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COSMIC MICROWAVE BACKGROUND PROBES MODELS OF INFLATION

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ABSTRACT: Inflation creates both scalar (density) and tensor (gravity wave) metric perturbations. We find that the tensor mode contribution to the CMB anisotropy on large-angular scales can only exceed that of the scalar mode in models where the spectrum of perturbations deviates significantly from scale invariance (e.g., extended and power-law inflation models and extreme versions of chaotic inflation). If the tensor mode dominates at large-angular scales, then the value of $\Delta T/T$ predicted on 1° is less than if the scalar mode dominates, and, for cold dark matter models, $b > 1$ can be made consistent with the COBE DMR results.

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The recent COBE DMR [1] measurements of large-angular-scale anisotropy in the cosmic microwave background (CMB) provide important experimental support for the hot big bang model. Perhaps the most striking conclusion to be drawn from the COBE DMR data is that it is consistent with a scale-invariant spectrum of primordial density (scalar) perturbations extending well outside the horizon at the epoch of last scattering.

A scale-invariant spectrum is consistent with inflation, which predicts perturbations generated by quantum fluctuations [2], and also with models that generate perturbations by classical effects, such as theories with cosmic strings, textures, global monopoles, and non-topological excitations. Inflation also produces a spectrum of gravity waves (tensor metric fluctuations) with wavelengths extending beyond the horizon, providing a possible means for distinguishing it from the other scenarios. Recently it was even speculated that the anisotropy detected by the COBE DMR might be largely due to inflation-produced tensor rather than scalar perturbations [3]. In this *Letter*, we show that tensor dominance of the CMB quadrupole anisotropy is indeed possible for a class of inflationary models. We find that the ratio of tensor to scalar contributions is directly tied to the rate of inflationary expansion and the "tilt" of the spectrum of density perturbations away from scale invariance. Models that permit tensor dominance include extended inflation, power-law inflation and extreme versions of chaotic inflation. While the COBE DMR results alone cannot distinguish tensor from scalar perturbations, we show how additional measurements on small-angular scales may distinguish the two. We also discuss the implications for large-scale structure.

CMB temperature anisotropies on large-angular scales ($\gtrsim 1^\circ$) are produced by metric fluctuations through the Sachs-Wolfe effect [4]. These temperature fluctuations can be decomposed into spherical-harmonic amplitudes; for scale-invariant scalar-mode fluctuations, the quadrupole is given by [5]

$$S \equiv \left\langle a_2^2 \right\rangle_S \equiv \left\langle \sum_{m=-2}^{m=2} |a_{2m}|^2 \right\rangle = \frac{1}{60\pi} \frac{H^4}{\dot{\phi}^2} = \frac{128\pi^2}{45} \frac{V^3}{V'^2 m_{Pl}^6}, \tag{1}$$

where H is the Hubble parameter, ϕ is the scalar field that rolls during inflation, $V(\phi)$ is its potential, $m_{Pl} = 1.22 \times 10^{19}$ GeV is the Planck mass, and the final expression follows from the slow-roll equation of motion, $3H\dot{\phi} = -V'$. The rhs is to be evaluated $N \sim 60$ e-foldings before the end of inflation, when fluctuations on CMB length scales crossed outside the horizon [6]. The corresponding formula for tensor fluctuations is [7]:

$$T \equiv \left\langle a_2^2 \right\rangle_T = 7.74 \frac{V}{{m_{\rm Pl}}^4}, \tag{2}$$

The ratio of tensor to scalar quadrupole anisotropies is, therefore,

$$\frac{T}{S} \equiv \frac{\langle a_2^2 \rangle_T}{\langle a_2^2 \rangle_S} \approx 0.28 \left(\frac{V' m_{\rm Pl}}{V} \right)^2 \Bigg|_{N \sim 60}. \tag{3}$$

Note that the coefficients in Eqs. (1, 2) were derived assuming strict scale invariance. Since we will find below that models with $T/S \gtrsim 1$ deviate from scale invariance, we have numerically computed the coefficients in Eqs. (1, 2) for "tilted" spectra and find that the numerical coefficient in Eq. (3) changes very little ($\lesssim 10\%$) for the tilts consistent with the COBE DMR results.

Extended [8] and power-law [9] inflation models can be described in terms of a potential of the form, $V(\phi) = V_0 \exp(-\beta\phi/m_{\rm Pl})$, where β is constant or slowly time-dependent. In extended inflation ϕ is related to a field that is coupled to the scalar curvature (e.g., a dilaton or Brans-Dicke field), which leads to a modification of Einstein gravity. The modified gravity action can be re-expressed via a Weyl transformation as the usual Einstein action plus a minimally coupled scalar field (ϕ) with an exponential potential. In the simplest example of extended inflation [8], $\beta = \sqrt{64\pi/(2\omega+3)}$, where ω is the Brans-Dicke parameter. For an exponential potential, Eq. (3) implies:

$$\frac{T}{S} \approx 0.28\beta^2 = \frac{56}{2\omega+3} \tag{4}$$

The ratio $T/S \gtrsim 1$ for $\omega \lesssim 26$ ($\beta \gtrsim 1.9$). Interestingly, $\omega \lesssim 26$ is almost precisely what is required to avoid unacceptable inhomogeneities from big bubbles in extended inflation [10]. (Though $\omega \lesssim 26$ is inconsistent with solar-system limits for Brans-Dicke theory, these constraints are evaded by giving the Brans-Dicke field a mass.)

Chaotic inflation models [11] typically invoke a potential of the form, $V(\phi) = \lambda\phi^p$, where $\phi \gg m_{\rm Pl}$ initially, and rolls to $\phi = 0$. The ratio of tensor to scalar anisotropies can be expressed in terms of ϕ_N, the value of the scalar field $N \sim 60$ e-foldings before the end of inflation. Using the relation,

$$N(\phi) = \int_{t_{\rm end}}^{t_N} H dt = \frac{8\pi}{{m_{\rm Pl}}^2}\int_{\phi_{\rm end}}^{\phi_N}\frac{V}{V'}d\phi = \frac{4\pi}{p}\frac{\phi^2}{{m_{\rm Pl}}^2} - \frac{p}{12}, \tag{5}$$

where $\phi_{\rm end}^2 = p^2 {m_{\rm Pl}}^2/48\pi$, we find that [12]:

$$\frac{T}{S} \approx \frac{p}{17.4}\left[1 + \frac{p}{720}\right]^{-1}, \tag{6}$$

where we have set $N = 60$. For the chaotic-inflation models usually discussed, $p = 2$ and 4, the scalar mode dominates: $T/S = 0.11$ and 0.23; however, for extreme models, $p \gtrsim 18$, the tensor mode could dominate.

New inflation models [13] entail slow-roll from $\phi \approx 0$ to $\phi = \sigma$ down flat potentials of the Coleman-Weinberg form, $V(\phi) = B\sigma^4/2 + B\phi^4\,[\ln(\phi^2/\sigma^2) - \frac{1}{2}]$, where $B \simeq 10^{-15}$ for density perturbations of an acceptable size. In new inflation T/S also depends upon ϕ_N; paralleling the previous analysis,

$$N(\phi) = \frac{8\pi}{{m_{\rm Pl}}^2}\int_{\phi_{\rm end}}^{\phi_N}\frac{V}{V'}d\phi \approx \frac{\pi}{2|\ln(\phi_N^2/\sigma^2)|}\frac{\sigma^4}{\phi_N^2 {m_{\rm Pl}}^2}; \tag{7}$$

$$\frac{T}{S} \approx \frac{3.2\times 10^{-4}}{|\ln(\phi_N^2/\sigma^2)|}\left(\frac{\sigma}{m_{\rm Pl}}\right)^4 . \qquad (8)$$

Scalar dominates tensor for $\sigma \lesssim 10m_{\rm Pl}$, and, naively, it would appear that T/S can be made greater than unity for $\sigma \gtrsim 10m_{\rm Pl}$. However, one finds that ϕ_{60} is very close to σ for $\sigma \gtrsim 10m_{\rm Pl}$, violating the implicit assumption, $\phi_{60} \ll \sigma$. That is, for $\sigma \gg m_{Pl}$, ϕ rolls down the steeper (harmonic) part of the potential close to the minimum, so that $V(\phi) \simeq 4B\sigma^2(\phi-\sigma)^2$, just as in chaotic inflation with $p = 2$. In this case, the tensor mode does not dominate $(T/S \simeq 0.11)$ [14].

We will now show that T/S cannot be arbitrarily large by deriving model-independent relations between T/S, the rate of inflation, and the tilt of the density perturbation spectrum away from scale-invariance [16]. The ratio of tensor to scalar perturbations is controlled by the steepness of the potential, $V'm_{\rm Pl}/V$; cf. Eq. (3). During inflation, this quantity also determines the ratio of the kinetic to potential energy of the scalar field [15], $\frac{1}{2}\dot\phi^2/V \simeq (V'm_{\rm Pl}/V)^2/48\pi$, which in turn determines the effective equation of state $(p=\gamma\rho)$ and the evolution of the cosmic-scale factor $(R \propto t^m)$: $\gamma = [\frac{1}{2}\dot\phi^2 - V]/[\frac{1}{2}\dot\phi^2 + V]$ and $m = 2/3(1+\gamma)$ (during inflation γ and m can vary). It is simple to show that the tensor perturbations are characterized by a power spectrum $|\delta_k^T|^2 \propto k^{n-1}$ and the scalar (density) perturbations by $|\delta_k^S|^2 \propto k^n$, where $n = (m-3)/(m-1)$. In the limit of exponential inflation, $\frac{1}{2}\dot\phi^2/V \to 0$, $m \to \infty$, the tensor and scalar perturbations are scale invariant $(n = 1)$ [17].

The relationships between $\frac{1}{2}\dot\phi^2/V$ and m, m and n, together with Eq. (3), allow us to express the expansion-rate index m and the power-spectrum index n (for $N \sim 60$) in terms of T/S:

$$m = 14\left(\frac{S}{T}\right) + 1/3 \simeq 14\left(\frac{S}{T}\right); \qquad n = 1 - \frac{3\,(T/S)}{21 - \,(T/S)} \simeq 1 - \frac{1}{7}\left(\frac{T}{S}\right). \quad (9)$$

(We remind the reader that the numerical coefficients here depend upon that in Eq. (3), which depends weakly on the ratio T/S for $n \gtrsim 0.5$.) If the tensor mode is to dominate—i.e., $T/S \gtrsim 1$—then m must be less than about 14 and n must be less than about 0.85. The converse is also true: In models where the expansion is exponential and the spectrum is scale invariant, the ratio of tensor to scalar is very small. From the fact that inflation must be "superluminal" ($m > 1$), we can use Eq. (9) to derive an approximate *upper* bound, $T/S \lesssim 20$ [18]. However, the COBE DMR [1] bound on the power-spectrum index n, $n = 1.1 \pm 0.6$, which implies that $n \gtrsim 0.5$ when $T/S \gtrsim 1$, leads to the stronger limit, $T/S \lesssim 3$ (and $m \gtrsim 5$). (Doubtless, there are yet stronger bounds on n based upon structure formation).

We can now apply these results for the specific models for which we found $T/S \gtrsim 1$, extended and chaotic inflation. In extended (or power-law) inflation, the power spectrum is tilted according to $n \simeq (2\omega - 9)/(2\omega - 1)$ and $m = (2\omega + 3)/4$. Using the COBE DMR limit, $n \gtrsim 0.5$, we find a plausible range, $26 \gtrsim \omega \gtrsim 9$. For chaotic inflation, $n \approx 1 - p/120$ and $m \approx 240/p$, leading to a somewhat extreme range, $60 \gtrsim p \gtrsim 18$.

Tensor contributions have significant implications for CMB measurements. First, the COBE DMR results alone do not distinguish scalar from tensor contributions to the anisotropy; see Fig. 1. However, the COBE DMR results, combined with measurements on smaller-angular scales, might distinguish the two. The COBE DMR measurement implies $\langle a_2^2 \rangle = (4.53 \pm 2.5) \times 10^{-10}$, where we should keep in mind that this is a measurement of $\langle a_2^2 \rangle_T + \langle a_2^2 \rangle_S$. Going to smaller-angular scales, the scalar contribution to the CMB anisotropy grows relative to the tensor, but the net contribution to small-angle measurements is diminished compared to no tensor mode at all; see Fig. 2. (We are assuming that no late re-ionization washes out fluctuations on small-angular scales.) Hence, comparing large- and small-angle anisotropy measurements can, in principle, separate the scalar and tensor contributions. (Another possibility for separating the two is to measure the polarization of the CMB anisotropy as the tensor modes lead to a slight polarization [19].)

The tensor mode can seriously affect the interpretation of CMB measurements for large-scale structure, regardless of the form of dark matter. As an example, the best fit cold dark-matter (CDM) model to the COBE DMR results assuming $T/S \ll 1$ has a bias factor $b \simeq 1$. (The bias factor $b \equiv 1/\sigma_8$, where σ_8 is the *rms* mass fluctuation on the scale $8h^{-1}$ Mpc.) If, however, the tensor contribution to the CMB quadrupole is significant, then the extrapolated density perturbation amplitude at $8h^{-1}$ Mpc is reduced, and the best-fit CDM model has $b > 1$; see Fig. 2. Two related effects combine to increase b: the power spectrum is tilted (less power on small scales for fixed quadrupole anisotropy), and scalar perturbations only account for a fraction of the quadrupole anisotropy. We find, very roughly,

$$b \simeq 100^{(1-n)/2}\sqrt{1+T/S} \simeq 10^{(T/S)/7}\sqrt{1+T/S}, \tag{10}$$

where "100" is the ratio of the scale relevant to the quadrupole anisotropy, $\lambda \sim 1000h^{-1}$ Mpc, to the scale $8h^{-1}$ Mpc. For $T/S = 0.53, 1.4, 2.5$, and 3.3, the bias factor $b = 1.4, 2.4, 4.6$, and 7.8 (and $n = 0.92, 0.78, 0.59$ and 0.44). While these numbers should only be taken as rough estimates, the trend is clear: larger T/S permits larger bias.

In sum, if small-angular-scale measurements find $\Delta T/T$ significantly lower than that extrapolated from the COBE DMR quadrupole (see e.g., [20]), there are now at least two possible explanations consistent with inflation. Either re-ionization has washed out the small-angle fluctuations, or tensor fluctuations contribute significantly to the COBE DMR observations. In the latter case, what can CMB studies tell us about inflation? Our analysis suggests a remarkable conclusion—COBE DMR combined with small-angular-scale measurements can directly relate the key cosmological parameters that govern large-scale structure, such as the bias factor b in CDM models and the power-spectrum index n, to the microphysical parameters that control inflation.

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[17] By scale invariance, we mean that density perturbations and gravity waves cross back inside the horizon with amplitudes that are independent of their wavelength; with the usual definitions [7], this corresponds to power spectra with index $n = 1$. Our formula $n = (m-3)/(m-1)$ *only strictly applies to the tensor perturbations:* Since the ratio of scalar to tensor perturbations is proportional to $V/V' \propto \sqrt{m}$ and m varies during inflation (except in exact power-law inflation), m is a function of scale and introduces additional scale noninvariance for the scalar modes. For all the models considered in this paper the difference between n_S and n_T is not significant [e.g., in chaotic inflation, $(1 - n_S) = (1 + m/120)(1 - n_T)$, where $m = 240/p$]. It is possible to construct models where n_S and n_T differ significantly—and even to have $n_S > 1$ and $n_T < 1$—however, the bound we derive, $T/S \lesssim 3$, still holds.

[18] As T/S approaches this upper bound, $m \to 1$ and $n \to -\infty$. In this limit, our assumptions, slow roll ($\dot{\phi} = -V'/3H$) and the coefficient in Eq. (3), break down.

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FIGURE CAPTIONS

1. Temperature auto-correlation function (from the Sachs-Wolfe effect) for tensor and scalar modes each normalized to the COBE DMR quadrupole anisotropy using a scale-invariant spectrum and the COBE DMR window function [1]. CDM predictions [21] for the scalar contribution to $C(0)$ are also shown.

2. Constraints to the CMB anisotropy from various experiments (from [1]) and predictions for the South Pole anisotropy experiment on 1° for CDM models ($\Omega = 1$, $\Omega_B = 0.1$, $h = 0.5$), using the filter function from [22]: Open circle, CDM with $b = 1$, the best-fit CDM model to the COBE DMR if $T/S \ll 1$; Open triangle, CDM with $b = 2$, consistent with the COBE DMR only if $T/S \gtrsim 1$; Closed triangle, upper bound if COBE DMR were detecting pure tensor mode ($T/S \gg 1$).

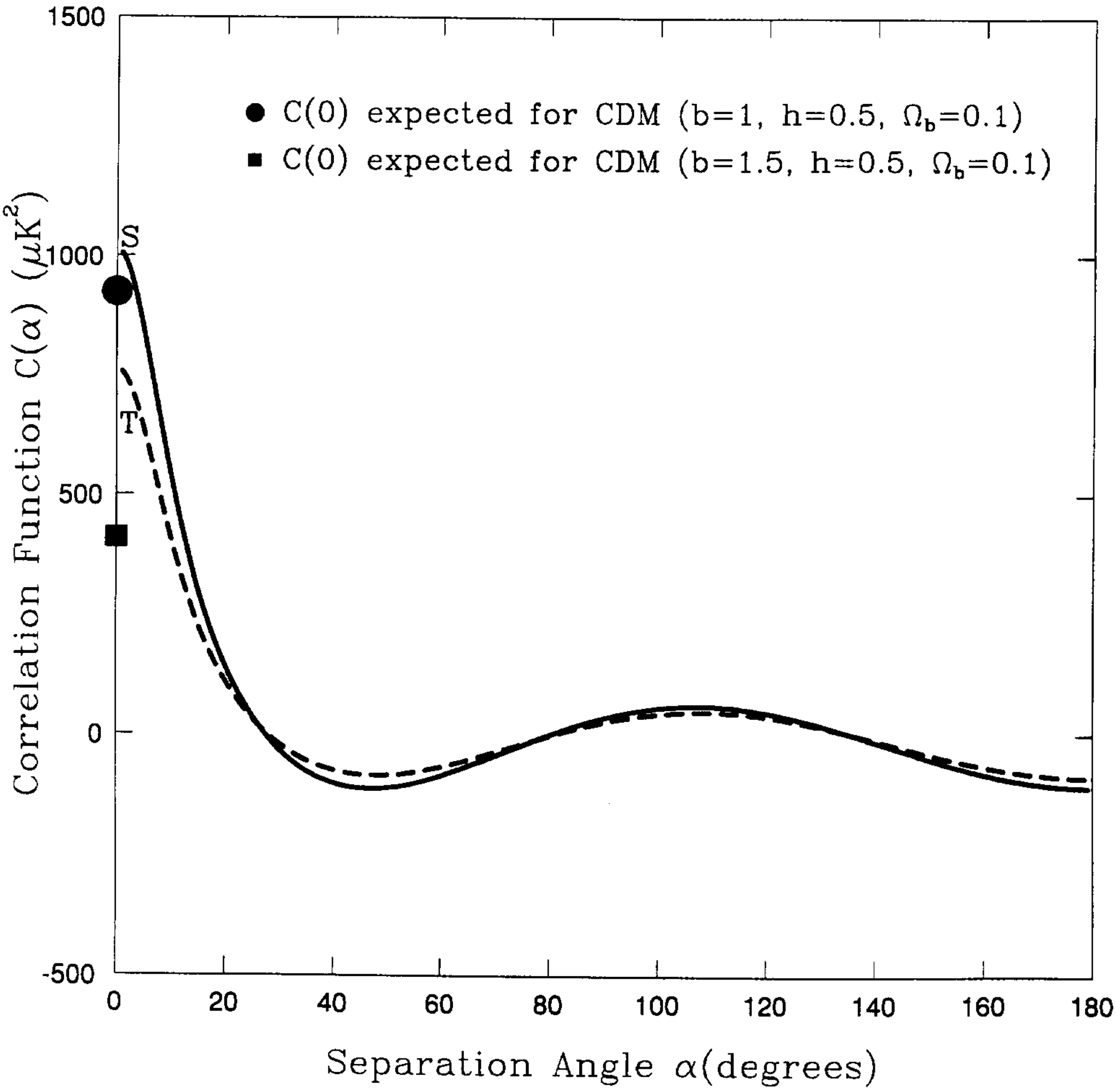
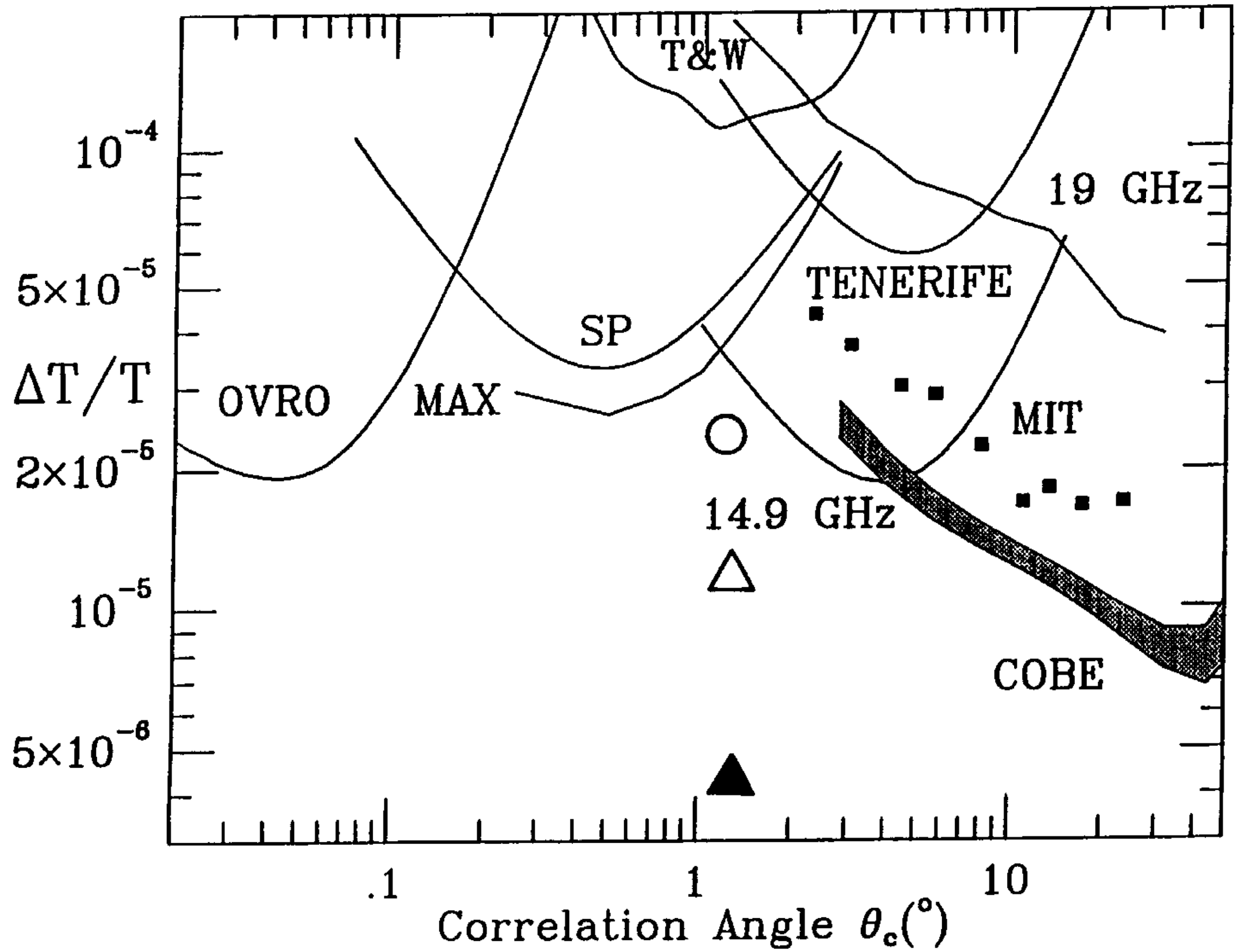

C(0) expected for CDM (b=1, h=0.5, Ωb=0.1)
C(0) expected for CDM (b=1.5, h=0.5, Ωb=0.1)
S
T
1500
1000
500
0
-500
Correlation Function C(α) (μK²)
0
20
40
60
80
100
120
140
160
180
Separation Angle α(degrees)

Figure 1.



10^-4
5×10^-5
ΔT/T
2×10^-5
10^-5
5×10^-6
OVRO
MAX
SP
T&W
TENERIFE
19 GHz
MIT
14.9 GHz
COBE
.1
1
10
Correlation Angle θc(°)

Figure 2.