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MBR ANISOTROPY FROM SCALAR FIELD GRADIENTS[1]

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ABSTRACT. Analytic calculations of the cosmological density fluctuations and MBR anisotropies induced by gradients in a topologically trivial scalar field are presented. This analytic solution should provide a good test for numerical simulations of microwave anisotropy from scalar fields. To the extent that these results generalize to other scalar field models and configurations, they imply 1) MBR measurements limit large scale primordial variations greater than about 5×10^{16}GeVwithin our horizon, 2) The total scalar field variation seems to be a good predictor for of the magnitude of the MBR anisotropy produced in an oscillation, 3) Scalar fields as well as other models of seeded produce a few times more $\Delta T/T$ for a given $\delta\rho/\rho$ (on the the same scale) than do primordial adiabatic perturbations, 4) Models of scalar field seeds which produce a scale-invariant spectrum of perturbations seem to require galaxies to be more clustered than the mass on small scales, 5) Scalar fields do not "tilt" the universe.

I. INTRODUCTION

There have been several recent suggestions that the observed cosmological inhomogeneities could have been seeded by the dynamics of a massless or ultralight scalar field. Proposed models include the breaking of a global continuous symmetry in the early universe, leading to Kibble gradients (Kibble 1976) in the remnant Goldstone boson. Large scale gradients induced by quantum fluctuations during inflation may also lead to structure. Such models include various $O(N)$ (non)linear σ-models [$N = 2$: global strings (Vilenkin and Everett 1982); $N = 3$: global monopoles (Barriola and Vilenkin 1989, Bennett and Rhie 1990); $N = 4$: cosmic textures (Turok 1989); large-N (Turok 1989)]. Even fluctuations in a topologically trivial scalar field may produce inhomogeneities (§II). Such theories are acceptable only if they are able to reproduce the observed density and velocity inhomogeneities while also respecting the observations of MBR anisotropies. The recent detection by the satellite COBE of large angular scale anisotropy (Smoot *et al.* 1992,

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hereafter COBE92-04) in conjunction with observations of galaxy clustering and peculiar velocities will provide a stringent test for these theories.

All of the models mentioned above are similar to inflationary models in that they produce an approximately "scale-invariant" Harrison-Zel'dovich spectrum of perturbations, although the temperature patterns may be significantly non-Gaussian. Scale-invariance implies that significant MBR anisotropies should exist at angular scales ($\gtrsim 10°$) where they cannot be erased by reionization of the universe, i.e. just the scales observed by COBE. Thus the anisotropy produced in these models should be measurable even if the universe is reionized at a high redshift.

While several calculations exist of the density perturbations produced, especially in the $N = 4$ texture model (*e.g.* Gooding, Spergel, and Turok 1991; Park, Spergel, and Turok 1991; Cen *et al.* 1991), calculations of MBR anisotropy are only now underway (D. Bennett private communication, D. Spergel private communication). Turok and Spergel (1990) have calculated the temperature boost/decrement given to photons passing by a collapsing texture ($N = 4$) in Minkowski space. Such a collapse is a rather dramatic oscillation of a scalar field, involving energy concentrated towards a point in space, and it is also very rare. Turok (1989) and Turok *et al.* (1991) showed that collapsing knots of global texture occur in only 4% of horizon volumes. In regions where texture collapse does not occur there will still be Kibble gradients of similar magnitude, but these will be smoothed out by a gentler, less dramatic type of oscillation.

In this paper we consider MBR anisotropy produced by the oscillations of a sine-wave configuration of a massless scalar field, which has a topologically trivial "flat" vacuum manifold. The calculation is done analytically in an expanding universe. The anisotropies in this simple oscillation should be indicative of those to be found between the "texture spots" in the $N = 4$ texture model, and may also be indicative of the magnitude of anisotropy in other models where the dynamics are not driven by the vacuum manifold geometry. While detailed numerical calculations will be necessary in order to test the nonlinear σ-models with observations, this exact calculation provides a simple framework in which to understand the various physical effects, and provides a potential check of numerical techniques as well.

We should note that it has been suggested that Planck-scale physics may not allow scalar field masses as small as is required in the models considered here (Kim and Lee 1989, Holman *et al.* 1992, Kamionkowski and March-Russell 1992). We feel that the present understanding of Planck-scale physics is not on a sufficiently firm basis to take these arguments as a constraint on cosmological theories. If, however, any of these scalar field theories were to provide good agreement with all cosmological observations then one might consider using this as an empirical constraint on Planck-scale physics.

II. METRIC FLUCTUATIONS

To estimate the effect of gradients in a scalar field we consider the simplest possible

configuration where gradients occur, i.e. a single minimally-coupled massless scalar field with a sinusoidal oscillation. We are most interested in the very large scale gradients in the scalar field, on the scale of $100 - 1000$ Mpc, which produce the large-angular scale MBR anisotropies. These scales entered the cosmological horizon during the matter-dominated era. Since gradients on scales larger than the horizon are frozen in, all of the interesting oscillations occur in the matter era. Thus it is sufficient for our purposes to consider a pure matter era cosmology. It is natural to consider a spatially flat cosmology. The metric is,

$$g_{\mu\nu} = a^2(\eta_{\mu\nu} + h_{\mu\nu}) \qquad \eta_{\mu\nu} = \mathrm{diag}[-1,1,1,1] \qquad a(\eta) \propto \eta^2, \tag{1}$$

where we use conformal time, $\eta = x^0 = \int a^{-1}dt$, comoving coordinates, $\mathbf{x}$, and gravitational units ($G = c = 1$). Recall that a scalar field, ϕ, is dimensionless in these units. Here $\eta_{\mu\nu}$ is the vacuum metric and $h_{\mu\nu}$ is the metric perturbation. We use the formalism and notation of Veeraraghavan and Stebbins (1990, hereafter VS) to calculate the linear perturbations produced by the scalar field. Thus we choose synchronous coordinates, $h_{00} = h_{0i} = 0$.

The equation of motion of the scalar field in an expanding universe is

$$\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} - \nabla^2\phi = 0 \qquad \dot{} \equiv \frac{\partial}{\partial\eta}. \tag{2}$$

At early times when the wavelength is much larger than the horizon, changes in ϕ are strongly damped by the expansion. Thus the appropriate initial condition is $\dot{\phi}(\mathbf{x},0) = 0$. Solving for the evolution of the sinusoidal ϕ-field we find

$$\phi(\mathbf{x},\eta) = \phi_0 \sin(\mathbf{q}\cdot\mathbf{x})\, f(|\mathbf{q}|\eta), \quad f(x) = 3\frac{j_1(x)}{x}, \quad f(0) = 1, \quad f'(0) = 0. \tag{3}$$

Here j_1 is a spherical Bessel function. The functional dependence of f follows from dimensional arguments. The quantity ϕ_0 determines the amplitude of the initial field gradient.

To derive the gravitational field we need the stress-energy of the ϕ-field which is given by

$$\Theta_{\mu\nu} = \phi_{,\mu}\phi_{,\nu} + \frac{1}{2}\eta_{\mu\nu}(|\dot{\phi}|^2 - |\nabla\phi|^2) \tag{4}$$

and is quadratic in ϕ. Thus the inhomogeneous part of $\Theta_{\mu\nu}$ will have a wavenumber $\vec{\kappa} = 2\mathbf{q}$. The metric perturbations which respond to the stress-energy will also have this higher spatial frequency. The relevant components of $\Theta_{\mu\nu}$ are

$$\begin{aligned} \Theta_{00} =& \frac{1}{16}\kappa^2\phi_0^2[(f^2 + f'^2) + (f^2 - f'^2)\cos\vec{\kappa}\cdot\mathbf{x}] \\ \Theta_+ \equiv& \Theta_{00} + \Theta_{ii} = \frac{1}{4}\phi_0^2 f'^2(1 - \cos\vec{\kappa}\cdot\mathbf{x}). \end{aligned} \tag{5}$$

The stress energy consists only of scalar terms in the scalar-vector-tensor decomposition of 3-dimensional tensor fields. This is a peculiarity of our sinusoidal configuration. If we had considered a configuration with two or more sine-waves whose wavenumbers were not parallel, their cross term in the stress-energy tensor would contain scalar, vector, and tensor parts.

Now we will assume that the cosmology is initially unperturbed, i.e. $h_{ij}(\mathbf{x},0)=0$, as would occur if the scalar field were produced in an initially homogeneous universe (see VS). In order to obey the the energy constraint equation (VS-3.8) we must set up an initial growing mode underdensity in the matter

$$\lim_{\eta_i\to 0}\delta_g(\mathbf{x},\eta_i) = -\lim_{\eta_i\to 0}\frac{2\pi\eta_i{}^2}{5}\Theta_{00}(\mathbf{x},\eta_i) = -\frac{\pi}{40}\phi_0^2(\kappa\eta_i)^2(1+\cos\vec{\kappa}\cdot\mathbf{x}) \tag{6}$$

which compensates the positive energy density of the ϕ-field. Since the momentum density of the ϕ-field is initially zero the momentum constraint equation is trivially satisfied by setting the matter velocity to zero. We take the matter to be effectively pressureless since, for the large scales we are interested in, even a light neutrino species would be effectively cold and pressureless. The subsequent evolution is given by (VS-5.5)

$$\delta(\mathbf{x},\eta) = -\pi\phi_0^2\left[(A(\kappa\eta)-B(\kappa\eta))+(A(\kappa\eta)+B(\kappa\eta))\cos\vec{\kappa}\cdot\mathbf{x}\right] \tag{7a}$$

where

$$A(x)=\frac{x^2}{40} \quad\text{and}\quad B(x)=\frac{x^2}{40}+18\frac{24+8x^2-x^4-4(6-x^2)\cos x-24x\sin x}{x^6}. \tag{7b}$$

The evolution of the trace of the metric perturbation $h\equiv h_{ii}$ is tied to the matter underdensity via $h=-2\delta$. As for the traceless part, $\tilde{h}_{ij}\equiv h_{ij}-\frac{1}{3}\delta_{ij}h$, it is fully determined by the energy constraint equation, (VS-3.18) since the metric is purely scalar. We finally obtain,

$$\begin{aligned} h_{ij}(\mathbf{x},\eta) = 2\pi\phi_0^2\Bigg[&\frac{1}{3}\delta_{ij}\left[(A(\kappa\eta)-B(\kappa\eta))+(A(\kappa\eta)+B(\kappa\eta)-C(\kappa\eta))\cos\vec{\kappa}\cdot\mathbf{x}\right] \\ &+\hat{\kappa}^i\hat{\kappa}^j C(\kappa\eta)\cos\vec{\kappa}\cdot\mathbf{x}\Bigg] \end{aligned} \tag{8a}$$

where

$$C(x)=\frac{3}{2}+\frac{36}{x^4}-\frac{18}{x^2}+\frac{x^2}{20}-\frac{36\cos x}{x^4}. \tag{8b}$$

Long after the wave enters the horizon the perturbation takes the form of a normal growing mode

$$\begin{aligned} h_{ij}(\mathbf{x},\eta) &= \frac{\pi}{10}\phi_0^2\left[\hat{\kappa}^i\hat{\kappa}^j((\kappa\eta)^2+30)-10\delta_{ij}\right]\cos\vec{\kappa}\cdot\mathbf{x}+\mathcal{O}\left(\frac{1}{\kappa\eta)^2}\right) \\ \delta(\mathbf{x},\eta) &= -\frac{1}{20}\pi\phi_0^2(\kappa\eta)^2\cos\vec{\kappa}\cdot\mathbf{x}+\mathcal{O}\left(\frac{1}{\kappa\eta)^2}\right) \\ \Phi(\mathbf{x},\eta) &= -4\pi\nabla^{-2}a^2\bar{\rho}\,\delta(\mathbf{x},\eta) = \frac{3}{10}\pi\phi_0^2\cos\vec{\kappa}\cdot\mathbf{x}+\mathcal{O}\left(\frac{1}{\kappa\eta)^2}\right) \end{aligned} \tag{9}$$

where Φ is the gravitational potential.

A conserved cosmological energy and momentum is given in §V of VS. To see how energy conservation works here, consider the homogeneous component of the energy. The $-(A-B)$ term which gives the homogeneous matter underdensity starts as a negative growing mode, $\sim -\frac{1}{40}(\kappa\eta)^2$ but as the ϕ-field oscillates it "transfers" all its energy to the matter via $\Theta_+ \propto 2\dot{\phi}^2$, leaving no net perturbation: $-(A-B) \to 0$. The $A+B$ term which is the inhomogeneous part of the matter underdensity starts out as a growing mode, $\sim \frac{1}{40}(\kappa\eta)^2$, and becomes an even larger growing mode $\sim \frac{1}{20}(\kappa\eta)^2$. As the ϕ-field oscillates the inhomogeneous part of the ϕ-field energy, $\propto |\nabla\phi|^2$, gets redistributed via the momentum density, $\propto -\dot{\phi}\nabla\phi$. The energy transport is from the regions where $|\nabla\phi|^2$ was initially large to regions where it was initially small. The total energy transport is actually twice that needed to smooth out the energy distribution, so the net effect is to transfer the energy away from regions of large negative compensation to regions where there is little compensation. Thus when the energy is transferred to the matter, via the $\dot{\phi}^2$ term, it adds to the initial compensation rather subtracting from it.

In the absence of a compensating underdensity, the scalar field cannot transfer its energy and the net perturbation would be halved. Contrary to naive expectations, the compensation actually increases the final inhomogeneity rather than decreasing it, in accordance with with Veeraraghavan and Stebbins (1992) who find that compensation can, in some cases, increase the observable effects of the perturbations (note however that compensation of the homogeneous part of the perturbation is crucial because without it the cosmology would be closed and not spatially flat as initially assumed). This effect of compensation increasing inhomogeneities is probably generic in scalar field models, because energy transfer, which is proportional to the square of the time derivative of the field, is likely to be largest where the initial field configuration takes its extremal values, and these are just the regions where the initial gradients and hence the initial scalar field density are small. Energy is transported away from regions of large compensating energy densities to regions where they are small, before it is transfered to the matter, increasing the net inhomogeneity.

III TEMPERATURE FLUCTUATIONS

Since we are interested in large-scale perturbations, much bigger than the horizon at last-scattering, we have ignored the initial radiation-dominated era. In the same spirit we will take the surface of last scattering to be at $\eta = 0$. For our solution the initial density inhomogeneity, outside the horizon, is zero. Thus for adiabatic initial conditions the initial temperature perturbation to the photons should also be zero, and the temperature deviation as seen by a given observer at event $(\mathbf{x}_o, \eta_o)$, in direction $\hat{\mathbf{n}}$ is given by,

$$\frac{\Delta T}{T}(\hat{\mathbf{n}}) = -\frac{1}{2}\int_0^{\eta_o} d\eta\, \hat{n}^i \hat{n}^j \dot{h}_{ij}(\mathbf{x}_\gamma(\eta), \eta) \tag{10}$$

to first order in the synchronous gauge, where we use the unperturbed trajectory $\mathbf{x}_\gamma(\eta) = \mathbf{x}_o + \hat{\mathbf{n}}(\eta_o - \eta)$. The deviation in temperature is relative to that expected at the observer *time* in an unperturbed universe. Of course, we have no *a priori* way of determining the MBR temperature at this *time*, and only differences in $\Delta T/T$ in different directions are measurable. Substituting our metric into equation (10) and integrating by parts we find

$$\frac{\Delta T}{T}(\hat{\mathbf{n}}) = -\pi\phi_0^2 \Bigg[\left[\frac{1}{3}(A-B) + C'\,\hat{\mathbf{n}}\cdot\hat{\vec{\kappa}}\,\sin\vec{\kappa}\cdot\mathbf{x}_\gamma - C''\,\cos\vec{\kappa}\cdot\mathbf{x}_\gamma\right]_{\eta=0}^{\eta=\eta_o} + \int_0^{\eta_o} \kappa d\eta\, D(\kappa\eta')\,\cos\vec{\kappa}\cdot\mathbf{x}_\gamma \Bigg] \tag{11a}$$

where

$$D(x) = C'''(x) + \frac{1}{3}(A'(x) + B'(x) - C'(x)). \tag{11b}$$

As we shall see below this expression has much the same form as the classical Sachs-Wolfe anisotropy (Sachs and Wolfe 1967). It is straightforward but tedious to perform the integration and the general result is given in Appendix A. Below we will present the results for two cases corresponding to large and smaller scale anisotropies.

Small Scale Anisotropies

Consider the anisotropy pattern when the wavelength is much smaller than the horizon at the time the photons are observed, i.e. $\kappa\eta \gg 1$, but also much larger than the horizon at last scattering. This will produce anisotropies with a small angular scale $\sim (\kappa\eta)^{-1}$. Using (11) we find

$$\begin{aligned}\frac{\Delta T}{T}(\hat{\mathbf{n}}) = -\pi\phi_0^2 \Bigg[& \frac{\kappa\eta_o}{10}\hat{\mathbf{n}}\cdot\hat{\vec{\kappa}}\,\sin\vec{\kappa}\cdot\mathbf{x}_o - \frac{1}{10}\,\cos\vec{\kappa}\cdot\mathbf{x}_o \\ & + \left(\frac{1}{10} + H_c(\hat{\mathbf{n}}\cdot\hat{\vec{\kappa}})\right)\cos\vec{\kappa}\cdot(\mathbf{x}_o + \hat{\mathbf{n}}\eta_o) \qquad \kappa\eta_o \gg 1. \\ & + H_s(\hat{\mathbf{n}}\cdot\hat{\vec{\kappa}})\,\sin\vec{\kappa}\cdot(\mathbf{x}_o + \hat{\mathbf{n}}\eta_o)\Bigg]\end{aligned} \tag{12a}$$

where

$$\begin{aligned} H_c(a) &= \int_0^\infty dx\, D(x)\,\cos ax \\ H_s(a) &= \int_0^\infty dx\, D(x)\,\sin ax. \end{aligned} \tag{12b}$$

The integral terms, H_c and H_s, are not present in the classical Sachs-Wolfe effect, but the boundary terms in Eq. (12) have an exact correspondence with the classical formulae:

$$\begin{aligned} -\pi\phi_0^2\frac{\kappa\eta_o}{10}\hat{\mathbf{n}}\cdot\hat{\vec{\kappa}}\sin\vec{\kappa}\cdot\mathbf{x}_o &= (\hat{\mathbf{n}}\cdot\mathbf{v}_o - \hat{\mathbf{n}}\cdot\mathbf{v}_{ls}), \\ -\pi\phi_0^2\left(\frac{1}{10}\cos\vec{\kappa}\cdot(\mathbf{x}_o+\hat{\mathbf{n}}\eta_o) - \frac{1}{10}\cos\vec{\kappa}\cdot\mathbf{x}_o\right) &= \frac{1}{3}(\Phi_{ls}-\Phi_o) \end{aligned} \quad . \tag{13}$$

The Doppler term on the surface of last scattering is zero, and the Doppler term due to the motion of the observer leads to a dipole pattern much larger than the rest of the anisotropies. Note that even though the ϕ-field is initially compensated the gravitational potential is not initially zero.

To calculate the rms small scale anisotropy, we drop the Doppler and Φ_o terms (which only contribute to large-scale anisotropies), and average over all observer locations $\mathbf{x}_o$ and lines of sight $\hat{\mathbf{n}}$ to get,

$$\frac{\Delta T_{\rm rms}}{T} \approx 1.07\,\phi_0^2. \tag{14}$$

We may compare this to the classical Sachs-Wolfe anisotropy which we would get for an adiabatic primordial perturbation density perturbation given by Eq (9). In the same approximation (i.e. ignoring the dipole and monopole terms at the observer), we find

$$\frac{\Delta T_{\rm rms}^{\rm classical}}{T} = \frac{1}{3}\sqrt{\langle|\Phi|^2\rangle} \approx 0.222\,\phi_0^2. \tag{15}$$

Thus, for a given δ today, the anisotropy produced by this ϕ-field configuration is on average about 5 times greater than that obtained in adiabatic models. This higher efficiency in producing MBR anisotropies for a given density perturbation may be what is required by the data. Normalizing the standard CDM model of primordial adiabatic perturbations with the anisotropy reported in COBE92-04 one finds that the small scale inhomogeneity is too large (see Gelb 1992). Small scale power may be less of a problem if the perturbations are seeded by scalar fields.

Ignoring the slow variation of H_s and H_c with angle, the pattern on the sky is sinusoidally varying. Thus the rms is not dominated by a few rare peaks, and there is no evidence of a strongly non-Gaussian pattern. On large scales ($\gtrsim 4°$), which are unlikely to be affected by reionization, measurements constrain $\Delta T/T \lesssim 1-3\times10^{-5}$ (Meyer *et al.* 1991) which implies that scalar field inhomogeneities of $\gtrsim 4\times10^{-3} \sim 5\times10^{16}$GeV with coherence scales $400\,h^{-1}$Mpc are excluded. If scalar field gradients are the source of inhomogeneities in the universe then the COBE92-04 detection implies $\Delta\phi \sim 3\times10^{-3} \sim 3\times10^{16}$GeV on the 10° or $10^3 h^{-1}$Mpc scale.

Large Scale Anisotropies

Scalar field gradients on the scale of our present horizon may lead to very large angular scale anisotropy. The canonical measure of this large angular scale anisotropy is the rms amplitude of the quadrupole anisotropy, Q, which gives the contribution to the rms $\Delta T/T$ from the quadrupole component of the temperature pattern. The value of Q will depend sinusoidally on the position of the observer, with rms,

$$Q_{\rm rms} = \frac{5}{8\sqrt{2}}\pi\phi_0^2\,|P(\kappa\eta_o)|, \qquad \begin{aligned} P(x) =& \frac{1}{5}\,E(x) + \int_0^x dy\,D(y)\,E(x-y) \\ E(x) =& 4\frac{(3-x^2)\sin x - 3x\cos x}{x^3}. \end{aligned} \tag{16}$$

A plot of $Q_{\rm rms}/(\pi\phi_0^2)$ is given in the figure. The maximal value occurs when $\kappa\eta_o \sim 6$, yielding $Q_{\rm rms} \sim 0.7\pi\phi_0^2$ while values of $Q_{\rm rms} \sim 0.03\pi\phi_0^2$ are possible even for $\kappa\eta_o$ as large as ~ 23.5. Since Q varies sinusoidally with position we again find that the rms is a good estimate of the "typical" anisotropy. Previously the upper limit on Q was 1.9×10^{-5} from the Relikt experiment (Klypin *et al.* 1987). In COBE92-04 a positive detection was announced at the level of $Q = 4.8(\pm 1.5) \times 10^{-6}$. Comparing the COBE detection and the maximal $Q_{\rm rms}$ above we find $\phi_o \lesssim 1.5 \times 10^{-3} \sim 2 \times 10^{16}$ GeV.

Finally we note that for wavelengths on the scale of the horizon or greater, the dipole anisotropy is never much greater than the quadrupole anisotropy. In fact for superhorizon perturbations the opposite is true. Thus isothermal perturbations in the form of adiabatically compensated scalar field gradients cannot lead to a "tilted universe" (Turner 1992) where the observed dipole is generated on superhorizon scales. In order to "tilt" the universe one needs to separate the photons and dark matter initially, that is, to consider non-adiabatic initial conditions.

IV. IMPLICATIONS

We have calculated analytically the MBR anisotropy generated by the oscillation of an initially compensated, simple, scalar field configuration. We find that any very large-scale gradient in a massless scalar field will produce inhomogeneous stress-energy and thus MBR anisotropies as well as density perturbations. COBE92-04 has reported anisotropies at the $\sim 1 \times 10^{-5}$ level from $10°$ up to the quadrupole. Our calculation thus indicates that horizon-sized variations in a simple scalar field of $\sim 3 \times 10^{-3} M_{\rm Planck} \approx 4 \times 10^{16}$GeV would be required to produce these anisotropies, and that larger variations are excluded by the COBE measurement.

Can we generalize this to limits on the VEV of $O(N)$ models? (By VEV we mean the radius of the spherical vacuum manifold.) We would like to stress that scalar field gradients will be present in these models whether or not topologically non-trivial effects exist in a region. The gradients are due to causality and not due to topology. The average difference between scalar field values (as measured along the geodesic of the vacuum

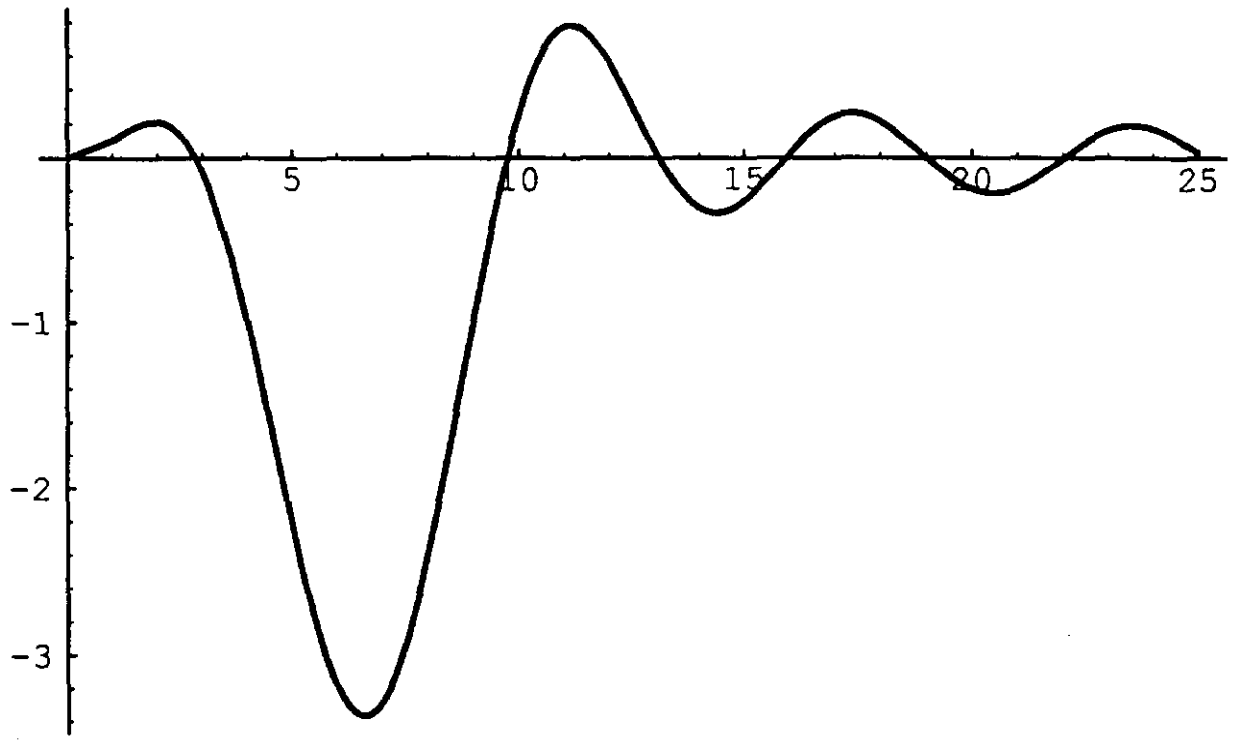

Figure. Here we plot $8\sqrt{2}\,P(\kappa\eta_o)/5$ vs. $\kappa\eta_o$. The absolute value of the ordinate gives the rms quadrupole moment in units of the $\pi\phi_0^2$, while the abscissa gives the wavenumber of the perturbation in units of the inverse horizon scale at the time of observation.

manifold) at causally disconnected points for these $O(N)$ models is $\frac{\pi}{2}\times$VEV, independent of N. Equating this variation to ϕ_0 in our sine wave model, we would expect from Eq. 14 to find rms $\Delta T/T$'s of $\sim 10.\,\mathrm{VEV}^2$. Bennett and Rhie (1992) have numerically calculated the anisotropy pattern from global monopoles ($N=3$) and cosmic textures ($N=4$) for cosmologically realistic configurations. Expanding their temperature pattern in spherical harmonics they find contribution to the mean square $\Delta T/T$ from each l to be well approximated by $\frac{2}{5}\alpha_N^2\,\mathrm{VEV}^4\,(2l+1)/(l(l+1))$ for $l>2$, where $\alpha_3\approx 8.7$ and $\alpha_4\approx 4.7$. For small angles this gives $\sqrt{\frac{4}{5}}\,\alpha_N\,\mathrm{VEV}^2$ as the rms contributed by each logarithmic interval in l, or $7.7\,\mathrm{VEV}^2$ for $N=3$ and $4.2\,\mathrm{VEV}^2$ for $N=4$, which are reasonably close to what we would expect from our exact calculation.

We may also compare to another exact calculation. For a collapsing spherical texture in Minkowski space, Turok and Spergel (1991) found fractional variations in temperature up to $8\pi^2\,\mathrm{VEV}^2$ where the initial configuration had a variation in the scalar field of π VEV. Taking a spherical average around the collapsing texture (see Appendix B, Eq B3) we find an rms $\Delta T/T$ of $3.9\,(\pi\,\mathrm{VEV})^2$ which is close to what we might guess from our exact calculation where the total variation in the field was $2\phi_0$. Note that this collapsing

knot gives a larger $\Delta T/T$ than would be expected for a typical variation of $\frac{\pi}{2}$VEV, but this can be "explained" because the variation is twice the mean and one would naively expect $\Delta T/T$ four times as large. Combining this with the fact that the peak anisotropy is roughly twice the rms, we conclude that the picture outlined in Turok and Spergel (1990) of "isolated spots on the sky" should have some validity if these collapsing spherical knots are as common as is claimed.

For the cases we have considered we have found that the relation between the magnitude of a scalar field variation and the magnitude of MBR anisotropy seems to be fairly universal. It does not seem to vary much between topologically trivial fields with flat vacuum manifold geometries and fields with topologically non-trivial and curved vacuum manifold geometries. Nor does it vary much between simple sinusoidal oscillation damped by cosmological expansion and exactly spherical topologically-driven collapse in Minkowski space where there is no damping. Thus generalizing our COBE92-04 limit on the scalar field variation to $O(N)$ models we find limits on the VEV a factor of $\frac{\pi}{2}$ smaller than the limits on the scalar field variation, or VEV$\lesssim 2 \times 10^{-3} M_{\rm Planck} \sim 2.5 \times 10^{16}$GeV. Particularly strong oscillations such as collapsing texture knots, or possibly annihilating monopoles, may lead to even stronger limits. It is interesting to note that our rough estimate of the VEV is about 3 times smaller than that required for textures to give unbiased density perturbations (Park *et al.* 1991) in agreement with Bennett and Rhie (1992).

Now let us return to the physical system for which we have actually done the calculation, i.e. a real, minimally-coupled scalar field. The COBE92-04 results indicate that anisotropies are roughly scale-invariant so the scalar field should also have a scale-invariant form, i.e. the power spectrum of the scalar field should be of the form

$$\langle|\tilde{\phi}_{\mathbf{k}}|^2\rangle \propto k^{-3}. \tag{17}$$

Thus one might consider a theory of a simple scalar field with this spectrum. How could one get such a spectrum? One might first consider the "equivalent" theory of an $N = 1$ σ-model. The dynamics of a scalar field with a linear configuration space is the same as that for an $N = 1$ σ-model which has a circular configuration space. It is true that the circular vacuum manifold comes back on itself, but this makes no difference for the dynamics of the field, since a circle has no intrinsic curvature. The Kibble gradients on such a manifold would have a spectrum roughly equivalent to Eq. (17). However, if one has Kibble gradients in this theory, one should also have global strings and it is these strings which should dominate the dynamics. Thus our calculation is not completely appropriate for this model. One could imagine that inflation diluted the strings, but this would also have diminished the Kibble gradients to a negligible level.

Quantum fluctuations during inflation will introduce new fluctuations of the scalar field, of the exact form as Eq. (17) (see Kolb and Turner 1990 §8.4). However these scalar-field fluctuations cannot be of sufficient amplitude to result in the COBE92-04 anisotropies.

The reason is simple. At the same time that quantum fluctuations are induced in the scalar field, they are also being induced in the graviton (gravitational wave) field. The amplitude of these fluctuations is,

$$\langle \phi^2 \rangle \sim \langle h_{\rm gw}^2 \rangle \sim \left(\frac{\hbar H}{M_{\rm Planck}} \right)^2 \tag{18}$$

where $h_{\rm gw}$ is the dimensionless amplitude of the gravitational wave, $\hbar$ is Planck's constant/2π, and H is the Hubble constant during inflation. The gravity waves, being metric perturbations, can cause anisotropies directly, producing anisotropies of amplitude $\sim h_{\rm gw} \sim \hbar H/M_{\rm Planck}$ at the horizon. Hence from COBE there exists a limit $H \lesssim 10^{-5} M_{\rm Planck}/\hbar$. The scalar field, on the other hand, can only create metric perturbations indirectly through its stress energy tensor which is quadratic in the amplitude ϕ. Thus from the limit from gravity waves we could conclude that quantum fluctuations in the scalar field during inflation will only produce anisotropies of order $(\hbar H/M_{\rm Planck})^2 \lesssim 10^{-10}$ which is totally unobservable. Furthermore the density perturbations induced by such scalar field fluctuations would be totally negligible in comparison with the observations. Thus according to conventional wisdom we cannot create interesting inhomogeneities in the universe via quantum fluctuations induced in massless (non-inflaton) scalar fields during inflation. However given the lack of any empirical evidence on quantum production of gravity waves, one might imagine that the gravitons are "special", being part of the rather special force, gravity, and are not emitted via the Hawking mechanism. Under this assumption the quantum production of density perturbations via a massless, non-inflaton, scalar field may be a viable mechanism for the generation of structure in our universe.

On scales $\gg 10\,$Mpc the MBR anisotropies in almost all models are primarily due to gravitational "Sachs-Wolfe-like" effects which scale as

$$\frac{\Delta T}{T} \propto \left(\frac{\lambda H_0}{c} \right)^2 \frac{\delta \rho}{\rho}. \tag{19}$$

With the discovery of the MBR anisotropies an important observational discriminator between theories is the proportionality constant in this relationship. We have already calculated this proportionality constant for the sinusoidal scalar field in equation (15) of this paper. It is interesting to compare this constant with that for other models. We do so in Table 1 using various published results to obtain "benchmark" numbers for various models. The models of seeded perturbations give significantly larger values than adiabatic perturbations. Thus while COBE seems to require a low bias ($\lesssim 1$) for primordial adiabatic perturbations such as canonical CDM models (Wright *et al.* 1992), they probably require a larger bias (> 1) for models of seeded perturbations which produce a similar (scale-invariant) spectrum of perturbations.

To summarize, scalar field gradients lead to scalar field oscillations which lead to time-varying gravitational fields and finally to MBR anisotropy. Limits on MBR anisotropies give us constraints on possible scalar field configurations and thus on various

model	$\Omega_0 \frac{\frac{\Delta T}{T}}{\frac{1}{3}\Phi_0} \propto \frac{\frac{\Delta T}{T}}{\frac{\delta\rho}{\rho}}$
$\Omega_0 = 1$	1
$\Omega_0 < 1$	0.4 -1
$\Omega_0 > 1, \dot{a} > 0$	0.2 -1
scalar field	4.5
collapsing texture	2.5
cosmic strings	2.5

Table. Here we compare "benchmark" ratios of MBR anisotropy to matter inhomogeneity for various models using various published computations. The first 3 are standard cosmological results and can be found in Peebles (1980). The "scalar field" model refers to this paper. The "collapsing texture" compares calculations of Turok and Spergel (1990) to Gooding *et al.* (1991) while the "cosmic string" model compares calculations of Bouchet *et al.* (1988) to Albrecht and Stebbins (1992). The details of the of how these ratios are calculated can be found in Appendix B.

models of structure formation involving scalar fields. We have analytically calculated the anisotropies for particular configurations. Our results imply that generating the present structure via scalar fields leads to somewhat larger anisotropies than in the standard "primordial adiabatic" scenario. The MBR anisotropies reported in COBE92-04 could be created by a scale invariant distribution of scalar field gradients, such as predicted in nonlinear σ-models with Kibble gradients. For $N = 4$ textures the amplitude of the VEV which is consistent with COBE seems to indicate that the model is only viable if galaxies are a highly biased tracer of the mass. The analytic results obtained here may provide a useful test for numerical calculations of anisotropies and density fluctuations in scalar field models.

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Appendix A: Exact Solutions

The integrals of equation (12) are

$$
\begin{aligned}
I_c(x,a) &\equiv \int^x dx'\, D(x') \cos ax' \\
&= \frac{3}{5}\Bigg[\left(\frac{-720}{x^6} + \frac{6}{x^4}(10+9a+6a^2) + \frac{a}{x^2}(1-a)(1+3a+6a^2)\right)\cos(1-a)x \\
&\quad + \left(\frac{-720}{x^6} + \frac{6}{x^4}(10-9a+6a^2) - \frac{a}{x^2}(1+a)(1-3a+6a^2)\right)\cos(1+a)x \\
&\quad + \left(\frac{1440}{x^6} - \frac{24}{x^4}(5+3a^2) + \frac{4a^2}{x^2}(5+3a^2)\right)\cos ax \\
&\quad + \left(-\frac{72}{x^5}(5+2a) + \frac{2a}{x^3}(1+3a+6a^2) - \frac{a}{x}(1-a)^2(1+3a+6a^2)\right)\sin(1-a)x \\
&\quad + \left(-\frac{72}{x^5}(5-2a) - \frac{2a}{x^3}(1-3a+6a^2) + \frac{a}{x}(1+a)^2(1-3a+6a^2)\right)\sin(1+a)x \\
&\quad + \left(\frac{-288a}{x^5} + \frac{8a}{x^3}(5+3a^2) - \frac{4a^3}{x}(5+3a^2)\right)\sin ax \\
&\quad + a(1-a)^3(1+3a+6a^2)\mathrm{Ci}((1-a)x) \\
&\quad - a(1+a)^3(1-3a+6a^2)\mathrm{Ci}((1+a)x) \\
&\quad + 4a^4(5+3a^2)\mathrm{Ci}(ax)\Bigg]
\end{aligned}
\tag{A1a}
$$

and

$$
\begin{aligned}
I_s(x,a) &\equiv \int^x dx'\, D(x')\, \sin ax \\
&= \frac{3}{5}\Bigg[-\left(\frac{72}{x^5}(5+2a) - \frac{2a}{x^3}(1+3a+6a^2) + \frac{a}{x}(1+a+a^2-9a^3+6a^4)\right)\cos(1-a)x \\
&\quad + \left(\frac{72}{x^5}(5-2a) + \frac{2a}{x^3}(1-3a+6a^2) - \frac{a}{x}(1-a+a^2+9a^3+6a^4)\right)\cos(1+a)x \\
&\quad + \left(\frac{288a}{x^5} - \frac{8a}{x^3}(5+3a^2) + \frac{4a^3}{x}(5+3a^2)\right)\cos ax \\
&\quad + \left(\frac{720}{x^6} - \frac{6}{x^4}(10+9a+6a^2) - \frac{a}{x^2}(1-a)(1+3a+6a^2)\right)\sin(1-a)x \\
&\quad - \left(\frac{720}{x^6} - \frac{6}{x^4}(10-9a+6a^2) + \frac{a}{x^2}(1+a)(1-3a+6a^2)\right)\sin(1+a)x \\
&\quad + \left(\frac{1440}{x^6} - \frac{24}{x^4}(5+3a^2) + \frac{4a^2}{x^2}(5+3a^2)\right)\sin ax \\
&\quad -a(1-a)^3(1+3a+6a^2)\mathrm{Si}((1-a)x) \\
&\quad -a(1+a)^3(1-3a+6a^2)\mathrm{Si}((1+a)x) \\
&\quad +4a^4(5+3a^2)\mathrm{Si}(ax)\Bigg].
\end{aligned}
\tag{A1b}
$$

These functions have limiting values

$$
\begin{aligned}
I_c(0,a) = \frac{3}{5}\Big(&1 + 3a^2(-1+8a^2) + a(1-a)^3(1+3a+6a^2)\ln(1-a) \\
&- a(1+a)^3(1-3a+6a^2)\ln(1+a) + 4a^4(5+3a^2)\ln|a|\Big)
\end{aligned}
\tag{A2a}
$$

and

$$
I_c(\infty,a) = 0 \qquad I_s(0,a) = 0 \qquad I_s(\infty,a) = -\frac{3\pi}{5}a\,(1+15a^4-2|a|^3(5+3a^2)). \tag{A2b}
$$

Small Scale Anisotropies

The angular average of equation (14) is given by the integrals

$$\frac{1}{2}\int_{-1}^{1} da \left[\left(\frac{1}{5} - I_c(0,a)\right)^2 + I_s^2(\infty,a)\right]. \tag{A3}$$

which can be performed using

$$\begin{aligned}
\frac{1}{2}\int_{-1}^{1} da\, I_s^2(\infty,a) &= \frac{107\pi^2}{10010} \\
\frac{1}{2}\int_{-1}^{1} da\, I_c^2(0,a) &= \frac{4973920 - 3515\pi^2 - 14511616\ln 2 + 10719744\ln^2 2}{300300} \\
\frac{1}{2}\int_{-1}^{1} da\, I_c(0,a) &= \frac{2(43 - 60\ln 2)}{35}
\end{aligned} \tag{A4}$$

which combine to give

$$\sqrt{\left\langle \left|\frac{\Delta T}{T}\right|^2 \right\rangle} = \pi\phi_0^2 \sqrt{\frac{4838356 - 305\pi^2 - 14305696\ln 2 + 10719744\ln^2 2}{600600}}. \tag{A5}$$

Eq. 15 comes from

$$\frac{1}{3}\sqrt{\langle|\Phi|^2\rangle} = \frac{1}{10\sqrt{2}}\pi\phi_0^2. \tag{A6}$$

Large Scale Anisotropies

We may decompose the anisotropy pattern as follows

$$\frac{\Delta T}{T}(\hat{\mathbf{n}}) = \sum_{l=0}^{\infty}\sum_{m=-l}^{l} a_{(l,m)} Y_{(l,m)}(\hat{\mathbf{n}}) \qquad a_{(l,m)} = \int d^2\hat{\mathbf{n}}\, Y^*_{(l,m)}(\hat{\mathbf{n}})\, \frac{\Delta T}{T}(\hat{\mathbf{n}}) \tag{A7}$$

where the $Y_{(l,m)}$'s are spherical harmonics. Since the anisotropy pattern is circularly symmetric around the $\mathbf{k}$ direction, if we choose this as the North Pole then $a_{(l,m)} = 0$ for $m \neq 0$. With this simplification the dipole and quadrupole moments of the anisotropy are given by

$$d = \sqrt{\frac{3}{4\pi}}\, |a_{(1,0)}| \qquad Q = \sqrt{\frac{5}{4\pi}}\, |a_{(2,0)}| \tag{A8}$$

from which follow Eqs (16) and (A9) for $Q_{\rm rms}$ and d respectively. We know of no analytic form for $P(x)$ which appears in $Q_{\rm rms}$, or for $R(x)$ which appears in d (see below).

Super-Horizon Perturbations

Now consider the dipole and quadrupole perturbations in the case that $\kappa\eta_o \ll 1$. The dipole is given by

$$d = -\pi\phi_0^2 \sin\vec{\kappa}\cdot\mathbf{x}_o \left[C'(\kappa\eta_o) + \frac{3}{5}R(\kappa\eta_o) + 3\int_0^{\kappa\eta_o} dy\, D(y)\, R(x-y)\right] \tag{A9}$$

where

$$R(x) = \frac{x\cos x - \sin x}{x^2} = -\frac{1}{3}x + \frac{1}{30}x^3 + \mathcal{O}(x^5). \tag{A10}$$

In the limit $\kappa\eta_o \ll 1$ this becomes

$$d = -\frac{73\pi}{1400}\phi_0^2\,(\kappa\eta_o)^3 \sin\vec{\kappa}\cdot\mathbf{x}_o + \mathcal{O}\left((\kappa\eta_o)^5\right) \tag{A11}$$

while the super-horizon limit of Q is

$$Q = \frac{\pi}{30}\phi_0^2\,(\kappa\eta_o)^2 \cos\vec{\kappa}\cdot\mathbf{x}_o + \mathcal{O}\left((\kappa\eta_o)^4\right). \tag{A12}$$

Since in this limit $Q \gg d$ we can conclude that superhorizon perturbations in scalar field gradients do not "tilt" the universe.

Appendix B: Comparison of Anisotropy and Density Production

Here we use various published results to calculate the ratio the amplitude of MBR anisotropies to density perturbations, in particular the ratio

$$\Omega_c \frac{\overline{\left|\frac{\Delta T}{T}\right|}}{\overline{\left|\frac{1}{3}\Phi\right|}} \propto \frac{\frac{\Delta T}{T}}{\frac{\delta\rho}{\rho}} \tag{B1}$$

for various theories. Here Φ is the Newtonian gravitational potential of the perturbations produced (remember $c = 1$) which gives the perturbation in density. Ω_c gives the fraction of the critical density in the matter which makes the perturbation, hence Φ/Ω_c gives the perturbation in overdensity, $\delta\rho/\rho$. If $^-$ indicates a root mean square average then for adiabatic perturbations this ratio would be unity. For the scalar field configuration considered here the universe is CDM dominated and hence $\Omega_c = 1$. Using Eq. 14 and 15, a ratio of 4.8 on small angular scales. Now let us consider results from other papers.

Turok and Spergel (1990) showed that for photon passing by a spherical collapsing texture in Minkowski space, the fractional energy shift ($\equiv$ fractional temperature shift) is give by

$$\left|\frac{\Delta T}{T}\right| = 8\pi^2 \mathrm{VEV}^2 \sqrt{\frac{r^2 - b^2}{r^2 + b^2}} \qquad b \le r \tag{B2}$$

where b is the impact parameter of the photon trajectory from the center of collapse, r is the distance of the photon from the center of collapse at the time of collapse, and we have dropped the sign of the anisotropy. Gooding *et al.* (1991) have estimated the linear density profile generated around a collapsing texture in an expanding $\Omega = 1$ universe. Translating this density profile into a gravitational potential, we find

$$\Phi = -0.9\frac{3}{5}8\pi^2 \mathrm{VEV}^2 \left(1 - \frac{r}{\eta_i}\right)^2 \tag{B3}$$

where r is the comoving distance from the center of collapse and η_i is the comoving horizon at the time of collapse. The factor of 0.9 is a correction that was found for the expanding universe at small distances. Since there is no correction for expansion in Eq. (B2) we will ignore this 0.9 when taking the ratio. To determine the ratio of Eq. (B1) we should divide (B2) by (B3). Here it is natural to identify the two r's in the two equations. The ratio still depends on b and η_i. For $r \ll \eta_i$ the ratio varies from 0 for $b = r$ to 5 for $b = 0$, with an angle weighted rms of 2.5. This is the number we will take as our benchmark.

Cosmic strings is a theory of seeded perturbations which does not involve ultralight scalar fields. Bouchet *et al.* (1988) have estimated the mean square anisotropy produced by the strings during a logarithmic interval of expansion

$$\frac{d}{d\ln a}\left|\frac{\Delta T}{T}\right|^2 = \frac{(6\mu)^2}{\ln 2}. \tag{B4}$$

Albrecht and Stebbins (1992) have estimated the power spectrum of the density fluctuations produced in during an interval of expansion in an $\Omega = 1$ universe. One can translate this into an equation for the mean square potential fluctuation produced per unit interval of time. Taking the matter-era limit of this equation

$$\begin{aligned} \frac{d}{d\ln a}\Phi^2 &= \frac{2304\pi}{25}\mu^2\overline{\beta^2\Sigma}\frac{\chi^2}{\xi^2}\int_0^\infty \frac{1}{\eta(1+2(k\chi)^2)(k^2+k_c 2)}\,dk \\ &= \frac{1152\pi^2}{25}\mu^2\overline{\beta^2\Sigma}\,\frac{\chi\eta}{\xi^2}\frac{1}{k_c\eta\,(1+\sqrt{2}\,k_c\chi)} \end{aligned}, \tag{B5}$$

where χ/η, χ/ξ, $\overline{\beta^2\Sigma}$, and $k_c\eta$ parameterize the string network. If we take parameters for the "I model", which is meant to approximate the simulations of Bennett and Bouchet simulations we find

$$\frac{d}{d\ln a}\Phi^2 = 76\,\mu^2 \tag{B6}$$

and the ratio (B1) is 2.5, which is curiously exactly what we got for textures!

Next consider an open universe with adiabatic perturbations on large scales. On angular scales $(1+z_{ls})^{-\frac{1}{2}} \ll \theta \ll \Omega_0^{-1}$, the anisotropy is given by the usual Sachs-Wolfe

formula $\frac{1}{3}\Phi_{ls}$, except in this case the gravitational potential decays with time so that the value at last scattering, Φ_{ls}, is not the value today, Φ_0. Using the formulae of Peebles (1980) §§11 we see

$$\delta \propto \frac{2\Omega+1}{1-\Omega} + \frac{3\Omega\left(\ln(1-\sqrt{1-\Omega}) - \frac{1}{2}\log\Omega\right)}{(1-\Omega)^{\frac{3}{2}}} \qquad a \propto \frac{1-\Omega}{\Omega} \tag{B7}$$

and since the gravitational potential decays as $\Phi \propto \delta/a$ we find that the ratio (B.1) is given by

$$\frac{2}{5}\frac{(1-\Omega_0)^{\frac{5}{2}}}{(2\Omega_0+1)\sqrt{1-\Omega_0} + 3\Omega_0\left(\ln(1-\sqrt{1-\Omega_0}) - \frac{1}{2}\ln\Omega_0\right)} \tag{B8}$$

which varies from unity when Ω_0 is near unity to $\frac{2}{5}$ for $\Omega_0 \ll 1$. For $\Omega_0 = 0.2$ we find the value 0.6.

We may also consider an expanding closed universe on angular scales $(1+z_{ls})^{-\frac{1}{2}} \ll \theta \ll \Omega_0^{-1}$. In this case (Peebles (1980) §§11)

$$\delta \propto \frac{2\Omega+1}{\Omega-1} - \frac{3\Omega\tan^{-1}\sqrt{\Omega-1}}{(\Omega-1)^{\frac{3}{2}}} \qquad a \propto \frac{\Omega-1}{\Omega} \tag{B9}$$

so the ratio of (B1) is

$$\frac{2}{5}\frac{(\Omega_0-1)^{\frac{5}{2}}}{\Omega_0\left((2\Omega_0+1)\sqrt{\Omega_0-1} - 3\Omega_0\tan^{-1}\sqrt{\Omega_0-1}\right)} \tag{B10}$$

which varies from 1 to $\frac{1}{5}$ as Ω_0 varies from 1 to ∞. Note that $\Omega_0 > 1$ is not currently popular because it gives a very small age for the universe.

References

A. Albrecht and A. Stebbins, *Perturbations from cosmic strings in cold dark matter*, Phys. Rev. Lett. **68** (1992), 2121-2124.

M. Bariola and A. Vilenkin, *Gravitational Field of a Global Monopole*, Phys. Rev. Lett. **63** (1989), 341-343.

D. Bennett and S. Rhie, *Cosmological Evolution of Global Monopoles and the Origin of Large-Scale Structure*, Phys. Rev. Lett. **65** (1990), 1709-1712.

D. Bennett and S. Rhie, *COBE Constraints on the Global Monopole and Texture theories of Cosmic Structure Formation*, preprint.

F. Bouchet, D. Bennett, and A. Stebbins, *Patterns of the cosmic microwave background from evolving string networks*, Nature **335** (1988), 410-414.

R. Cen, J. Ostriker, D. Spergel, and N. Turok, *A Hydrodynamic Approach to Cosmology: Texture-Seeded Cold Dark Matter and Hot Dark Matter Cosmogenies*, *Astrophys. J.* **383** (1991), 1-18.

J. Gelb, "*Large Simulations of Gravitational Clustering in the Universe*", MIT Ph.D. Thesis, 1992.

A. Gooding, D. Spergel, and N. Turok, *The Formation of Galaxies and Quasars in a Texture-Seeded Cold Dark Matter Cosmogony*, *Astrophys. J.Lett.* **372** (1991), L5-L8.

R. Holman, S. Hsu, E. Kolb, R. Watkins, and L. Widrow, *Cosmological Texture is Incompatible with Planck-scale Physics*, preprint NSF-ITP-92-04.

M. Kamionkowski and J. March-Russell, *Are Textures Natural?*, preprint PUPT-92-1304.

T. Kibble,, J. Phys. **A9** (1976), p. 1387.

J. Kim and K. Lee, *The Scale Problem in Wormhole Physics*, Phys. Rev. Lett. **63** (1989), p. 20.

E. Kolb and M. Turner, "*The Early Universe*", Addison-Wesley, New York, 1990.

A. Klypin, M. Sazhin, and I. Strukov, *Anisotropy of the Microwave Background: Results of the "Relikt" Experiment*, *Sov. Astron. Lett.* **13** (1987), 104.

S. Meyer, E. Cheng, and L. Page, *A Measurement of the Large-Scale Cosmic Microwave Background Anisotropy at 1.8 Millimeter Wavelength*, *Astrophys. J. Lett.* **371** (1991), L7-L9.

C. Park, D. Spergel, and N. Turok, *Large-Structure in a Texture-Seeded Cold Dark Matter Cosmogeny*, *Astrophys. J.* **372** (1991), L53-L57.

J. Peebles, "*The Large Scale Structrure of the Universe*", Princeton University Press, Princeton, 1980.

R. Sachs and A. Wolfe, *Perturbations of a Cosmological Model and Angular Variations of the Microwave Background*, *Astrophys. J.* **147** (1967), 73-90..

G. Smoot *et al.* **(COBE92-04)**, *Structure in the COBE DMR First Year Maps*, preprint COBE 92-04.

M. Turner, *The Tilted Universe*, *Gen. Rel. & Grav.* **24** (1992), 1-8.

N. Turok, *Global Texture as the Origin of Cosmic Structure*, *Phys. Rev. Lett.* **63** (1989), 2652-2628.

N. Turok and D. Spergel, *Global Texture and the Microwave Background*, *Phys. Rev. Lett.* **64** (1990), 2736-2739.

N. Turok and D. Spergel, *Scaling Solution for Cosmological σ Models at Large N*, *Phys. Rev. Lett.* **66** (1991), 3093-3096.

N. Turok, D. Spergel, W. Press, and B. Ryden, *Global Texture and the Microwave Background*, *Phys. Rev. D* **43** (1991), p. 1038.

S. Veeraraghavan and A. Stebbins **(VS)**, *Causal Compensated Perturbations in Cosmology*, *As-*

trophys. J. **365** (1990), 37-65.

A. Vilenkin and A. Everett, *Cosmic Strings and Domain Walls in Models with Goldstone and Pseudo-Goldstone Bosons*, *Phys. Rev. Lett.* **48** (1982), 1867-1870.

E. Wright *et al.*, *Interpretation of the CMB Anisotropy Dectected by the COBE DMR*, preprint COBE 92-06.