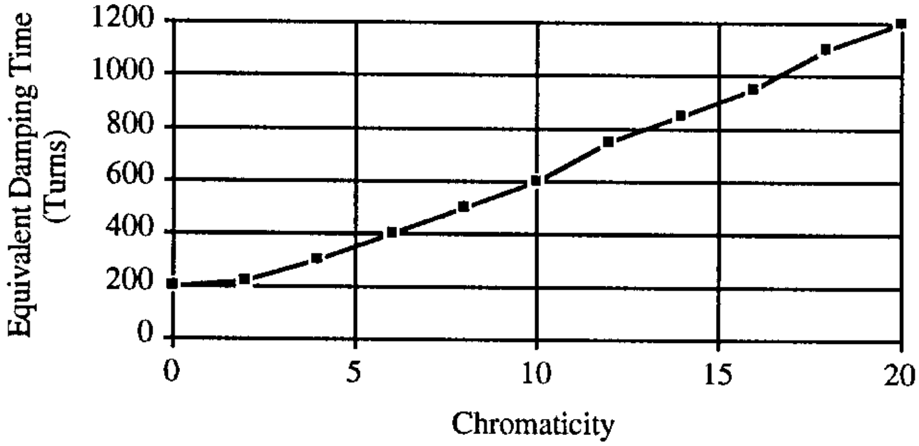

Figure 15: By adjusting the exponential decay time to match the amplitude of the second amplitude recoherence peak with the theoretical curve, the above plot of equivalent damping time (in turns) vs. chromaticity is generated.

To better quantify this result, it is necessary to understand the effective damping rate as a function of chromaticity. First, for zero chromaticity the damping time of a g=0.01 feedback system is 200 turns. For a short bunch, the time evolution of the coherent betatron amplitude vs. turn number is fitted with a function which is composed of equation (2) multiplied by an exponential decay factor. By finding the decay time in the exponent which generates the best agreement between this function and the data, an equivalent damping time dependence on chromaticity can be found. Figure 14 shows an example of such a best fit. Figure 15 shows the measured relationship between this equivalent damping time and chromaticity. Oddly enough, this dependence is a straight line with an asymptotic minimum of 200 turns (the value of 2/g). Therefore, for a short bunch and the default machine conditions, the equivalent damping time is 600 turns.

Often the gain of the feedback system is easier to change than the chromaticity. Chromaticity sometimes must be kept high due to instabilities or drifting sextupole moment around the ring due to time dependent persistent currents in superconducting dipole magnets. For fixed chromaticity, the evolution of emittance with turn number as a function of gain is shown in figure 16. Compiling the results of equilibrium emittance for both long and short bunch simulations for a wide range of gains, the plot in figure 17 is generated.

There are a number of interesting features in figure 17. Again, it is clear that a higher gain system is required for long bunches. The surprising feature of these results is the necessity of very short damping times (10 turns for long bunches and 20 turns for short bunches) to keep the emittance growth down to less than 10%.

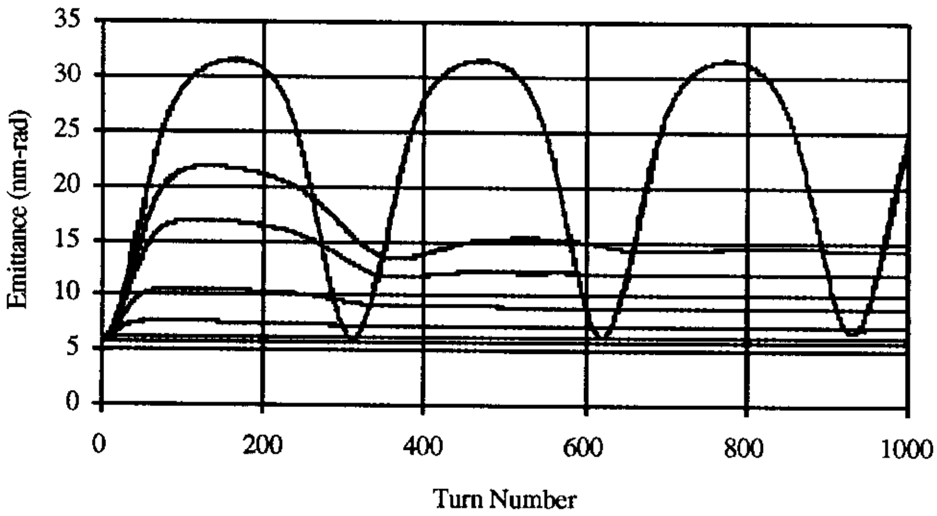

Figure 16: Time evolution of emittance for a number of gains. The top, fully modulated curve is for no damping. The lower curves are for gains of 0.01, 0.02, 0.05, 0.10, 0.20, and 0.50. The curves were generated with a long bunch.

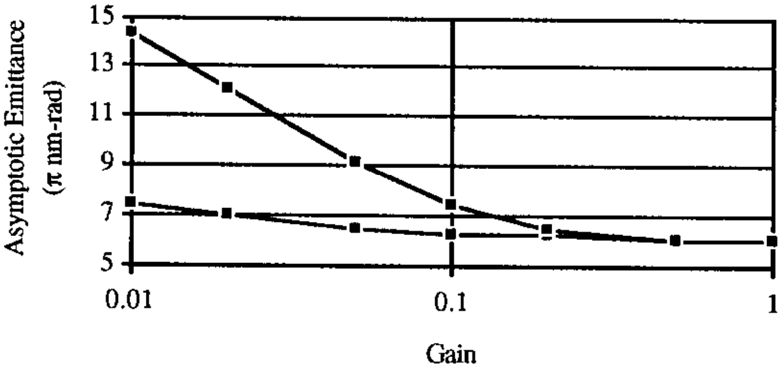

Figure 17: Asymptotically approached emittance as a function of gain for the data displayed in figure 16. The top curve is for a long bunch, the lower curve for a short bunch.

5. Extrapolation to SSC and Elosiatron

Tables 2 and 3 contain the SSC[3] and Eloisatron[4] values of the parameters shown in table 1. The initial geometric emittance and beam size are design values. The energy width and synchrotron tune are also design values. The chromaticity was chosen to keep the modulation amplitude $\sigma_\delta \xi_x / \nu_s$ at the value of 0.82. The injection error was chosen to be the same amplitude relative to the beam size as in the Tevatron case. The feedback gain is adjusted so that the damping time is the same number of synchrotron periods as in the Tevatron case. The RF period is calculated from the design RF frequencies, while the bunch lengths are adjusted to be the Tevatron fraction of the RF bucket occupied.

Table 2: Standard parameters used in the computer simulation runs reported in this paper. The values are chosen to approximate injection into the SSC Storage Ring.

Parameter	Value	Units
Initial Geometric Emittance	0.5×10^{-9}	m-rad
Initial Beam Size	0.34	mm
Fractional RMS Energy Width	0.1×10^{-3}	
Synchrotron Tune	2.2×10^{-3}	
Chromaticity	18	
$\sigma_\delta \xi_x / \nu_s$	0.82	
Initial Injection Error	1	mm
Feedback Gain	0.0067	
RF Period	2.8	nsec
RMS Bunch Length (Short)	0.12	nsec
RMS Bunch Length (Long)	0.47	nsec

Table 3: Standard parameters used in the computer simulation runs reported in this paper. The values are chosen to approximate injection into the Eloisatron.

Parameter	Value	Units
Initial Geometric Emittance	0.1×10^{-9}	m-rad
Initial Beam Size	0.1	mm
Fractional RMS Energy Width	0.03×10^{-3}	
Synchrotron Tune	1.0×10^{-3}	
Chromaticity	27	
$\sigma_\delta \xi_x / \nu_s$	0.82	
Initial Injection Error	0.3	mm
Feedback Gain	0.003	
RF Period	2.2	nsec
RMS Bunch Length (Short)	0.091	nsec
RMS Bunch Length (Long)	0.37	nsec

Applying the results found for the Tevatron parameters, for the parameters in table 2 the SSC ring must keep the chromaticity below 2 units for long bunches and 4 units for short bunches to restrict the injection emittance growth down to under 10%. Alternatively, the absolute damping time of the injection feedback system must be at most 30 turns for short bunches and 15 turns for long bunches. These damping times correspond to gains of 0.067 and 0.133 respectively, compared with 0.05 and 0.10 in the Tevatron.

In the case of the Eloisatron (a 100x100 Tev proton-proton collider), for the parameters in table 3 the main ring must keep its injection chromaticity below 3 units for a long bunch and 6 units for a short bunch. Varying the damping gain, the absolute damping time of the injection feedback system must be at most 60 turns for short bunches and 30 turns for long bunches (corresponding to damping gains of 0.033 and 0.067).

6. Conclusions

In this paper the dependence of emittance growth at injection into a high energy hadron accelerator is explored using a multiparticle computer simulation. Assuming both a short and long bunch, the reduction of emittance growth due to a damper system is shown to be compromised by large chromaticities. The damper gain required for reasonable choices of chromaticities is surprisingly large, meaning that the coherent betatron injection oscillation must be damped with a time constant much shorter than the decoherence time due to chromaticity.

Acknowledgments

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