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A. Klatchko, B. Choudhary and T. Huehn

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

University of California
Riverside, California 92521

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A.Klatchko, B.C. Choudhary and T. Huehn
University of California Riverside CA 92521
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Abstract

Reconstruction of particles that traverse an environment of large number of radiation lengths (X_o) is subjected to a substantial loss of information due to Multiple Colulomb Scattering (MS). The stochastic nature of the process makes it difficult to propagate local errors across the scattering material. A random walk (RW) approach to the calculation of the MS errors of the track parameters is described for such an environment. The simulated RW solution yields an estimate of the full end-to-end error matrix.

In a previous paper [1] we treated the errors due to MS in the framework of local tracking. Let us recall the main features of this approach. The particle trajectory traversing the material in the detector is broken into a series of semi straight lines. The local error variance and covariance of the track parameters are parameterized in terms of the scattering angle Θ^{MS} [2]. The calculated error matrix is then propagated by the track model matrix across the detector. However, estimation of the MS error matrix in an environment that comprise a large number of scatterers impose extra difficulties. As will be shown below, the estimation of errors in such an environment involves non trivial integrations over correlated track parameters in their entire phase space.

It is not very unusual in high energy physics detectors that a particle traverses dense material before its position (and thus direction) is directly measured (inferred). For example, consider the muon system in the **D0** detector [3]; where a magnetized iron toroid of circa 1 meter thickness in the central region and 1.5 meter in the ends ($X_o \approx 57 - 85$) is located between three groups of proportional drift tubes (denoted as **A**, **B** and **C** stations) which are set up to measure its trajectory. In many other detectors there are direct measurements of the particle position before and after a calorimetric device. Calorimeters are usually designed to contain a large portion of the shower giving rise to a large number of radiation lengths. The particle traversing the active and non active material of the detector is multiply scattered by the atoms that comprise these materials. Naturally, such devices introduce errors which have to be accounted for in the track reconstruction. Since we do not know of any measurement of MS errors in such an environment, it is the aim of this article to suggest an estimate of the full error matrix based on the stochastic nature of the process.

The MS process introduces small deviations into the track parameters in a random fashion usually referred to as stochastic. Assuming the scattering events are independent from each other, and that the deviations that they introduce are Gaussian distributed; the problem of a particle traversing the detector material can be described by a RW. The 'step length' of the 'walking' particle is determined by the largest experimental measured step length which is yet small enough in comparison with the total distance after n steps. The errors on the directions (and thus on the position) are parameterized by Moliere's theory. To our knowledge, the thickest material for which the measured MS angle was found to agree with Moliere's theory is of the order of 0.1 of a radiation length [4]. Applying Moliere's theory, one usually estimates this error by the Highland formula [5]. The RW scheme is consisted of breaking the particle trajectory in the scattering material into n steps, each step approximated by a quasi linear curve of 0.1 of a radiation length. The total trajectory sums up to an n-step walk. With each step is associated a random direction and thus a position error (displacement). These errors sum up to an end-to-end error on the position and the direction of the emerging particle, compared to a particle traversing the vacuum. It is this error that we wish to estimate.

At a given step, i, the 4 dimensional vector $\mathbf{v_i} = (x, y, \hat{x}, \hat{y})$ describes the local position and direction of the particle at that step. An n step walk is thus described by a sum over the local random vectors $\mathbf{v_i}$:

$$\mathbf{V} = \sum_{i=1}^{n} \mathbf{v_i} \tag{1}$$

where $\mathbf{V}$ is the 4-dimensional vector that emerges from the n-step walk. The probability density of finding $\mathbf{V}$ between $\mathbf{V}$ and $\mathbf{V} + d^4\mathbf{V}$ can be described by a Gaussian distribution:

$$g(\mathbf{V})d^4\mathbf{V} = \frac{1}{(2\pi n|\mathbf{M}|)^2} \exp[\frac{1}{2n}(\mathbf{V} - < \mathbf{V} >)^T M^{-1}(\mathbf{V} - < \mathbf{V} >)] \tag{2}$$

where $\mathbf{M}$ is the end-to-end covariance error matrix given by,

$$M_{\alpha\beta} = < \mathbf{v}_\alpha \mathbf{v}_\beta > - < \mathbf{v}_\alpha >< \mathbf{v}_\beta > \tag{3}$$

with α, β running over the 4 indices of the local vector $\mathbf{v_i}$. The moments , $< \mathbf{v}_\alpha >$ and $< \mathbf{v}_\alpha \mathbf{v}_\beta >$, are given by the following integrals:

$$< \mathbf{v}_\alpha > = \int \mathbf{v}_\alpha \rho(\mathbf{v}) d^4\mathbf{v}$$

$$< \mathbf{v}_\alpha \mathbf{v}_\beta > = \int \mathbf{v}_\alpha \mathbf{v}_\beta \rho(\mathbf{v}) d^4\mathbf{v} \tag{4}$$

where $\rho(\mathbf{v})d^4\mathbf{v}$ is the joint probability that the components of a single step vector, $\mathbf{v}_\alpha$, fall in the interval $\mathbf{v}_\alpha + d\mathbf{v}_\alpha$. The integration over these correlated variables is done in the entire 4 dimensional parameter space and is thus non trivial.

The RW approach allows one to calculate these integrals and obtain the end-to-end error matrix, $M_{\alpha\beta}$, based on the knowledge of $\rho(\mathbf{v})d^4\mathbf{v}$. The problem is thus reduced to the parameterization of the local errors inflicted on each of the entries $\mathbf{v}_\alpha$ in a single step. Througout this paper we use the parameterization of the local MS errors of [1] where:

$$\delta\hat{x}_i = -\frac{\theta_s}{\sqrt{2}}\sqrt{1 - \hat{x}_i^2} \tag{5}$$

with θ_s parameterized in the Highland fashion [5]:

$$\sigma_{\hat{x}} = \frac{0.013}{\sqrt{6}p}\sqrt{X_\circ}[1. + 0.088\log_{10}(X_\circ)] \tag{6}$$

The error on the position for each step is determined by equation (5) and the step length. Following the concept of [1] for the parameterization of the errors in a local step, we simulate a RW of a particle with a given momentum through a large number of radiation lengths. The end-to-end error matrix is estimated by the *rms* of the position and direction errors of the particle as it emerges out of the material.

In this study 1000 tracks are stepped, in a RW manner, through 50 and 100 of radiation lengths with different momenta of 5 and 40 GeV, accounting for the energy loss by a parameterization based on a fit to the data in [6]. The number of radiation lengths is chosen to describe the muon tracking environment in the **D0** detector, where about 100 - 150 radiation lengths separate between the central tracking to the muon chambers, and 50 - 85 radiation lengths are found in the toroid between the **A** and **B**,**C** muon modules [3]. The range of the momenta reflects the range of muon momenta associated with **B**, **W/Z** and **top** physics.

In figures 1-4 we plot the errors on the position and the direction after traversing 50 and 100 radiation lengths both for 5 GeV and 40 GeV particles. Note that the mean error is consisted with zero but the *rms* is non zero as expected from a stochastic process. To check that the results are in agreement with what one expects from a mean angular error as parameterized in [5] we calculate the quantity:

$$\tilde{\sigma_x} = \theta^{RW}_{rms} \cdot L \tag{7}$$

where θ^{RW}_{rms} is the *rms* MS angle in the plane defined as:

$$\theta^{RW}_{rms} = cos^{-1}(\sum_{\alpha=1}^{3} \hat{x}^{in}_\alpha \cdot \hat{x}^{out}_\alpha)$$

in/out stand for going into/out of the scattering material, and L is the shortest distance that a particle with the direction cosines, $\hat{x}^{in}$, would have traversed if MS was not present. $\tilde{\sigma_x}$ should be comparable in magnitude to the *rms* position error, σ_x, a feature which is seen in our results confirming the overall approach. In table 1 we compare the quantity, $\tilde{\sigma_x}$, with the *rms* position error. Some differences of ≈(17-25)% are seen between the two quantities. The discrepancy may stem from the nontrivial way the local errors sum up to the end-to-end error, compared to the calculation of equation (6), but statistic fluctuations can not be rulled out.

In figure 5 we plot the end-to end direction error, $\sigma^{RW}_{\hat{x}}$, of 5 and 40 GeV particles traversing various numbers of radiation lengths ($X_\circ$:10 to 150). Overlaid is a parameterization of this error after the Highland formula. As in the case of the position error, where $\tilde{\sigma_x}$ agrees with the *rms* of the accumulated error, the accumulated direction error complies nicely with the local parameterization of the direction error [1]. This agreement provides a direct way to calculate the diagonal errors ($M_{\alpha\alpha}$) of the end-to-end error matrix.
Finally, table 2 contains the off diagonal *rms* elements ($M_{\alpha\beta}$) of the covariance error matrix.

To conclude, we have used a parameterization of the MS errors associated with 0.1 of a radiation length , to simulate a RW type of solution for the estimation of the full end-to-end

$X_\circ$	P [GeV]	$\tilde{\sigma_x}$ [cm]	σ_x [cm]
50	5	0.40	0.47
100	5	1.2	1.5
50	40	0.05	0.05
100	40	0.14	0.17

Table 1: *rms* of the position error, σ_x, compared with $\tilde{\sigma_x}$ of equation (6).

$X_\circ$	P [GeV]	$\sigma_{(x,y)}$ [cm^2]	$\sigma_{(\hat{x},\hat{y})}$	$\sigma_{(x,\hat{x})}$ [cm]	$\sigma_{(y,\hat{x})}$ [cm]
50	5	0.14	0.6E-04	0.34E-02	0.26E-02
100	5	1.4	0.15E-03	0.18E-01	0.14E-01
50	40	0.24E-02	0.9E-06	0.6E-04	0.5E-04
100	40	0.19E-01	0.19E-05	0.26E-03	0.20E-03

Table 2: *rms* of the error matrix off-diagonal elements

error matrix that is associated with a particle traversing a large number of radiation lengths. The results are in good agreement with what is expected from a direct calculation and yet the scheme allows to estimate the error covariance (off-diagonal elements) of the track parameters. The RW approach fits very well with the stochastic nature of the MS process and can be customized for the specific geometry of each detector. We also draw the readers attention to the fact that the RW solution offers an estimation for the position - direction covariance which Moliere's theory does not provide. Naturally, a full detector simulation, as can be found in the **GEANT** package, would be a better software environment for a more detailed study.

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Figure Captions

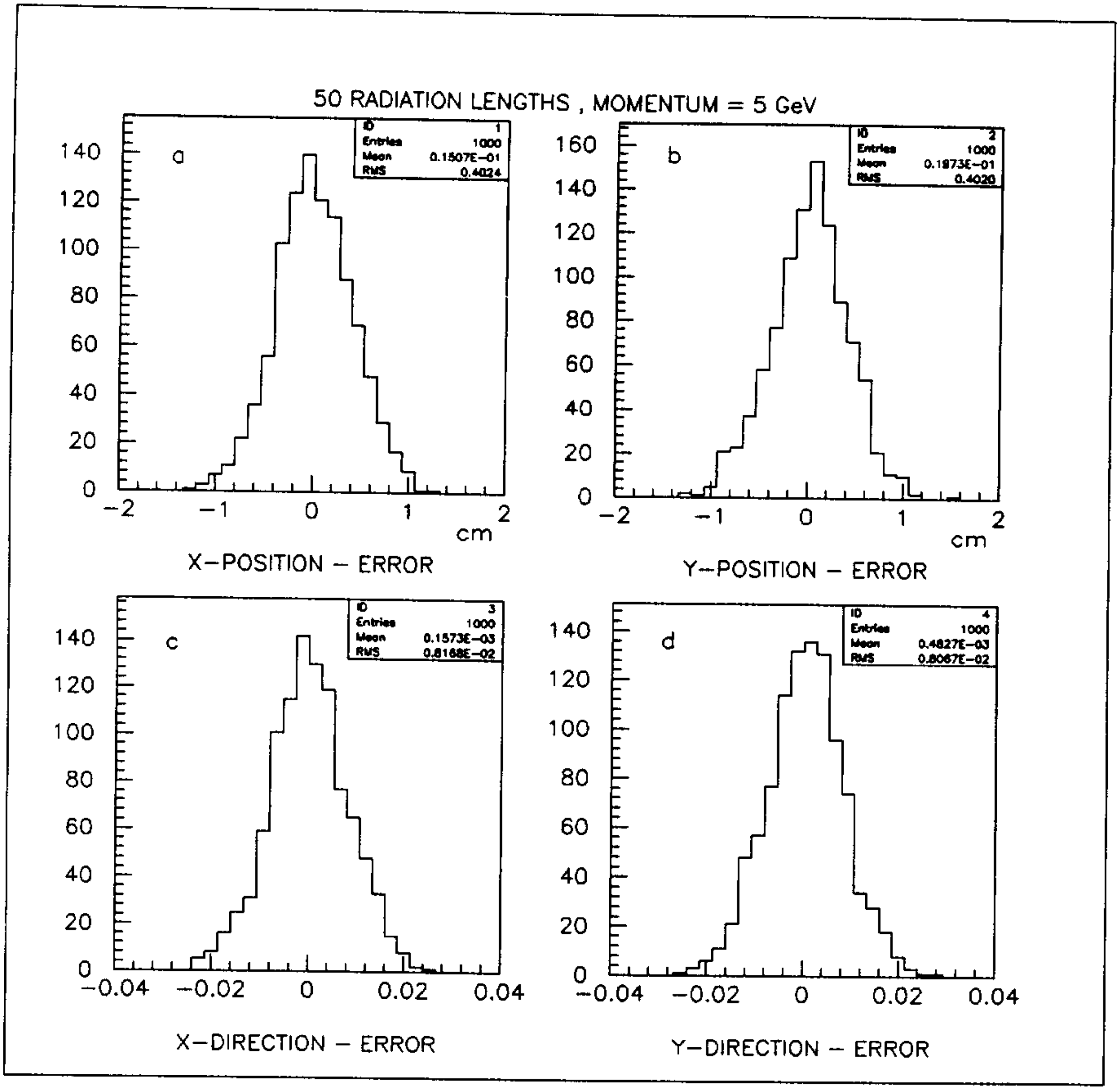
Fig. 1 Distributions of the x (a) and y (b) position errors, and $\hat{x}$ (c) $\hat{y}$ (d) direction errors after an n-step RW, simulated for 5 GeV particles in a scattering material of $X_{\circ} = 50$. Compare the rms of the position error to $\tilde{\sigma_x} = 0.47$ cm

Fig. 2 Distributions of the x (a) and y (b) position errors, and $\hat{x}$ (c) $\hat{y}$ (d) direction errors after an n-step RW, simulated for 5 GeV particles in a scattering material of $X_{\circ}$=100. Compare the rms of the position error to $\tilde{\sigma_x} = 1.5$ cm

Fig. 3 Distributions of the x (a) and y (b) position errors, and $\hat{x}$ (c) $\hat{y}$ (d) direction errors after an n-step RW, simulated for 40 GeV particles in a scattering material of $X_{\circ}$=50. Compare the rms of the position error to $\tilde{\sigma_x} = 0.05$ cm

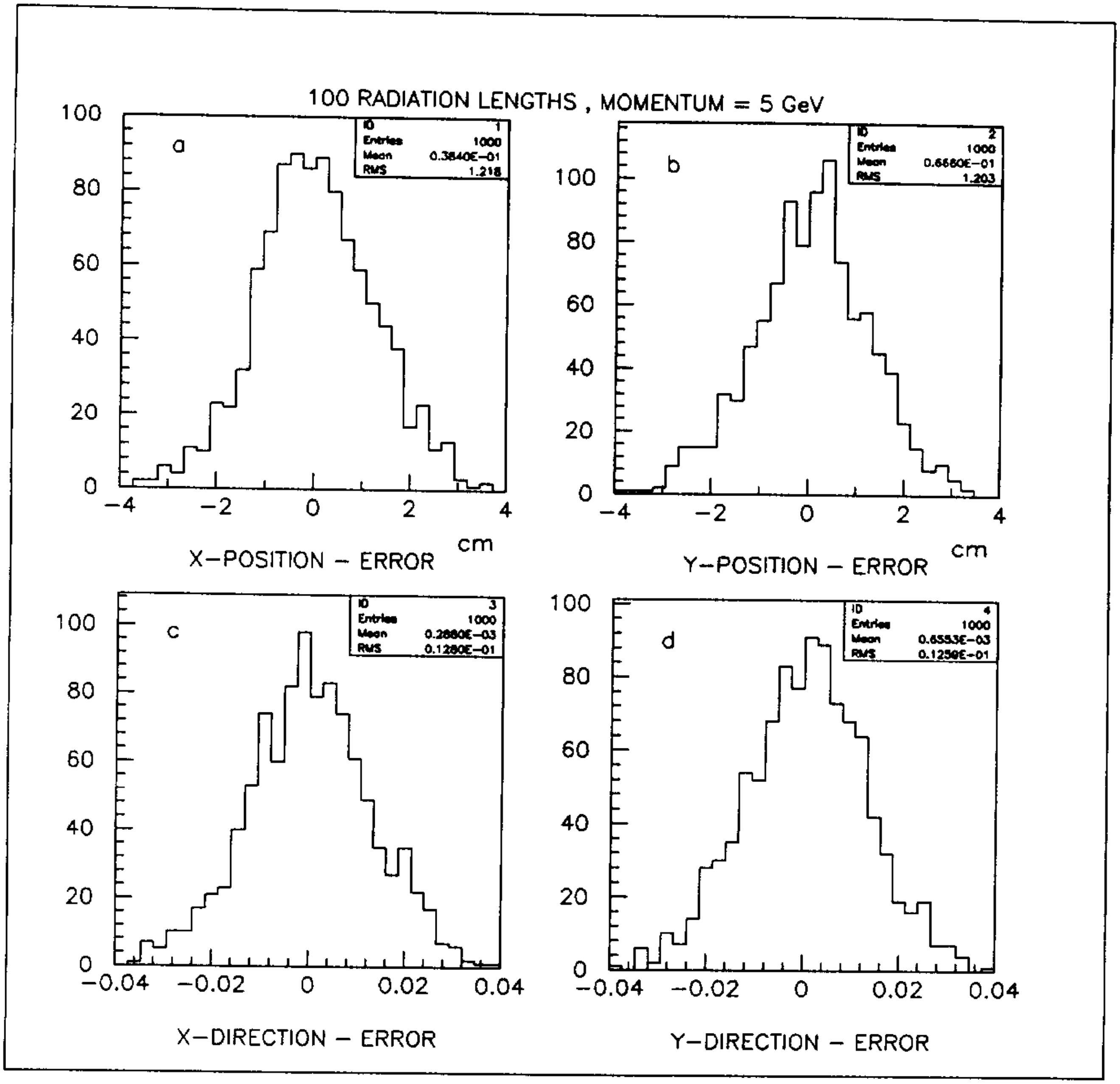
Fig. 4 Distributions of the x (a) and y (b) position errors, and $\hat{x}$ (c) $\hat{y}$ (d) direction errors after an n-step RW, simulated for 40 GeV particles in a scattering material of $X_{\circ}$=100. Compare the rms of the position error to $\tilde{\sigma_x} = 0.17$ cm

Fig. 5 Direction errors $(\hat{x}, \hat{y})$ after an n-step RW, simulated for 5 (a) and 40 (b) GeV particles as a function of $X_{\circ}$, overlaid is the Highland parameterization (see text).



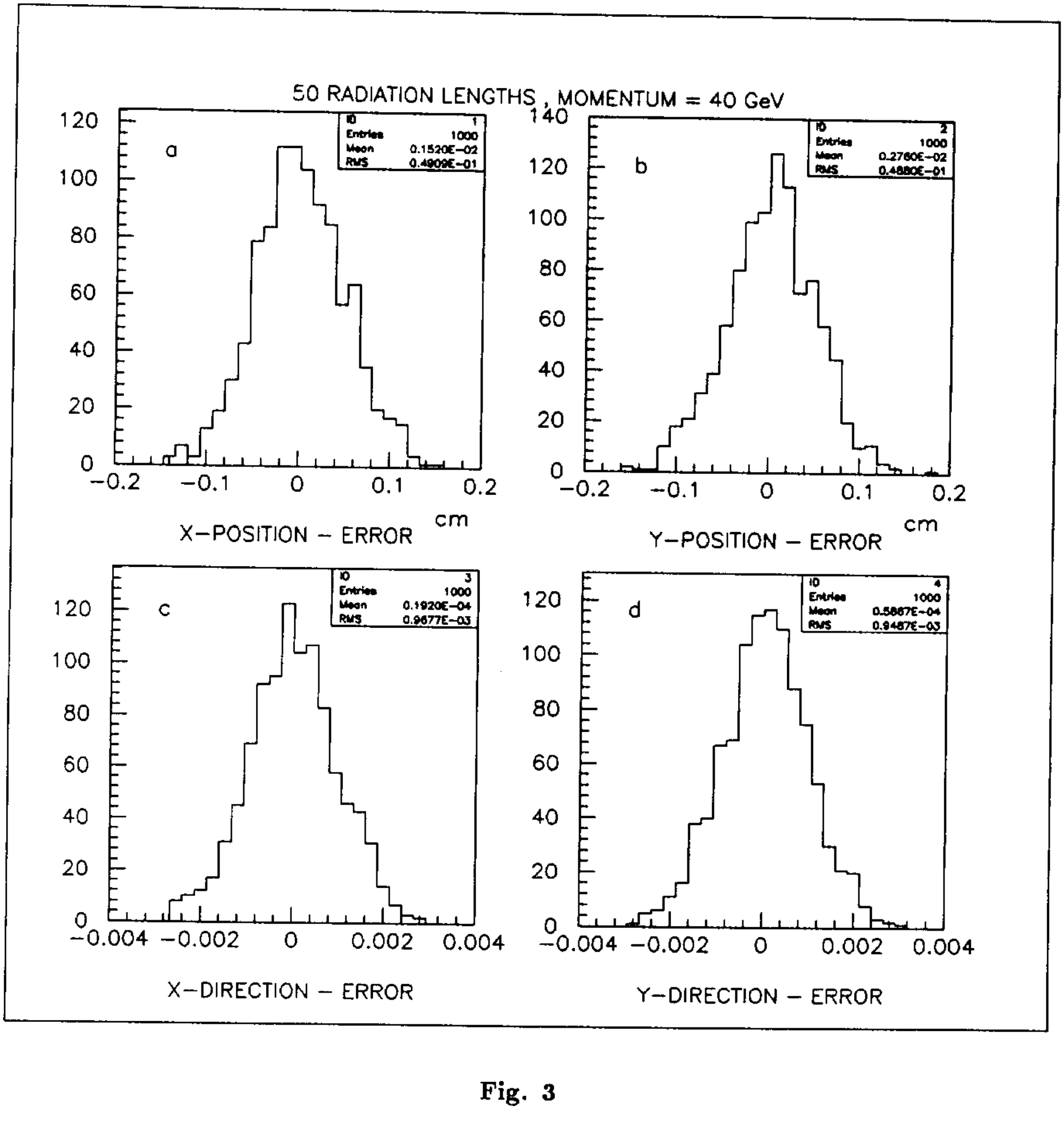
50 RADIATION LENGTHS , MOMENTUM = 5 GeV
a
b
c
d
cm
X-POSITION - ERROR
Y-POSITION - ERROR
X-DIRECTION - ERROR
Y-DIRECTION - ERROR

Fig. 1



100 RADIATION LENGTHS , MOMENTUM = 5 GeV
a
ID 1
Entries 1000
Mean 0.3840E-01
RMS 1.218
X-POSITION - ERROR
cm
b
ID 2
Entries 1000
Mean 0.6660E-01
RMS 1.203
Y-POSITION - ERROR
cm
c
ID 3
Entries 1000
Mean 0.2880E-03
RMS 0.1280E-01
X-DIRECTION - ERROR
d
ID 4
Entries 1000
Mean 0.6553E-03
RMS 0.1259E-01
Y-DIRECTION - ERROR

Fig. 2



50 RADIATION LENGTHS , MOMENTUM = 40 GeV
X–POSITION – ERROR
Y–POSITION – ERROR
X–DIRECTION – ERROR
Y–DIRECTION – ERROR
cm
cm
a
b
c
d

Fig. 3

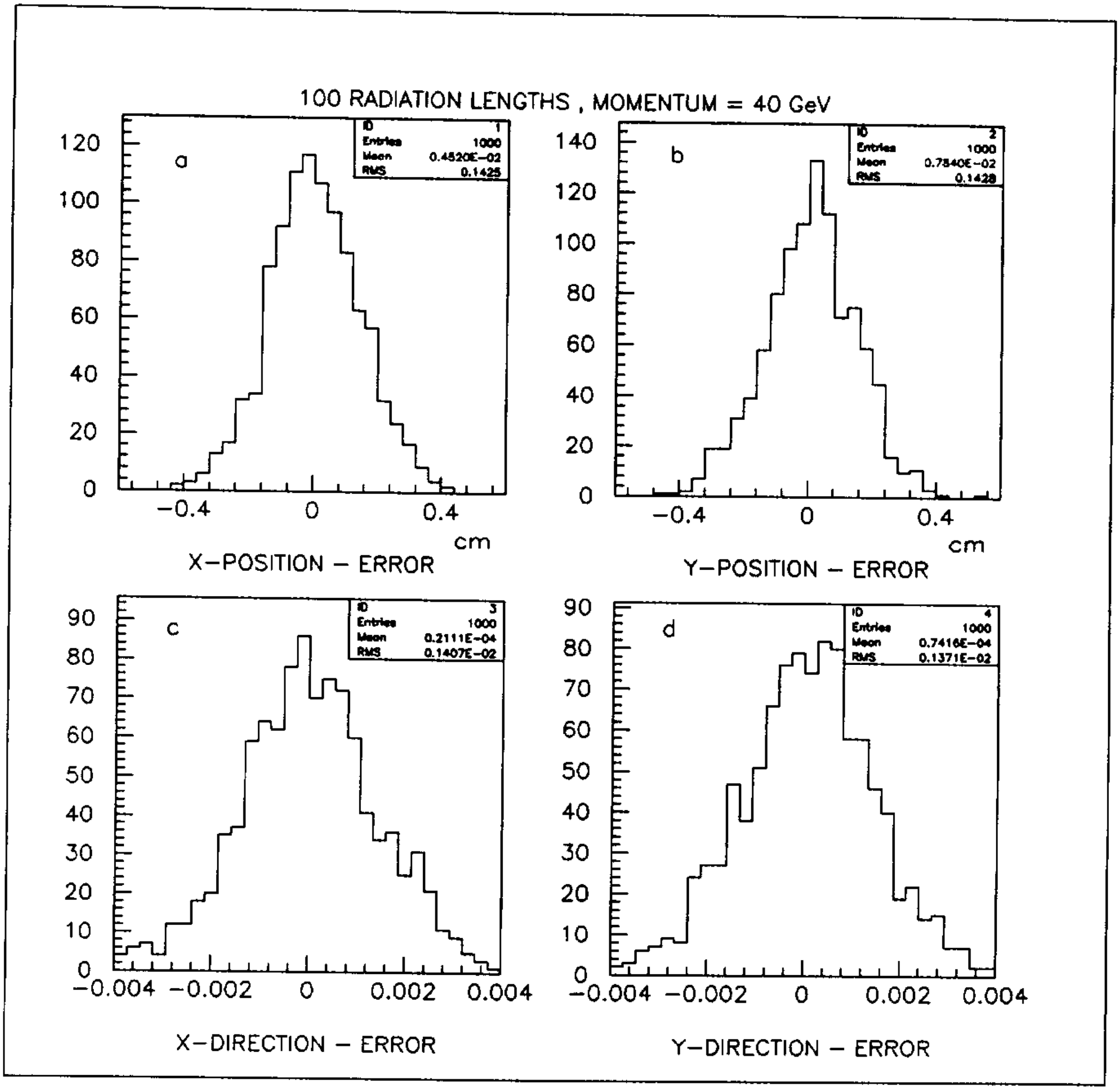
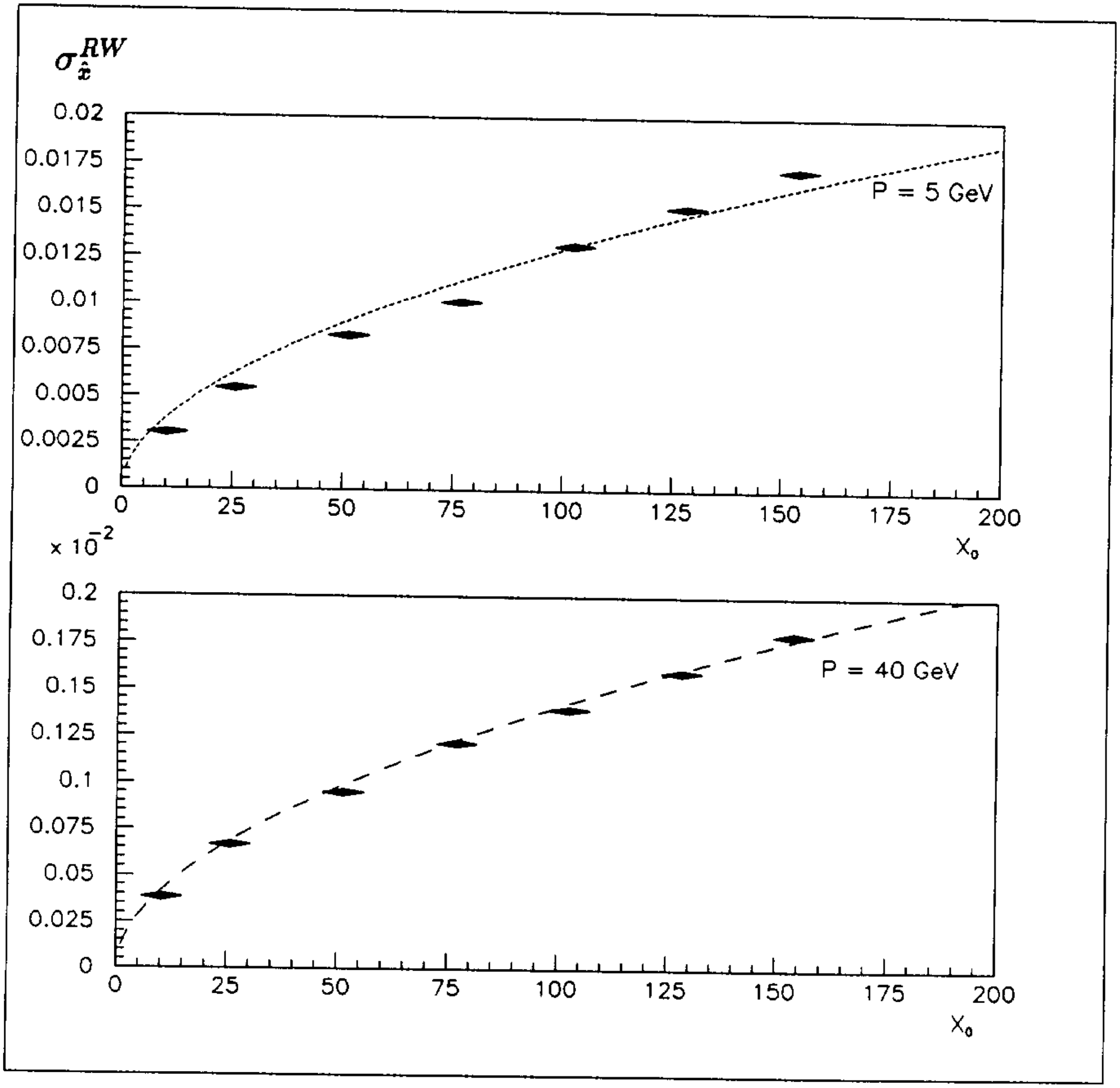

100 RADIATION LENGTHS , MOMENTUM = 40 GeV
a
ID 1
Entries 1000
Mean 0.4520E-02
RMS 0.1425
X-POSITION - ERROR
cm
b
ID 2
Entries 1000
Mean 0.7840E-02
RMS 0.1428
Y-POSITION - ERROR
cm
c
ID 3
Entries 1000
Mean 0.2111E-04
RMS 0.1407E-02
X-DIRECTION - ERROR
d
ID 4
Entries 1000
Mean 0.7416E-04
RMS 0.1371E-02
Y-DIRECTION - ERROR

Fig. 4



$\sigma_{\hat{x}}^{RW}$
0.02
0.0175
0.015
0.0125
0.01
0.0075
0.005
0.0025
0
P = 5 GeV
0
25
50
75
100
125
150
175
200
X_0
$\times 10^{-2}$
0.2
0.175
0.15
0.125
0.1
0.075
0.05
0.025
0
P = 40 GeV
0
25
50
75
100
125
150
175
200
X_0

Fig. 5