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Anomalous dimensions for all dimension 8 CP odd operators with only photons and gluons

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We calculate the anomalous dimensions of the complete set of CP odd (dimension 8) operators involving only field strengths of the photon and the gluon. Some of our results disagree with those in the literature. The phenomenology of these operators are also briefly discussed.

The complete set of P odd, T odd, dimension eight independent operators involving only gluonic and photonic field strengths can be listed1·2 as:

$$
\begin{aligned}
O_{8,1} &= \tfrac{1}{12} g^4 \tilde{G}^a_{\mu\nu} G^{a\mu\nu} G^b_{\alpha\beta} G^{b\alpha\beta} \quad , \\
O_{8,2} &= \tfrac{1}{12} g^4 \tilde{G}^a_{\mu\nu} G^{b\mu\nu} G^a_{\alpha\beta} G^{b\alpha\beta} \quad , \\
O_{8,3} &= \tfrac{1}{12} g^4 d^{abe} d^{ecd} \tilde{G}^a_{\mu\nu} G^{b\mu\nu} G^c_{\alpha\beta} G^{d\alpha\beta} \quad , \\
O_{8,4} &= \tfrac{1}{3} e_Q g^3 d^{abc} \tilde{F}_{\mu\nu} G^{a\mu\nu} G^b_{\alpha\beta} G^{c\alpha\beta} \quad , \\
O_{8,5} &= \tfrac{1}{3} e_Q g^3 d^{abc} \tilde{G}^a_{\mu\nu} G^{b\mu\nu} G^c_{\alpha\beta} F^{\alpha\beta} \quad , \\
O_{8,6} &= \tfrac{1}{2} e_Q^2 g^2 \tilde{F}_{\mu\nu} F^{\mu\nu} G^a_{\alpha\beta} G^{a\alpha\beta} \quad , \\
O_{8,7} &= \tfrac{1}{2} e_Q^2 g^2 \tilde{G}^a_{\mu\nu} G^{a\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \quad , \\
O_{8,8} &= \tfrac{1}{2} e_Q^2 g^2 \tilde{F}_{\mu\nu} G^{a\mu\nu} F_{\alpha\beta} G^{a\alpha\beta} \quad , \\
O_{8,9} &= \tfrac{1}{12} e_Q^4 \tilde{F}_{\mu\nu} F^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \quad ,
\end{aligned}
\tag{1}
$$

where d^{abc} is the totally symmetric tensor of $SU(3)$, $F^{\mu\nu}$ is the photonic field strength and $G^{a\mu\nu}$ is the gluonic field strength. $(\tilde{F}_{\mu\nu}, \tilde{G}^a_{\mu\nu}) \equiv \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}(F^{\alpha\beta}, G^{a\alpha\beta})$ with $\epsilon^{0123} = +1$. The gauge couplings of QED and QCD are e and g respectively. We define e_Q as the charge unit of the heavy quark Q multiplying the coupling $e\,(e > 0)$. In renormalizable gauge theories, these operators are in general induced by two–loop diagrams when all the heavy particles in the intermediate states together with their CP violating interactions are integrated out.

In general, these operators may not be competitive with other operators of lower dimensions because higher dimensional operators are suppressed by higher powers of $1/M$ for some large mass scale M. However, in some models M may not be very high (such as b or c quark masses)1·2. In that case, these operator could become important due to the renormalization group effects or enhanced hadronic matrix elements or both. Therefore it is interesting to investigate what experimental constraints one can impose on such operators.

The anomalous dimensions of the three purely gluonic operators $O_{8,i}$ $(i = 1, 2, 3)$ were calculated by Morozov3 some years ago and were recently rederived by Booth4. were confirmed. The phenomenology of these operators has also been discussed1. In this paper we investigate the remaining operators involving the photon. In particular we calculate their anomalous dimensions and discuss their potential low energy consequences and experimental constraints.

For the two operators, $O_{8,i}$ $(i = 4, 5)$, involving one photonic and three gluonic field strengths, their anomalous dimensions have been reported before 5. Our results however disagree with theirs. We found that the anomalous dimensions are less negative than previously reported values and thus the corresponding renormalization group (RG) suppression is less severe. The RG equations for $O_{8,4}$ and $O_{8,5}$ can be written as

$$\mu\frac{d}{d\mu}O_{8,4} = -\frac{\alpha_s}{4\pi}(23O_{8,4} + O_{8,5}) \quad ,$$
$$\mu\frac{d}{d\mu}O_{8,5} = -\frac{\alpha_s}{4\pi}(O_{8,4} + 23O_{8,5}) \quad . \tag{2}$$

The combinations $O^{\pm} = O_{8,4} \pm O_{8,5}$ thus evolve diagonally according to the eigenvalues, -24 and -22, which differ from those values $(-25 \pm \sqrt{6})$ given in Ref.5. The main constraint on these operators can be obtained from the experimental bound on the neutron electric dipole moment $D_N < 10^{-25}$ e-cm. The contribution from $O_{8,4}$ and $O_{8,5}$ has been analyzed in Ref.5. Due to the weaker RG suppression, the constraint is actually a bit stronger than those obtained in Ref.5. This constraint can be used to put limits on the electric dipole moment C'_Q (EDM) and the chromoelectric dipole moment C_Q (CEDM) of various quarks Q. C_Q and C'_Q are defined by an effective Lagrangian at the scale just above the heavy quark threshold,

$$\mathcal{L} = \cdots + \frac{i}{2}\bar{Q}\sigma^{\mu\nu}\gamma_5(gC_Q G^a_{\mu\nu}T^a + e_Q C'_Q F_{\mu\nu})Q \quad . \tag{3}$$

The dimension 8 operators arise when the heavy quark degree of freedom is removed. The low energy Lagrangian is given by

$$\mathcal{L} = -\frac{1}{256\pi^2 M_Q^3}\left[K^+(3C_Q + C'_Q)O^+ + K^-(C_Q - C'_Q)O^-\right] + \cdots \quad , \tag{4}$$

with the operators defined at the low energy scale where $g = 4\pi/\sqrt{6}$ 6. The RG factors are

$$K^{\pm} = \left[\frac{33-2n_f}{18}\ln\frac{M_Q^2}{\Lambda^2}\right]^{-\frac{k^{\pm}}{(33-2n_f)}} \quad , \quad k^+ = 36, \quad k^- = 33 \quad . \tag{5}$$

The naive dimensional analysis (NDA)7 estimates that

$$D_N \simeq \frac{M_\chi^3}{16\pi^2}\frac{g^3}{256\pi^2 M_Q^3}\left\{(3K^+\xi^+ + K^-\xi^-)eC_Q + (K^+\xi^+ - K^-\xi^-)e_Q C'_Q\right\} \quad , \tag{6}$$

with the unknown nonperturbative factors $\xi^{\pm}$ naively of order about 1. Assuming no accidental cancellation between the EDM's and the CEDM's, it is enough to look at the less RG suppressed term proportional to ξ^-. The limits obtained for c and b quarks are

$$\begin{aligned} C_b &< \quad 52\ G_F m_b/(16\pi^2\xi^-) \quad , \\ C_c &< \quad 3\ G_F m_c/(16\pi^2\xi^-) \quad , \\ C'_b &< \quad 160\ G_F m_b/(16\pi^2\xi^-) \quad , \\ C'_c &< \quad 4\ G_F m_c/(16\pi^2\xi^-) \quad . \end{aligned} \tag{7}$$

The above bounds on C_b and C_c are less stringent than those deduced from the purely gluonic dimension 8 operators in Ref.1 because of the QCD enhancement effect in $O_{8,1}$ at low energy.

For the remaining four operators it is clear that $O_{8,6}, O_{8,7}$, and $O_{8,9}$ should have vanishing anomalous dimensions. The renormalization group equation for $O_{8,8}$ can be written as

$$\begin{aligned} &\mu\tfrac{d}{d\mu}O_{8,6} = 0 \quad , \\ &\mu\tfrac{d}{d\mu}O_{8,7} = 0 \quad , \\ &\mu\tfrac{d}{d\mu}O_{8,8} = \tfrac{\alpha_s}{4\pi}(2O_{8,6} + 2O_{8,7} - 12O_{8,8}) \quad . \end{aligned} \tag{8}$$

Therefore the RG evolution of the operator $O_{8,8}$ will have the suppressive effect on the operator itself while inducing the operators $O_{8,6}$ and $O_{8,7}$ along the way. Among different models of CP violation, just as the other $O_{8,i}$, these operators have the largest impact on those in which they are induced at b or c quark scale if these quarks carry electric or chromoelectric dipole moments due to higher energy CP violation. The resulting coefficients in various models of CP violation have been worked out before[1,2]. A typical value for them is $(\frac{eg_s}{16\pi^2})^2(\frac{\delta}{M_W})(\frac{1}{M^3}) \simeq 10^{-7}\delta(\frac{\text{GeV}}{M})^3$ GeV^{-4} for some mass scale M. The CP violating phase δ is of order 10^{-3} or smaller. So far we have not been able to find any significant experimental constraints on the coefficients of the three hybrid operators $O_{8,i}$ $(i = 6, 7, 8)$ and the purely photonic operator $O_{8,9}$ that can reach anywhere near that level.

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