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Vector-Boson versus Gluon Fusion at Hadron Colliders

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Abstract

We use chiral perturbation theory to show that pseudo-Goldstone boson scattering and gluon fusion probe different aspects of electroweak symmetry breaking at hadron colliders. In particular, the physics responsible for unitarizing the lowest-order pseudo-Goldstone boson scattering amplitudes need not significantly affect the gluon fusion process. We first show this within the context of strict chiral perturbation theory, and then discuss it using the language of resonances.

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In a recent series of papers, we have used chiral perturbation theory to study electroweak symmetry breaking at high-energy hadron colliders [1,2,3]. Our analysis is motivated by technicolor theories, in which the Higgs sector of the standard model is replaced by some sort of new, strongly-interacting physics that dynamically breaks the electroweak symmetry. The new physics induces strong interactions between the would-be Goldstone bosons that form the longitudinal components of the W and Z. The interactions can be parametrized quite generally in terms of a low-energy effective Lagrangian. The effective Lagrangian describes the interactions of the would-be Goldstone bosons at energies below the scale of symmetry breaking, and below the masses of any other particles associated with the symmetry breaking sector.

When the global symmetry group is larger than the usual $SU(2) \times SU(2)$ of the standard model, the effective Lagrangian contains new scalar fields, in addition to the usual would-be Goldstone bosons. In most models, these pseudo-Goldstone bosons are relatively light, so they should be copiously produced at the SSC or LHC. In general, these particles affect the scattering of longitudinal W's and Z's. If some carry color, as is typically the case, they also enhance the gluon fusion production of longitudinal vector bosons.

To lowest order in an energy expansion, the effective Lagrangian contains a single term:

$$\begin{aligned} \mathcal{L}^{(2)} &= \frac{f^2}{4}\,\mathrm{Tr}\Big(D_\mu \Sigma^\dagger D^\mu \Sigma\Big) \\ \Sigma &= \exp\left(\frac{2iT^a\pi^a}{f}\right)\,, \end{aligned} \tag{1}$$

where f is the pseudo-Goldstone decay constant. In this expression, the T^a are the (broken) generators associated with a global symmetry group G, and $\mathrm{Tr}(T^aT^b) =$

$\frac{1}{2}\delta^{ab}$. For $G = SU(2) \times SU(2)$, the fields π^a are the would-be Goldstone bosons associated with the W and Z. For larger groups, they also include the pseudo-Goldstone bosons discussed above.

The covariant derivative $D_\mu\Sigma$ describes the embedding of $SU(3) \times SU(2)_L \times U(1)_Y$ into the symmetry group G. For $G = SU(2N)\times SU(2N)$, we find $f = v/\sqrt{N}$ in terms of the usual $v \simeq 250$ GeV. The covariant derivative is given by

$$D_\mu\Sigma \;=\; \partial_\mu\Sigma \;+\; \frac{i}{2}g_s G^\alpha_\mu[T^\alpha,\Sigma] \;+\; \frac{i}{2}\frac{g}{\sqrt{N}}W^i_\mu T^i\Sigma \;-\; \frac{i}{2}\frac{g'}{\sqrt{N}}B_\mu\Sigma T^3\;, \qquad (2)$$

where G^α_μ is the $SU(3)$ color gauge field, and W^i_μ and B_μ are the $SU(2)_L \times U(1)_Y$ gauge bosons. In this expression, the color coupling is written as a commutator because color $SU(3)$ is contained in the diagonal subgroup of G. In contrast, $SU(2)_L$ is purely left-handed, while $U(1)_Y$ acts on the right. The covariant derivative determines the lowest-order couplings of the Goldstone particles to the standard-model gauge bosons.

The lowest-order effective Lagrangian can be used to describe pseudo-Goldstone boson scattering to order p^2 and gluon-gluon scattering to order p^4 in the energy expansion. To this order, the predictions are universal, in the sense that they depend only on the symmetry group G and on the decay constant f. At order p^4, the pseudo-Goldstone scattering amplitudes depend on the next-to-leading-order effective Lagrangian, which contains four operators:

$$
\begin{aligned}
\mathcal{L}^{(4)} \;&= \frac{L_1}{16\pi^2}\mathrm{Tr}\Big(D_\mu\Sigma^\dagger D^\mu\Sigma\Big)\mathrm{Tr}\Big(D_\nu\Sigma^\dagger D^\nu\Sigma\Big) \\
&+\; \frac{L_2}{16\pi^2}\mathrm{Tr}\Big(D_\mu\Sigma^\dagger D^\nu\Sigma\Big)\mathrm{Tr}\Big(D_\mu\Sigma^\dagger D^\nu\Sigma\Big) \\
&+\; \frac{NL_3}{16\pi^2}\mathrm{Tr}\Big(D_\mu\Sigma^\dagger D^\mu\Sigma D_\nu\Sigma^\dagger D^\nu\Sigma\Big) \\
&+\; \frac{NL_4}{16\pi^2}\mathrm{Tr}\Big(D_\mu\Sigma^\dagger D^\nu\Sigma D_\mu\Sigma^\dagger D^\nu\Sigma\Big)\ .
\end{aligned}
\tag{3}
$$

The coefficients L_i are expected to be of order one; they are determined by the dynamics that underlie the symmetry breaking.

It has been known for quite some time that the chiral expansion breaks down at a scale of order $\Lambda \lesssim 4\pi v/N$ [4]. This value is based on the naive argument that loop corrections must not dominate tree-level results. Of course, the argument does not fix the scale very precisely. For the purist, this is irrelevant, since the expansion only makes sense at energies much lower than the scale of symmetry breaking. From a practical point of view, however, one must decide where to trust a given calculation. It is often suggested that the cutoff be placed at the scale where the lowest-order pseudo-Goldstone boson scattering amplitudes violate perturbative (two-body-elastic) unitarity, a scale which is also of order $4\pi v/N$.

Since the lowest-order effective Lagrangian is unique, its predictions do not distinguish between different dynamical models of symmetry breaking. These distinctions appear at order p^4, in the form of the unknown coefficients L_i. For practical applications, one would like to identify a region where the energies are high enough for these terms to be significant, and yet low enough for one to trust the energy expansion.#1

#1 Unitarization prescriptions have been proposed as a way to extend the region of validity

As a practical answer to this question, we have advocated using the **next-to-leading-order** chiral Lagrangian in the vicinity of the scale where the **lowest-order** partial waves first violate unitarity [6]. The motivation for this choice is simple:

1. The energy is large enough for the L_i to be important.

2. The corrections stay within a factor of two of the lowest-order amplitudes.

Of course, we must now decide where to stop trusting the next-to-leading results. We assert that it is reasonable to use the order p^4 chiral Lagrangian in the region where the $\mathcal{O}(p^4)$ partial waves preserve unitarity. Alternatively, we can also add resonances to the model [7] and preserve unitarity beyond the naive counting scale Λ. In this case, the formalism should not be trusted beyond the scale $\Lambda \sim 4\pi v/N$, about 800 GeV for $N = 4$.

Although chiral perturbation theory gives rigorous low-energy results, it becomes more model-dependent when used at higher energies. We can still use it as a qualitative guide from which we can infer general trends, but we should not take any particular numbers very seriously. If we wish to make detailed predictions for all energies, we must construct (and solve) explicit models. Given the large number of possibilities associated with electroweak symmetry breaking, we believe that our more modest goals are both practical and reasonable.

Let us now compute the pseudo-Goldstone-boson scattering amplitudes within this framework. For $G = SU(2N) \times SU(2N)$, spontaneously broken to $SU(2N)$, there are three channels that grow with N [8]. They are the singlet, the symmetric adjoint and the antisymmetric adjoint. The $J = 0$ partial wave in the singlet

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of the calculations [5]. This is contrary to the spirit of effective Lagrangians, where the fundamental physics is parametrized by the L_i.

channel is

$$\begin{aligned}
\mathrm{Re}\; a_0^S &= \frac{1}{32\pi}\left\{\frac{sn}{f^2} + \frac{s^2}{2\pi^2 f^4}\left[L_1^r(\mu)\left(n^2 - \frac{1}{3}\right) + L_2^r(\mu)\left(1 + \frac{n^2}{3}\right)\right.\right. \\
&\left. + \frac{L_3^r(\mu)}{6}\left(4n^2 - 5\right) + \frac{L_4^r(\mu)}{6}\left(2n^2 - 5\right)\right] \\
&\left. - \frac{n^2 s^2}{3456\pi^2 f^4}\left(150\log\left(\frac{s}{\mu^2}\right) - 11\right)\right\} ,
\end{aligned} \tag{4}$$

while the $J = 1$ partial wave in the antisymmetric adjoint channel is

$$\begin{aligned}
a_1^{AA} &= \frac{1}{192\pi}\left[\frac{ns}{f^2} - \frac{s^2}{2\pi^2 f^4}\left(2L_1^r(\mu) - L_2^r(\mu)\right.\right. \\
&\left.\left. + \frac{n^2}{4}(L_3^r(\mu) - 2L_4^r(\mu))\right) - \frac{n^2 s^2}{576\pi^2 f^4}\right]
\end{aligned} \tag{5}$$

and the $J = 0$ partial wave in the symmetric adjoint channel is

$$\begin{aligned}
\mathrm{Re}\; a_0^{SA} &= \frac{1}{64\pi}\left[\frac{ns}{f^2} + \frac{2s^2}{3\pi^2 f^4}\left(L_1^r(\mu) + 2L_2^r(\mu)\right.\right. \\
&\left. + \frac{n^2-5}{2}L_3^r(\mu) + \frac{n^2-10}{4}L_4^r(\mu)\right) \\
&\left. - \frac{n^2 s^2}{3456\pi^2 f^4}\left(60\log\left(\frac{s}{\mu^2}\right) + 1\right)\right] .
\end{aligned} \tag{6}$$

In these formulae, $n = 2N$ and we have treated the pseudo-Goldstone bosons as massless. Our amplitudes agree with those of Ref. 9, once we account for the fact that we have chosen a different renormalization scheme:

$$\begin{aligned}
L_1 &= L_1^r(\mu) - \frac{1}{32}\frac{1}{\hat{\epsilon}} - \frac{1}{16} \\
L_2 &= L_2^r(\mu) - \frac{1}{16}\frac{1}{\hat{\epsilon}} - \frac{1}{8} \\
L_3 &= L_3^r(\mu) - \frac{1}{24}\frac{1}{\hat{\epsilon}} - \frac{5}{72} \\
L_4 &= L_4^r(\mu) - \frac{1}{48}\frac{1}{\hat{\epsilon}} - \frac{1}{18} .
\end{aligned} \tag{7}$$

Our prescription is such that all constants that appear at order p^4 in the singlet, the antisymmetric adjoint and the symmetric adjoint amplitudes are absorbed into the renormalized coefficients.

In Figure 1 we have plotted these partial waves for some particular values of the $L_i^r(\mu)$. We see that there can be a considerable difference between the energies at which the $\mathcal{O}(p^2)$ and the $\mathcal{O}(p^4)$ partial waves first violate unitarity. We associate the unitarity violation with the appearance of some structure in the fundamental theory that cannot be properly described by the low-energy constants (such as a resonance).[#2] The chiral Lagrangian is most useful when these structures are pushed as high as possible, roughly, to the naive counting scale Λ. For comparison we also present in this figure the result of a model with vector and scalar resonances coupled via the Lagrangian

$$\mathcal{L} \;=\; i\frac{g_\rho f^2}{m_\rho}\mathrm{Tr}\Big(\rho_{\mu\nu}\xi^\dagger D_\mu \Sigma D_\nu \Sigma^\dagger \xi\Big) + \frac{g_\sigma f^2}{m_\sigma}\sigma \mathrm{Tr}\Big(D_\mu \Sigma D^\mu \Sigma^\dagger\Big)\;. \tag{8}$$

The couplings of the resonances to the pseudo-Goldstone bosons can be written in terms of the resonance widths.[#3]

Let us next turn our attention to gluon fusion. To lowest order in chiral perturbation theory, $\mathcal{L}^{(2)}$ does not induce the production of longitudinal gauge boson pairs by gluon fusion. This process first occurs at order p^4 in the chiral expansion, that is, at one-loop with $\mathcal{L}^{(2)}$ and tree-level with $\mathcal{L}^{(4)}$. To this order, the gluon fusion process is independent of the L_i. This is in sharp contrast to

#2 If there is such low energy structure, it is clear that it should be studied directly and *not* by an energy expansion.

#3 ξ is such that $\xi \cdot \xi = \Sigma$. The vector resonance has been introduced as an antisymmetric tensor field following Gasser and Leutwyler, Ref. 4.

the pseudo-Goldstone boson scattering amplitudes, which depend on the L_i. The amplitude for gluon fusion production of $Z_L Z_L$, for example, is given by:

$$
\begin{aligned}
&M\Big(g_\mu^\alpha(q_1) g_\nu^\beta(q_2) \rightarrow Z_L(p) Z_L(p') \Big) \\
&\qquad\qquad = \epsilon^\mu(q_1)\epsilon^\nu(q_2)\delta_{\alpha\beta} \sum_R T(R) \frac{\alpha_s}{\pi v^2} \left(-\frac{s}{2} g_{\mu\nu} + q_{2\mu} q_{1\nu} \right) ,
\end{aligned}
\tag{9}
$$

where $T(R)$ is $1/2$ for each color triplet and 3 for each color octet.

For the $SU(8)$ Farhi-Susskind model, the lowest-order pseudo-Goldstone boson scattering amplitudes violate unitarity [10] at about 440 GeV. To order p^4, however, this scale depends on the L_i. For some choices of the L_i, unitarity violation can be pushed near the naive counting scale of $\Lambda \lesssim 800$ GeV. Therefore we claim that to order p^4, it is reasonable to use Eq. (9) above threshold, up to about 800 GeV. Above 440 GeV, the result becomes more model-dependent, and as we have said previously, it is only intended to give a rough guide.

In a recent paper [10], it has been argued that our glue-glue amplitude is an obvious overestimate of the physical amplitude because the singlet $SU(8)$ partial wave violates unitarity above 440 GeV. We wish to stress that this violation of unitarity occurs in the pseudo-Goldstone boson channels and *not* in the gluon fusion channel, and that the precise energy at which it occurs is model-dependent. As we have emphasized, to order p^4 in chiral perturbation theory, the glue-glue prediction is universal, while the scale of unitarity violation in pseudo-Goldstone scattering is not. To illustrate our point, we will first discuss the related process $\gamma\gamma \rightarrow \pi^0\pi^0$. We will then construct an explicit model that does *not* violate unitarity below $\Lambda \sim 800$ GeV in the pseudo-Goldstone boson scattering amplitudes, and yet still results in a gluon fusion production of $Z_L Z_L$ pairs as large as the one we presented in Ref. 2.

We show in Figure 2 the analogous QCD process $\gamma\gamma \to \pi^0\pi^0$, and the prediction from chiral perturbation theory [11,12]. We have presented the lowest-order chiral perturbation theory result up to a very high energy, 1.4 GeV, where we have no reason to believe this prediction. However, we see that it does give a reasonable qualitative picture, accurate to a factor of two, below the naive counting scale of 1.2 GeV.[#4]

We now turn to our second and more important point, that the physics of vector-boson scattering is very different from that of gluon fusion. Within pure chiral perturbation theory, this is clear when one goes to order p^6. At this order, the gluon fusion process receives many contributions, some of which do not directly affect pseudo-Goldstone-boson scattering. For example, gluon fusion receives new contributions from the operator:

$$\mathcal{L}^{(6)} = \frac{\alpha_s L}{4\pi\Lambda^2}\mathrm{Tr}\left(D_\alpha \Sigma D^\alpha \Sigma^\dagger\right) G^{a\mu\nu} G^a_{\mu\nu} \, . \tag{10}$$

These terms clearly do *not* contribute to pseudo-Goldstone-boson scattering except at one loop in QCD. Given the size of α_s, these contributions can be safely neglected when compared to the potential strongly-interacting electroweak symmetry breaking sector.

Unfortunately, at order p^6, chiral perturbation theory is not practical due to the large number of operators that appear. We can, however, resort to specific models; for example, including resonances. Above 440 GeV, it is possible for a resonance

#4 Recall that the lowest-order $J = 0$, $I = 0$, partial wave amplitude in $\pi - \pi$ scattering violates unitarity at about 500 MeV.

like the techni-rho to unitarize the pseudo-Goldstone scattering amplitudes, as shown in Figure 1. However, the techni-rho does not appear as a resonance in the gluon-fusion process. Although the techni-rho certainly affects glue-glue scattering, this mode is much more sensitive to resonances which couple directly to the glue-glue intial state, as a techni-sigma or techni-f_2 [13]. For example, the following operator induces a direct coupling between the techni-sigma and the gluons,

$$\mathcal{L} \;=\; \frac{\alpha_S h_\sigma}{m_\sigma} \sigma G^a_{\mu\nu} G^{a\mu\nu} \; . \tag{11}$$

In Figure 3 we show the contribution from a color octet of pseudos to $Z_L Z_L$ production via gluon fusion at the SSC. We compare this to the contribution from the techni-sigma of Eqs. (8), (11). We have chosen the same values of mass and width used in Figure 1, as well as $h_\sigma = 0.8$, a number of the same order as those found in the model of Cahn and Suzuki [8]. We see that in this model, pseudo-Goldstone boson scattering does not violate unitarity below $\Lambda \lesssim 800$ GeV, and yet, the production of $Z_L Z_L$ pairs in gluon fusion is comparable to that obtained in lowest-order chiral perturbation theory.

Direct couplings as that in Eq. (11) are completely unconstrained by the unitarity of the pseudo-Goldstone scattering process. For example, in QCD, the f_2 gives a major contribution to photon-photon scattering, as shown in Figure 2. At energies near 1 GeV, the resonant production of the f_2 provides the dominant contribution to the scattering amplitude. The only constraint on the f_2 coupling to photons comes from the $\gamma\gamma \to \pi^0\pi^0$ process itself.

In this letter we have argued that pseudo-Goldstone boson scattering and gluon fusion probe different aspects of electroweak symmetry breaking. We have emphasized that different physics couples to the two initial states. In chiral perturbation theory, this shows up in the different operators that contribute to each process at a given order in the energy expansion. In models with resonances this shows up in the direct couplings of resonances to the glue-glue state. We have performed our analysis for chiral $SU(2N) \times SU(2N)$, however, it is clear that the same conclusions hold for the more usual case $N = 1$. The only difference being that $N = 1$ does not allow additional colored pseudo-Goldstone bosons. Although we have presented results for massless pseudo-Goldstone bosons, we have checked that including a small mass does not affect the qualitative results of this paper.

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REFERENCES

1. S. Dawson and G. Valencia, *Nucl. Phys.* **B352** (1991) 27; J. Bagger, S. Dawson and G. Valencia, *Effective Field Theory Calculation of $pp \to V_L V_L X$*, Fermilab-Pub-92/75-T.
2. J. Bagger, S. Dawson and G. Valencia, *Phys. Rev. Lett.* **67** (1991) 2256.
3. T. Appelquist and C. Bernard, *Phys. Rev.* **D22** (1980) 200; A. Longhitano, *Nucl. Phys.* **B188** (1981) 118; M. Chanowitz and M. Gaillard, *Nucl. Phys.* **B261** (1985) 379; M. Chanowitz, H. Georgi and M. Golden, *Phys. Rev.* **D36** (1987) 1490; A. Dobado and M. Herrero, *Phys. Lett.* **228B** (1989) 495; J. Donoghue and C. Ramirez, *Phys. Lett.* **234B** (1990) 361; B. Holdom, *Phys. Lett.* **258B** (1991) 156; A. Falk, M. Luke and E. Simmons, *Nucl. Phys.* **B365** (1991) 523.
4. J. Gasser and H. Leutwyler *Ann. Phys.* **158** (1984) 142; M. Soldate and R. Sundrum *Nucl. Phys.* **B340** (1990) 1.
5. See for example A. Dobado, M. Herrero, and T. Truong, *Phys. Lett.* **235B** (1990) 129 and 134; R. Willey, *Phys. Rev.* **D44** (1991) 3646.
6. J. Bagger, S. Dawson and G. Valencia, *Proceedings of the 1990 Snowmass Study on Physics in the 90's*, Snowmass, CO.
7. S. Weinberg, *Phys. Rev.* **166** (1968) 1568; S. Coleman, J. Wess and B. Zumino, *Phys. Rev.* **177** (1969) 2239; C. Callan, S. Coleman, J. Wess and B. Zumino, *Phys. Rev.* **177** (1969) 2246; Applications to hadron colliders can be found in J. Bagger, T. Han and R. Rosenfeld, *Proceedings of the 1990 Snowmass Study on Physics in the 90's*, Snowmass, CO. and references therein.

8. R. Cahn and M. Suzuki, *Phys. Rev. Lett* **67** (1991) 169.

9. R. S. Chivukula, M. J. Dugan and M. Golden, HUTP-92/A025, BUHEP-92-18.

10. R. S. Chivukula, M. Golden and M. V. Ramana, *Phys. Rev. Lett.* **68** (1992) 2883.

11. The Crystal Ball data are from, H. Marsike et. al., *Phys. Rev.* **D41** (1990) 3324.

12. J. Bijnens and F. Cornet, *Nucl. Phys.* **B296** (1988) 557; J. F. Donoghue, B. R. Holstein, and Y. C. Lin, *Phys. Rev.* **D37** (1988) 2423; J. Bijnens, S. Dawson and G. Valencia, *Phys. Rev.* D44 (1991) 3555; C. Im, SLAC-Pub-5627; M. R. Pennington, DTP-92-32 (1992) and references therein.

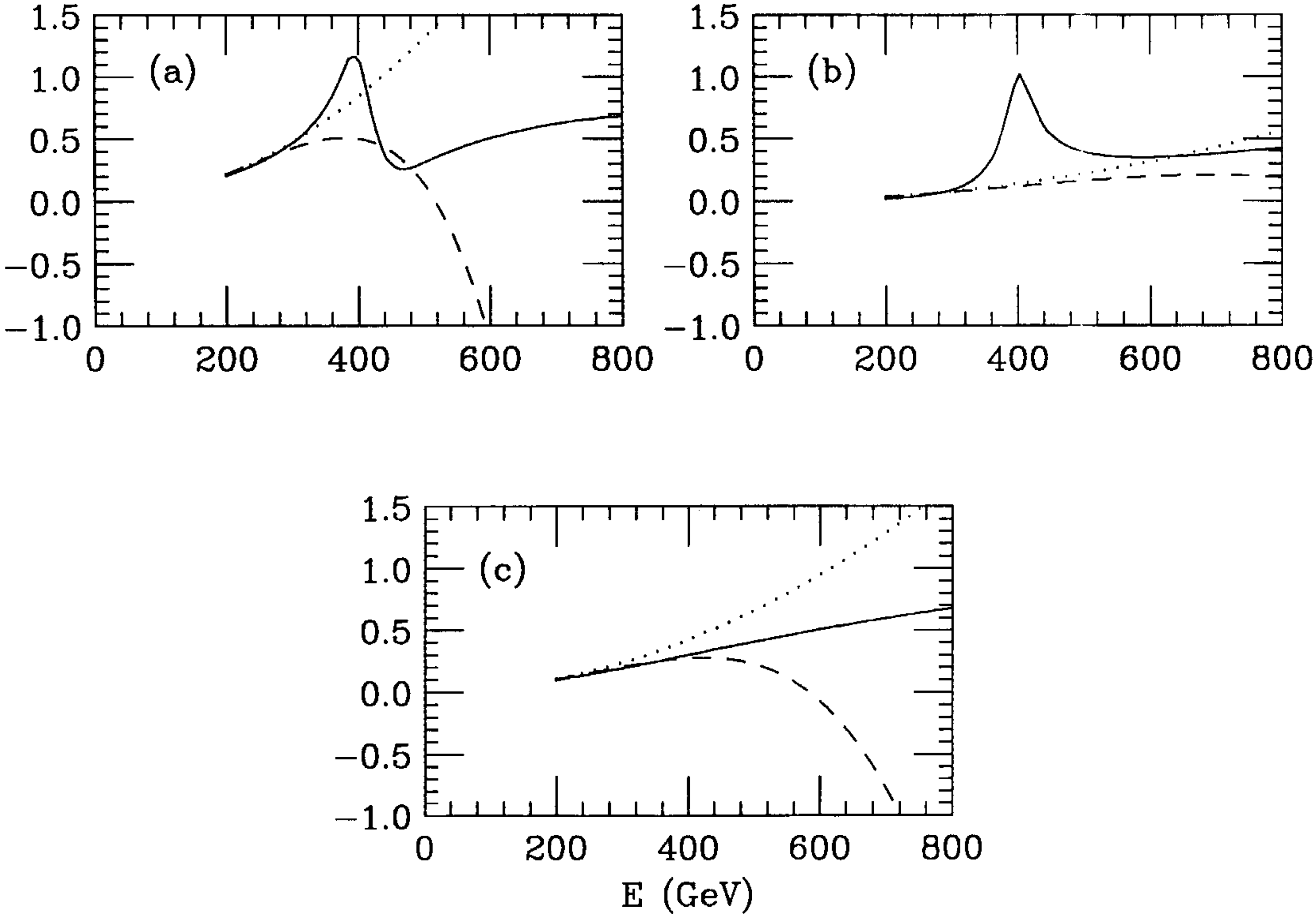
13. J. Bagger, S. Dawson, E. Poppitz and G. Valencia, in preparation.

FIGURE CAPTIONS

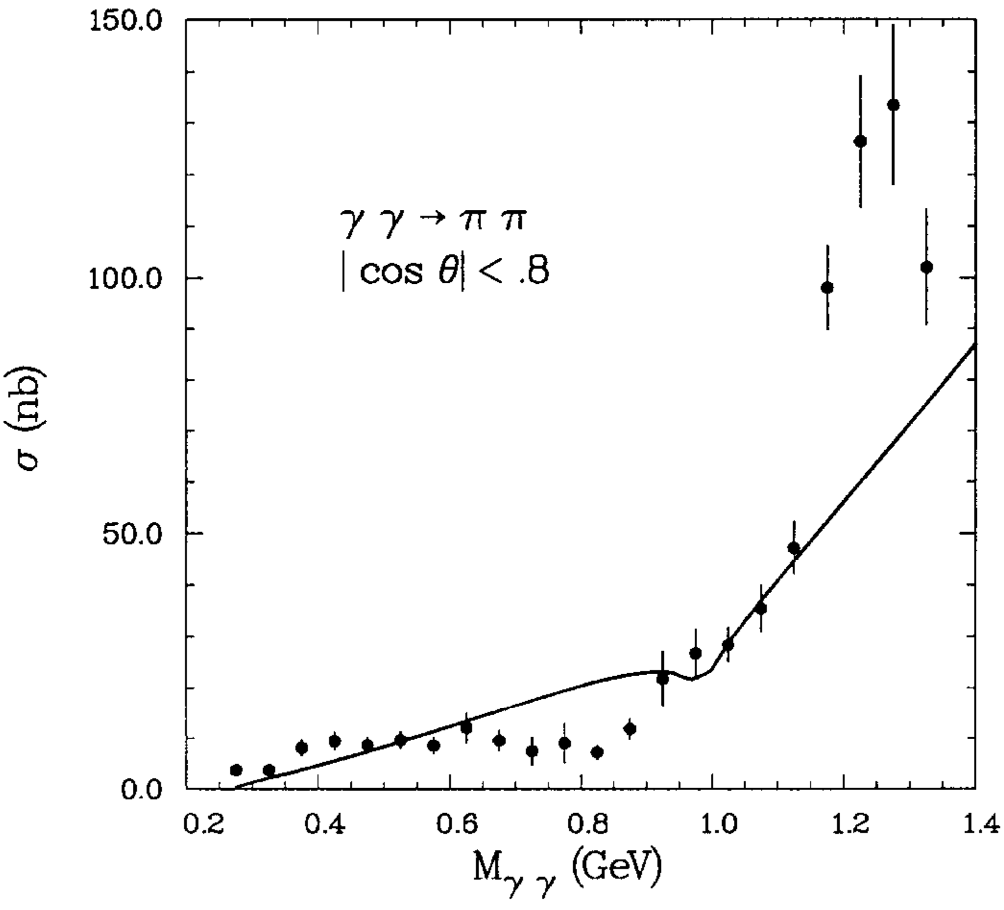
1) We show the partial wave amplitudes for the singlet channel (a); the antisymmetric adjoint (b); and the symmetric adjoint (c). The dotted curves represent the real part of the lowest-order results, and the dashed curves represent the real part of the order p^4 results for $L_1^r(\mu) = -0.10$, $L_2^r(\mu) = 0.30$, $L_3^r(\mu) = -0.09$ and $L_4^r(\mu) = -0.13$, all at $\mu = 384$ GeV. The solid line is the modulus of the amplitude for a tree-level model with a vector resonance of mass 400 GeV and width 40 GeV and a scalar resonance of mass 400 GeV and width 50 GeV.

2) The cross section for $\gamma\gamma \to \pi^0\pi^0$, from Ref. 11. The solid line gives the $\mathcal{O}(p^4)$ prediction in $SU(3)$ chiral perturbation theory.

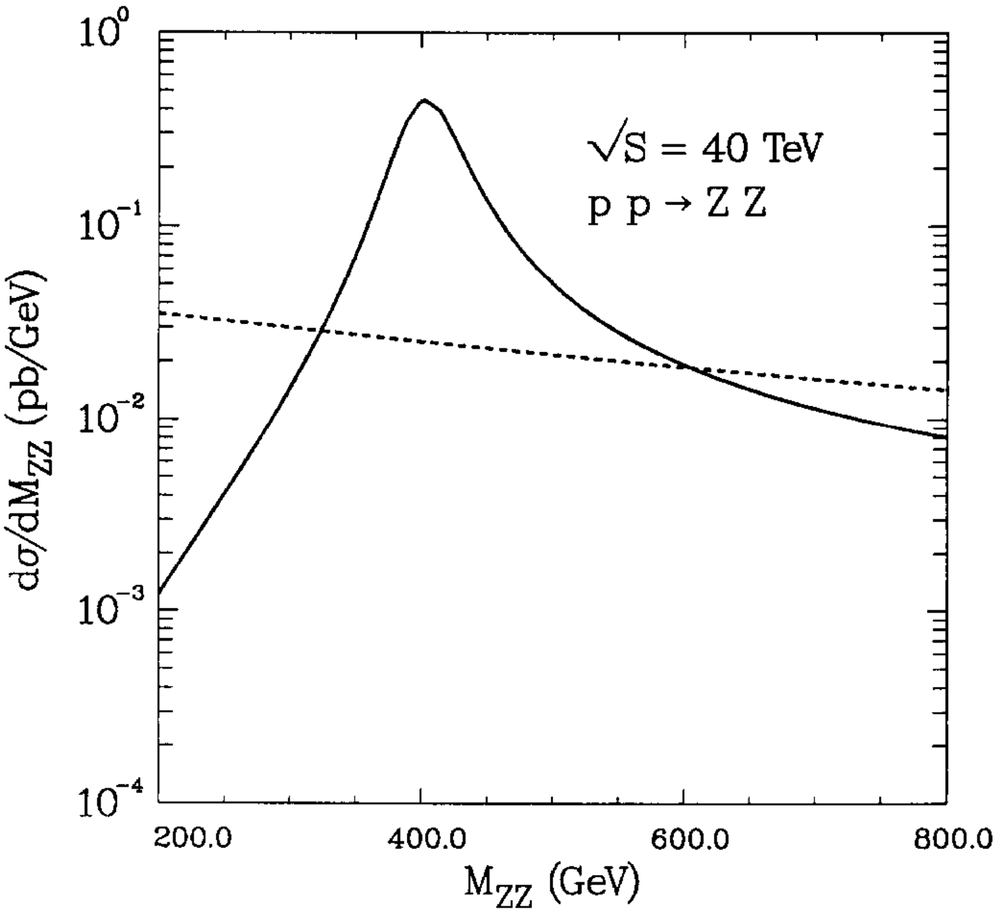
3) Production of $Z_L Z_L$ pairs by gluon fusion at the SSC with a rapidity cut $|y| < 2.5$. The dotted line corresponds to one color octet of pseudo-Goldstone bosons. The solid line corresponds to a techni-sigma of mass 400 GeV, width 50 GeV and $h_\sigma = 0.8$.



(a)
(b)
(c)
1.5
1.0
0.5
0.0
−0.5
−1.0
0
200
400
600
800
E (GeV)



150.0
100.0
50.0
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
σ (nb)
$M_{\gamma\gamma}$ (GeV)
$\gamma\gamma \to \pi\pi$
$|\cos\theta| < .8$



10^0
10^{-1}
10^{-2}
10^{-3}
10^{-4}
$d\sigma/dM_{ZZ}$ (pb/GeV)
$\sqrt{S}$ = 40 TeV
p p → Z Z
200.0
400.0
600.0
800.0
M_{ZZ} (GeV)