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Biasing and Hierarchical Statistics in Large-scale Structure

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ABSTRACT

In the current paradigm there is a non-trivial bias expected in the process of galaxy formation. Thus, the observed statistical properties of the galaxy distribution do not necessarily extend to the underlying matter distribution. Gravitational evolution of initially Gaussian seed fluctuations predicts that the connected moments of the matter fluctuations exhibit a hierarchical structure, at least in the limit of small dispersion. This same hierarchical structure has been found in the galaxy distribution, but it is not clear to what extent it reflects properties of the matter distribution or properties of a galaxy formation bias.

In this paper we consider the consequences of an arbitrary, effectively local biasing transformation of a hierarchical underlying matter distribution. We show that a general form of such a transformation preserves the hierarchical properties and the shape of the dispersion in the limit of small fluctuations, i.e. on large scales, although the values of the hierarchical amplitudes may change arbitrarily. We present expressions for the induced hierarchical amplitudes $S_{g,j}$ of the galaxy distribution in terms of the matter amplitudes S_j and biasing parameters for $j = 3$–7. For higher order correlations, $j > 2$, restricting to a linear bias is not a consistent approximation even at very large scales. To draw any conclusions from the galaxy distribution about matter correlations of order j, properties of biasing must be specified completely to order $j-1$.

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1 Introduction

There is accumulating observational evidence that the large scale galaxy j-point correlation functions exhibit a hierarchical structure. Averaged over a sphere of radius R, this means the order j connected moments obey

$$\bar{\xi}_j(R) = S_j\, \bar{\xi}_2(R)^{j-1}, \tag{1}$$

where the S_j are constants over the range of R where the variance or dispersion $\bar{\xi}_2$ has a constant slope, $\bar{\xi}_2(R) \propto R^{-\gamma}$. This relation can follow from the scaling symmetry $\xi_j(\lambda \boldsymbol{x}_1, \ldots, \lambda \boldsymbol{x}_j) = \lambda^{-(j-1)\gamma} \xi_j(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_j)$ or from the multi-point expression

$$\xi_j(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_j) = \sum_{\alpha} Q_{j,\alpha} \sum_{\{ab\}} \prod^{j-1} \xi_2(r_{ab}) \tag{2}$$

(Fry 1984b). In the standard graphical notation of field theory, associated with each term in equation (2) there is a graph, such that vertices, or nodes, correspond to the points $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_j$, and edges, or lines, between node a and node b correspond to factors $\xi_2(x_{ab}) = \langle\, \delta(\boldsymbol{x}_a)\delta(\boldsymbol{x}_b)\,\rangle$ that connect all points. Thus the hierarchy (2) is composed of "tree" graphs (connected with no cycles) of j vertices and $j-1$ edges. The sum over α denotes topologically distinct graphs; the sum over $\{ab\}$ is over relabelings within α. If all $Q_{j,\alpha}$ are identical, there are in total j^{j-2} terms, corresponding to all possible reassignments of the labels a, $b = 1, \ldots, j$, and, up to geometrical factors usually very close to 1, $S_j = j^{j-2} Q_j$.

Observations suggest that the hierarchy, equation (1) or equation (2), holds both on mildly linear, $\bar{\xi}_2(R) \lesssim 1$, and nonlinear, $\bar{\xi}_2(R) \gtrsim 1$, scales, at least for the lower values of j, and has been found in angular catalogs of optical (e.g. Groth & Peebles 1977 Fry & Peebles 1978; Szapudi *et al.* 1992) and IRAS (Meiksin *et al.* 1992) galaxies. Similar results have been reported for redshift samples of IRAS galaxies (Bouchet *et al.* 1992) and in the CfA and SSRS optical catalogs (Gaztañaga 1992).

Remarkably, this same hierarchical structure is predicted for the matter distribution evolved gravitationally in perturbation theory when the initial fluctuations are Gaussian (e.g. Peebles 1980, Fry 1984b, Goroff *et al.* 1986, Bernardeau 1992) and also in the highly nonlinear regime of gravitational clustering (Davis & Peebles 1977, Peebles 1980, Fry 1984a, Hamilton 1988). But, in order to relate theory with the observations, we have to address the problem of how well galaxies trace the matter fluctuations. Are the observed hierarchical properties of the galaxy distribution a consequence of the hierarchical properties of matter? Or, are they an accident or conspiracy of galaxy-matter biasing? If the galaxy distribution is determined physically by the mass distribution, then we expect that the number density of galaxies should be given as a functional of the mass density, $n_g(\boldsymbol{x}) = F[\rho(\boldsymbol{x})]$. Linear biasing, that the galaxy fluctuations are proportional to the matter fluctuations, $\delta_g = b\delta_\rho$, is often assumed as an approximation at large scales. For this case, up to scalings, all statistical properties are preserved by the biasing, and the observed galaxy properties do reflect the matter distribution. However, in the general case, we expect it is highly unlikely that the relation is both local and linear. Below, we study how an arbitrary nonlinear biasing affects statistical studies on large scales, $R \gtrsim 10\, h^{-1}\,\mathrm{Mpc}$ (Hubble's constant $H_0 = 100\, h\, \mathrm{km\, s^{-1}\, Mpc^{-1}}$), where $\bar{\xi}_2(R) \lesssim 1$. We compute the resulting correlation amplitudes directly for low order correlations, and we show that the results extend to all orders. We argue finally that there may be observational evidence that biasing must be a nonlinear transformation, and it is not clear whether the linear approximation is good, or even consistent, at large scales.

2 Biasing and hierarchical distributions at large scales

2.1 One-point statistics

Let us first consider the statistics for one random variable, the (smoothed) density contrast $\delta_W(\boldsymbol{x})$:

$$\delta_W(\boldsymbol{x}) = \int d^3x'\, \delta(\boldsymbol{x}')\, W(\boldsymbol{x} - \boldsymbol{x}'), \tag{3}$$

with $\delta(\boldsymbol{x}) = [\rho(\boldsymbol{x}) - \overline{\rho}]/\overline{\rho}$, where $\rho(\boldsymbol{x})$ is the local density, $\overline{\rho}$ the mean density and $W(\boldsymbol{x})$ a normalized window function. For a top-hat window, $\delta_W(\boldsymbol{x})$ is just the volume average of $\delta(\boldsymbol{x})$ over a sphere of radius R. To simplify notation, we use δ for $\delta_W(\boldsymbol{x})$. The statistical average is over different realizations of $\delta(\boldsymbol{x})$ and corresponds to the average over position in a fair sample of the universe.

As the result of biasing, we assume that the (smoothed) galaxy density can be written as a function of the mass density, $\delta_g = [n(\boldsymbol{x}) - \overline{n}]/\overline{n} = f(\delta)$, and express f as a Taylor series:

$$\delta_g = f(\delta) = \sum_{k=0}^{\infty} \frac{b_k}{k!} \delta^k. \tag{4}$$

The linear term b_1 corresponds to the usual linear bias factor b. To have $\langle \delta_g \rangle = 0$ we must fix $b_0 = -\sum_{k=2}^{\infty} b_k \langle \delta^k \rangle / k!$. The value of b_0 is irrelevant for the connected moments for $j \geq 1$, and we will make no further mention of b_0. Equation (4) is not the most general possibility; we could conceive of a relation involving δ at all points such as

$$\delta_g(\boldsymbol{x}) = b_0 + \int d^3x'\, b_1(\boldsymbol{x}')\, \delta(\boldsymbol{x} - \boldsymbol{x}') + \int d^3x' d^3x''\, b_2(\boldsymbol{x}', \boldsymbol{x}'')\, \delta(\boldsymbol{x} - \boldsymbol{x}')\delta(\boldsymbol{x} - \boldsymbol{x}'') + \cdots. \tag{5}$$

However, to lowest order in δ, equation (5) would give the two-point function

$$\xi'(x_{12}) = \int d^3x_1' d^3x_2'\, b(\boldsymbol{x}_1') b(\boldsymbol{x}_2') \xi(|\boldsymbol{x}_{12} - \boldsymbol{x}_{12}'|). \tag{6}$$

The observational suggestion that for groups and clusters the correlations of selected objects are proportional to those of galaxies, $\xi_{cc}(r) = b^2 \xi_{gg}(r)$, is an indication that if the relation is nonlocal, the range is relatively short. For the windowed field (3) smoothed over large scales, equation (4) should provide an adequate first approximation.

If the matter density δ has hierarchical irreducible correlations or cumulants as in equation (2), we show next that in the limit of small ξ_2, the local biasing transformation in equation (4) preserves the hierarchical structure.

2.2 Expressions for the first orders

For the first few low order correlations we can compute directly correlations of the biased field in terms of those of the original matter field and the biasing parameters. We consider the case of one-point statistics to simplify notation. We assume that the matter distribution has hierarchical connected moments

$$\overline{\xi}_j = \langle \delta^j \rangle_c = S_j\, \overline{\xi}_2^{\,j-1}, \tag{7}$$

where $\overline{\xi}_2 = \langle \delta^2 \rangle_c$. We use the generating function method for calculating $\langle \delta_g^j \rangle_c$ from

$$\overline{\xi}_{g,j} = \langle \delta_g^j \rangle_c = \frac{d^j}{dt^j} \ln \langle e^{t\delta_g} \rangle |_{t=0}, \tag{8}$$

(Fry 1985), where the biased field δ_g is given by equation (4). Following this procedure gives the following for $\bar{\xi}_{g,j}$ for j = 2-5:

$$
\begin{aligned}
\bar{\xi}_{g,2} &= b^2\bar{\xi}_2 + b^2\bar{\xi}_2^2(c_2S_3 + c_3 + c_2^2/2) + \mathcal{O}(\bar{\xi}_2^3) \\
\bar{\xi}_{g,3} &= b^3\bar{\xi}_2^2(S_3 + 3c_2) + b^3\bar{\xi}_2^3(3c_2S_4/2 + 9c_3S_3/2 + 6c_2^2S_3 + 3c_4/2 + 6c_2c_3 + c_2^3) + \mathcal{O}(\bar{\xi}_2^4) \\
\bar{\xi}_{g,4} &= b^4\bar{\xi}_2^3(S_4 + 12c_2S_3 + 4c_3 + 12c_2^2) + b^4\bar{\xi}_2^4[2c_2S_5 + 8c_3S_4 + 18c_2^2S_4 + (6c_3 + 12c_2^2)S_3^2 \\
&\quad +(12c_4 + 78c_2c_3 + 36c_2^3)S_3 + 2c_5 + 18c_2c_4 + 12c_3^2 + 36c_2^2c_3 + 3c_2^4] + \mathcal{O}(\bar{\xi}_2^5), \\
\bar{\xi}_{g,5} &= b^5\bar{\xi}_2^4[S_5 + 20c_2S_4 + 15c_2S_3^2 + (30c_3 + 120c_2^2)S_3 + 5c_4 + 60c_3c_2 + 60c_2^3] \\
&\quad +b^5\bar{\xi}_2^5[5c_2S_6/2 + (25c_3/2 + 40c_2^2)S_5 + (25c_3 + 70c_2^2)S_3S_4 + (25c_4 + 230c_2c_3 + 180c_2^3)S_4 \\
&\quad +(75c_4/2 + 330c_2c_3 + 240c_2^3)S_3^2 + (25c_5 + 310c_2c_4 + 210c_3^2 + 1020c_2^2c_3 + 240c_2^4)S_3 \\
&\quad +5c_6/2 + 40c_2c_5 + 70c_3c_4 + 180c_2^2c_4 + 240c_2c_3^2 + 240c_2^3c_3 + 12c_2^5] + \mathcal{O}(\bar{\xi}_2^6),
\end{aligned}
\tag{9}
$$

where we write $c_k = b_k/b$ for $k \geq 2$. We have obtained, but do not display, results up to order $\mathcal{O}(\bar{\xi}_2^6)$ for $\bar{\xi}_{g,j}$ up to $j = 7$. The leading term in equation (9) for $\bar{\xi}_{g,2}$ is the linear bias result, $\bar{\xi}_{g,2} = b^2\bar{\xi}_2$. To leading order in $\bar{\xi}_2$, the remaining results, $\bar{\xi}_{g,j}$ for $j \geq 3$, are hierarchical, $\bar{\xi}_{g,j} = S_{g,j}\,\bar{\xi}_{g,2}^{j-1}$, with amplitudes $S_{g,j}$ given by

$$
\begin{aligned}
S_{g,3} &= b^{-1}(S_3 + 3c_2) \\
S_{g,4} &= b^{-2}(S_4 + 12c_2S_3 + 4c_3 + 12c_2^2) \\
S_{g,5} &= b^{-3}[S_5 + 20c_2S_4 + 15c_2S_3^2 + (30c_3 + 120c_2^2)S_3 + 5c_4 + 60c_3c_2 + 60c_2^3] \\
S_{g,6} &= b^{-4}[S_6 + 30c_2S_5 + 60c_2S_3S_4 + (60c_3 + 300c_2^2)S_4 + (90c_3 + 450c_2^2)S_3^2 \\
&\quad +(60c_4 + 900c_2c_3 + 1200c_2^3)S_3 + 6c_5 + 120c_4c_2 + 90c_3^2 + 720c_3c_2^2 + 360c_2^4] \\
S_{g,7} &= b^{-5}[S_7 + 42c_2S_6 + 105c_2S_3S_5 + (105c_3 + 630c_2^2)S_5 + 70c_2S_4^2 \\
&\quad +(420c_3 + 2520c_2^2)S_3S_4 + (140c_4 + 2520c_2c_3 + 4200c_2^3)S_4 \\
&\quad +(105c_3 + 630c_2^2 + 315c_4 + 5670c_2c_3 + 9450c_2^3)S_3^2 \\
&\quad +(105c_5 + 2520c_2c_4 + 1890c_3^2 + 18900c_2^2c_3 + 12600c_2^4)S_3 \\
&\quad +7c_6 + 210c_2c_5 + 420c_3c_4 + 2100c_2^2c_4 + 3150c_2c_3^2 + 8400c_2^3c_3 + 2520c_2^5]
\end{aligned}
\tag{10}
$$

The numerical factors are determined by combinatorics and, as in perturbation theory, can be related to a counting of tree graphs. This is especially evident in the terms induced solely by the c_k (cf. Fry 1984*b*), where the sum of coefficients is just j^{j-2}, the total number of labeled tree graphs. Equivalent results were first derived using a different technique by James & Mayne (1962), who present contributions up to S_6, or $\mathcal{O}(\bar{\xi}_2^5)$. Notice that the parameters b_j in the biasing function can be chosen arbitrarily at each order, and thus can modify the matter amplitudes S_j into arbitrary galaxy amplitudes $S_{g,j}$.

Our formulation does not apply to the popular model of bias as a sharp threshold clipping (Kaiser 1984, Politzer & Wise 1985, Bardeen *et al.* 1986, Szalay 1988), where $\delta_g = 1$ for $\delta > \nu\sigma$ and $\delta_g = 0$ otherwise; this biasing function does not have a series representation around $\delta = 0$. However, such a clipping applied to a Gaussian background still produces a hierarchical result with $S_{g,j} = j^{j-2}$ in the limit $\nu \gg 1$, $\sigma \ll 1$. Remarkably, this is the same result as we obtain from equation (10) for an exponential biasing of a Gaussian matter distribution, $\delta_g = \exp(\alpha\delta/\sigma)$ (cf. Bardeen *et al.* 1986, Szalay 1988). This bias function has an expansion $\delta_g = \sum_k (\alpha\delta/\sigma)^k/k!$ and thus $c_j = b^{j-1}$, independent of α and σ. With $S_j = 0$, the terms induced by $c_j = b^{j-1}$ in equation (10) also give $S_{g,j} = j^{j-2}$. We speculate that the threshold bias applied to a hierarchical matter distribution will give results similar to equation (9) and equation (10).

In a similar way one could compute the multipoint correlations and the biased multi-point amplitudes $Q_{g,j}$ in terms of the local matter amplitudes Q_j in equation (2). The calculation in this case will be identical to the one for the smoothed fluctuations above, with S_j effectively replaced by $j^{j-2}Q_j$, but with additional attention required for topologically distinct configurations.

2.3 General results: One point statistics

The results summarized in equation (10) involve the cancellation of an increasing number of lower order terms; the raw moments $\langle \delta_g^j \rangle$ are of order $\bar{\xi}_2^{j/2}$. Thus, that the cumulants of the biased distribution are also hierarchical is likely to be more than an accident. This was proved in general in the following theorem by James (1955) and James and Mayne (1962):

> THEOREM 1: If a variate δ possesses finite cumulants of all orders with $\langle \delta^j \rangle_c = \mathcal{O}(\nu^{-j+1})$, and if the cumulants of $\delta_g = f(\delta)$ are calculated on the basis of a (possibly formal) Taylor expansion (4) where the b_k do not depend upon ν, i.e. they are $\mathcal{O}(\nu^0)$, then $\langle \delta_g^j \rangle_c = \mathcal{O}(\nu^{-j+1})$.

As noted above, this is by no means obvious for $j > 2$, as the raw j-moment of δ_g is of order $\nu^{-j/2}$, but on taking the connected part the terms up to $\mathcal{O}(\nu^{-j+1})$ always seem to cancel. The explanation for this cancelation, i.e. the proof of the theorem, is based on an adoption of the Fisher rules for obtaining the sampling cumulants of k-statistics (Kendall, Stuart and Ord 1987) to statistics of polynomial symmetric functions.

To prove the theorem, James (1955) first considers the variables $z_r = b_r\delta^r$. It is straightforward to see that $\langle \delta_g^j \rangle_c = \sum_{r_1} \cdots \sum_{r_j} \langle z_{r_1} \cdots z_{r_j} \rangle_c$. Therefore it is sufficient to show that $\langle z_{r_1} \cdots z_{r_j} \rangle_c$ is of order ν^{-j+1}. Now consider a sample $\delta_1, \ldots, \delta_n$ of n independent values of δ to define the general statistics $z_r = b_r(\sum \delta_i)^r$, $r = 1, 2, \ldots$. James now uses the Fisher rule that states that to find the cumulants of the z-statistics in terms of population cumulants, we can neglect an array which splits up into two or more disjoint blocks. Finally, to conclude the proof, it is necessary to use that each $\langle \delta^j \rangle = \mathcal{O}(\nu^{-j+1})$; a different structural relation is not preserved under the general transformation in equation (4).

This theorem applies directly to the large scale distribution. From the results of perturbation theory we can assume that the matter distribution, δ, follows the hierarchical relation $\bar{\xi}_j = \langle \delta^j \rangle_c = S_j \bar{\xi}_2^{j-1}$ and so we have the required conditions for the theorem with $\nu = \bar{\xi}_2^{-1}$. If biasing can be described by a local transformation, so that the galaxy field δ_g can be expressed as in equation (4) with $\delta_g = f(\delta)$ then we conclude from the theorem above that the galaxy distribution will also be hierarchical for small values of ξ_2, i.e. large scales. Reversely, if the galaxy distribution is hierarchical and if $\delta_g = f(\delta)$ then the underlying matter statistics must be hierarchical at large scales.

2.4 General results: Multi-point statistics

James (1955) also considers a more general result using multivariate sampling rules. He proposed and proved the following theorem:

> THEOREM 2. If $\delta_{g,1} = f_1(\delta_1, \ldots, \delta_N)$, $\delta_{g,2} = f_2(\delta_1, \ldots, \delta_N)$, ... are functions of the variates $\delta_1, \ldots, \delta_N$ formally expansible in the forms:
>
> $$f_k(\delta_1, \ldots, \delta_N) = (b_k)_0 + \sum_i (b_k)_{1,i}\, \delta_i + \frac{1}{2} \sum_{i,j} (b_k)_{2,ij}\, \delta_i \delta_j + \cdots, \qquad (11)$$
>
> and if the j-cumulant, $\langle \delta_{i_1} \cdots \delta_{i_j} \rangle_c = \mathcal{O}(\nu^{-j+1})$, with $i_1, \ldots, i_j = 1, \ldots, N$ and $j = 1, 2, \ldots$, then the same holds for the cumulants $\langle \delta_{g,i_1} \cdots \delta_{g,i_j} \rangle_c$ of $\delta_{g,N}$.

For the case of spatial distribution we can interpret these variates as corresponding to the density contrast at different points, $\delta_k = \delta(\boldsymbol{x}_k)$ and $\delta_{g,p} = \delta_g(\boldsymbol{x}_p)$, so that the multivariate cumulants above are the standard correlation functions, $\langle \delta(\boldsymbol{x}_1) \cdots \delta(\boldsymbol{x}_j) \rangle_c$. From the hierarchy (2) above, the j-correlation for matter is of order ν^{-j+1} with ν the inverse amplitude of the two-point function. Therefore a local biasing transformation, equation (4): $\delta_g(\boldsymbol{x}_k) = f(\delta(\boldsymbol{x}_k))$, will produce $\langle \delta_g(\boldsymbol{x}_1) \ldots \delta_g(\boldsymbol{x}_j) \rangle_c = \mathcal{O}(\nu^{-j+1})$ and consequently the hierarchy (2) for galaxies.

Theorem 2 applies even when the coefficients b_k are functions of position, an inhomogeneous, nonlocal biasing transformation, $\delta_g(x_j) = F[x_j, \delta(x_1), \ldots, \delta(x_N)]$. In this case, the induced correlations can have little in common with the underlying matter correlations. Nevertheless we still have $\langle \delta_g(\boldsymbol{x}_1) \ldots \delta_g(\boldsymbol{x}_j) \rangle_c = \mathcal{O}(\nu^{-j+1})$, but now with local or scale-dependent values of Q_j.

2.5 A bias transformation Group

In a practical situation, only the lower moments of the observed galaxy distribution can be determined. We will define two spatial distributions to be equivalent to order N if their moments agree up to order N; a class of equivalent distributions will be called an N-order distribution. We can also define the equivalance relation for bias transformations: two biasing transformations over an N-order distribution are equivalent if, and only if, the first N coefficients of expansion (4) are equal. The set of equivalent classes of transformations will be called N-order biasing or N-order transformations. With this nomenclature, equation (10) shows that an N-order transformation can arbitrarily change one N-order hierarchical distribution to another. It is easy to see that N-order transformations, $\{b \,;\, \ldots \,;\, c_N\}$, form a ***non-Abelian Group*** of transformations. The composition (or group operation) of the transformation $\{b_B \,;\, c_{B,2} \,;\, c_{B,3} \,;\, \ldots\}$ following $\{b_A \,;\, c_{A,2} \,;\, c_{A,3} \,;\, \ldots\}$ yields the transformation:

$$\{b_A b_B \,;\, c_{A,2} + b_A\, c_{B,2} \,;\, c_{A,3} + 3\, b_A\, c_{A,2}\, c_{B,2} + b_A^2\, c_{B,3} \,;\, \ldots\}. \tag{12}$$

The neutral element is $\{1 \,;\, 0 \,;\, \ldots \,;\, 0\}$ and the inverse is:

$$\{b \,;\, c_2 \,;\, c_3 \,;\, \ldots\}^{-1} = \{b^{-1} \,;\, -b^{-1} c_2 \,;\, b^{-2}(3c_2^2 - c_3) \,;\, \ldots\}, \tag{13}$$

so that (10) can be easily inverted to give S_j in terms of $S_{g,j}$. These properties will be useful when comparing models with observations. For example, consider that the distribution of both optical (O) and IRAS (I) selected galaxies are related to the matter distribution by $\{b_O \,;\, c_{O,2} \,;\, c_{O,3} \,;\, \ldots\}$ and $\{b_I \,;\, c_{I,2} \,;\, c_{I,3} \,;\, \ldots\}$. Under the group properties, there will also be a biasing transformation between the optical and IRAS distributions, $\delta_I = f_{IO}(\delta_O)$, with $\{b_{IO} \,;\, c_{IO,2} \,;\, c_{IO,3} \,;\, \ldots\}$ given by

$$\begin{aligned} b_{IO} &= b_I / b_O \\ c_{IO,2} &= b_O^{-1}(c_{I,2} - c_{O,2}) \\ c_{IO,3} &= b_O^{-2}(c_{I,3} - c_{O,3}) + 3\, b_O^{-1}\, c_{O,2}\, c_{IO,2}, \end{aligned} \tag{14}$$

and so on. Let us apply these properties to the observations.

2.6 Biasing between optical and IRAS distributions

The relations obtained above can be used to fit a phenomenological bias between optical (O) and IRAS (I) selected galaxies because, as pointed out in the introduction, both optical and IRAS distributions are hierarchical at large scales, at least to the lower orders. Direct comparison of the dispersion at different scales gives $\langle \delta^2 \rangle_I = b_{IO}^2 \langle \delta^2 \rangle_O$, with $b_{IO} = 0.7 \pm 0.1$ (e.g. Strauss *et al.* 1992, Saunders *et al.* 1992; although this is the value of b quoted at $R \simeq 8\, h^{-1}$ Mpc, there is no significant different for larger scales), in agreement with a local biasing transformation. We will look for a class of transformations $\{b_{IO} ; c_{IO,2} ; c_{IO,3} ; \dots \}$ to relate optical, δ_O, and IRAS, $\delta_I = f(\delta_O)$ distributions. From (10) we have:

$$\begin{aligned} S_{I,3} &= b_{IO}^{-1}(S_{O,3} + 3\,c_{IO,2}) \\ S_{I,4} &= b_{IO}^{-2}(S_{O,4} + 12\,c_{IO,2}S_{O,3} + 4\,c_{IO,3} + 12\,c_{IO,2}^2). \end{aligned} \quad (15)$$

We apply this expression to values for amplitudes found from optical and IRAS samples:

- for optical galaxies: Szapudi *et al.* (1992) from the Lick sample obtain $S_{O,3} = 4.32 \pm 0.21$ and $S_{O,4} = 31 \pm 5$.
- for IRAS galaxies: Meiksin *et al.* (1992) obtain $S_{I,3} = 2.19 \pm 0.18$ and $S_{I,4} = 10.1 \pm 2.9$.

(We have taken $\gamma = 1.8$, $S_3 = 3Q_3$ and $S_4 = 16Q_4$.) These values are extracted from angular distributions and corrected for projection using the same techniques for IRAS and optical galaxies. With these amplitudes and the value of b_{IO} above we use equation (15) to find obtain

$$\begin{aligned} c_{IO,2} &= -0.93 \pm 0.06, \\ c_{IO,3} &= 2.95 \pm 0.65, \end{aligned} \quad (16)$$

incompatible, within the estimated errors (added in quadrature) with a linear biasing between optical and IRAS distributions, which would imply $c_{IO,2} = c_{IO,3} = 0$. By using the group composition properties (14) one can conclude that $c_{O,2} \neq c_{I,2}$, so that both can not be zero at the same time. Thus, a linear biasing from matter for both optical and IRAS galaxies is inconsistent with the observations cited.

3 Discussion

We do not observe the full matter distribution, but at best just part of the visible galaxy distribution, and, as shown above, in designing a bias prescription one must address the problem beyond linear order to extract meaningful information from higher order galaxy correlations. We can think of several distinct stages where a nonlinear processing may enter between one and the next. First, the matter field evolves gravitationally from initial conditions, a process that is well known to be nonlinear and that from Gaussian seed fluctuations produces hierarchical statistics, as in equation (1) or equation (2) (cf. Fry 1984*b*, Goroff *et al.* 1986, Bernardeau 1992). This alone guarantees that the matter distribution is not Gaussian. At some point, physics determines how matter is processed into candidates for observation, luminous stars, galaxies, and so on. Evidence for dark matter suggests that this does not happen uniformly. The light produced is collected by telescopes, recorded by instruments, photographic plates or CCD's. Finally, the astronomer applies further selection criteria to the images she obtains in order to create a catalog of galaxies or of clusters of galaxies. By equation (12), the end result is some effective transformation, likely to be different for each different category of objects observed.

Previous models that attempt to relate the statistical properties of biased galaxy and matter distributions (Kaiser 1984, Politzer & Wise 1985, Bardeen *et al.* 1986, Szalay 1988, and references therein) have assumed Gaussian underlying matter fluctuations. The basic assumption in all these models is that the physical processes involved in galaxy formation can be described by a transformation of the matter field, $\delta_r(\boldsymbol{x})$ smoothed over a galactic scale r. In the notation by Szalay (1988), a local transformation of the matter fluctuations, δ, leads to the galaxy fluctuations, $\delta_g = f(\delta) = G(y) - 1$, where $y = \delta/\sigma$ is a normalized matter fluctuation and G is the 'luminosity density.' Equation (10) shows that assuming the matter fluctuations are Gaussian is inadequate for a gravitationally evolved field: at each order, the terms arising from gravity and from bias are of comparable amplitude. Fry (1986) has considered the more general case of biasing from hierarchical matter fluctuations, but only up to the three-point galaxy correlation function, which was also found hierarchical.

In this paper we have considered the more general case of hierarchical, rather than Gaussian, matter fluctuations. We have shown that any sequence of local biasing transformations gives a contribution comparable to that from nonlinear gravitational evolution at each order in $\bar{\xi}_2$, As argued in §2.3, this result, that a very general nonlinear bias preserves the hierarchical structure in the limit of small ξ_2, involves a remarkable cancelation which results from the statistical properties of connected moments. In a sense gravity evolution for large scales is similar to local biasing, but as pointed out by Fry (1984*b*) the self similar time evolution is a unique feature of gravity, and what might serve to distinguish gravity from any other transformation is the characteristic values of the amplitudes S_j, which can be calculated explicitly in gravitational instability. This, in turn, can allow us to determine properties of the bias function from $S_{g,j}$.

To what extent can we say that the observed galaxy properties are a consequence of the initial Gaussian conditions? If we consider gravitational evolution in perturbation theory the problem of the initial conditions is very simple. If the initial correlations are in leading order $\langle \delta^j \rangle_c \propto \langle \delta^2 \rangle^\alpha$,

- $\alpha < j/2$ implies non-Gaussian and non-hierarchical initial conditions that dominate the evolution during the regime in which $\langle \delta^2 \rangle$ is small.
- $j/2 < \alpha < j-1$ implies quasi-Gaussian but non-hierarchical initial conditions. In this case, evolution will produce two contributions to ξ_j: a dominant non-hierarchical term that grows as $A^j(t)$ and a hierarchical term with characteristic amplitude S_j that grows as $A^{2(j-1)}(t)$ but may not become significant until $\xi \sim 1$.
- $\alpha > j-1$ implies strongly-Gaussian initial conditions. In this case, the leading order effect of evolution will produce hierarchical statistics with characteristic amplitudes S_j for all times.

That is, the initial conditions for large-scale structure formation could be Gaussian if, and only if, the evolved matter distribution is observed to be hierarchical at large scales.

The explicit relations between galaxy and matter amplitudes presented in equation (10) show that if we allow biasing to be an arbitrary function, then the observed galaxy amplitudes can be arbitrarily different from the matter ones. On the other hand, one can use these relations to learn about biasing by comparing galaxy amplitudes with theoretical matter predictions. It is also clear from (10) that a linear biasing approximation is consistent only for the two-point correlation function. In general, even in the limit of weak fluctuations on very large scales, the j-point galaxy amplitudes have biasing contributions not only from the linear term but from all orders up to $j-1$.

Are the observed hierarchical properties of the galaxy distribution a consequence of the hierarchical properties of matter? Or, are they an accident or conspiracy of galaxy-matter biasing? We have shown here that in the case of local biasing, the observed galaxy hierarchy at large scales can only be a consequence of hierarchical properties of the smoothed matter distribution and thus suggests that the initial conditions were indeed hierarchical or Gaussian.

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