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Seesaw Neutrino Mass Ratios with Radiative Corrections

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ABSTRACT

Unlike neutrino masses, the ratios of neutrino masses can be predicted by up-quark seesaw models using the known quark masses and including radiative corrections, with some restrictive assumptions. The uncertainties in these ratios can be reduced to three: the type of seesaw (quadratic, linear, etc.), the top quark mass, and the Landau-triviality value of the top quark mass.

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The inconclusive but suggestive results of recent solar and atmospheric neutrino and beta decay experiments [1] lead to the possibility of neutrino masses, which additionally may have important application to cosmology, astrophysics and laboratory searches for neutrino oscillations. The most economical model of light neutrinos is the so-called “seesaw” of the grand-unified type, which requires a superheavy right-handed neutrino for each ordinary neutrino and arises naturally in partially or completely unified theories with left-right symmetry, such as SO(10) [2,3,4]. These grand unified seesaw models predict small but non-zero Majorana masses for the ordinary neutrinos in terms of the Dirac masses of the up-type quarks (u, c, t) and the superheavy right-handed Majorana masses. These predictions are made uncertain, however, by the unknown right-handed masses and by radiative corrections. But the *ratios* of neutrino masses are more definite in seesaw models, under some necessary and minimal assumptions (printed below in *italics*) about the physics underlying the seesaw [5]. The uncertainties in the mass ratios can then be narrowed to a handful.

The general tree-level form of the seesaw model mass matrix for three families is:

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix}, \quad (1)$$

in the left- and right-handed neutrino basis, where each entry is a 3×3 matrix. We assume that the upper left corner is zero, as a non-zero Majorana mass for left-handed ν generally requires an $SU(2)_L$ Higgs triplet, an unnatural addition to the Standard Model in light of known electroweak neutral-current properties [6]. The Dirac matrix m_D is both an $SU(2)_L$ and an $SU(2)_R$ doublet. The symmetric superheavy Majorana mass matrix M_N for the right-handed neutrinos N violates lepton number, but is a Standard Model gauge singlet. M_N must be a remnant of a broken $SU(2)_R$ or larger symmetry. Assuming the eigenvalues of M_N are much greater than those of m_D , the light neutrinos acquire a symmetric Majorana mass matrix $m_\nu = m_D M_N^{-1} m_D^T$ and the superheavy neutrinos a

mass matrix M_N upon block diagonalization of (1). The superheavy neutrinos have masses equal to the eigenvalues $(M_{N_1}, M_{N_2}, M_{N_3})$ of M_N . The matrix M_N can have a variety of sources [3,5]. In models with tree-level breaking of $SU(2)_R$, the right-handed mass requires an $SU(2)_R$ Higgs triplet — in $SO(10)$ models, a Higgs **126**. In models with minimal Higgs content ($SU(2)_{L,R}$ singlets and doublets only, as in superstring models), the matrix M_N must arise either from loop effects [3,7] or from non-renormalizable terms, presumably induced by gravity [8].

Making predictions from the seesaw matrix m_ν requires additional assumptions. To obtain simple scaling dependence of light neutrinos masses on the eigenvalues of m_D requires the assumptions *that the matrix m_D can be freely diagonalized and that the intergenerational mixings in M_N are no larger than the ratios of eigenvalues between generations*. We then need to know the eigenvalues of m_D : *here the simple grand-unified seesaw is assumed, so that $m_D \propto m_u$, the up-type quark mass matrix*. † For predictiveness, *the eigenvalues of M_N are assumed proportional to a power p of the eigenvalues of m_u* . The $p = 0$ and $p = 1$ cases are the “quadratic” and “linear” seesaws, respectively, because of the dependence of $m_{\nu,i}$ on $m_{u,i}$ [4,5]. ‡

The family kinship of quarks and leptons in order of ascending mass is assumed; a different kinship merely requires relabelling the neutrinos appropriately. Forming the ratio of any two light neutrino masses,

$$\frac{m_{\nu,i}}{m_{\nu,j}} = \frac{m_{u,i}^2}{m_{u,j}^2} \cdot \frac{M_{N,j}^p}{M_{N,i}^p}, \quad (2)$$

† The Dirac mass matrix for the charged leptons m_l is proportional to the down-type quark (d, s, b) mass matrix m_d in the simplest grand unified seesaw models. The matrix proportionalities of m_D and m_u , and m_l and m_d , require that each pair of Dirac masses be generated by only or mainly one Higgs representation. Otherwise, a specific ansatz of Dirac masses is needed.

‡ If the eigenvalues of M_N increase no more than linearly with the hierarchy of eigenvalues in m_u ($p \leq 1$ for a simple power law), and $m_l \propto m_d$, then, additionally, the neutrino mixing matrix is identical to quark CKM mixing matrix, at least at the putative unification scale. For reasonable values of the top quark mass, this equality approximately holds at low energies [4].

we obtain the power-law dependence of the seesaw, with exponent $2 - p$. Taking the ratios of neutrino masses eliminates the overall unknown scale in M_N . However, the form (2) requires radiative corrections to the fermion masses to arrive at predictions. The tree-level result (2) is taken to be exact at some scale $\mu = M_X$, typically the grand unification scale; the masses $m_\nu(\mu)$, $m_u(\mu)$, and $M_N(\mu)$ are then run down to low energies and related to the physical masses to yield radiatively modified seesaw predictions. The leading logarithm approximation is sufficient for our purposes and is evaluated here in the \overline{MS} scheme. As a number of authors have noted, much of the uncertainty in these corrections cancels out in fermion mass ratios, if some general conditions hold about the physics that produces the corrections [5,9].

Corrections to the fermion masses are assumed to come from two sources, Higgs-Yukawa couplings and gauge couplings. *A generalized family symmetry is assumed for the gauge interactions, so that, apart from differences in mass thresholds, the gauge corrections are "family-blind"*. The mass matrices can then be diagonalized and corrections applied to individual eigenvalues. Higgs corrections to the masses are proportional to their underlying Yukawa couplings. For the light ν , these are negligible, as they are for the up-type quarks, except for the top quark.* For the superheavy N , the eigenvalues $M_{N,i}(X)$ are proportional to the power p of the eigenvalues $m_{u,i}(X)$.

Considering only gauge corrections first, the \overline{MS} renormalization group equations for the fermion masses and gauge couplings 1...n... are standard [10]:

$$\begin{aligned} \frac{d \ln m(\mu)}{d \ln \mu} &= \sum_n b_m^{(n)} \cdot g_n^2(\mu), \\ \frac{d g_n^2(\mu)}{d \ln \mu} &= -2b_n \cdot g_n^4(\mu), \end{aligned} \tag{3}$$

with the general solution

$$m(\mu)/m(\mu_0) = \prod_n [g_n(\mu)/g_n(\mu_0)]^{-b_m^{(n)}/b_n}. \tag{4}$$

* The large top quark Yukawa coupling also leads to renormalization group corrections to the first-third and second-third family CKM quark mixings.

The ν mass ratios at the scale M_X are the same as the physical ratios:

$$m_{\nu,i}(X)/m_{\nu,j}(X) = m_{\nu,i}/m_{\nu,j}. \quad (5)$$

The equality holds because the known and unknown gauge corrections to light neutrino masses are due to heavy, flavor-blind interactions that begin to run only at the W boson mass, far above any neutrino mass. The gauge corrections to the up-type quark mass ratios are substantial, because they partly arise from QCD and because the quark masses have a large hierarchy in the presence of massless gauge bosons. To evaluate these corrections completely requires the assumption that *there are no new particles of mass between the Z boson and top quark masses with Standard Model gauge couplings. The gauge corrections require the top quark mass to logarithmic accuracy, which we take from the best neutral-current data to be $m_t = 160$ GeV [6].* (Powers of the top quark mass are left explicit.) Apart from differences in mass thresholds, the gauge corrections from QCD, QED and the hypercharge $U(1)_Y$ are the same for all up-type quarks. The weak isospin $SU(2)_L$ corrections to the quark masses are zero, since these masses are of the Dirac type, mixing left- and right-handed fields. Corrections due to new gauge couplings would begin at scales above m_t and would cancel in the ratios. With $\kappa = 1$ GeV and taking $m_u(\kappa) = 5$ MeV, $m_c(\kappa) = 1.35$ GeV [10], and m_t as free if it occurs as a power,

$$\begin{aligned} m_c(X)/m_u(X) &= m_c(\kappa)/m_u(\kappa) = 270 \\ m_t(X)/m_c(X) &= (1.90)m_t/m_c(\kappa) = 140(m_t/100\text{GeV}) \\ m_t(X)/m_u(X) &= (1.90)m_t/m_u(\kappa) = 38000(m_t/100\text{GeV}). \end{aligned} \quad (6)$$

The top quark mass is defined by $m_t = m_t(m_t)$. Since $M_{N,i}(X) \propto m_{u,i}^p(X)$, the gauge corrections to M_N are accounted for in the gauge corrections to $m_u(X)$. *Any corrections to M_N due to new gauge interactions either cancel in the ratios or are assumed to be weakly coupled and thus small.*

The other set of corrections are due to the fermions' couplings to the Higgs sector. The Yukawa couplings and fermion masses are simultaneously diagonal. In the neutrino mass

ratios, under our assumptions, only the Yukawa coupling to the top quark is important. The renormalization group equation for the top quark mass is modified from (3) to

$$\frac{d \ln m_t(\mu)}{d \ln \mu} = \sum_n b_m^{(n)} \cdot g_n^2(\mu) + b_m^H \cdot [m_t(\mu)/M_W]^2, \quad (7)$$

where the factor b_m^H depends on the Higgs sector. The solution to (7) can be written as $m_t(\mu) = f(\mu) \cdot m_t(\mu)_0$, where $m_t(\mu)_0$ is the solution to (3). Taking $f(m_t) = 1$,

$$1 - \frac{1}{f^2(X)} = 2b_m^H \int_{m_t}^{M_X} \frac{d\mu}{\mu} \cdot \frac{m_t^2(\mu)_0}{M_W^2}. \quad (8)$$

The numerical evaluation of $f(X)$ requires the function $m_t(\mu)_0$ over the full range from the top quark mass to unification. However, our ignorance of this function and of the Higgs sector can be collapsed into a single number, the Landau-triviality value of the top quark mass, m_{tL} . This is the top quark mass for which, with a fixed M_X , the right-hand side of (8) is unity and $f(X)$ diverges. That is, $f(\mu)$ diverges before μ reaches M_X , if m_t exceeds m_{tL} . The triviality value m_{tL} is the upper limit of the top quark mass:

$$\frac{1}{m_{tL}^2} = 2b_m^H \int_{m_t}^{M_X} \frac{d\mu}{\mu} \cdot \frac{m_t^2(\mu)_0}{M_W^2 m_t^2}, \quad (9)$$

with the presence of the unknown m_t as the lower bound inducing only a small logarithmic error. (The r.h.s. of (9) contains no powers of the top quark mass.) Then

$$f^2(X) = \frac{1}{1 - m_t^2/m_{tL}^2}. \quad (10)$$

For example, in the minimal Standard Model, with $M_X = M_{Pl} \simeq 1 \times 10^{19}$ GeV, $m_{tL} \simeq 760$ GeV; in the supersymmetric (SUSY) case, with the same M_X , $m_{tL} \simeq 190$ GeV. Of particular interest because of its successful prediction of the weak mixing angle, the SUSY SU(5) grand unified model yields $m_{tL} \simeq 180$ GeV, with $M_X \simeq 2 \times 10^{16}$ GeV. The non-SUSY SO(10) model, breaking through an intermediate left-right model, gives $m_{tL} \simeq 380$ GeV [4,6].

With the aforementioned assumptions, the final mass ratios for the light neutrinos are

$$\begin{aligned}
m_{\nu_\mu}/m_{\nu_e} &= (270)^{2-p} \\
m_{\nu_\tau}/m_{\nu_\mu} &= \frac{1}{(1 - m_t^2/m_{tL}^2)^{1-p/2}} \cdot [140 \cdot m_t/100\text{GeV}]^{2-p} \\
m_{\nu_\tau}/m_{\nu_e} &= \frac{1}{(1 - m_t^2/m_{tL}^2)^{1-p/2}} \cdot [38000 \cdot m_t/100\text{GeV}]^{2-p}.
\end{aligned} \tag{11}$$

For a given ν_e or ν_μ mass, the ν_τ mass can be sensitive to the top quark mass beyond the naive seesaw dependence, because of the triviality factor.

It would be interesting to check how varying these assumptions changes the neutrino mass ratios. Unfortunately, most of the assumptions cannot be changed without losing predictiveness. The flavor-blindness of the gauge interactions is especially crucial. However, switching to a leptonic seesaw, with $m_D \propto m_l$, does lead to predictive neutrino mass ratios, if the eigenvalues $M_{N,i}(X) \propto m_{l,i}^p(X)$ and all neutrinos and charged leptons are subject only to family-blind, weakly-coupled gauge interactions. Then

$$m_{\nu,i}/m_{\nu,j} = [m_{l,i}/m_{l,j}]^{2-p} \tag{12}$$

is a good approximation.

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