

## Metastable Cosmic Strings in Realistic Models

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ABSTRACT: We investigate the stability of the electroweak Z-string at high temperatures. Our results show that while finite temperature corrections can improve the stability of the Z-string, their effect is not strong enough to stabilize the Z-string in the standard electroweak model. Consequently, the Z-string will be unstable even under the conditions present during the electroweak phase transition. We then consider phenomenologically viable models based on the gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  and show that metastable strings exist and are stable to small perturbations for a large region of the parameter space

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for these models. We also show that these strings are superconducting with bosonic charge carriers. The string superconductivity may be able to stabilize segments and loops against dynamical contraction. Possible implications of these strings for cosmology are discussed.

Over the last two decades, cosmic strings have evoked a great deal of interest both as possible remnants of a Grand Unified era in the early universe as well as a possible mechanism for structure formation in the universe<sup>1</sup>. However, no compelling particle physics models exist that give rise to such defects. Recently, a defect that is closely related to the “ordinary” cosmic string has been found<sup>2,3</sup> in what may be the most compelling of all particle physics models - the standard electroweak model. The defect is identical in its structure to the cosmic string solution found by Nielsen and Olesen<sup>4</sup> and may be thought of as a cosmic string embedded in the standard electroweak model<sup>5</sup>. The difference now is that the defect does not owe its existence to topology and consequently may not be stable. The stability of the defect depends on the parameters of the electroweak model<sup>6</sup>.

In Ref. 7 the stability of the string in the standard electroweak model was analyzed. This resulted in a map of parameter space demarcating the regions in which the string is stable to small perturbations from the regions in which it is unstable. Given the known value of the Weinberg angle,  $\sin^2 \theta_W \approx 0.23$ , and the constraints on the Higgs mass,  $m_H > 57$  GeV, it is clear that the electroweak string is unstable. However, the analysis in Ref. 7 was limited to the bare electroweak model. The issue of stability must be reconsidered when one takes quantum and thermal corrections to the potential into account. In essence, the question is whether the strings can be stable at temperatures close to the electroweak phase transition temperature. If this is true, the strings may be relevant to cosmology. We answer this question in Sec. 2 where we map the parameter space as in Ref. 7 for the case when thermal and quantum corrections are taken into account. The results show that these corrections increase the region of stability, but not to the extent of allowing for stable electroweak strings in the standard model even near the electroweak phase transition.

We then consider the question of whether there is any realistic particle physics model in which one might expect stable embedded strings. We show that left-right models<sup>8</sup> are good candidates. In Sec. 3 we consider an  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  model. The parameters

may be chosen such that this model gives acceptable particle physics and also contains *stable* strings. This gives us a concrete example of a realistic particle physics model with stable embedded strings.

The cosmology of embedded strings will be very different from that of topological strings. The basic reason for this has to do with the metastability of the embedded strings versus the complete stability of topological strings. In Sec. 4 we speculate on the cosmology of embedded strings. The results of this section should not be thought of as firm conclusions but only as first guesses intended to inspire future work. Section 5 contains our conclusions.

## 2. Stability of the Z-String at Finite Temperature

The addition of quantum corrections to the Higgs potential can have a drastic effect on the vacuum structure of the standard model<sup>9</sup>. The most important correction is in the form of a  $\phi^4 \log(\phi/M)$  term, which can destabilize the potential at large  $\phi$ . However, our interests lie at relatively small  $\phi$  where this term is quite small. We have done a stability analysis including this term and found that there is very little effect. Therefore for the remainder of this discussion we shall ignore quantum corrections and concentrate on those induced by finite temperature effects.

The one loop finite temperature effective potential for the Higgs field can be written as<sup>10</sup>

$$V_T(\phi) = \lambda \left( \phi^\dagger \phi - \frac{\eta^2}{2} \right)^2 + DT^2 \phi^\dagger \phi - ET(\phi^\dagger \phi)^{3/2} \quad (2.1)$$

where  $D$  and  $E$  are functions of the particle masses, and can be approximated by

$$D = \frac{1}{8\eta^2} \left( 2m_W^2 + m_Z^2 + 2m_t^2 \right), \quad (2.2)$$

$$E = \frac{1}{4\pi\eta^3} \left( 2m_W^3 + m_Z^3 \right) \quad (2.3)$$

Here  $\eta = 246$  GeV is the expectation value of  $\phi$  at the minimum of the zero-temperature potential. Here we have chosen to ignore temperature corrections to  $\lambda$ , which are logarithmic and should not effect our results significantly.

As in the zero temperature case, Z-string solutions will take the form

$$W^{\mu 1} = W^{\mu 2} = A^\mu = 0, \quad Z^\mu = -\frac{v(r)}{r} \hat{e}_\theta$$

$$\phi = f(r) e^{i\theta} \Phi, \quad \Phi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad (2.4)$$

where we have assumed the string to be aligned along the  $z$  axis, and  $r$  and  $\theta$  are polar coordinates in the  $xy$ -plane. The functions  $f$  and  $v$  are determined by the equations of motion:

$$f'' + \frac{f'}{r} - \left(1 - \frac{\alpha}{2}v\right)^2 \frac{f}{r^2} - 2\lambda \left(f^2 - \frac{\eta^2}{2}\right) f + DT^2 f - \frac{3}{2}ET f^2 = 0 \quad (2.5)$$

$$v'' - \frac{v'}{r} + \alpha \left(1 - \frac{\alpha}{2}v\right) f^2 = 0 \quad (2.6)$$

where primes denote differentiation by  $r$ . Here  $\alpha$  is given by  $g = \alpha \cos(\theta_W)$ .

The functions also satisfy the boundary conditions:

$$f(0) = v(0) = 0, \quad f(\infty) = f_\infty, \quad v(\infty) = \frac{2}{\alpha} \quad (2.7)$$

where  $f_\infty$  is the magnitude of the global minimum of  $V_T$ .

In order to study the solutions to these equations numerically, it is convenient to introduce the dimensionless quantities:

$$P \equiv f/f_\infty, \quad V \equiv \frac{\alpha}{2}v, \quad R \equiv \frac{\alpha\eta}{2\sqrt{2}}r \quad (2.8)$$

so that the equations take the simple form

$$P'' + \frac{P'}{R} - (1 - V)^2 \frac{P}{R^2} - \beta(P - 1)(P - P_e)P = 0. \quad (2.9)$$

$$V'' - \frac{V'}{R} + (1 - V)P^2 = 0 \quad (2.10)$$

where  $\beta = \frac{8\lambda}{\alpha^2}$  and

$$P_e = -1 + \frac{2ET}{ET + \sqrt{T^2 \left( E^2 - \frac{4\beta\alpha^2}{9} D \right) + \frac{\beta^2\alpha^4}{18} \eta^2}} \quad (2.11)$$

The parameter  $P_e$  carries all of the information about finite temperature effects in the rescaled potential. It takes on values between  $-1$  at  $T = 0$  up to  $0.5$ , above which  $P = 0$  becomes the true vacuum. For  $0 < P_e < 0.5$ ,  $P = 0$  is a local minimum, separated from the global minimum at  $P = 1$  by a potential barrier. This assumes the phase transition to be first order; in models with a second order transition  $P_e \leq 0$ .

Eqns. (2.9-2.10) can be solved using standard methods. The string configurations that result, even in the extreme  $P_e = 0.5$  case, are not qualitatively different than  $T = 0$  strings. In particular, both  $P(r)$  and  $V(r)$  remain monotonically increasing functions of  $r$ .

The stability of electroweak strings at finite temperature can be determined in a similar manner to that for zero temperature strings as described in Ref. 7. Here we will give a short review of the procedure, referring the reader to Ref. 7 for details.

The energy functional for two-dimensional static solution in the electroweak model may be written in the standard notation of Ref. 11:

$$E = \int d^2x \left[ \frac{1}{4} G_{ij}^a G_{ij}^a + \frac{1}{4} F_{Bij} F_{Bij} + (D_j \phi)^\dagger (D_j \phi) + V_T(\phi) \right] \quad (2.12)$$

where,  $i, j = 1, 2$  and  $a = 1, 2, 3$ . The string solution that extremizes this energy functional is given in (2.4) and we now perturb around that solution.

It can be shown that the only relevant perturbations are those in which the upper component,  $\phi_1$ , of the Higgs doublet and the  $W$  fields are perturbed. These fields can be expanded in modes:

$$\phi_1 = \chi^m(r) e^{im\theta} \quad (2.13)$$

for the  $m^{\text{th}}$  mode where  $m$  is any integer. For the  $n^{\text{th}}$  mode of the gauge fields we have,

$$\vec{W}^1 = \left[ \{ \bar{f}_1^n(r) \cos(n\theta) + f_1^n \sin(n\theta) \} \hat{e}_r + \frac{1}{r} \{ -\bar{h}_1^n \sin(n\theta) + h_1^n \cos(n\theta) \} \hat{e}_\theta \right] \quad (2.14)$$

$$\vec{W}^2 = \left[ \{ -\bar{f}_2^n(r) \sin(n\theta) + f_2^n \cos(n\theta) \} \hat{e}_r + \frac{1}{r} \{ \bar{h}_2^n \cos(n\theta) + h_2^n \sin(n\theta) \} \hat{e}_\theta \right] \quad (2.15)$$

Of these, the  $m = 0, n = 1$  mode is the most dangerous and it is sufficient to look at this mode alone. Further, it turns out that the barred variables separate from the unbarred ones and it is sufficient to look at the problem in the unbarred variables alone.

Define the quantities

$$F_\pm = \frac{f_2 \pm f_1}{2} \quad (2.16)$$

$$\xi_\pm = \frac{h_2 \pm h_1}{2} \quad (2.17)$$

$$\zeta = (1 - \gamma v)\chi + \frac{1}{2}gf\xi_+ \quad (2.18)$$

Then, after substituting the mode expansions in the energy functional and a lot of algebra, we find that the energy variation around the unperturbed solution is,

$$\delta E = 2\pi \int dr r \left[ \left\{ \frac{\zeta'^2}{P_+} + U(r)\zeta^2 \right\} + \right. \\ \left. \text{sum of whole squares} \right] \quad (2.19)$$

where primes denote differentiation with respect to  $r$ ,

$$P_+ = (1 - \gamma v)^2 + \frac{1}{2}g^2r^2f^2, \quad (2.20)$$

$$U(r) = \frac{f'^2}{P_+f^2} + \frac{2S_+}{g^2r^2f^2} + \frac{1}{r} \frac{d}{dr} \left( \frac{rf'}{P_+f} \right), \quad (2.21)$$

and,

$$S_+ = \frac{g^2f^2}{2} - \frac{\gamma^2v'^2}{P_+} + \gamma r \frac{d}{dr} \left[ \frac{v'(1 - \gamma v)}{r P_+} \right]. \quad (2.22)$$

The energy variation in (2.19) is minimized if the sum of squares is chosen to vanish. This fixes the modes  $F_\pm$  and also requires  $\xi_- = 0$ . Then we are simply left with a problem in

$\zeta$ . (Note that although the explicit form of the Higgs potential does not appear anywhere in the  $\zeta$  dependent terms, it does appear implicitly in the unperturbed solution given by  $P$  and  $V$ .)

By performing an integration by parts, we can now rewrite the  $\zeta$  part of  $\delta E$  as,

$$\delta E[\zeta] = 2\pi \int dr r \zeta \mathbf{O} \zeta \quad (2.23)$$

where,  $\mathbf{O}$  is the differential operator:

$$\mathbf{O} = -\frac{1}{r} \frac{d}{dr} \left( \frac{r}{P_+} \frac{d}{dr} \right) + U(r) . \quad (2.24)$$

Now we have to determine if the differential equation

$$\mathbf{O} \zeta = \omega \zeta \quad (2.25)$$

has any negative eigenvalues ( $\omega$ ). The boundary conditions on  $\zeta$  are:  $\zeta(r = 0) = 1$  and  $\zeta(r = \infty) = 0$ .

In this way, the whole stability analysis has been reduced to the single differential equation (2.25). This equation can be solved numerically using the shooting method.

For a given  $P_e$  and  $\beta$ , one can use (2.25) to find the value of  $\theta_W$  for which there exists a zero eigenmode. This determines the critical value of  $\theta_W$  which marks the boundary between stability and instability for the strings. In Fig. 1 we plot a number of curves showing the regions of stability for different values of  $P_e$ . In Fig. 2, we plot regions of stability for different temperatures, assuming fixed values of  $D/\alpha^2$  and  $E/\alpha^2$ .

These results show a number of significant features. First, we see that the region of stability grows significantly as the temperature increases, including regions where  $\beta > 1$ . Thus finite temperature effects have a stabilizing effect on the strings. This allows one to consider scenarios where strings only exist during a specific epoch in the evolution of the universe. Stable strings can form in a phase transition, but then become unstable and

disappear as the temperature drops below some critical value. It is important to note, however, that even at finite temperature the standard model value of  $\sin^2(\theta_W) = 0.23$  and  $m_H > 57\text{GeV}$  is still deep within the region of instability.

Another feature of the plot is that there appears to be a lower bound to the  $\sin^2(\theta_W)$  at which there are stable strings. The stability region for small  $\beta$  is unknown, due to the fact that in this limit the strings become very thick and are difficult to treat numerically. For the  $T = 0$  case it was unclear whether the region of stability extended down to small  $\sin^2(\theta_W)$  at small  $\beta$ . For the finite temperature case, all of the stability curves corresponding to different  $P_e$  should converge at  $\beta = 0$ . This is because the potential becomes unimportant in the lagrangian in this limit. From the figure it seems likely that the critical value of  $\sin^2(\theta_W)$  at  $\beta = 0$  is around 0.92. The absolute lower bound for stable strings is thus given by the  $P_e = 0.5$  curve, and is  $\simeq 0.91$ .

### 3. Metastable Strings in Left-Right Models

As we have seen in Sec. 2, the Z-strings of the electroweak theory are unstable to small perturbations for physically reasonable values of the Higgs mass and the Weinberg angle, even when quantum and finite temperature corrections are taken into account. This leads us to wonder if there are *any* well motivated particle physics models that admit (meta) stable, embedded strings.

A hint as to how to go about finding such a model comes from the analysis in Sec.2. There we were essentially trying to stabilize the string through modification of the Higgs potential. Our failure to obtain stable strings was due in some part to the small value of  $\sin^2(\theta_W)$  required by the standard model. Thus, we are motivated to look for extensions of the standard model where the gauge sector of the theory is enlarged, allowing for the presence of other Weinberg-type angles which can take somewhat larger values.

A well-known extension of the gauge sector of the standard model is the left-right model based on the gauge group  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ <sup>8</sup>. One of the interesting features of this class of theories is that they can be compatible with known experimental results even if the scale of  $SU(2)_R \times U(1)_{B-L}$  breaking to  $U(1)_Y$  is quite low i.e. 500 GeV - 1 TeV.<sup>12</sup>

The field content of the model we consider is as follows (the quantum numbers are the  $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$  representation assignments). The left handed quarks and leptons transform as  $(2, 1; 1/3)$ ,  $(2, 1; -1)$  respectively while their right-handed companions transform as  $(1, 2; 1/3)$ ,  $(1, 2; -1)$  (note that we have added a right handed neutrino state). The minimal Higgs content required for the phenomenological viability of the model is:  $\phi \sim (1, 2; -1)$ ,  $\chi \sim (1, 3; 2)$ ,  $\Delta \sim (2, 2; 0)$ . The right handed triplet  $\chi$  is required to give the right handed neutrino a large Majorana mass so as to implement the see-saw mechanism<sup>13</sup>, while  $\phi$  is needed to yield the correct pattern of symmetry breaking and  $\Delta$  induces the Dirac masses of all other fermions. The phenomenology of this model was considered in Ref.12. We assume that the vacuum expectation values (VEV's)  $f_\phi$ ,  $f_\chi$ ,  $v$  of  $\phi$ ,  $\chi$  and  $\Delta$  respectively satisfy the following hierarchy:  $v \ll f_\chi \ll f_\phi$ .

From the viewpoint of constructing models with embedded metastable strings, the advantages of the left-right model described above are clear. We can just take the Z-string found in the electroweak model and embed it in the *right handed* sector. To the extent that  $v$ ,  $f_\chi$  are much smaller than  $f_\phi$ , we can neglect the backreaction of  $\Delta$  and  $\chi$  on the string configuration described by  $\phi$  and the right-handed neutral gauge boson  $Z_R$  (these effects are proportional to  $f_\chi/f_\phi$ ). Thus the stability analysis of Sec.2 goes through without any changes except for the replacement  $(g, g') \rightarrow (g_R, g_{B-L})$ . This implies that there is a non-trivial region of the parameter space for which our model will admit metastable strings.

While the strings of the left-right model are stable to small perturbations, as it stands, it would appear that they are unstable to perturbations along the string. In other words, they are unstable to contraction (which leads to the annihilation of the monopole-antimonopole

pair at the ends of the string). Since we expect that the contraction time scale will be at most a Hubble time, if these strings are to be of any cosmological significance, we must find a way to make them more stable against this mode of instability. One possibility is that if the strings are *superconducting*, a standing wave of charge carriers can be set up along the string, which reflects off the monopoles at either end. While the reflection coefficient is not unity, we can imagine that it is large enough so that it will take some time before the string can rid itself of enough current so as to allow for it to contract away. This mechanism for preventing the dynamical collapse of string *loops* has been studied in some detail in earlier work and it has been shown that there is a region of parameter space where current carrying loops can form static rings, or, vortons. As we will see below, it is these loops that will be most relevant from a cosmological perspective.

To show that the string is superconducting we start by displaying the Higgs potential for the coupled  $\phi - \chi$  system (note that this system is remarkably similar to the triplet majoron model<sup>14,15</sup>):

$$\begin{aligned}
V(\phi, \chi) = & \lambda_1(\phi^\dagger\phi - f_\phi^2/2)^2 + \lambda_2(\text{tr}(\chi^\dagger\chi) - f_\chi^2/2)^2 + \lambda_3(\phi^\dagger\phi - f_\phi^2/2)(\text{tr}(\chi^\dagger\chi) - f_\chi^2/2) \\
& + \lambda_4(\phi^\dagger\phi\text{tr}(\chi^\dagger\chi) - \phi^\dagger\chi\chi^\dagger\phi) + \lambda_5((\text{tr}(\chi^\dagger\chi))^2 - \text{tr}(\chi^\dagger\chi\chi^\dagger\chi))
\end{aligned}
\tag{3.1}$$

We parametrize  $\phi$  and  $\chi$  in the following fashion:

$$\phi = \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix}, \tag{3.2}$$

$$\chi = \begin{pmatrix} \chi^+/\sqrt{2} & \chi^{++} \\ \chi^0 & -\chi^+/\sqrt{2} \end{pmatrix}, \tag{3.3}$$

where the factors of  $\sqrt{2}$  in  $\chi$  have been chosen so that if  $D_\mu\chi$  is the  $\chi$  covariant derivative, then  $\text{tr}((D_\mu\chi)^\dagger D^\mu\chi)$  is the correctly normalized  $\chi$  kinetic term. Note that  $\chi$  can also be written as  $\chi = \sqrt{2}(T_3\chi^+ + T_+\chi^{++} + T_-\chi^0)$ , where  $\{T_+, T_-, T_3\}$  are the generators of  $SU(2)_R$ , satisfying;  $[T_3, T_\pm] = \pm T_\pm$ ,  $[T_+, T_-] = T_3$ . The action of  $SU(2)_R$  on  $\chi$  is via commutator:  $T^a W_\mu^a \cdot \chi \equiv [T^a, \chi] W_\mu^a$

If  $\lambda_{1,2,4,5} > 0$  and  $|\lambda_3| < 2\sqrt{\lambda_1\lambda_2}$ , then  $V(\phi, \chi)$  is positive definite and  $\phi, \chi$  acquire the following VEV's:  $\langle \phi^0 \rangle = f_\phi/\sqrt{2}$ ,  $\langle \chi^0 \rangle = f_\chi/\sqrt{2}$ .

We now claim that given this potential, there are large regions of the  $\lambda_{1,2,3,4,5}$  parameter space for which  $\chi^{+,++}$  act as bosonic charge carriers on the  $\phi$  string. To show this, we use the following argument, first given by Witten<sup>16</sup>. First we show that it can be energetically favorable for the components of  $\chi$  to be nonzero in the core of the string where  $\phi = 0$ . If  $\phi = 0$ , the potential for  $\chi$  reads:

$$V(\phi = 0, \chi) = -\lambda_1 f_\phi^2/2 + \lambda_2 (\text{tr}(\chi^\dagger \chi) - f_\chi^2/2)^2 - \lambda_3 (f_\phi^2/2)(\text{tr}(\chi^\dagger \chi) - f_\chi^2/2) + \lambda_5 ((\text{tr}(\chi^\dagger \chi))^2 - \text{tr}(\chi^\dagger \chi \chi^\dagger \chi)) \quad (3.4).$$

This is extremized if (i)  $\chi = 0$ , or (ii) either  $|\chi^0|^2 = |\chi^{++}|^2$  or  $2\lambda_2(\text{tr}(\chi^\dagger \chi) - f_\chi^2/2) = \lambda_2 f_\phi^2/2$ . It is easy to see that if  $\lambda_2 f_\chi^2 + \lambda_3 f_\phi^2/2 > 0$ , then  $\chi = 0$  is a *maximum* of the potential. Thus, in this case, non-zero values of  $\chi$  are energetically preferred in the string core.

The above analysis is not sufficient to show the existence of bosonic charge carriers. We must check to see that the kinetic term for  $\chi$  also allows for a nonzero value of  $\chi$  in the string. We do this by showing that the equations of motion for  $\chi$ , linearized around  $\chi = 0$ , admit growing solutions. This will then show that in the background of the  $\phi$  string,  $\chi$  is unstable to the formation of a nonzero condensate on the string. Let us first consider the  $\chi^{++}$  equation of motion:

$$-\partial_\mu (\partial^\mu \chi^{++} - i\alpha_R \cos 2\theta_R Z_R^\mu \chi^{++}) = 2\lambda_2 (\text{tr}(\chi^\dagger \chi) - f_\chi^2/2) \chi^{++} + \lambda_3 (f_{NO}(\rho)^2 - f_\phi^2/2) \chi^{++} + \lambda_5 (\chi^{+2} + 2\chi^0 \chi^{++}) \chi^{0*}. \quad (3.5)$$

Here  $\alpha_R \equiv \sqrt{g_R^2 + g_{B-L}^2}$ ,  $\theta_R$  is the right-handed version of the Weinberg angle and  $f_{NO}(\rho)$  is the  $\phi$  part of the string configuration. In the string,  $Z_R$  takes the form  $Z_R(\rho) = -(v(\rho)/\rho) \hat{e}_\theta$ , where  $v(\rho)$  is the Nielsen-Olesen configuration for the vector field. We now linearize eqn.(3.5) around  $\chi = 0$  and take the following form for the perturbation of  $\chi^{++}$ :

$\delta\chi^{++} = \exp(-i\omega_{++}t)g_{++}(\rho)$ . The linearized equation of motion for  $g_{++}(\rho)$  reads:

$$-\nabla^2 g_{++}(\rho) + \mathcal{V}(\rho)g_{++}(\rho) = \omega_{++}^2 g_{++}(\rho) \quad (3.6),$$

where  $\mathcal{V}(\rho)$  is given by:

$$\mathcal{V}(\rho) = -\lambda_2 f_\chi^2 + \lambda_3 (f_{\text{NO}}(\rho))^2 - f_\phi^2/2 \quad (3.7),$$

and  $\nabla^2$  is the two dimensional Laplacian. We see that at  $\rho = 0$ ,  $\mathcal{V} = -(\lambda_2 f_\chi^2 + \lambda_3 f_\phi^2/2)$  and that  $\mathcal{V}$  increases monotonically with  $\rho$  until it reaches the asymptotic value of  $-\lambda_2 f_\chi^2$ . Thus, as long as  $\lambda_2 f_\chi^2 + \lambda_3 f_\phi^2/2 > 0$ ,  $\mathcal{V}$  is negative definite and, as in Witten's original analysis<sup>16</sup>, the two dimensional Schroedinger equation above for  $g_{++}$  will admit at least one bound state with  $\omega_{++}^2 < 0$ . Thus,  $\chi^{++}$  is unstable to forming a condensate on the string. A similar analysis can be repeated for the other components of  $\chi$ , with the result that under certain conditions, they too can condense onto the string (except for  $\chi^0$ , since it has a nonzero expectation value away from the string).

#### 4. Cosmological Speculations

Here we speculate as to the possible cosmological implications of embedded strings. There are many uncertainties in outlining any cosmological scenario involving these strings because of the model dependence of many of their characteristics. Here we will content ourselves with outlining one of several possible scenarios in which embedded strings might be cosmologically relevant. We will also try to compare and contrast the formation and evolution of embedded strings with that of standard topological strings.

Consider the production of strings in a phase transition in the early universe. At temperatures above the phase transition temperature, the thermal fluctuations in the fields will spontaneously produce string-like configurations which will, however, decay just as fast. As the temperature decreases and the universe goes through the phase transition, some of the

string-like configurations that were undergoing thermal fluctuations will freeze out and thus not decay. It is these string configurations that may survive the phase transition and be important for cosmology. This process of thermal production is the same for topological as well as embedded strings.

There is, however, an important difference between topological and embedded strings. This is that topological strings cannot end whereas embedded strings may end on monopoles. Hence, after the phase transition, topological strings can only occur as closed loops or infinite strings, while embedded strings can also occur as finite segments of strings with monopoles attached at their ends. This is the crucial difference between the two kinds of strings.

We now discuss the formation of embedded strings. The first question is: what is the size distribution of the embedded strings after the phase transition? This question cannot be answered with any certainty but some reasonable guesses can be made. As we discussed above, the production of the strings is thermal and is similar in some ways to the production of topological strings; hence, it is prudent to first look at topological strings. In this case<sup>17</sup>, the string network upon formation consists of a network of infinite strings that contain about 80% of the entire string length. The remaining 20% goes into a scale invariant distribution of closed loops. These results were obtained by using an argument first given by Kibble<sup>18</sup>. In this argument, if the boundary conditions on a spatial contour are fixed, they determine whether there is a string passing through the contour almost unambiguously.<sup>19</sup> So to detect the presence of a string, all one needs to check are the boundary conditions.

This “Kibble” mechanism does not apply in the case of embedded strings because the boundary conditions are not sufficient to determine the presence of a string. However, one might assume that there is a certain probability of a string passing through any given contour. On this basis, one could attempt to use the results of Ref. 20. In the case that the probability of string formation is sufficiently low, there is a population of loops whose length

distribution is given by,

$$dn(l) = a \frac{e^{-bl/\xi}}{\xi^2 l^2} dl . \quad (4.1)$$

The dimensionless parameters  $a$  and  $b$  will depend on the probability of string formation while  $\xi$  is the correlation length at the phase transition which we will assume is given roughly by  $T_c^{-1}$  where  $T_c$  is the temperature at which the phase transition occurs. If the phase transition is second order, the correlation length can actually exceed  $T_c^{-1}$  by orders of magnitude, leading to a small value of  $b$  in the above equation. Note that the exponent of  $l$  in the denominator is 2 and not 5/2 as might be expected from a scale invariant distribution. When the string formation probability is low, the number density of open strings will be similar to that in (4.1) but the overall amplitude will be suppressed by a factor  $\exp(-(m - \mu m^{-1})/T_c)$  where,  $m$  is the mass of the monopole necessary to terminate a string and  $\mu$  is the mass density of the string. (The exponent is derived by the following considerations: the energy cost in terminating a string is the mass of the monopole  $m$  but were the string not to terminate, the energy cost would have been the string density  $\mu$  multiplied by the size of the monopole  $\approx m^{-1}$ . ) If the mass of the monopole is large - that is, if the strings are stabilized by a large potential barrier - the open segments of string are negligible in number as compared to the closed loops. It should also be remarked that the exponential suppression of long loops (and open segments) may be viewed as a Boltzmann factor in the thermal production of embedded strings.

If the probability of string formation is large, the loop distribution will be given by a scale-invariant distribution

$$dn(l) = \alpha \frac{e^{-\beta l/\xi}}{\xi^{3/2} l^{5/2}} dl . \quad (4.2)$$

If strings cannot terminate, a network of infinite strings would also be present. However, since embedded strings can terminate, the length that would have been in infinite strings would now be in finite segments of string. The length distribution of the finite segments would also be exponential since, at every step, there is a certain probability for the string to

end. But, for large string formation probabilities, the total length in open segments would exceed the total length in loops. In the limit that the monopole becomes infinitely heavy, the open segments would be infinitely long and the fraction of length in open strings would approach 80%.

There is another important feature of the string network that we have ignored so far: the strings are superconducting. Then, during the phase transition, random currents will be induced on the strings. The net current on a loop of size  $l$  is expected to be proportional to  $T_c^2(l/\xi_i)^{1/2}$  where  $\xi_i \sim T_c^{-1}$  is the correlation length of the random currents. There will be currents on the open strings also. However, it is not clear what happens to the current when it encounters the monopole at the end of the string. We expect that the current could be reflected off the monopole and, in this way, a standing wave would be set up on the open string. (In addition to the reflection, there might be a small transmission amplitude and the current would slowly leak out from the string.) Another way of saying this is that the zero modes are a solution to the Dirac equation (or the Klein-Gordon equation for bosonic superconductivity) in the presence of the string. The string provides a potential well for the zero mode carriers. In the case of an open string, one might envisage the presence of standing wave solutions while in the case of a loop, one can imagine traveling waves going around the loop in addition to the standing waves.

In the following we will assume that, after the phase transition, there is a loop distribution given by (4.1) and a strongly suppressed open string distribution also given by the form in (4.1). In addition, all the strings carry currents in proportion to the square root of their length.

What is the evolution of this system?

Let us first consider the loops. The dynamics of the loop is governed by the tension of the string, frictional forces and the Hubble expansion. We are justified in ignoring the Hubble expansion since it is unimportant on scales much smaller than the horizon. (The

embedded string distribution is exponentially suppressed at long lengths and so it is unlikely to find strings that span the horizon. Therefore, the Hubble expansion has no significant effect on the dynamics of embedded strings). Initially the frictional forces are very large and so the string motion is highly damped. This would lead to a collapse of the loops under their own tension. However, the effective tension of the string is a sum of the bare tension and the square of the current on the string<sup>21,22</sup>. As the loop collapses, the current builds up and the effective tension becomes smaller. Now two possibilities can occur<sup>21,22,23,24</sup>: (i) the current is so large that the charge carriers can leave the string - that is, the current can saturate, and, (ii) the effective tension goes to zero and the loop does not collapse any further. If possibility (i) is realized, the loops continue to collapse and eventually disappear. Depending on the lifetime of the loops and their decay products, their cosmology may be of some interest. If possibility (ii) is realized, the loops form static ring configurations that can survive until some quantum tunneling event causes the charge to leak. In this case, the rings would have a magnetic dipole moment and perhaps some net electric charge and could survive for a very long time. Depending on the net charge that a ring carries, the rather severe constraints on charged dark matter (CHAMPS) would apply<sup>25</sup>.

The evolution of the open segments of strings is even less certain than that of the loops but we shall indicate some possible scenarios. The initial dynamics of the monopoles and open segments will be heavily damped due to the friction from the ambient plasma. The long range magnetic field of the monopoles will be frozen into the cosmological plasma. The tension in the open strings will shrink the segments, bringing the monopoles and antimonopoles at the ends together. One possibility is that the current and the charge in the segment would prevent the segment from shrinking any further as happens in the case of the loop. Then the segment would form a dumbbell and survive for a very long time. On the other hand, if the current leaks out through the monopoles, the segment would collapse rather quickly since the frictional forces cannot slow down the longitudinal motion of the string but only

the transverse motion. In this scenario, the open segments decay soon after forming and disappear. The disappearance of open segments (and loops) would also be hastened by the breaking up of long strings by the spontaneous nucleation of monopole and antimonopole pairs. However, we might assume that this process will be slow (compared to the direct collapse of a segment) since it requires a monopole pair to nucleate by a quantum process.

There is yet another alternative to this entire scenario which follows from the stability analysis in Sec. 2. From Fig. 2, we see that it is possible for the strings to be stable at high temperatures and unstable at low temperatures. Then the string network - the rings and dumbbells - would behave like unstable particles with a life-time given by the time it takes for the universe to cool down to the temperature of instability. Unstable particles have been considered on numerous occasions in cosmology, particularly as a means for generating additional entropy. It is amusing that embedded strings would be natural candidates for such unstable particles.

What may be the consequences of long-lived rings and dumbbells? The most obvious consequence is that these objects may be the dark matter of the universe and may still be around today. They may be lurking in stars and in galactic halos. On the other hand, since these objects are formed in the early universe, there is a chance that they will come to dominate the universe rather early (since they redshift as matter). In this case, their cosmology might be useful to constrain particle physics models - though, given the uncertainties, this promises to be a difficult task. Finally, the decay of the rings and dumbbells would produce energetic exotic particles. These decay products might lead to interesting effects. Finally, the presence of strings for some period of time could lead to baryogenesis<sup>26</sup>.

## 5. Conclusions

Topological strings can have dramatic consequences in the early universe but can occur only in certain specially constructed particle physics models. On the other hand, embedded strings are almost universal in their occurrence but their consequences depend on their stability. For the embedded string to have some affect on cosmology, it should survive for one Hubble time at the very least. This criterion makes it necessary to study the stability of the electroweak Z-string at high temperatures.

We have analyzed the stability of the Z-string at high temperatures and also taken quantum corrections to the scalar potential into account. The analysis shows that thermal corrections tend to enhance stability but the effect is too small to stabilize the Z-string in the standard electroweak model with  $\sin^2 \theta_W \approx 0.23$  and Higgs mass larger than 57 GeV. Hence, we come to the conclusion that the Z-string is unstable at all temperatures. Then, even if a Z-string configuration is formed during the electroweak phase transition, it will quickly decay into particles and the string will not survive for more than a Hubble time. This means that Z-strings are probably irrelevant for cosmology after the electroweak phase transition; their role during the electroweak phase transition is still unclear.

We found that it is possible to construct phenomenologically acceptable left-right models that also admit *stable* embedded strings ( $Z_R$ -strings). Due to its stability the  $Z_R$ -string may survive for a large number of Hubble times and may be cosmologically significant.

The cosmology of embedded strings was discussed in Sec. 4. Here, we pointed out that embedded strings would be produced thermally during the phase transition. The Boltzmann suppression of long string segments and large loops means that there is a possibility that all the loops and segments will collapse dynamically and decay into ordinary particles. On the other hand, the pressure from bosonic and fermionic zero modes on the strings might prevent this collapse and serve to stabilize the “rings” and “dumb-bells”. We considered the

more interesting possibility that some of these remnants may have survived for a few Hubble times and perhaps even until the present epoch.

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## Figure Captions

1. Regions of stability for various values of  $P_e$ . Plotted are the boundaries between stability and instability for (from left to right)  $P_e = 0.5, 0.4, 0.25, 0, -0.5, -1.0$ . Strings are (meta)stable for parameters to the right of the curves.
2. Regions of stability for various temperatures for fixed  $D/\alpha^2 = 0.224$  and  $E/\alpha^2 = 0.019$ . Strings are stable in regions below the solid lines and to the right of the dashed line. The dashed line corresponds to the boundary of stability at the phase transition.

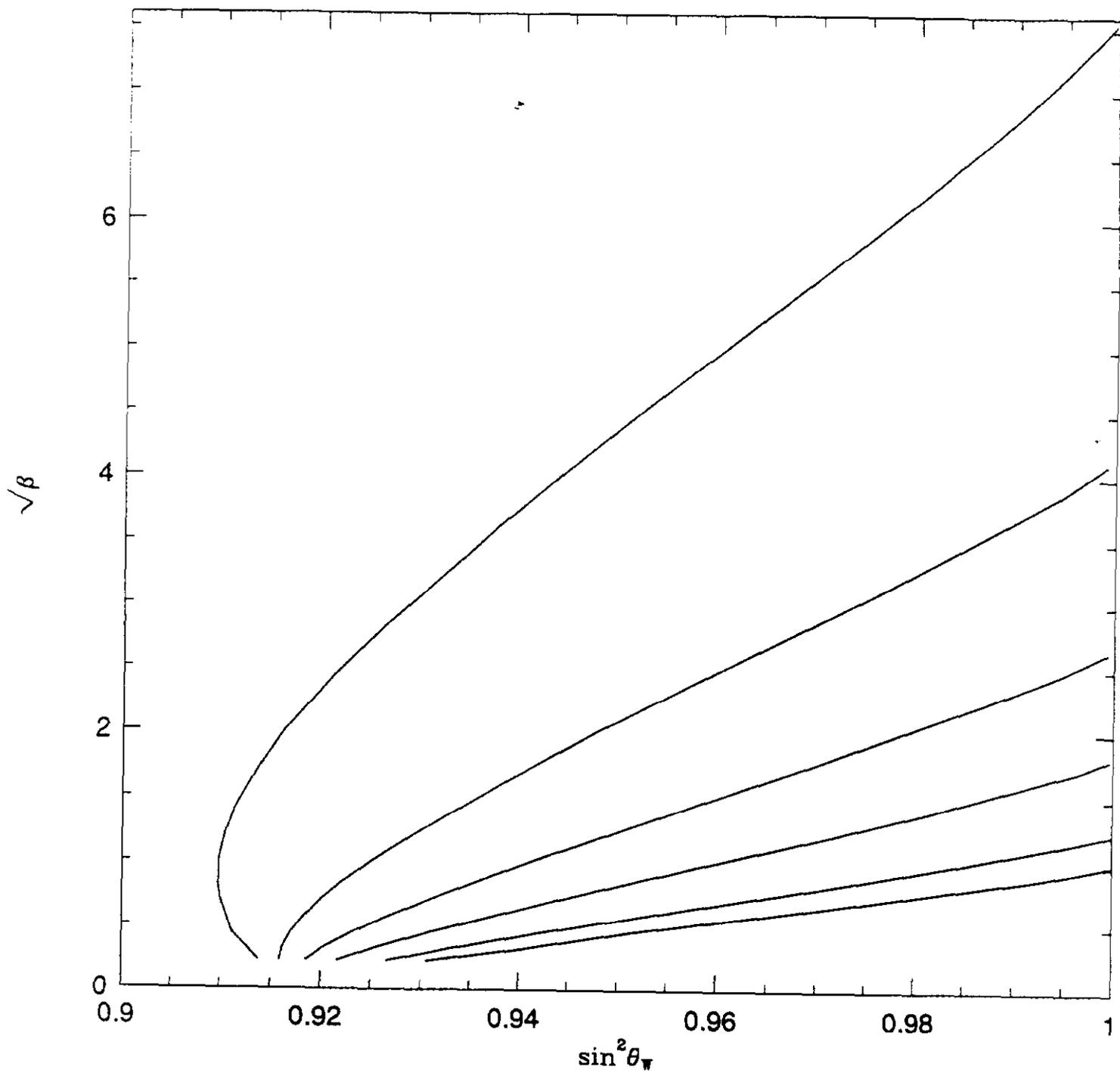


FIG. 1

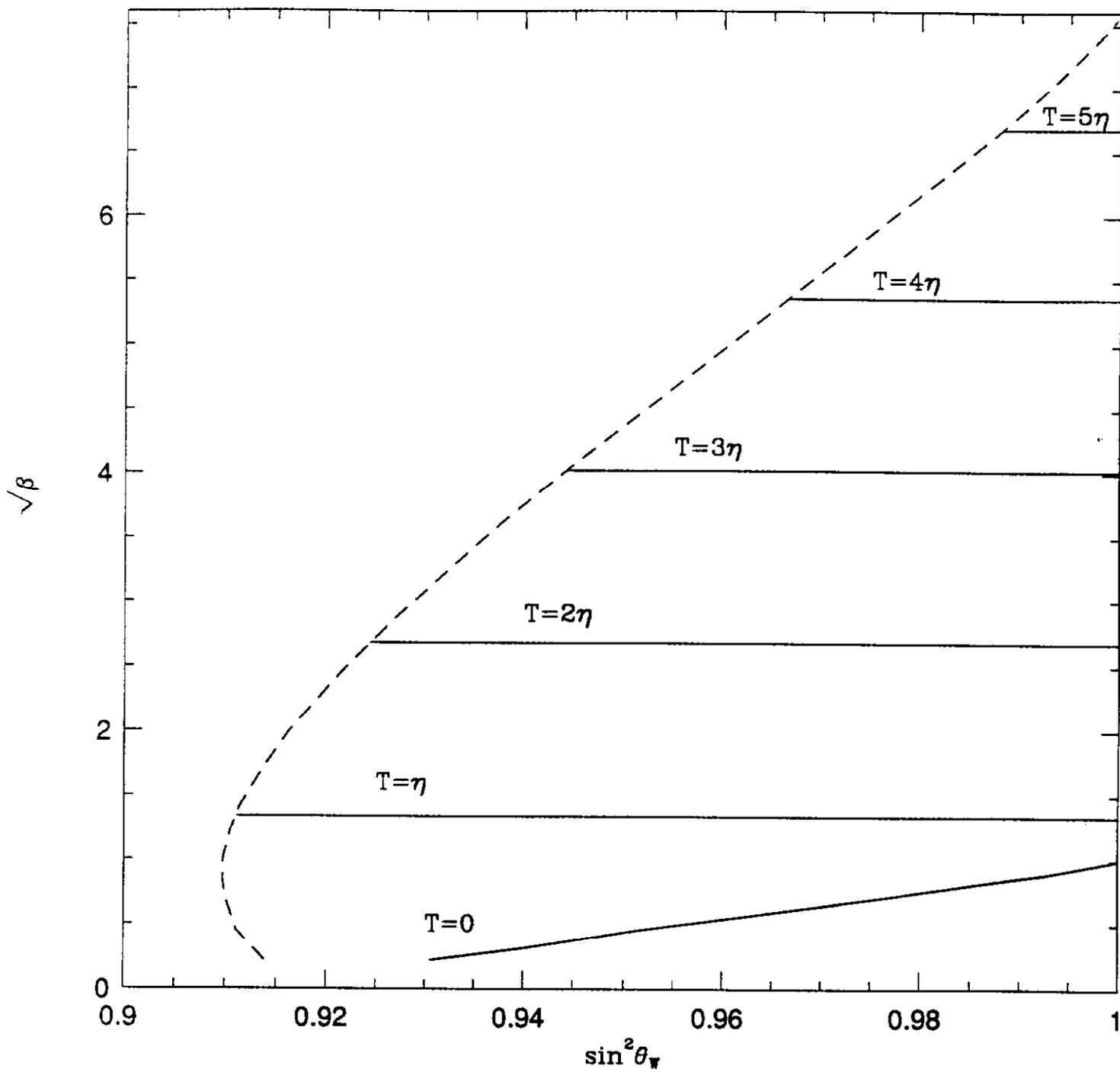


FIG. 2