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ANGULAR DIAMETERS AS A PROBE OF A COSMOLOGICAL CONSTANT AND Ω *

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Abstract

The lensing effect of curved space, which can cause the angular diameter of a fixed reference length seen on the sky to reach a minimum and then increase with redshift, depends sensitively on the value of the cosmological constant, Λ, in a flat universe. The redshift of an observed minimum and the asymptotic slope can in principle provide strong constraints on Λ. The sensitivity to a non-zero cosmological constant in a flat universe is compared to the sensitivity to q_0 in an open universe without a cosmological constant, and to inherent ambiguities due to uncertainties in distance measures and the possible effects of evolution. If evolutionary uncertainties can be overcome, the reported observations of the angular diameter of compact radio jets as a function of redshift, which appear to exhibit such a minimum, could provide the strongest available limit on the cosmological constant in a flat universe, and on Ω in an open universe.

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The Cosmological Constant may be at the same time the most strongly constrained and the poorest understood theoretical quantity in nature. The fact that we do not live in an identifiably de Sitter universe implies that the Cosmological Constant is over 120 orders of magnitude smaller than the value one might naively expect, namely $\Lambda \approx M_{Pl}^4$ (using units where $h/2\pi = c = 1$). It is therefore tempting to speculate that its value is identically zero. However, not only do we have no clear understanding of why this might be the case (although recently some progress on this issue has been made [Baum 1983;Hawking 1984; Coleman 1984]), but there are observational reasons which suggest that the Cosmological constant might *not* be zero, but could dominate the energy density of the universe today. If the oldest globular cluster stars are indeed older than 15 Gyr, as current analyses suggest, this will be incompatible with any value of Hubble constant greater than about 40 km/sec/Mpc in a flat universe without a cosmological constant, or about 70 km/sec/Mpc in an open universe. Similarly, structure formation arguments in a Cold Dark Matter dominated universe with scale invariant adiabatic density perturbations, normalized to the recent COBE report of a quadrupole anisotropy [Smoot et al 1992], also improve if there exists a non-zero cosmological constant.

Recently, a number of researchers have examined the possibility of using the optical depth for gravitational lensing of distant quasars by intervening galaxies to probe the geometry of the universe, and hence constrain the cosmological constant [Turner 1990; Fukugita, Futamase and Kasai 1990; Mao 1991; Krauss and White 1992; Fukugita and Turner 1992; Kochanek 1992]. The existing data, which is sparse, appears to constrain a cosmological constant contribution to Ω to be less than about .95, but remains compatible with the favored value of $\Omega_\Lambda \approx 0.8$ [i.e. see Krauss 1992]. The redshift distribution of galactic lenses is also a useful probe [Fukugita, Futamase and

Kasai 1990; Krauss and White 1992; Kochanek 1992], but here again, the available statistics are marginal.

We propose here to use another sort of gravitational lensing phenomenon for this purpose. It is well known that, due to its spatial curvature, the universe itself can act as a lens of large focal length. Nearby objects are not affected, but objects located at distances which approach the Hubble size can be greatly magnified (i.e. see [Misner, Thorne, and Wheeler 1973]). Traditionally this effect has been discussed as a possible way to distinguish between an open and closed universe. We describe here how it can be used in principle to effectively limit the cosmological constant, assuming the universe is flat, as both theoretical prejudice and several recent analyses suggest.

If we write the Robertson-Walker metric in the form:

$$ds^2 = dt^2 - R^2(t)\left(d\chi^2 + s_k^2(\chi)d\Omega^2\right) \tag{1}$$

where the form of s_k depends on the curvature ($k = 0, \pm 1$), the angular diameter $\delta \ll 1$ of a proper distance D located perpendicular to the line of sight at $\chi = \chi_1, t = t_1$ is given by [Weinberg 1972, Misner Thorne and Wheeler 1973]

$$\delta = \frac{D}{R(t_1)s_k(\chi_1)}. \tag{2}$$

In a flat $k = 0, \Omega = 1$ universe with non-zero cosmological constant contribution $\Lambda = 1 - \Omega_0$, $s_k(\chi) = \chi$, and the relationship between the co-ordinate χ and the redshift z is given by

$$\chi(z) = \int_1^{1+z} \frac{\mathrm{dy}}{(\Omega_0 y^3 - \Omega_0 + 1)^{1/2}} \tag{3}$$

This integral cannot be written in a simple analytical form in general, except for the extreme cases $\Omega_0 = 0, 1$. In these cases, normalizing $D/R(t_{today}) = 1$, one finds:

$$\delta = \begin{cases} \frac{(1+z)^{3/2}}{2[(1+z)^{1/2}-1]}; & \Omega_0 = 1 \\ \\ \frac{(1+z)}{z}; & \Omega_0 = 0 \end{cases} \tag{4}$$

As can be seen from (4), in the standard case of a flat universe when the cosmological constant is zero ($\Omega_0 = 1$) the angular diameter as a function of redshift has its well known minimum at $z = 5/4$. However when there is zero matter ($\Omega_0 = 0$) and the cosmological constant contributes all the energy density the minimum disappears. For intermediate cases, the integral (3) must be done numerically. Several such cases, along with the two extremes, are shown in figure 1, plotted on a log-log plot. As can be seen there, as long as $\Omega_0 \neq 0$ the minimum persists, although it gets less pronounced, and is pushed to higher redshifts as the matter density is decreased.

These features can be exploited in principle to constrain the cosmological constant, as can be seen explicitly by differentiating the angular diameter versus redshift curves. The differentiated curves are shown in figure 2. Here a reference line representing zero slope is displayed. As expected from the analytic results for the extreme cases (using our normalization for D), the asymptotic slope approaches 1/2 (0) for $\Omega_0 = 1$ (0) respectively. (In the former case, the slope actually overshoots its asymptotic value in the region $z < 5$). The redshift of the minimum angular diameter (zero slope) moves from $z = 1.25$ for $\Omega_0 = 1$ to $z \approx 2$ for $\Omega_0 = 0.1$, while the asymptotic slope moves from $\approx 1/2$ to ≈ 0.1 in this case. The specific redshifts for the minimum angular diameter, and the slope of the angular diameter-redshift function at z=4 are tabulated for various values of Ω_0 in table 1.

Of course, this test is not an unambiguous probe of Λ, unless we also know the geometry of the universe. For example, it is well known that if the universe is open, the angular diameter can also continue to decrease, although it is less well known that this need not be the case. To examine the sensitivity to Λ for a flat universe versus Ω for an open universe, we can repeat the above analysis for a $k = -1$ cosmology, with zero cosmological constant. In this case $s_k(\chi) = sinh(\chi)$. The formula for χ as a function of redshift is in this case (where q_0 is the deceleration parameter $=\Omega_0/2$ for $\Lambda = 0$):

$$\chi(z) = [1 - 2q_0]^{1/2} \int_1^{1+z} \frac{dy}{y[2q_0 y + (1 - 2q_0)]^{1/2}} \tag{5}$$

which can be done analytically. In the limit $q_0 = 0$, one finds, using (2) that the angular diameter decreases monotonically as a function of reshift. However, just as for the flat case with small non-zero matter density, if $q_0 \neq 0$ then one finds that this function has a minimum, which again becomes less pronounced and occurs at higher redshift as q_0 is decreased. In figure 1 the results for the case $q_0 = 0$ and $q_0 = 0.1$ ($\Omega \approx 0.2$) are shown. In figure 2 the differentiated curves are displayed for these two cases. As can be seen, the $q_0 = 0$ case is almost identical to the $\Lambda = 1$ case, which is not surprising. The case of $q_0 = 0.1$ is very similar to the $\Lambda = 0.9$ case. As a result, if an astrophysical source with fiducial proper length can be measured as a function of redshift out to redshifts of 2-4,*and if the possible evolutionary uncertainties discussed below can be removed* a sensitivity which can distinguish an $\Omega = 1$ flat universe from an $\Omega = 0.2$ open universe is equivalent to a sensitivity which can rule out a value of $\Lambda > 0.9$, if one assumes that the universe is flat. If no other cosmological information were utilized, one could not distinguish between these two possibilities using this test alone.

It is worth stressing that a sensitivity to smaller values of Λ is not unreasonable. The minimum of the angular diameter-redshift relation moves from $z \approx 1.2$ to $z \approx 1.6$ for $\Lambda = 0.7$. Such a distinction may prove plausible observationally.

These arguments would be primarily academic if there were not a realistic possible astrophysical fiducial probe. Recently a survey of the angular diameter of ≈ 100 compact parsec-scale radio jets (i.e. [Pearson 1990]) with size $O(40)$ pc ($H = 50$

km/s/Mpc) in AGN's using VLBI, extending out to $z \approx 3$ has been reported [Kellerman 1992], in which a minimum at $z \approx 1$ is apparently observed . As the parameters of this survey become established, the analysis described here could be used to provide in principle a robust limit on the cosmological constant in a flat universe which might plausibly exceed those presently available from the statistics of gravitational lensing of quasars. This of course depends in practice on how strongly such effects as evolution can be constrained.

Indeed, any mention of a potential candidate probe of the geometry of the universe would be incomplete without some discussion of the plagues on all such measurements: evolutionary effects, and uncertainty in distance measures. First, let us assume a minimum is observed. How can one be certain that the angular diameter-redshift relation that one observes is geometric, and not related to dynamical evolution of the source? While no evolutionary relationship would be expected to produce a minimum by itself, as we describe below, a specific, but plausible evolution of the size of compact jets could, when combined with the geometric effect, efficiently mask, or mimic the behavior of angular diameter vs redshift for all the cases we have examined.

Clearly, if the intrinsic size of compact jets varies monotonically with redshift, this could shift the redshift of any observed minimum. Unless this variation is relatively smooth however, the slope of the angular diameter-redshift away from the minimum might be expected to be altered due to such dynamical evolution in a distinguishable way. The simplest, and at the same time a physically plausible, possibility for an evolutionary trend which might mimic the geometric effects is to imagine that the intrinsic source size D varies with redshift in a way similar to the co-ordinate dependence, i.e. $D \approx D_0(1+z)^{\alpha}$, where D_0 is the source size in nearby objects.

We find that such a variation can either produce or remove any pre-existing minima, even for relatively mild values of α. To see that this is possible, consider a $\Lambda = 1$ flat universe, in which the slope of $\delta(z)$ approached zero from below at large redshift. If $\alpha = 1/2$ in the relation for D above, then this slope would approach instead $1/2$ at large z, thus mimicking the behavior of an $\Omega_0 = 1$ universe. In figures 3 and 4, we show the effect in the opposite case, namely assuming an $\Omega_0 = 1$ universe to begin with, we examine both the angular diameter-redshift relations, and their derivatives which result from several choices of $\alpha < 0$. As can be seen, $\alpha = -.25$ results in a curve whose minima, and asymptotic slope are very similar to the case of an $\Omega_0 = .25, \Lambda = .75$ universe. Alternatively, a similar magnitude, but positive value for α would result in the latter universe mimicking the former.

It is worth noting that the slopes of the curves, while similar at large redshift, differ somewhat at small redshift, where most of the data will in fact be expected to be accumulated. Could such a difference, if observed, be observationally significant? There is yet another instrinsic uncertainty which suggests that unless one measures sources out to significantly beyond the any observed minimum, comparison of the small redshift and large redshift slopes may be ambiguous. Recall that we normalized the angular diameter so $D/R_0 = 1$. This can only be done if we know the intrinsic value of D, or D_0 if D varies. However, our knowledge of this intrinsic magnitude suffers from our uncertainty in the distance scale of universe, coming from our uncertainty in H_0, at the factor of 2 level. Changing D by a factor of 2 will change the slope on either side of the minimum by a similar factor, thus tilting the overall curve. Unless one has a sufficiently large lever arm on both ends, it is unlikely therefore that one can overcome Hubble constant-based uncertainties in the slope of either side.

Finally, is a value $|\alpha| = .25$ reasonable? This suggests that at a redshift of 2 the size of compact jets would have changed by about 30% compared to their size in nearby galaxies. This is not an extreme variation. One could imagine plausible mechanisms which could produce such an effect. Radio jets presumably result from accretion onto a black hole. If the accretion rate varies over cosmological time then this would impact on the energy production which fuels the jets. If, for example, heating by matter accreting over time caused the material surrounding the hole to "puff up", this might lower the accretion rate. Alternatively, collapse of matter in the region of the hole over time might increase the accretion rate.

These arguments are not meant to be fatal to this method. There are merely presented as caveats. If a minimum *is* confirmed in the angular diameter-redshift relation at $z \approx 1$ for radio jets then it will imply one of three things: (a) the cosmic density parameter is greater than a certain minimum value which could exceed the value inferred from the dynamics of galaxies and clusters, (b) if the universe is flat, the cosmological constant is smaller than the amount required to account for the difference between virial estimates for clusters and galaxies and $\Omega = 1$, or (c) evolutionary effects have the same form and magnitude as we have assumed here. We expect the latter possibility should be addressable by more detailed modelling. If it can be invalidated, then "universal lensing" causing a magnification of angular diameters of objects at cosmological redshifts can not only give strong evidence for a flat universe, but also can potentially rule out a cosmological constant dominated one.

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Ω_0	Redshift of minimum δ	Asymptotic Slope (z=4)
0.0	—	-.062
0.1	2.08	.176
0.2	1.76	.254
0.3	1.60	.308
0.4	1.50	.353
0.5	1.44	.392
0.6	1.38	.426
0.7	1.34	.457
0.8	1.31	.486
0.9	1.28	.513
1.0	1.25	.538

Table 1: Redshifts at Minimum and Asymptotic Slope of Angular Diameter vs. Redshift curve for various values of Ω_0 in a Flat Universe

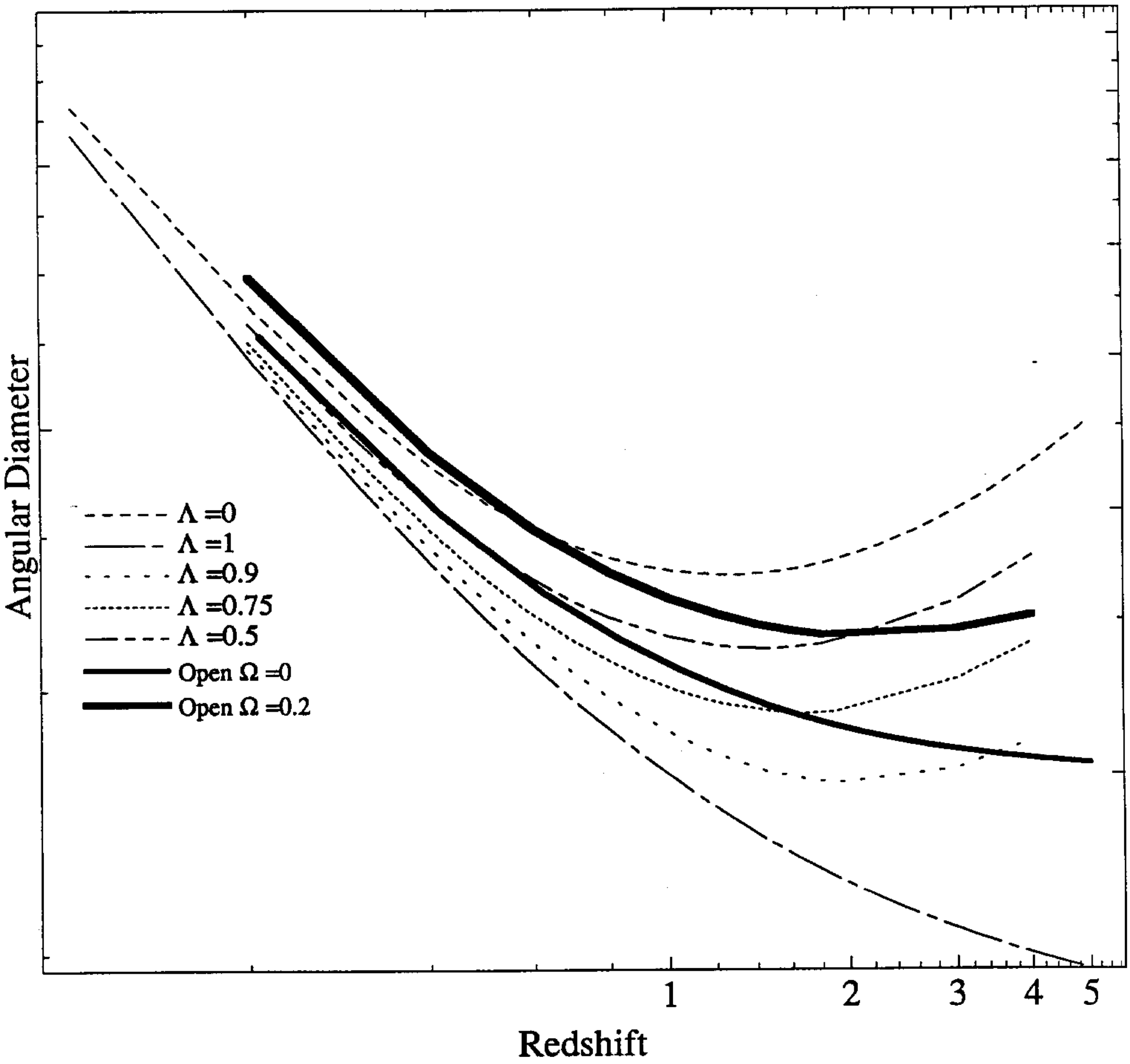
Figure Captions

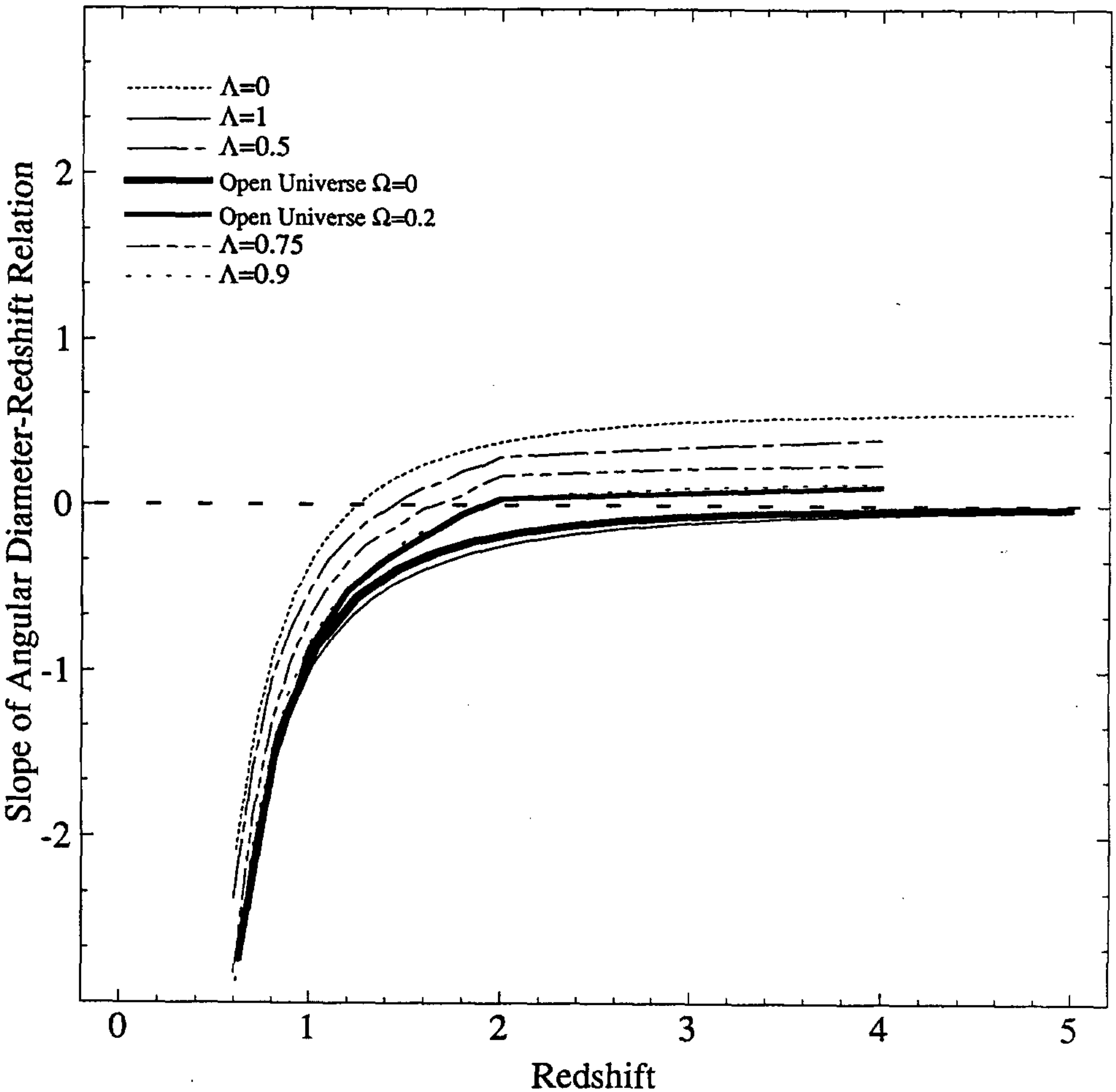
Figure 1a: The angular diameter versus redshift for a unit fiducial length perpendicular to the line of sight. Cases shown include flat universes with cosmological constant, and open universes with zero cosmological constant

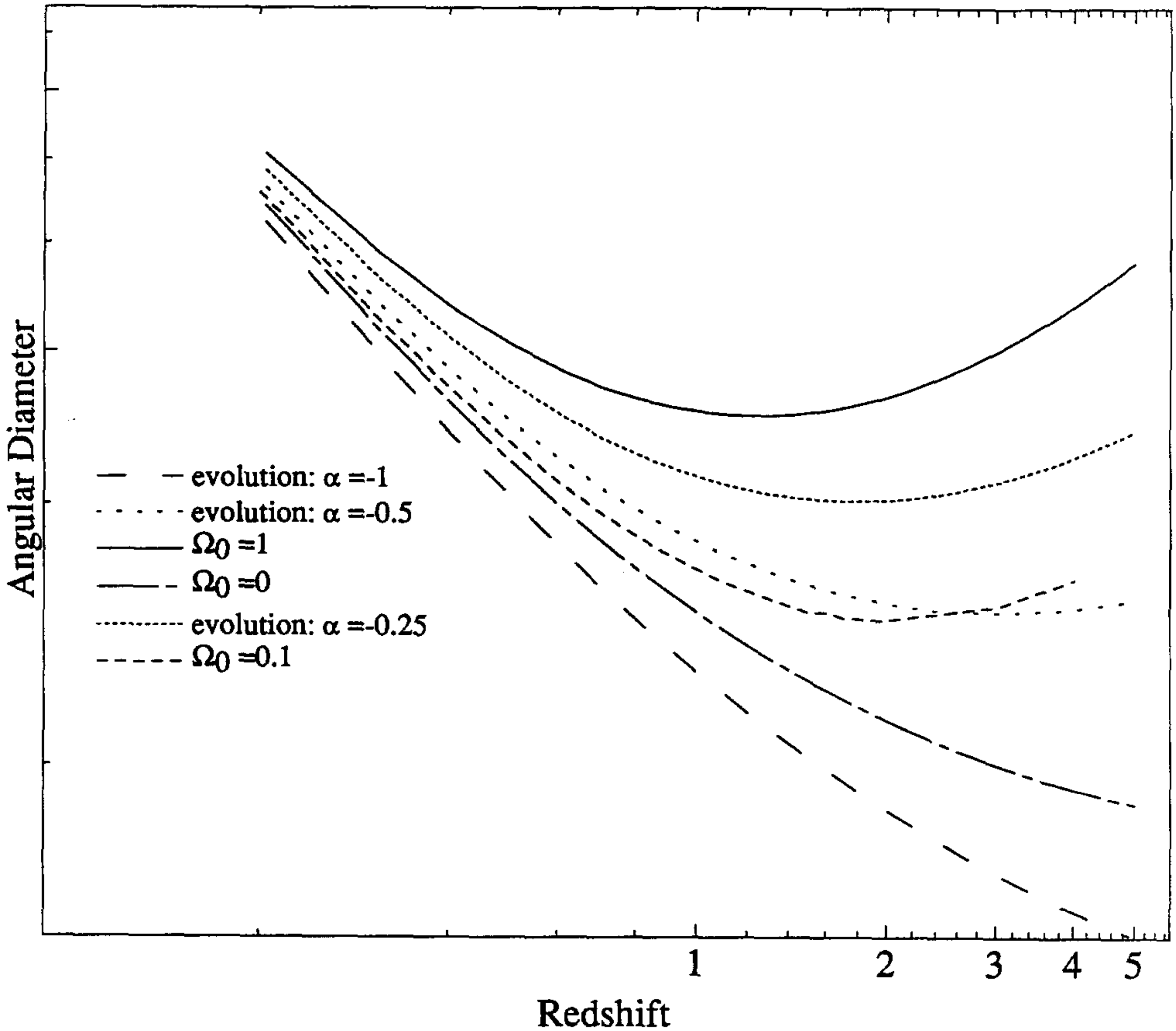
Figure 1b: The derivatives, with respect to redshift of the curves shown in figure 1a. The redshifts of zero slope (i.e minima) occur where these curves intersect the heavy dashed curve. Also important are the asymptotic values of the slopes.

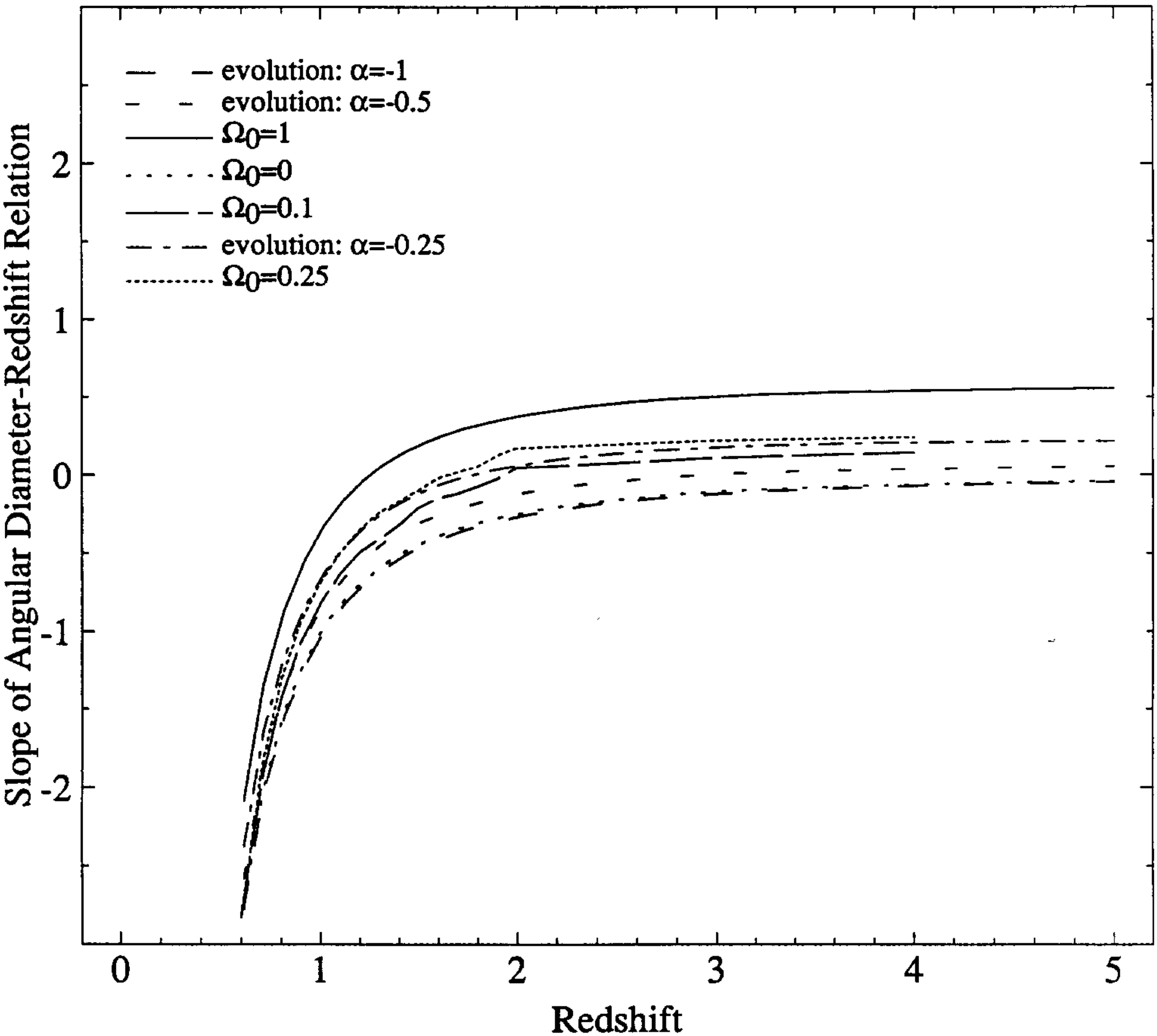
Figure 2a: Same as figure 1a for an $\Omega_0 = 1$ flat universe with evolutionary variation in fiducial refence probe going as $D = D_0(1+z)^\alpha$, in comparison to the cases $\Omega_0 = 0, 1$ without such variation.

Figure 2b: same as figure 1b, for the cases described in figure 2a


Angular Diameter
Λ =0
Λ =1
Λ =0.9
Λ =0.75
Λ =0.5
Open Ω =0
Open Ω =0.2
1
2
3
4
5
Redshift


Λ=0
Λ=1
Λ=0.5
Open Universe Ω=0
Open Universe Ω=0.2
Λ=0.75
Λ=0.9
Slope of Angular Diameter-Redshift Relation
2
1
0
-1
-2
0
1
2
3
4
5
Redshift


evolution: α =-1
evolution: α =-0.5
Ω0 =1
Ω0 =0
evolution: α =-0.25
Ω0 =0.1
Angular Diameter
Redshift
1
2
3
4
5



evolution: α=-1
evolution: α=-0.5
Ω0=1
Ω0=0
Ω0=0.1
evolution: α=-0.25
Ω0=0.25
Slope of Angular Diameter-Redshift Relation
Redshift
2
1
0
-1
-2
0
1
2
3
4
5