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BREAKDOWN OF DIMENSIONAL REGULARIZATION IN THE SUDAKOV PROBLEM.

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ABSTRACT

An explicit example is presented (a one-loop triangle graph) where dimensional regularization fails to regulate the infra-red singularities that emerge at intermediate steps of studying large- Q^2 Sudakov factorization. The mathematical nature of the phenomenon is explained within the framework of the theory of the As -operation.

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1 Introduction. The power of dimensional regularization [1] in applied Quantum Field Theory is nothing short of miraculous. Its notational and ideological economy makes it a perfect tool for a wide range of perturbative calculations¹ as well as a convenient instrument for theoretical studies of factorization problems².

Practical calculations in perturbative QCD typically exploit the presence of a large kinematic variable Q and deal with (the leading terms of) the corresponding asymptotic expansion of the cross section. Results describing the leading terms of such expansions generalize the familiar Wilson short-distance operator product expansion and are known as factorization theorems (for a review see e.g. [6]).

The starting point of the construction of such an asymptotic expansion is a formal Taylor expansion of the integrand with respect to the asymptotic parameter of the problem. Such an expansion typically generates infrared singularities, in addition to the ultraviolet divergences of the original diagrams. Even though there exist well-defined prescriptions for eliminating the infrared singularities in the (Wilson) coefficient functions of an expansion, the singularities are important at intermediate stages.

The main advantage of dimensional regularization is that it simultaneously regulates both ultraviolet and infrared divergences while preserving Lorentz and gauge invariance. Situations where dimensional regularization works well comprise all expansion problems of a Euclidean type [4],[7] and many problems of an inherently non-Euclidean (Minkowskian) nature [5].

The aim of the present note is to show that there are singularities that arise in the asymptotic analysis of Minkowski space problems and that cannot be regulated by dimensional continuation. We will pinpoint the origin of the difficulties by means of an explicit one-loop example: a form factor graph. We will show that the nature of the problem is very general so that the same difficulty is bound to arise in many Minkowski space situations.

A subsidiary aim is to popularize the concepts of a new and very general method of analyzing asymptotic behavior of Feynman graphs—the method of the As -operation for products of singular functions [7]. The theory of the As -operation has been worked out in detail for the Euclidean problems [7], [8], where it provides a simple and clean method for treating such problems as renormalization, the operator product expansion, and the large-mass expansion. While the principles of As -operation [7] are very general, there are several technical problems that one has to overcome before a fully satisfactory extension of the method to the most general non-Euclidean situations is achieved. The results we describe represent a first step in that direction.

2 The example. Consider the one-loop diagram of Fig. 1. We will see that neither the possible presence of UV divergences nor the structure of the numerator of the cor-

¹See, e.g., the record-setting 5-loop calculations in the $\varphi_{D=4}^4$ model [3]; the multiloop calculations in QCD [4]; the industrial-scale QCD calculations of multijet hadronic scattering processes [5].

²See, e.g., the derivation of the general Euclidean asymptotic expansions of perturbative Green functions [7]-[8].

responding integrand are relevant; only the structure of the denominators is important. Therefore, it is irrelevant whether the corresponding particles are scalars, spinors or vectors. For concreteness, one can imagine that the horizontal fat line is a gluon in Feynman gauge while the side lines are quarks, and we will use these names for the lines. We consider the asymptotic behavior when $Q^2 \equiv -q^2 = -(p_1 - p_2)^2$ gets large with all other invariants— p_1^2 , p_2^2 and the masses—held fixed.

To focus on the analytical effect we wish to describe, we will choose the quarks to be massless, and the gluon to have mass m ,³ although the choice of masses does not affect the general principles of our method. The external quarks are on-shell: $p_1^2 = p_2^2 = 0$.

We assume that the MS scheme [10] is used for UV renormalization; the dependence on the renormalization parameter μ thus introduced is logarithmic and known explicitly (the renormalized diagram has the form of μ -independent terms plus constant $\times \log \mu$). Anyhow, since we will eventually concentrate on studying the *integrand* prior to loop integration, the UV behavior is also inessential.

There are two essential dimensional parameters in the problem: Q^2 and m^2 . The third parameter is the renormalization scale μ . The Sudakov asymptotic regime is $Q^2 \rightarrow \infty$, with fixed m^2 and μ . By dimensional analysis this is equivalent to $m^2, \mu^2 \rightarrow 0$ with fixed Q^2 . (For definiteness, we will consider the case of a space-like momentum transfer q^μ .) Since the dependence on μ is known explicitly, it is sufficient to consider the expansion at $m^2 \rightarrow 0$ with Q^2 and μ fixed (cf. [12]). This is the most convenient way to proceed within the techniques of the *As*-operation [7].

The integrand for the graph is

$$I(k; p_1, p_2, m) \equiv \frac{1}{k^2 - m^2 + i\eta} \times \frac{1}{(k - p_1)^2 + i\eta} \times \frac{1}{(k - p_2)^2 + i\eta}. \quad (1)$$

Considering contributions to the integral from various regions of integration space is equivalent to considering various integrals of the form

$$G[\phi; p_1, p_2, m] \equiv \int d^D k I(k; p_1, p_2, m) \phi(k), \quad (2)$$

where ϕ is an arbitrary test function (i.e. a smooth function which is non-zero only in a finite subregion of the integration space). So, one arrives at the basic problem of expanding arbitrary integrals of the form (2) in $m \rightarrow 0$ with p_1 and p_2 fixed and light-like. This is exactly the same as to say that one has to expand the *integrand* (1) in powers and logarithms of the small parameter m^2 in the sense of distributions. Since such expansions commute with multiplication by polynomials [7], one can forget about numerators of propagators as well as possible vertex factors.

Construction of an expansion in the sense of distributions involves the following steps [7]: formal Taylor expansion of the product (1) in powers of m ; classification of singularities of such a formal expansion; construction of counterterms to be added to the

³Note that a non-zero gluon mass may have a dynamical origin [9]. Gauge invariance, however important for studying combinatorial properties of entire perturbation series, is of no relevance to the analytical problem of asymptotic expansions of individual graphs. If one wishes, one can consider the graph to be one in an abelian gauge theory, where the gluon can consistently be given a mass.

formal expansion to transform it into a correct expansion in the sense of distributions. Construction of counterterms proceeds in an iterative fashion: from singularities of simpler nature—i.e., to which fewer singular factors contribute—to the more complicated ones; construction of counterterms for the latter involves simpler counterterms obtained at previous iterations.

For practical reasons it is convenient, whenever possible, to employ a regularization throughout the entire procedure. Dimensional regularization is the prime candidate for that role. However, as we will see shortly, it fails to regulate a singular expression at an intermediate step of constructing the expansion in the sense of distributions for (1).

3 Geometry of singularities of the formal expansion. We follow the general procedure that gives the As -operation. First we make a formal expansion of the integrand in powers of m . This gives a correct leading order expansion of the integral with a test function, provided that the test function is zero in a neighborhood of all singularities of the expanded integrand. That is

$$G[\phi; p_1, p_2, m] = \int d^D k \phi(k) \frac{1}{k^2 + i\eta} \times \frac{1}{(k - p_1)^2 + i\eta} \times \frac{1}{(k - p_2)^2 + i\eta} + o(1), \quad \text{if } \phi \text{ is zero near singularities.} \quad (3)$$

Here the remainder is a power of m smaller than the leading term.

To get a complete expansion, valid for all test functions, one should examine a small neighborhood of the singularities of the first term on the right of Eq.(3). The principles of the As -operation prescribe that we should start with the highest dimension singular surface and then treat successively lower dimension surfaces.

Since the integration contours may be deformed away from the light-cone singularity of any single propagator (except at its apex), such a singularity is integrable. Hence the only singularities that we need be concerned with are the apexes of the light cones and the intersections of the light-cone singularities of different propagators,

Now, the first denominator in Eq.(3) generates a singularity localized on the light-cone $k^2 = 0$. The second factor is singular on $(k - p_1)^2 = 0$ which is nothing but the light-cone $k^2 = 0$ shifted so that its apex is at the point $k = p_1$. Since p_1 is a light-like vector, the two light cones intersect on the straight line

$$A = \{k = z_1 p_1, \quad -\infty < z_1 < +\infty\}. \quad (4)$$

This intersection is non-trivial because the light cones are not transverse at the intersection points. Similarly, the singularities of the first and third factor overlap on the line

$$B = \{k = z_2 p_2, \quad -\infty < z_2 < +\infty\}. \quad (5)$$

The singularities of the second and third factors intersect on a smooth manifold and are transverse there; this implies that in a small neighborhood of each intersection point

the integral factorizes, so that the singularity is integrable and such an intersection is harmless. The exception is the point $k = 0$ where the first denominator is also zero, and we consider this point separately.

There are three points—namely, $k = 0$, $k = p_1$, and $k = p_2$ —where the analytical nature of singularities is more complicated than at the generic points of the lines (4)–(5). In particular, the singularity

$$S = \{k = 0\}, \quad (6)$$

is where the effect we are after takes place.

The geometrical pattern of singularities can be visualized as in Fig. 2.

4 Structure of singularities at $k \propto p_1$. Consider singularities near a generic point on the line A , (4). Fix $z \neq 0, 1$, and consider a small neighborhood \mathcal{O} of the point zp_1 . The third factor of the integrand is smooth in \mathcal{O} and, therefore, can be effectively relegated to the test function. So, within \mathcal{O} it is sufficient to study the expansion of the product of only the first two factors that contribute non-trivially to the singularity in \mathcal{O} . It is convenient to choose light-cone coordinates, $k = (k_+, k_-, k_\perp)$, so that $p_1 = (p_{1+}, 0, 0_\perp)$, $p_2 = (0, p_{2-}, 0_\perp)$. Our conventions will be such that $k^2 = k_+k_- - k_\perp^2$, $2k \cdot p_1 = p_{1+}k_-$, and $d^D k = \frac{1}{2}dk_+dk_-d^{D-2}k_\perp$.

Then the formal expansion of the first two factors takes the form:

$$\begin{aligned} & \frac{1}{zp_{1+}k_- - k_\perp^2 - m^2 + i\eta} \times \frac{1}{(z-1)p_{1+}k_- - k_\perp^2 + i\eta} \\ &= \frac{1}{zp_{1+}k_- - k_\perp^2 + i\eta} \times \frac{1}{(z-1)p_{1+}k_- - k_\perp^2 + i\eta} + o(1). \end{aligned} \quad (7)$$

One can see that the singularities of the product on the r.h.s. are localized at the origin of the space of the variables k_- and k_\perp . This means that (7) holds in the sense of distributions on test functions that are zero near $k_- = k_\perp = 0$. The situation here is very similar to what one has in the case of a single Euclidean propagator treated in section 7 of [7]—see especially eqs. (7.25) and (7.26). The only important difference is that the singular functions on the r.h.s. are not homogeneous. This is remedied by the change of variable $k_- = \pm t^2$ after which the expressions on the r.h.s. of (7) become homogeneous with respect to simultaneous scaling in t and k_\perp , and simple power counting shows that the singularity is logarithmic (at $D = 4$).

This allows one to repeat the reasoning of section 7 of [7] to write down the following analogue of eq. (7.25) of [7]:

$$\begin{aligned} \text{The l.h.s. of eq.(7)} &= \frac{1}{zp_{1+}k_- - k_\perp^2 + i\eta} \times \frac{1}{(z-1)p_{1+}k_- - k_\perp^2 + i\eta} \\ &+ c_A(m^2, z) \frac{1}{p_{1+}} \delta(k_-) \delta^{(D-2)}(k_\perp) + o(1), \end{aligned} \quad (8)$$

with

$$\begin{aligned}
c_A(m^2, z) &= p_{1+} \int dk_- d^{D-2} k_{\perp} \left[\frac{1}{z p_{1+} k_- - k_{\perp}^2 - m^2 + i\eta} \times \frac{1}{(z-1) p_{1+} k_- - k_{\perp}^2 + i\eta} \right. \\
&\quad \left. - \frac{1}{z p_{1+} k_- - k_{\perp}^2 + i\eta} \times \frac{1}{(z-1) p_{1+} k_- - k_{\perp}^2 + i\eta} \right] \\
&= p_{1+} \int dk_- d^{D-2} k_{\perp} \frac{1}{z p_{1+} k_- - k_{\perp}^2 - m^2 + i\eta} \times \frac{1}{(z-1) p_{1+} k_- - k_{\perp}^2 + i\eta}. \quad (9)
\end{aligned}$$

In the second form of this integral we have used the fact that the integral of a homogeneous function⁴ is zero within dimensional regularization.

At this point dimensional regularization works. Recall [7] that the role of the counterterm c_A is two-fold: first, it cancels the divergence of the first singular expression (after integration with test functions); second, it ensures that the expansion is asymptotic ($\mathcal{O}(1)$) on any test function. The expansion (8)–(9) holds in the sense of distributions on any test function (in the variables k_-, k_{\perp}). The simplification gained is that, whereas the l.h.s. of (8) has a complicated dependence on m , the first two terms on the right have a simple power dependence, which becomes logarithmic at $D = 4$. This we will verify from the explicit calculation of C_A in the next section.

5 Explicit expressions for counterterms. It is not difficult to perform the integrations in (9) explicitly⁵ to obtain

$$c_A(m^2, z) = 2i \theta(0 < z < 1) \Gamma(\epsilon) \pi^{2-\epsilon} [(1-z)m^2]^{-\epsilon}, \quad (10)$$

where $\epsilon = (4 - D)/2$.

There are a few points worth making here. First, the counterterm is zero for $z > 1$ and $z < 0$. This agrees with the fact that according to the Landau equations⁶ the singularities at those values of z are not pinched. In the language of distribution theory, this says that for $z < 0$ and $z > 1$ the product in the first term of the r.h.s. (8) is (*i*) well-defined in the

⁴Recall that the reasoning that leads to this expression is, strictly speaking, done in the “straightened” coordinates (t, k_{\perp}) with homogeneous functions.

⁵Recall that the definition of dimensional regularization [1] prescribes that we should perform the integrations over the transverse components first. In the present case, however, the same result is reproduced in a slightly easier way if one first performs integration over k_- . At this point one should be aware of the fact that the underlying definition of the integral is in terms of the homogeneous coordinates, and the cutoffs that are present at intermediate stages (cf. [7]) are symmetric with respect to the homogeneous coordinates and become asymmetric in the coordinates k_-, k_{\perp} . Such subtleties stress the need for a meaningful rigorous definition of dimensional regularization in momentum/coordinate space representations. Some of the issues involved will be discussed in a forthcoming publication [15].

⁶See e.g. [11]. Note that the Landau equations are usually associated with studying analyticity properties of Feynman diagrams. Finding cuts etc., however, is equivalent to determining when an expansion near the corresponding value of external momenta and masses contains non-trivial (i.e. non-analytic) contributions. In the context of the expansion problem proper this issue was reconsidered by Libby and Sterman [12]. A reinterpretation of the Landau equations from the point of view of asymptotic expansions of distributions is presented in [13].

sense of distributions and (ii) does not require additional counterterms to approximate the l.h.s. in the sense of distributions to $o(1)$. (In general, (ii) does not necessarily follow from (i).)

Second, the counterterm (10) is the only term in (8) that contains a non-analytic dependence on m (cf. the discussion of the role of such counterterms in [7]). After expansion in ϵ the dependence becomes logarithmic.⁷

Third, the expression (10) and all its derivatives in z have well-defined limits as $z \rightarrow +0$. This conclusion does not change if the second (quark) propagator in (1) and, correspondingly, in the first term of (7) contains a non-zero mass, say, m_1 , because then the last factor in (10) will simply be replaced by $[zm_1^2 + (1-z)m^2]^{-\epsilon}$. Note, however, that the presence of mass in the first factor (the bottom line or gluon in Fig. 1) is important.

Finally, it is not difficult to rewrite the above expansion in an explicitly covariant form:

$$\begin{aligned} & \frac{1}{k^2 - m^2 + i\eta} \times \frac{1}{(k - p_1)^2 + i\eta} \\ &= \frac{1}{k^2 + i\eta} \times \frac{1}{(k - p_1)^2 + i\eta} + \int_0^1 dz \delta^{(D)}(k - zp_1) c_A(m^2, z) + o(1). \end{aligned} \quad (11)$$

We have explicitly taken into account that c_A vanishes outside the interval $0 < z < 1$. Note that the δ -function on the r.h.s. is D -dimensional. This expression represents the leading power term in a correct asymptotic expansion in the sense of distributions on test functions that are zero in small neighborhoods of $k = 0$ and $k = p_1$. Note that a similar expansion is obtained for the expansion of the product of the first and third factors of (1) (with p_1 replaced by p_2).

6 Taking into account the third factor. Let us now consider the entire expression (1). In order to transform its formal expansion, the integrand of (3), into a well-defined expansion in the sense of distributions, the general recipe of the Extension Principle of [7] tells to add counterterms localized at singular points of the formal expansion in (3) with properly chosen coefficients. Let us show that the expansion that is valid on the test functions that vanish in neighborhoods of the points $k = 0$, $k = p_1$ and $k = p_2$ is given by the following formula:

$$\begin{aligned} & \frac{1}{k^2 - m^2 + i\eta} \times \frac{1}{(k - p_1)^2 + i\eta} \times \frac{1}{(k - p_2)^2 + i\eta} \\ &= \frac{1}{k^2 + i\eta} \times \frac{1}{(k - p_1)^2 + i\eta} \times \frac{1}{(k - p_2)^2 + i\eta} \\ & \quad + \frac{1}{(k - p_2)^2 + i\eta} \times \int_0^1 dz \delta^{(D)}(k - zp_1) c_A(m^2, z) \\ & \quad + \frac{1}{(k - p_1)^2 + i\eta} \times \int_0^1 dz \delta^{(D)}(k - zp_2) c_B(m^2, z) + o(1). \end{aligned} \quad (12)$$

⁷It is possible to avoid the use of dimensional regularization in expressions like (8)–(9)—cf. [14] where the results of [7] are presented in a regularization-independent form.

Indeed, any test function $\varphi(k)$ that vanishes in small neighborhoods of the points $k = 0$, $k = p_1$ and $k = p_2$, can be represented as a sum $\varphi_1 + \varphi_2$ where φ_1 is zero around the line $k \propto p_2$ and φ_2 is zero around the line $k \propto p_1$. On φ_1 , the second counterterm vanishes and we are left with the product of the expansion (11) times the third factor. Since the product of the third factor and φ_1 is a valid test function $\tilde{\varphi}_1$, one arrives at a correct expansion to $o(1)$. A similar reasoning is applied to φ_2 (note that the function c_B coincides with c_A in our example).

It follows that the expansion (12) is actually correct for all test functions that vanish in small neighborhoods of the points $k = 0$, $k = p_1$ and $k = p_2$. To make the expansion valid on *all* test functions, one has to add appropriate counterterms localized at the points $k = 0$, $k = p_1$, and $k = p_2$. Such counterterms are, in general, linear combinations of δ -functions and their derivatives localized at those points with coefficients depending on the expansion parameter in a non-analytic (logarithmic) manner [7].

To construct the additional counterterms, one must (i) perform an appropriate power counting in order to determine the strength of the singularity; (ii) introduce an intermediate regularization to make the singularity manageable (in Euclidean problems [7] dimensional regularization automatically regulates all singularities) or perform an explicit subtraction (as in [14]); (iii) determine an explicit form of the counterterms that need to be added in order to ensure the approximation property of the resulting expansion.

It will be at step (ii) that dimensional regularization fails in our example.

7 Singularity at $k = 0$. Let us focus on the point $k = 0$. One has to study the singularity of the entire r.h.s. of (12) at $k \rightarrow 0$. The r.h.s. of (12) contains contributions that are analytic in m , and those that are not. It is sufficient to consider the latter since they cannot be affected by how the analytic contributions are treated, and it is the non-analytic terms that will exhibit the failure of dimensional regularization.

The terms with non-analytic dependence in m are known explicitly in our case:

$$\begin{aligned} & \left[\frac{1}{(k - p_2)^2 + i\eta} \times \int_0^1 dz \delta^{(D)}(k - zp_1) c_A(m^2, z) + (1 \leftrightarrow 2) \right] \\ & = \left[\frac{1}{Q^2} \int_0^1 dz \frac{1}{z - i\eta} \delta^{(D)}(k - zp_1) c_A(m^2, z) + (1 \leftrightarrow 2) \right]. \end{aligned} \quad (13)$$

One can immediately see that:

- (a) One has to deal with the product of a one-dimensional distribution $1/(z - i\eta)$, which itself is well-defined if integrated with *smooth* test functions, times $\theta(z)$. The resulting expression is singular and ill-defined at $z = 0$. The distribution $1/(z - i\eta)$ is generated from the propagator $1/[(k - p_2)^2 + i\eta]$ when we set $k = zp_1$.
- (b) Dimensional regularization does not regulate this singularity because the form of the product is independent of ϵ and there remain no “unused” extra dimensions.

- (c) There are no cancellations at $k = 0$ between contributions from the two counterterms corresponding to the two singular lines.

It remains to note that the effect of breakdown of dimensional regularization in the above example persists (even if the formulas become more cumbersome) if one introduces masses of order m —say, m_1 and m_2 —into the two quark propagators, or allows the external quarks to be off-shell by $O(m^2)$. One can also see that the same configuration of singularities emerges e.g. in the studies of the large- s limit (see Fig. 3). All this points to universality of the phenomenon of the breakdown of dimensional regularization in expansion problems in non-Euclidean asymptotic regimes.

8 Conclusions. We have considered a rather typical non-Euclidean expansion problem (a one-loop form factor graph in the Sudakov asymptotic regime) within the framework of the theory of As -operation, and we saw some significant differences from Euclidean problems. In particular, we have discovered a class of singularities which are not regulated by dimensional regularization.

The origin of the dimensionally-nonregularizable singularities in non-Euclidean asymptotic expansion problems is completely general:

First, the non-trivial (“pinched”) singularities of the expanded integrands in the case of non-Euclidean asymptotic regimes may be localized on manifolds with boundaries (which is never the case for Euclidean regimes, where singularities are always localized on linear subspaces of the space of integration momenta).

Second, construction of a complete expansion requires introduction of counterterms that contain the non-analytic dependence on the expansion parameter and are localized on such manifolds; such counterterms may have coefficients that, together with their derivatives, possess finite non-zero limiting values at the boundaries of such manifolds when the boundaries are approached from within the manifold ($z \rightarrow +0$ in our case) while being identically zero outside the boundary. In other words, if the boundary is described by the equation $z = 0$ in local coordinates with $z > 0$ corresponding to the pinched submanifold, then the coefficients near the boundary have the form $\theta(z) \times$ (a smooth function of z)—even for $D \neq 4$.

Third, light-cone singularities of the factors that do not contribute to such counterterms may pass over the boundaries of the corresponding manifolds; when projected onto such manifolds (which is exactly what happens when one introduces the counterterms into the entire product—cf. (13)), they take the form $1/(z \pm i\eta)$.

Fourth, such manifolds may be geometrically positioned so that the extra dimensions that are instrumental in the mechanism of dimensional regularization are “used up” in the counterterms and do not provide any suppression for the resulting singular product of the type $\theta(z) \times 1/(z \pm i\eta)$.

It should be emphasized that the problem here is not an ambiguity as in the case of γ_5 , but a failure of dimensional regularization to regulate a particular class of infra-

red singularities. Moreover, since expansions in the sense of distributions in powers and logarithms of the expansion parameter are unique, one has to conclude that it is impossible to get rid of the problem by choosing a different “factorization scheme”.

The problem is certainly associated with our insistence on strict power-and-logarithm expansions: The $1/z$ singularity gives a problem because we have expanded everything else in the integrand in powers of a small variable. However, the requirement that the expansions to be constructed run in powers and logarithms of the expansion parameter (the requirement of “perfect factorization” [7]) cannot be relaxed for both phenomenological and technical reasons. In particular, such expansions possess the property of uniqueness which greatly facilitates iterative construction of the expansions, relieving one of having to worry about unitarity and gauge invariance of the final results etc. [7]. This is particularly true in non-Euclidean problems, since it is only at the leading twist level that we get simple factorization theorems.⁸

On the other hand, the existence and nature of the anomalies one may have to deal with as a result of the effect we have described is not obvious. One thing is clear, however: whether one opts for other regularizations (e.g. analytic regularization which would replace $1/(z \pm i\eta)$ in the above expressions by $(z \pm i\eta)^{-\lambda}$), or chooses to combine dimensional regularization with a formalism involving direct subtractions as in [14]—or to forgo dimensional regularization altogether in favor of the latter—the consequences may be rather unpleasant both for practical calculations and for the general theory of higher-twist factorization.

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⁸In the method one of us has given [16] for treating the Sudakov form factor at the complete leading twist level, non-dimensionally-regularizable singularities are avoided either by the use of axial gauge or by the use of an equivalent trick in Feynman gauge. One consequence of the resulting lack of “perfect factorization” is an annoying proliferation of remainder terms in Ward identities. These are especially tricky to handle in a non-abelian theory.

References

- [1] G. 't Hooft and M. Veltman: Nucl. Phys. B44 (1972) 189; for a review see e.g. [2].
- [2] J. C. Collins: "Renormalization", Cambridge University Press, 1984.
- [3] F. V. Tkachov: Phys. Lett. 100B (1981) 65; K. G. Chetyrkin, S. G. Gorishny, S. A. Larin and F. V. Tkachov: Phys. Lett. 132B (1983) 351; S. G. Gorishny, S. A. Larin and F. V. Tkachov: Phys. Lett. 101A (1984) 120.
- [4] O. V. Tarasov, A. A. Vladimirov and A. Yu. Zharkov: Phys. Lett. 93B (1980) 429; S. A. Larin, F. V. Tkachov, and J. A. M. Vermaseren: Phys. Rev. Lett. 66 (1991) 862; S. G. Gorishny, A. L. Kataev and S. A. Larin: Phys. Lett. 259B (1991) 144.
- [5] R. K. Ellis, M. A. Furman, H. E. Haber and I. Hinchliffe: Nucl. Phys. B173 (1980) 397; T. Matsuura, S. van der Marck and W. van Neerven: Nucl. Phys. B319 (1989) 570; W. T. Giele and E. W. N. Glover: FERMILAB-PUB-91/100-T (to be published in Phys. Rev. D).
- [6] J. C. Collins, D. E. Soper and G. Sterman: "Factorization of hard processes in QCD", in: "Perturbative QCD" (A. H. Mueller, ed.), World Scientific, Singapore, 1989.
- [7] F. V. Tkachov: "Euclidean Asymptotic Expansions of Green Functions of Quantum Fields. (I) Expansions of Products of Singular Functions", preprint FERMILAB-PUB-91/347-T; Int. J. Mod. Phys. A (1992) (in print) and refs. therein.
- [8] G. B. Pivovarov and F. V. Tkachov: "Euclidean Asymptotic Expansions of Green Functions of Quantum Fields. (II) Combinatorics of the As -operation", preprint FERMILAB-PUB-91/345-T; Int. J. Mod. Phys. A (1992) (in print); A. N. Kuznetsov, F. V. Tkachov and V. V. Vlasov: "Techniques of distributions in perturbative quantum field theory", preprint PSU-TH-108/92.
- [9] J. M. Cornwall: Phys. Rev. D26 (1982) 1453.
- [10] G. 't Hooft: Nucl. Phys. B61 (1973) 455.
- [11] R. J. Eden, P. V. Landshoff, D. I. Olive and J. C. Polkinghorne: "The Analytic S-Matrix", Cambridge University Press, 1966.
- [12] S. Libby and G. Sterman: Phys. Rev. D18 (1978) 3252, 4737.
- [13] F. V. Tkachov: in preparation.
- [14] A. N. Kuznetsov, F. V. Tkachov and V. V. Vlasov: "Techniques of Distributions in Perturbative Quantum Field Theory. (I) As -operation for products of singular functions", preprint PSU/TH/108, June 1992 (to be published).
- [15] A. N. Kuznetsov and F. V. Tkachov: (in preparation).
- [16] J. C. Collins, Phys. Rev. D22 (1980) 1478; "Sudakov Form Factors" in: "Perturbative QCD" (A. H. Mueller, ed.), World Scientific, Singapore, 1989.

Figure captions.

- Fig. 1. The triangle graph corresponding to our example (1). The bottom line corresponds to a gluon with non-zero mass m , and the side lines to massless quarks.
- Fig. 2. The geometrical pattern of singularities due to the three denominators of the formal expansion, (3).
- Fig. 3. A configuration of propagators (and singularities) essentially similar to that in Fig. 1 emerges in the large- s small- t problem.

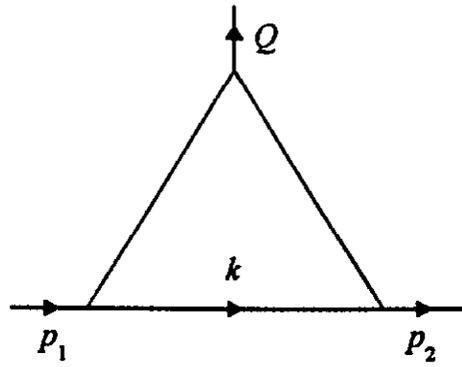


Fig. 1

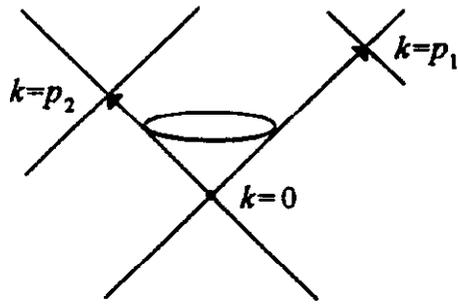


Fig. 2

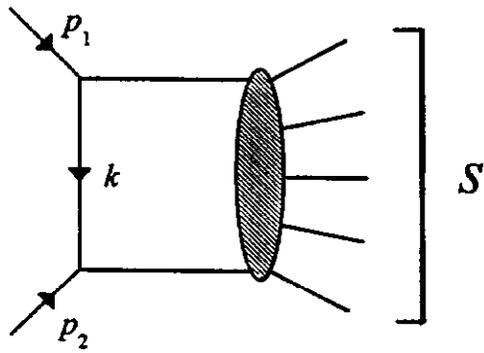


Fig. 3