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Nonperturbative Corrections to Inclusive Beauty and Charm Decays : QCD versus Phenomenological Models

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Abstract

We present a selfconsistent method for treating nonperturbative effects in inclusive nonleptonic and semileptonic decays of heavy flavour hadrons. These effects give rise to powerlike corrections $\propto 1/m_Q^n$, $n \geq 2$ with m_Q denoting the heavy quark mass. The leading correction to the semileptonic branching ratio occurs for n=2. It is expressed in terms of the vector-pseudoscalar mass splitting: $\delta BR_{sl}/BR_{sl} \simeq BR_{nl} \cdot 6((M_V^2 - M_P^2)/m_Q^2) \cdot (c_+^2 - c_-^2)/2N_c$ and yields a *reduction* of BR_{sl}. This nonperturbative correction contributes to the nonleptonic width with a sign opposite to that of the perturbative terms that are non-leading in $1/N_c$. In beauty decays the former reduces the latter by 20 % whereas in charm decays they more or less cancel. This leads to a *reduction* of BR_{sl} by no more than 10 % in beauty decays and by a factor of roughly two in charm decays. We confront these results with those obtained from phenomenological models of heavy flavour decays and find that such models are unable to mimic these leading corrections by a specific choice of quark masses or by invoking Fermi motion.

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Strong interactions affect weak decays of hadrons in two ways:

(i) They modify the *quark level* weak Lagrangian.

(ii) They introduce bound state dynamics in the initial state and drive hadronization in the final state.

It is in particular effect (ii) that we are at present unable to treat in a satisfactory way. Yet it has always been expected that the *inclusive* weak decays of hadrons containing a sufficiently heavy quark Q can be dealt with quantitatively [1]. The inclusive decay widths for heavy flavour hadrons H_Q is usually computed by considering the decay of a 'quasifree' quark Q in complete analogy to the decays of muons and τ leptons. Such a treatment is refined by computing the 'ultraviolet' renormalization of the effective $|\Delta Q| = 1$ Lagrangian; this will incorporate the main effects listed under point (i) above. (One can also include next-to-leading corrections due to perturbative gluon bremsstrahlung.) This decay mechanism is usually referred to as the 'spectator ansatz'. Among other things it predicts uniform lifetimes and semileptonic branching ratios for hadrons of a given flavour Q. This procedure appears to work reasonably well in beauty decays. In charm decays on the other hand the agreement of such expectations with the data is much poorer as lifetime ratios of two to three emerge. This shows the presence of large 'preasymptotic' corrections; yet it is fair to point out that even this represents a vast improvement over the situation in strange decays. Furthermore some "non-spectator" effects have been identified and were found to be indeed of order unity in charm decays and thus in qualitative agreement with the pattern in the data; the analogous corrections produce only percent level effects in inclusive beauty decays [2, 4, 5].

There are however serious shortcomings to this simple approach:

• Bound state effects as well as hadronization – listed above under (ii) – are not included in such a simple treatment. This severely limits the accuracy level one can hope for.

• Various *phenomenological* decay models have been employed to include these long distance forces. Yet they invariably introduce new *a priori* free input parameters that have to be fitted from the data. This limits the predictive power of the theoretical treatment considerably. Furthermore it is quite unclear how various systematic uncertainties inherent in these models can be estimated in a reliable way.

The goal of this letter is to outline in some detail a general procedure that

(a) enables us to calculate *inclusive* nonleptonic as well as semileptonic decay rates in terms of the fundamental parameters like the KM parameters;

(b) allows for a *systematic* evaluation of the theoretical uncertainties that can be refined *successively;*

(c) is *intrinsically* connected to *QCD* without having to call upon a phenomenological *"deus ex machina"*.

The numbers we are going to present in this paper are intended to illustrate the method and show the trend of the effects. They should not be taken as the final numerical results. We will discuss how a consistent application of our approach can lead to more precise numbers in the future.

The remainder of the paper will be organized as follows: in Sect.1 we introduce the heavy mass expansion we are going to employ in our analysis; in Sect.2 we discuss

semileptonic branching ratios before presenting our conclusions and an outlook in Sect.3.

1 Heavy Mass Expansion

In our analysis we will follow the general method outlined by Shifman and Voloshin in ref.[4]. It was suggested there to obtain the inclusive decay widths of heavy flavour hadrons H_Q from the absorptive parts of the forward amplitude $Q \to Q$ for the heavy quark Q to second order in the weak Lagrangian; the simplest such diagram is shown in Fig.1. Integrating out the fields in the intermediate state one can expand this absorptive part into a series of inverse powers of the heavy quark mass m_Q.

The leading contribution is obtained from the absorptive part of the two-loop diagram in Fig.1: it is given by an effective *operator* of the form

$$\hat{\Gamma}_o = \frac{G_F^2}{192\pi^3} \cdot N_c \cdot |KM|^2 \cdot m_Q^5 \, \bar{Q}Q \quad . \tag{1}$$

N_c denotes the number of colours and KM the appropriate KM factors; the masses of q_1, $\bar{q}_2$ and q_3 have been ignored here for simplicity. The standard quasifree decay width alluded to above is obtained when one evaluates the *matrix element* of this operator between nonrelativistic (with respect to Q) hadronic states that contain the heavy quark Q [4, 5]; this is explained in somewhat more detail later on.

When the intermediate (anti)quark lines are cut and treated as *external* quark fields – see Fig.2 a,b – one obtains contributions that are non-leading in $1/m_Q$. This procedure generates $SU(2)$ and $SU(3)$ *non-singlet* operators that create lifetime differences between the different kinds of mesons H_Q with a given *heavy* flavour Q, namely $\bar{Q}u$, $\bar{Q}d$ and $\bar{Q}s$. The diagram in Fig.2a [2b] is usually referred to as 'Weak Annihilation' or 'W Exchange' ['Pauli Interference']; similar diagrams also describe 'Weak Scattering' in baryons. Attention in the literature has so far focused on these kinds of preasymptotic effects [6, 7, 4, 5, 8].

At tree level one obviously obtains four-fermion operators of dimension *six* in this way

$$\hat{\Gamma}_{\text{WA, PI}} \propto \frac{G_F^2}{12\pi} \cdot |KM|^2 \cdot m_Q^2 \, (\bar{Q}\Gamma Q)(\bar{q}\Gamma q) \tag{2}$$

where q is one of the light quark fields and Γ denotes a combination of γ-matrices. These operators thus yield a contribution of order $1/m_Q^3$ relative to the spectator result stated in eq.(1). In ref.[2] this classification was justified for *inclusive* transitions also in the presence of gluon emission and it was shown that the latter merely renormalizes – in a calculable way – the coefficients of these operators.

The $1/m_Q$ expansion enables us to incorporate successively all possible nonperturbative corrections to the decay widths. There are also preasymptotic contributions that affect *uniformly* all hadrons H_Q of a given heavy flavour Q, at least within an isomultiplet; they lead actually to larger corrections than those stated in eq.(2) as will be shown now. These contributions come from the quasifree two-loop amplitude of Fig.1 albeit with the interaction of quarks with soft gluons included. Those effects

are most conveniently dealt with by calculating the $Q \to Q$ amplitude in the background gluon field: the inclusive width is then obtained as an expansion in terms of *local* operators that contain $\bar{Q}Q$ and the gluon field. As the first byproduct one sees that there cannot be terms *linear* in $1/m_Q$. For only operators of dimension *four* that contain $\bar{Q}Q$ could induce them. Yet such an operator is either a total derivative and thus has to vanish; or it can be reduced – via the equations of motion – to the original quasifree operator $\bar{Q}Q$ thus merely redefining the overall coefficient entering eq.(1).

The first nonleading operator in this expansion carries dimension *five* and is the chromomagnetic dipole operator for the heavy quark Q:

$$D_G = \bar{Q}\, i\sigma_{\mu\nu}\hat{G}_{\mu\nu}\, Q \tag{3}$$

where $\hat{G}_{\mu\nu} = gG^a_{\mu\nu}\lambda^a/2$. It is interesting to note that the same operator was used in ref.[9] to determine the leading corrections to the factorizable amplitude for the *exclusive* decay $B \to D\pi$.

As already stated there are dimension *six* four-fermion operators that appear explicitly by cutting one of the light quark propagators. We would like to add here that dimension six operators that do not contain light quark fields as *external* legs (and hence cannot cause a splitting among the decay widths of hadrons within the same isomultiplet) are in general of little practical relevance. For such operators can arise only by embedding the light quark lines into a vacuum condensate. Yet for purely left-handed weak vertices such effects are proportional to the current quark masses; thus they will be small and in general insignificant (unless one studies the question of $SU(3)$ breaking).

Operators containing additional gluon fields appear on the dimension *seven* level and thus can give corrections of order $1/m_Q^4$ only. Calculating them is in principle straightforward, yet probably of little practical value since their matrix elements are mostly unknown. Furthermore there are quite a few of them and for this reason it is unlikely that they can be directly related to observable quantities. These contributions could therefore well represent the intrinsic limit on the numerical accuracy of our approach. Yet since they are of order $1/m_Q^4$ they are expected to induce only rather small corrections. A presumably conservative estimate of the accuracy of the expansion described here could be obtained by considering only the factorizable contributions which are expected to be dominant. It should be possible to estimate their impact since there are fewer operators that contain them and these contributions have a simpler structure.

Once the *operator product expansion* has been obtained up to a certain order one has to address the delicate question of how to evaluate the *matrix elements* of the various local operators. For even the quasifree operator $\bar{Q}Q$ receives nonperturbative contributions. This is most clearly seen by again employing a heavy quark expansion. The situation here is actually simpler than the one encountered in a general application of Effective Heavy Quark Theory [10] (hereafter referred to as *EHQT*): for we are here interested only in 'zero recoil' amplitudes. We can then simply use a nonrelativistic expansion

$$\bar{Q}Q = v_\mu\bar{Q}\gamma_\mu Q - 1/4m_Q^2\,\bar{Q}\,i\sigma G\,Q - 1/2m_Q^2\,\bar{Q}\,(D^2 - (v_\mu D_\mu)^2)\,Q\ +\ O(1/m_Q^3) \tag{4}$$

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As before, linear terms in $1/m_Q$ vanish due to the equation of motion.

Concerning the matrix elements three observations can be made:

(i) The first operator on the right hand side of eq.(4) is the generator of the conserved charge that is associated with the heavy flavour Q. Its matrix element taken between heavy flavour hadrons H_Q therefore yields *exactly* unity.

(ii) The matrix elements of the second operator between meson states can be directly expressed in terms of the mass splitting between the vector and the pseudoscalar $\bar{Q}q$ boundstates; it is thus extracted from the data. The matrix element between the heavy flavour baryon Λ_Q on the other hand vanishes.

(iii) The third term represents the kinetic energy of the heavy quark in the presence of the gluon background field. Its matrix element is quite possibly different when taken between baryon rather than meson states. So far we have not found a reliable way for extracting the size of these matrix elements from a direct phenomenological analysis. This operator enters the subleading $1/m_Q$ corrections to the masses of hadrons H_Q:

$$M_{H_Q} = m_Q + \mu_o + \mu_1^2/m_Q + \ldots \quad .$$

Therefore one piece of information can be gained here by considering the masses of charm and beauty hadrons and actually that combination from which the chromomagnetic contribution drops out:

$$(3M_D + M_{D^*} - 4M_{\Lambda_c}) - (3M_B + M_{B^*} - 4M_{\Lambda_b}) \simeq$$

$$\simeq 2(1/M_D - 1/M_B) \cdot (\langle \text{baryon}|\bar{Q}\vec{D}^2 Q|\text{baryon}\rangle - \langle \text{meson}|\bar{Q}\vec{D}^2 Q|\text{meson}\rangle) \quad . \tag{5}$$

(The vanishing of the anomalous dimension for the kinetic energy has been taken into account here). Once a precise value for the Λ_b mass has been obtained, we can then extract one combination of matrix elements for the $\bar{Q}\vec{D}^2 Q$ operator; the typical scale here however is probably only about 50 MeV . We believe that an analysis based on *QCD* sum rules could be developed to estimate this term both in mesons and in baryons or at least a complementary combination of matrix elements.

Fortunately many interesting quantities do not depend on this operator. For it appears only in the nonrelativistic expansion of the operator $\bar{Q}Q$ and as such contributes with the same weight to all decays of a given hadron H_Q; accordingly it will not affect the semileptonic *branching ratio* which will be discussed next.

2 Semileptonic Branching Ratio

As a topical application of this general approach let us consider the semileptonic branching fraction for beauty hadrons. As explained before the $1/m_Q^2$ nonperturbative corrections to this quantity are given by the matrix element of the chromomagnetic operator. A rather straightforward calculation yields for nonleptonic transitions in the external gluon field (hereafter we omit the obvious KM factors)

$$\hat{\Gamma}_{nl} = \frac{G_F^2 m_b^5}{192\pi^3} \cdot N_c \left\{ A_0 z_0 \cdot (\bar{b}b - \frac{1}{m_b^2} \bar{b} i\sigma G b) - A_2 z_2 \cdot \frac{4}{m_b^2} \bar{b} i\sigma G b) \right\} \tag{6}$$

where A_0, A_2 denote colour factors

$$A_0 = c_1^2 + c_2^2 + \frac{2}{N_c}c_1c_2 + O(\alpha_s(m_b^2))\,, \quad A_2 = \frac{2}{N_c}c_1c_2 + O(\alpha_s(m_b^2)) \tag{7}$$

$$c_1 = (c_+ + c_-)/2\,, \quad c_2 = (c_+ - c_-)/2 \tag{8}$$

and $z_0(m_c^2/m_b^2)$, $z_2(m_c^2/m_b^2)$ represent phase space factors that reflect the sizeable mass of the c quark:

$$z_0(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x\,, \quad z_2(x) = (1-x)^3\ . \tag{9}$$

Obviously $z_0(0) = z_2(0) = 1$ holds.

As stated before these calculations are most conveniently performed by using the explicit expressions for the intermediate quark propagators in the presence of a gluon background field. Employing specifically the Fock-Schwinger (fixed point) gauge for the gluon field (see ref.[11] for details) further facilitates such computations. The operator that emerges there is actually $p_\mu \tilde{G}_{\mu\nu}\gamma_\nu\gamma_5$; this expression is reduced to the chromomagnetic operator by applying the equations of motion to the b field (or by confining oneself to nonrelativistic beauty fields).

The two terms in eq.(6) with coefficients A_0 and A_2 show that there are two distinct sources for the dimension five operator D_G:

(a) The first emerges from the quasifree diagram of Fig.1 upon rewriting $\bar{b}(i\rlap{/}\partial)^5 b$ in terms of covariant derivatives $\bar{b}(i\rlap{/}D)^5 b$ that are simplified due to equation of motion; the D_G operator is then induced by Dirac commutators and thus reflects the conservation of colour.

(b) The second term represents the chromomagnetic interaction of the various quarks in the internal lines in Fig.1 with the external gluon field. It can actually be proven in general that only the antiquark, but not the quarks can contribute here when both weak vertices are purely left-handed.

For semileptonic transitions on the other hand one obtains

$$\hat{\Gamma}_{sl} = \frac{G_F^2 m_b^5}{192\pi^3} \cdot z_0 \cdot (\bar{b}\,b - \frac{1}{m_b^2}\,\bar{b}\,i\sigma G\,b) \tag{10}$$

i.e. there is no overall colour factor N_c, $c_+ = c_- = 1$ applies here and the second term on the right hand side of eq.(6) is absent.

It is thus the operator $\bar{b}\,i\sigma G\,b$ that can generate a shift in the predicted semileptonic branching ratio. Its matrix element between the Λ_b baryon vanishes. For B mesons on the other hand it does not vanish; it can actually be expressed in terms of the mass splitting between B and B^* mesons according to the following relations:

$$\langle B|\,\bar{b}\,i\sigma G\,b\,|B\rangle = -3\langle B^*|\,\bar{b}\,i\sigma G\,b\,|B^*\rangle \tag{11}$$

$$(M_{B^*}^2 - M_B^2)\langle B|\,\bar{b}\,b\,|B\rangle = -1/2\,(\langle B^*|\,\bar{b}\,i\sigma G\,b\,|B^*\rangle - \langle B|\,\bar{b}\,i\sigma G\,b\,|B\rangle) \tag{12}$$

(it should be noted that eq.(12) does not depend on the normalization of the operators and states.) The modification of the nonleptonic width is then given by the following factor:

$$r \simeq 1 - 6 \frac{M_{B^*}^2 - M_B^2}{m_b^2} \cdot \frac{A_2}{A_0} \cdot \frac{z_2}{z_0} \quad . \tag{13}$$

Since the colour factor c_2 and thus also A_2 are negative this nonperturbative correction actually enhances Γ_{nl} and thus *decreases* the semileptonic branching fraction. The scale for the reduction of $BR_{sl}(B)$ is set by the factor $6(M_{B^*}^2 - M_B^2)/m_b^2 \simeq 15\%$; that would translate to a reduction in $BR_{sl}(B)$ by about 1.5 percentage points, *i.e.* from, say, 12-13% (see *e.g.* ref.[12]) to 10.5-11.5% – if the remaining factor $A_2 z_2/A_0 z_0$ equaled unity.

The 'kinematic' ratio z_2/z_0 must actually exceed unity as seen from the following semi-quantitative argument: consider the limiting case when the mass of the charm quark approaches that of the beauty quark; the mass scale that in this case enters into the denominator of the chromomagnetic term will be the energy release $m_b - m_c$ rather than m_b. Numerically we obtain from eq.(9): $z_2/z_0 \simeq 1.5$. The ratio of colour factors A_2/A_0 on the other hand will reduce the size of the effect. For the antiquark can interact with the external gluon field in the linear approximation only if the colour flow through the two weak vertices is different. This leads to a reduction factor $1/N_c$ and also makes the effect proportional to the coefficient c_2. Its size is small in beauty decays and one obtains on the leading *log* level $A_2/A_0 \simeq -(0.15 - 0.2)$. Combining all these factors we obtain

$$\delta BR_{sl}/BR_{sl} \simeq BR_{nl} \cdot (1 - r) \simeq 3 - 4\%$$

It is quite conceivable however that the subleading perturbative corrections – namely those due to the emission of hard gluons with $|\vec{k}| \sim m_b$ – will increase this ratio further and thus soften colour suppression. For the presence of such gluons immediately invalidates both reasons for the suppression of the 'leading' correction: it enables all fields in the loops to contribute to the operator D_G and it can change the unfavorable colour flow. Such effects are only of order α_s and cannot bring any *log* enhancement. Therefore it is unlikely that they can completely eliminate the suppression; nevertheless it is conceivable that the next-to-leading perturbative correction to the coefficient function may essentially change the numerical result.

Two general remarks are in order here:

(i) It would not invalidate a perturbative treatment of the problem at hand if these higher order corrections indeed turned out to be larger than the lowest order one: for the suppression of the lowest order result is due to the specific structure of the weak interactions, namely their purely chiral nature, and has nothing to do with the strong forces.

(ii) Since the mass difference $m_b - m_c$ is still large compared to typical hadronic scales one can calculate the coefficient of the chromomagnetic operator D_G within perturbation theory; such a computation thus presents only a *technical* challenge, but not one of a principal nature.

It is quite intriguing to extrapolate this analysis down to the case of charm decays: since $6(M_{D^*}^2 - M_D^2)/m_c^2 \sim 2$ and colour suppression is less severe there we find that a very large nonperturbative reduction arises here for the semileptonic branching ratio of charm mesons. In addition the dimension six four-fermion operators produce a

large decrease in the width of $D^{\pm}$ through Pauli Interference [2, 5]. This yields the correct pattern as observed by experiment, however definite quantitative predictions seem to be unreliable here.

There is another semi-quantitative observation that should be noted: the nonperturbative term in eq.(6) that is responsible for increasing the nonleptonic B decay width is – through A_2 – suppressed by $1/N_c$; there is also a $1/N_c$ term – due to A_0 – appearing in the first quasifree term and it enters with the *opposite* sign there. Thus the two terms that are non-leading in $1/N_c$ tend to cancel each other. This provides another illustration of preasymptotic *QCD* corrections tending to *dynamically* suppress non-leading corrections in $1/N_c$ [2, 13, 9] and thus to imitate the phenomenological prescription of retaining only the leading terms [3]. It also shows on the other hand that such a recipe *cannot* be viewed as a fundamental and universal rule. This cancellation is relatively mild for $\Gamma_{nl}(B)$ – say up to 20%; for the non-leptonic width of D mesons on the other hand it could be complete. The situation here differs from what was found in ref.[2] for flavour-dependent effects: there the cancellation was due to *perturbative* corrections and for this reason it was strong both in charm and in beauty; in the present case the corrections are nonperturbative and their role decreases significantly from charm to beauty. It should also be noted that the authors of ref.[9] found that the terms that are non-leading in $1/N_c$ cancel almost completely for the exclusive mode $B \rightarrow D\pi$; as we have seen here this is not the case for the inclusive width.

Similar considerations can be applied to the inclusive width for $b \rightarrow c\bar{c}s$ transitions. The relative weight of the nonperturbative corrections could be quite sizeable there due to the small amount of energy release that is available. Of course the overall size of this width is also suppressed by phase space. We can also expect here even more important modifications to the corresponding colour factor A_2 in eq.(7) coming from the subleading perturbative corrections than we discussed for $b \rightarrow c\bar{u}d$. Yet these are 'just' technical problems that can and will be addressed in future work. Lastly forthcoming experimental studies will allow the reliable isolation of this class of transitions.

The approach presented here can also be applied to the lepton *spectra* in semileptonic beauty decays. The relevant operator expansion is then given in terms of inverse powers of $(p_b - p_l)^2$ with p_b [p_l] denoting the momentum of the b quark [lepton]. Thus the series blows up near the endpoint in the lepton energy spectrum. Yet there one can rely on the results obtained from the *EHQT* approach. Since $m_b > m_c \gg \mu_{had}$, $m_b - m_c \gg \mu_{had}$ hold there exists a regime for the lepton energy where both expansions are valid simultaneously. This argument is actually rather similar to the discussion of ref.[14].

To summarize: we have identified a correction to the inclusive nonleptonic decay width that has no counterpart in the semileptonic width. It may account for up to 15% of the total beauty width. The lowest loop estimate though yields only about 3%; this is roughly similar in size to the effect of interference coming from four-fermion operators of dimension six. That a dimension five and a dimension six operator have a similar impact here is due to two factors: the colour suppression of the dimension five operator as well as the fact that it appears in the two loop diagram and thus

contains an extra factor of $1/\pi^2$. Basically the same reason [7] enhances the weight of the standard corrections to the lifetimes. A more exact estimate requires calculation of the $O(\alpha_s)$ corrections; still it seems unlikely that in the Standard Model it could by itself shift the semileptonic fraction by more than 1% per lepton flavour.

3 Summary and Outlook

We have outlined here a general method that allows to calculate the inclusive transition rates for the weak decays of heavy flavour hadrons. It consists of four elements:

(i) The forward amplitude $Q \to Q$ (more exactly its absorptive part) is expanded into a series of local operators of increasing dimension whose coefficients are proportional to powers of $1/m_Q$. This operator expansion depends on the intermediate state, namely whether one is considering nonleptonic or semileptonic transitions, *i.e.* $Q \to q_1 \bar{q}_2 q_3 \to Q$ or $Q \to q_1 \bar{\nu} l \to Q$; it is also sensitive to the masses of the quarks q_i. On the other hand it is universal for all hadrons carrying the heavy flavour Q.

(ii) A nonrelativistic expansion is given for these local operators, again in powers of $1/m_Q$.

(iii) The inclusive decay rate is obtained from the matrix element of this operator expansion taken between the decaying meson or baryon state.

(iv) The matrix elements for the operators that appear in this final expansion are determined by symmetry arguments and/or by relating them to other observables like the masses of heavy flavour hadrons. The size of these matrix elements in general depends on the type of H_Q, *i.e.* whether it is a meson or a baryon, whether it is charged or not, whether it carries strangeness etc.

Our approach allows to incorporate nonperturbative effects in a selfconsistent way as corrections in a $1/m_Q$ expansion. We find:

• There are no corrections to the quasifree picture of order $1/m_Q$.

• The leading nonperturbative corrections arise on the $1/m_Q^2$ level. They are $SU(3)_{fl}$ invariant, *i.e.* affect the heavy flavour meson decays in a uniform way independent of the flavour of the light antiquarks. They enhance the nonleptonic decay width in mesons and lead to a corresponding *reduction* in the semileptonic branching ratio.

• The nonperturbative corrections that appear on the $1/m_Q^3$ level are not $SU(2)$ and $SU(3)$ invariant; thus they generate differences in the lifetimes and semileptonic branching ratios among all heavy flavoured hadrons.

• Due to the powerlike scaling behaviour in $1/m_Q$ all these preasymptotic effects are much larger in charm than in beauty decays.

• We have found some cases where nonperturbative and perturbative corrections that are non-leading in N_c contribute with the opposite sign and thus tend to cancel each other. This provides a dynamical explanation for some of the successes of the phenomenological prescription to drop terms that are non-leading in N_c. Yet it also shows that such a procedure cannot be expected to be of universal validity.

These findings can be stated in a more quantitative manner:

(1) As previously shown [2] lifetime differences in B meson decays cannot be expected to exceed the percent level while lifetime ratios of two emerge naturally for D

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meson decays.

(2) Nonperturbative corrections may reduce the semileptonic branching ratio *uniformly* in B decays by up to 10% relative to the value inferred from the naive spectator ansatz. For D decays they could well reduce it by a factor of two!

Point (2) is quite intriguing when comparing our approach with the usual one that involves modifying the quasifree result with phenomenological models. The one class of parameters that is quite uncertain there is the mass for the final state quarks, *i.e.* whether the small 'current' values should be adopted or the higher 'constituent' values. Yet increasing the final state quark masses over their 'current' values will necessarily *enhance* the semileptonic branching ratio! The *QCD* approach presented here on the other hand demands that it is the current masses that must be used in the calculations. Powerlike nonperturbative corrections for the colourless quark-antiquark loop itself in Fig.1 are known to *increase* the hadronic width. Some of these corrections could indeed effectively mimic the insertion of constituent masses into the propagators; yet being of order $1/m_b^6$ they are in fact strongly suppressed. The corrections to the quark loop discussed in ref.[4] for charm decays are subleading contributions resulting in terms that start with $1/m_b^4$. The leading effects are actually due to the interaction of the decay quarks with the light degrees of freedom present in the initial hadron. They induce $1/m_Q^2$ corrections that are explicitly calculable and in principle depend on the nature of the spectator (but not on its flavour).

We have outlined here a general procedure. It will be improved and extended in the future in four respects:

(i) Some of the numerical predictions stated above were somewhat tentative since not all the relevant calculations have been performed yet. Since the 'missing' computations involve perturbation theory this presents just a technical delay and not a stumbling block in principle.

(ii) The real accuracy that can be obtained in this approach is to be determined by calculating terms of order $1/m_Q^4$ and estimating the size of the relevant matrix elements.

(iii) The case of heavy flavour baryons can be and will be incorporated in a systematic way [15].

(iv) A natural approach to the problem of $SU(3)_{\mathrm{fl}}$ breaking in heavy flavour decays emerges from our treatment. It can be expected quite generally from the Heavy Quark Expansion outlined above that the apparent size of $SU(3)_{\mathrm{fl}}$ breaking scales like $m_s/\mu_{\mathrm{had}} \cdot 1/m_Q^2$; in other words $SU(3)_{\mathrm{fl}}$ breaking is expected to amount to no more than a few percent effects in beauty decays [15] – whereas a conventional application of $SU(3)$ arguments can do no better than allow for the usual 20-30% breaking effects.

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Figure Captions

Fig.1 Simplest two-loop diagram describing the total decay width $Q \rightarrow q_1 \, \bar{q}_2 \, q_3$

Fig.2 Diagrams representing flavour-dependent corrections to widths:
    a. "Weak Annihilation" in heavy meson decays
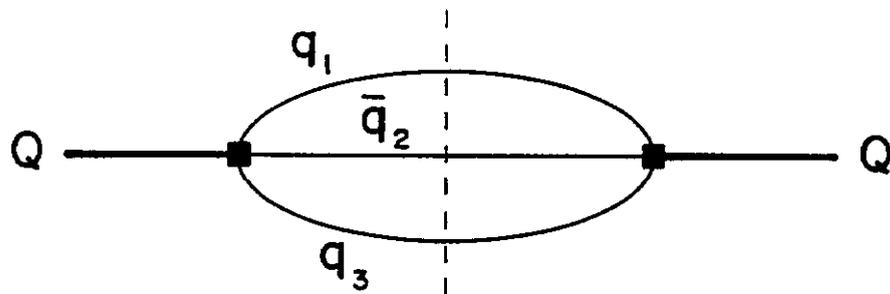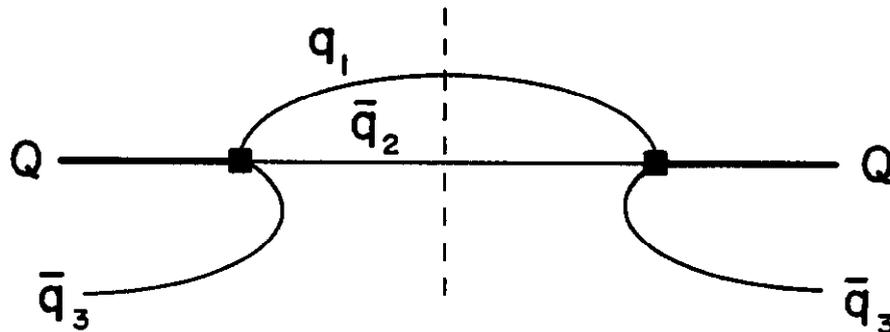   **b.** "Pauli Interference" effects in heavy meson decays
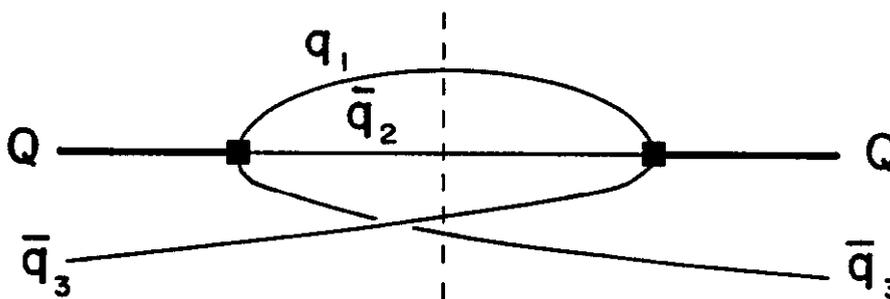
Fig. I

Fig. 2a

Fig. 2b