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ELECTRONIC NOISE CALCULATIONS FOR A FAST CURRENT AMPLIFIER WITH A WAVE-GUIDE LINE AT ITS INPUT.

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Abstract

A formula for the electronic noise of a fast current amplifier with a wave-guide line at its input and an integrator at the output is deduced. For the case of a short line the result is the same as if there was a capacitor of the line capacitance at the amplifier input instead of the line. However, as soon as the line delay time exceeds half of the integration time, the integrated noise charge ceases to depend on any of the wave-guide line parameters.

The calculations are compared with measurements.

The high pressure gas ionization calorimeter[1], which is under development for the SDC forward calorimetry, consists of tube ionization chambers. From the electronic point of view, each of these tubes is a 3 m long wave-guide line with impedance $z = 24\ \Omega$. A fast current amplifier with matched input impedance $\rho = z$ is supposed to be connected to each tube (or an amplifier with $\rho = z/n$ is to be connected to n tubes in parallel). The opposite end of the line is open. The aim of this paper is to calculate the electronic noise for such a detector.

We will examine a design where the amplifier output signal is integrated immediately by a charge sensitive analog to digital converter (ADC) during the gate time τ without any pulse shaping circuit. This simplification allows one to get analytical expressions for the registered noise charge and to understand its dependence on the design parameters. We believe, however, that the results obtained below will also be valid for a design with a shaping circuit if the gate time τ is replaced by an appropriate effective integration time. Besides, a shaping circuit is not necessary if one uses a flash ADC for signal processing.

The noise equivalent scheme of the design is shown in fig.1. There are two independent noise sources: voltage (series) noise and current (parallel) noise[2,3]. The values of **e** and **i** are the r.m.s.

amplitudes of the series and parallel noise sources, respectively, per 1 Hz of frequency bandwidth. We will assume that both sources produce "white" noise, so the values of **e** and **i** do not depend on the frequency. We will also assume the amplifier transition time is small compared to both the time of signal propagation along the wave-guide line (delay time) and the gate signal width. Therefore, the effect of the amplifier on the noise frequency spectra is negligible compared to those of the wave-guide line and the integrator.

Let us consider now a wave of frequency ω produced by the series noise source in a narrow frequency bandwidth $\Delta\omega$ with random amplitude a and random phase δ:

$$e_p = a \sin(\omega t + \delta). \tag{1}$$

The mean square voltage of this wave is $\langle e_p^2 \rangle = \langle a^2 \rangle /2$ and it is equal to $(e^2 \Delta\omega/2\pi)$ by definition. Hence:

$$\langle a^2 \rangle = e^2 \Delta\omega/\pi \tag{2}$$

The current induced at the amplifier input by this wave together with the wave reflected from the open end of the line of length ℓ is:

$$i_p = \frac{a}{2\rho} \{\sin(\omega t + \delta) - \sin(\omega t + \delta + 2\omega\ell/v)\} \tag{3}$$

where v is the wave speed in the line. At the integrator output this current produces a charge (reduced to the amplifier input) equal to:

$$q_p = \int_0^{\tau} i_p \, dt = \frac{2a}{\rho\omega} \sin(\omega\tau/2) \sin(\omega\ell/v) \cos(\omega\tau/2 + \omega\ell/v + \delta) \tag{4}$$

Averaging the square of this expression over random values of the phase δ and the amplitude a, one gets:

$$\langle q_p^2 \rangle = \frac{2e^2 \Delta\omega}{\rho^2 \omega^2} \sin^2(\omega\tau/2) \sin^2(\omega\ell/v) \tag{5}$$

Since the noise amplitudes of all the frequencies are statistically independent, the mean square of the total integrated noise charge from the series noise source Q_e can be calculated as:

$$< Q_e^2 > = \int_0^\infty \frac{2e^2\, d\omega}{\rho^2 \omega^2} \sin^2(\omega\tau/2)\, \sin^2(\omega\ell/v) \tag{6}$$

This integral is equal to[4]:

$$< Q_e^2 > = \begin{cases} \dfrac{e^2}{\rho^2}\,\dfrac{\tau}{4}, & \text{if} \quad \ell \geq \dfrac{v\tau}{2} \\[2ex] \dfrac{e^2}{\rho^2}\,\dfrac{\ell}{2v}, & \text{if} \quad \ell < \dfrac{v\tau}{2} \end{cases} \tag{7}$$

Similarly, the total integrated noise charge from the parallel noise source is:

$$< Q_i^2 > = \begin{cases} i^2\,\dfrac{\tau}{4}, & \text{if} \quad \ell \geq \dfrac{v\tau}{2} \\[2ex] \dfrac{i^2}{2}\,(\tau - \dfrac{\ell}{v}), & \text{if} \quad \ell < \dfrac{v\tau}{2} \end{cases} \tag{8}$$

The difference between the two expressions (7) and (8) comes from the difference in modes of mixing of the direct and reflected waves at the amplifier input for the two cases: they are subtracted one from the other in the case of the series noise source and summed up in the case of the parallel noise source.

The total noise charge from both noise sources is:

$$< Q^2 > = \begin{cases} \left(\dfrac{e^2}{\rho^2} + i^2\right) \dfrac{\tau}{4}, & \text{if} \quad \ell \geq \dfrac{v\tau}{2} \\[2ex] \dfrac{e^2}{\rho^2}\,\dfrac{\ell}{2v} + \dfrac{i^2}{2}\,(\tau - \dfrac{\ell}{v}), & \text{if} \quad \ell < \dfrac{v\tau}{2} \end{cases} \tag{9}$$

It is interesting to compare this result with the noise calculations for the case where a capacitor is connected to the amplifier input instead of a cable. To do this we need to notice that the wave (1) from the voltage noise source produce at the amplifier input shunted by a capacitance C a current equal to:

$$i_p = \frac{a}{\sqrt{\rho^2 + \frac{1}{\omega^2 C^2}}} \sin(\omega t + \delta'). \qquad (10)$$

The analogous formula for the input current induced by the parallel noise source has the form:

$$i_p = \frac{b}{\sqrt{1 + \rho^2 \omega^2 C^2}} \sin(\omega t + \delta''), \qquad (11)$$

where b is a random amplitude satisfying the requirement: $<b^2> = \mathbf{i}^2 \Delta\omega/\pi$.

Repeating now the procedures of integrating and averaging (4), (5) and (6) one can get the following expressions for the series and parallel noise charge:

$$< Q_e^2 > = \frac{e^2}{\rho} \frac{C}{2} (1 - e^{-\tau/\rho C}) \qquad (12)$$

$$< Q_i^2 > = \frac{i^2}{2} \{\tau - \rho C(1 - e^{-\tau/\rho C})\} \qquad (13)$$

These expressions can be compared with the corresponding formulas (7) and (8) if one take into account that the capacitance of the wave-guide line is equal to $C = \ell/v\rho$. The diagrams of functions (7) and (8) along with those of (12) and (13) are shown in fig.2. One can see that the results of the noise calculations for the cases of the wave-guide line and of the capacitance practically coincide if the line length $\ell < v\tau/2$ (or, equivalently, the line capacitance $C < \rho\tau/2$). However, as soon as the line length exceeds the value $\ell = v\tau/2$, the noise charge of the amplifier with the wave-guide line no longer depends on any of the line parameters, in contrast to the capacitance case.

The difference between these cases is more clearly evident when one compares their respective signal to noise ratios. While the registered signal amplitude practically does not depend on the cable length (we assume the cable is matched with the amplifier input impedance), it decreases with input capacitance increasing. For large values of the capacitance ($\rho C >> \tau$), the signal to noise ratio is inversely proportional to the capacitance value.

To check the above formulas we have made noise measurements of a fast low noise amplifier[5] connected to a 50 Ω impedance cable at its input. The noise characteristics of this amplifier were determined previously[6] by measuring the dependence of the amplifier noise on the value of an external resistor shunting the amplifier input (as in fig. 1, but resistors of different value were used instead of the cable). Those measurements yielded $\mathbf{e} = 0.62$ nV/√Hz and $\mathbf{i} = 3.8$ pA/√Hz. The amplifier input impedance was measured to be $\rho = 62\ \Omega$, the amplifier signal rise and decay times were less than 10 ns.

Results of the noise charge measurements of the amplifier with cables of different length at the input are shown in fig. 3. The formula (9) with free parameters **e** and **i** was fitted to the measured points (in the case 'without cable' the dependence (9) is reduced to the expression $<Q^2> = i^2\tau/2$). As one can see this dependence describes well the measured data with parameters **e** = 0.50 nV/√Hz and **i** = 3.1 pA/√Hz, which do not differ much from the values measured previously.

Acknowledgements

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List of Figures

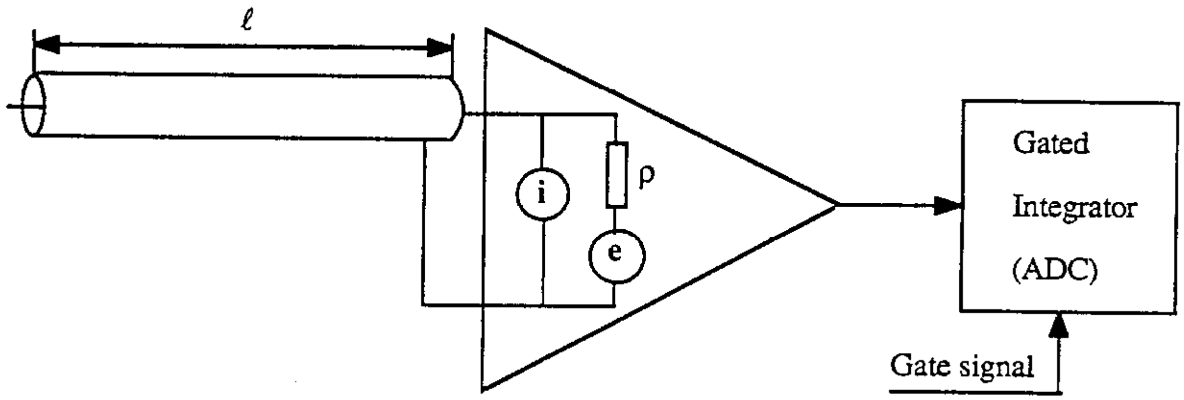

Figure 1

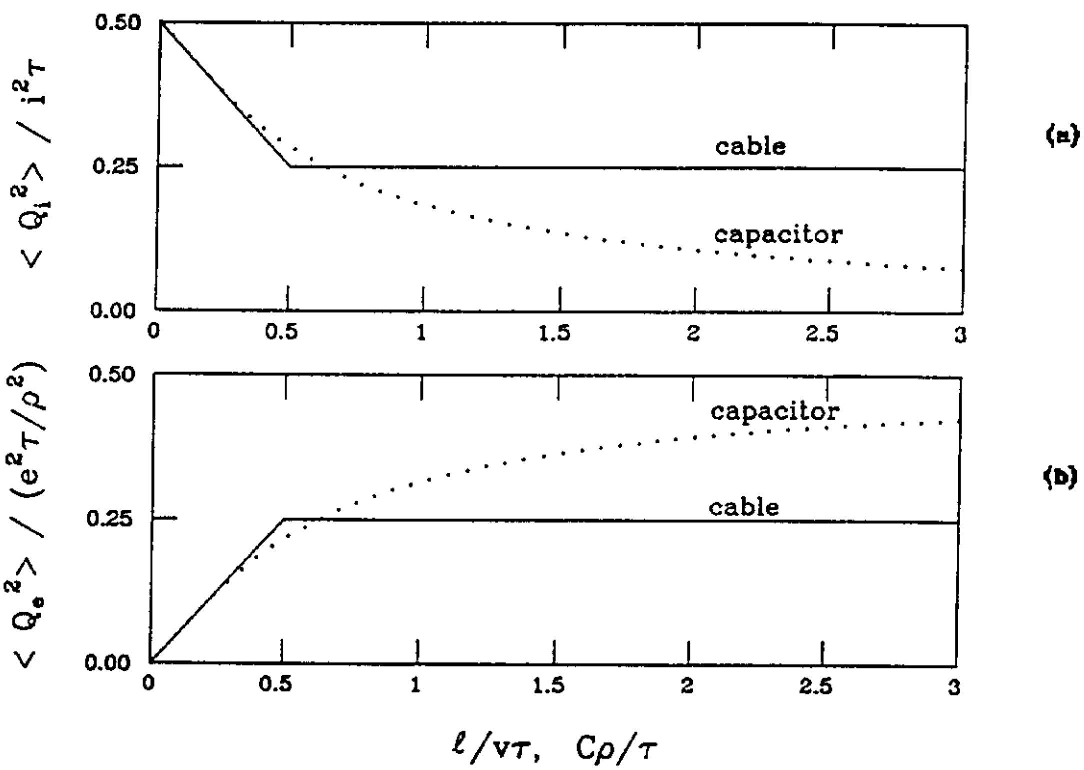
0.50
0.25
0.00
cable
capacitor
(a)
0
0.5
1
1.5
2
2.5
3
capacitor
cable
(b)
$< Q_i^2 > / i^2\tau$
$< Q_e^2 > / (e^2\tau/\rho^2)$
$\ell/v\tau, \quad C\rho/\tau$

Figure 2

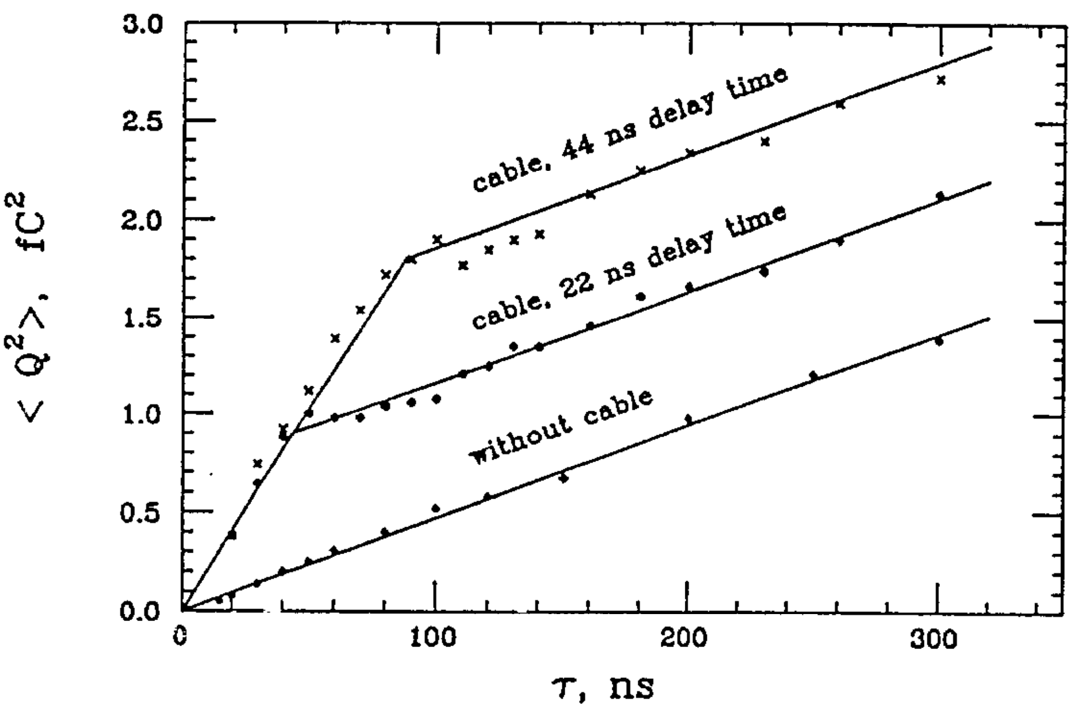
cable, 44 ns delay time
cable, 22 ns delay time
without cable
< Q² >, fC²
τ, ns
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
100
200
300

Figure 3