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## LIMITS TO THE PRIMORDIAL HELIUM ABUNDANCE IN THE BARYON INHOMOGENEOUS BIG BANG \*

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### Abstract

The parameter space for baryon inhomogeneous big-bang models is explored with the goal of determining the minimum helium abundance obtainable in such models while still satisfying the other light-element constraints. We find that the constraint of  $(D+{}^3\text{He})/\text{H} < 10^{-4}$  restricts the primordial helium mass fraction from baryon-inhomogeneous big-bang models to be  $\geq 0.231$  even for a scenario which optimizes the effects of the inhomogeneities and destroys the excess lithium production. Thus, this modification to the standard big bang as well as the standard homogeneous big bang model itself would be falsifiable by observation if the primordial  ${}^4\text{He}$  abundance were observed to be less than 0.231. Furthermore, a present upper limit to the observed helium mass fraction of  $Y_p^{\text{obs}} \leq 0.24$  implies that the maximum baryon-to-photon ratio allowable in the inhomogeneous models corresponds to  $\eta \leq 2.3 \times 10^{-9}$  ( $\Omega_b \leq 0.35$ ) even if all conditions are optimized.

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## 1. Introduction

An important prediction (Walker, *et al.* 1991) of standard homogeneous big bang nucleosynthesis is that the primordial helium mass fraction,  $Y_p$ , must be  $\geq 0.236$ . The purpose of this paper is to show to what degree this constraint can be relaxed if baryon inhomogeneities are present during primordial nucleosynthesis. Such inhomogeneities could have been produced during the cosmic quark-hadron phase transition or by other processes in the early universe (*cf.* Malaney & Mathews 1992). At the same time current observations extrapolated to zero of helium abundance as a function of metallicity tend to give values of  $Y_p = 0.23$  (Pagel *et al.* 1991; Walker *et al.* 1991) or even as low as  $Y_p = 0.225$  (Mathews, Boyd, & Fuller 1992). However, possible systematic errors still allow values as large as  $Y_p \sim 0.24$ . As possible systematic errors get constrained it may eventually be possible to truly resolve whether or not the observed  ${}^4\text{He}$  abundance is a fatal problem for standard homogeneous big bang nucleosynthesis. We will show in this paper that baryon number density fluctuations can be optimized to lower the allowed range on  $Y_p$  to 0.231 but no lower. Thus, if  $Y_p$  is ever proven to be in the range of 0.231 to 0.236, it could be a signature of such baryon inhomogeneities and possibly a probe of the quark-hadron transition. However, if  $Y_p$  is found to be  $< 0.231$ , then even optimized baryon inhomogeneities would not resolve the discrepancy. We will discuss possible implications of this eventuality.

There has been considerable recent discussion (Alcock, Fuller, & Mathews 1987; Applegate, Hogan, & Scherrer 1988; Fuller, Mathews, & Alcock 1988; Kurki-Suonio *et al.* 1988; Malaney & Fowler 1988; Terasawa & Sato 1989; 1990; Kajino & Boyd 1990; Mathews *et al.* 1990; Kawano *et al.* 1991) on the possibility that an inhomogeneous distribution of baryons in the early universe, together with the differential diffusion of neutrons and protons could lead to primordial nucleosynthesis constraints which are different from the standard homogeneous big bang model (Wagoner, Fowler, & Hoyle 1967; Schramm & Wagoner 1977; Yang *et al.* 1984; Boesgaard & Steigman 1985; Walker, *et al.* 1991). Initial excitement about these models focused on the possibility of obtaining a high baryon-to-photon ratio,  $\eta$ , while still fitting the observed light element abundances (Applegate *et al.* 1988; Alcock, *et al.* 1987). However, subsequent multizone treatments of the diffusion process (Kurki-Suonio *et al.* 1989; Terasawa and Sato 1989; 1990; Mathews *et al.* 1990) have shown that, although slightly higher values of  $\eta$  may be possible, it is not possible to increase  $\eta$  enough to have a present closure density in baryons and still satisfy the constraints from light element abundances even with optimized baryon inhomogeneities. We show below that, if all conditions are optimized for the inhomogeneous model, then a present upper limit to the observed helium mass fraction of  $Y_p^{obs} \leq 0.24$  implies a maximum allowable baryon-to-photon ratio of  $\eta \leq 2.3 \times 10^{-9}$  (or a maximum baryonic contribution to the closure density of  $\Omega_b \leq 0.35$ ).

Nonetheless, the possibility of using light element abundances as a signature for baryon inhomogeneities in the early universe remains important. While the formation of nuclei with  $A > 7$  has frequently been discussed in this regard (*cf.* Boyd & Kajino 1989; Kajino, Mathews, & Fuller 1990; Kawano, Malaney, & Fowler 1991), in this paper we focus on  ${}^4\text{He}$ .

## 2. The Calculations

The calculations presented here are based upon the multi-zone primordial nucleosynthesis plus baryon diffusion code described in Mathews *et al.* (1990) (hereafter MMFA90). In that paper, the baryon number density fluctuations were described as condensed spheres or spherical shells parameterized by an initial ratio  $R$  of baryon density in the high density region to low-density region, a fraction  $f_v$  of the total volume in the high density region, a separation distance  $l$  between the centers of the high-density regions, and the baryon-to-photon ratio  $\eta$ . Diffusion is allowed to occur through a comoving Eulerian grid of variable zone sizes defined so as to best resolve the boundary between the low-density and high-density regions.

It was found (MMFA90) that the lowest  ${}^4\text{He}$  abundance was obtained for a comoving separation distance of 43 m at  $T = 100$  MeV, independent of the other parameters of the model. The results were also relatively insensitive to the fluctuation amplitude as long as  $R \gtrsim 10^3$  producing only a 0.002 reduction in helium between  $R = 10^3$  and  $R = 10^6$ . The helium was also found to be relatively insensitive to the volume fraction or geometry, being slightly optimized for a condensed-sphere geometry with  $f_v^{1/3} \sim 0.25$ .

The present task, therefore, becomes simplified to a study of primordial nucleosynthesis yields as a function of  $\eta$  (or the present fraction of the critical density in baryons  $\Omega_b$ ) and  $l$  with the intention of finding the minimum helium abundance for which the constraints on the other light element abundances can be satisfied. For these calculations we fix the optimum values of  $R = 10^6$  and  $f_v^{1/3} = 0.25$  as  $\eta$  and  $l$  are varied.

## 3. Light Element Constraints

Since it is our intention to find the lowest possible helium abundance we chose the most liberal light element constraints within the context of the baryon inhomogeneous big bang model. For  ${}^7\text{Li}$ , for example, we will presume that the effects of late-time expansion (Alcock *et al.* 1990) of the proton rich region can be optimized so as to destroy  ${}^7\text{Be}$  and  ${}^7\text{Li}$  for any value of  $\eta$  and  $l$  such that the Pop II constraint ( $\text{Li}/\text{H} < 1.7 \times 10^{-10}$ , Rebolo *et al.* 1988; Walker *et al.* 1991) can be satisfied. The consequences of the overproduction of lithium on the allowed parameters for an inhomogeneous big bang have been discussed elsewhere (Kurki-Suonio *et al.* 1989; MMFA90). It is absolutely essential that a mechanism such as late time expansion or galactic evolution (Mathews, Alcock, & Fuller 1989) be invoked to destroy lithium. Since inhomogeneities always increase the primordial lithium abundance (Alcock *et al.* 1987), no inhomogeneous big bang model can be constructed which is consistent with the Pop II lithium abundance, unless a destruction mechanism is invoked.

The most significant constraints for our purposes thus arise from D and  ${}^3\text{He}$ . Since deuterium is converted to  ${}^3\text{He}$  in stars (Yang *et al.* 1984) the most stringent constraint comes from the sum  $(\text{D}+{}^3\text{He})/\text{H}$ . For this constraint we adopt  $(\text{D}+{}^3\text{He})/\text{H} < 10^{-4}$  from Walker *et al.* (1991). Our goal will therefore be to satisfy this constraint as a function of  $l$  and  $\eta$ , and then to identify which values lead to the lowest primordial helium abundance. Although we will presume that the excess lithium production is destroyed by the late time expansion, we will ignore the slight increase in  $(\text{D}+{}^3\text{He})/\text{H}$  which accompanies the lithium destruction (Alcock *et al.* 1990) in this mechanism. This has the effect of strengthening

our  $(D+{}^3\text{He})/H$  constraint.

#### 4. Results

Figure 1 summarizes the lowest helium abundances obtained in the standard big bang (SBB) and inhomogeneous big bang (IBB) models as a function of baryon-to-photon ratio ( $\eta_{10} = \eta \times 10^{10}$ ). The corresponding fraction of the closure density in baryons ( $\Omega_b$ ) can be obtained from  $\eta_{10}$  using the relation,  $\Omega_b = 0.015(T/2.75K)^3(H_0/50)^{-2}\eta_{10}$ , where  $T$  is the present microwave background temperature and  $H_0$  is the present Hubble parameter in units of  $km\ s^{-1}Mpc^{-1}$ . The two parallel curves above and below each thick line show the uncertainty in these results from  $\pm 2\sigma$  variations of the neutron half life. The central solid curve results from an adopted neutron half life of  $\tau_n = 888.6 \pm 7.0(2\sigma)s$ . The limits imposed by various light element abundances are indicated by hatched lines.

The central curves on Figure 1 show the helium abundance from the standard big bang model with 3 neutrino flavors. The limits which we derive are the same as in Walker *et al.* (1991), namely  $\eta_{10} \geq 2.8$  from  $(D+{}^3\text{He})/H \leq 10^{-4}$  which implies  $Y_p \geq 0.236$ . Similarly,  ${}^7\text{Li}/H \leq 1.7 \times 10^{-10}$  implies  $\eta_{10} \leq 4.0$  and  $Y_p \leq 0.243$ .

The lower set of curves on Figure 1 show the lowest primordial helium abundance as a function of  $\eta_{10}$  from the baryon inhomogeneous models. The result of imposing the constraint  $(D+{}^3\text{He})/H \leq 10^{-4}$  is that for  $\eta_{10} \leq 14$  the separation distance must be reduced below the value which minimizes the  ${}^4\text{He}$  abundance. The resultant minimum  ${}^4\text{He}$  abundance thus gradually increases until the lowest allowed  ${}^4\text{He}$  abundance for our chosen parameters can even be in excess of the standard homogeneous big bang value. If we also varied  $R$  or  $f_v$  in these calculations the curve indicating the  $(D+{}^3\text{He})/H$  constraint would have continuously gone over to the SBB minimum, but could not have become lower than the minimum value for the inhomogeneous model at  $\eta = 14$ .

Thus, we find that the absolute lowest  ${}^4\text{He}$  abundance possible in the baryon inhomogeneous big bang models without abandoning the  $(D+{}^3\text{He})/H$  constraint is  $Y_p \geq 0.231$ . This value is achieved for  $\eta_{10} = 14$  ( $\Omega_b = 0.21$ ). Hence, it is possible to reduce the primordial helium abundance (although only slightly) and allow for a larger baryon density in the inhomogeneous models. A present upper limit to the observed helium mass fraction (Pagel *et al.* 1991) of  $Y_p^{obs} \leq 0.24$  implies that the maximum baryon density allowable in the inhomogeneous models corresponds to  $\eta_{10} \leq 23$  ( $\Omega_b \leq 0.35$ ). Interestingly, this limit just corresponds to dynamical estimates of the closure density inferred from clusters of galaxies (*e.g.* Davis & Peebles 1983). Thus, baryon inhomogeneous models may allow for sufficient baryonic dark matter to account for the inferred gravitational mass up to the scale of galaxy clusters.

Of course, an even lower  ${}^4\text{He}$  abundance could be obtained both for the homogeneous and inhomogeneous models if the  $(D+{}^3\text{He})/H$  constraint could be relaxed. The critical point is whether  ${}^3\text{He}$  is produced or destroyed in stars. The observation of significant  ${}^3\text{He}$  in horizontal-branch stars (Hartoog 1979) and planetary nebulae (Rood, Bania, & Wilson 1992) shows that  ${}^3\text{He}$  is enriched and does indeed survive to the advanced stages of stellar evolution for low mass stars. Thus, the  $(D+{}^3\text{He})/H$  constraint must be taken as firm. This constraint may even become more stringent as more quantitative observations and models are made. This means that a firm observational low value for  $Y_p$  ( $< 0.231$ ) would

be able to falsify both the homogeneous and inhomogeneous big bang models.

If the  $(D+{}^3\text{He})$  constraint should become more stringent with time, however, it can not fall below  $(D+{}^3\text{He})/H \leq 5 \times 10^{-5}$  without the standard big bang becoming inconsistent with the constraint from  ${}^7\text{Li}$  (Walker *et al.* 1991). To illustrate the effect of a more stringent  $(D+{}^3\text{He})/H$  constraint on the baryon inhomogeneous models we have also shown on Figure 1 the effect of setting  $(D+{}^3\text{He})/H < 5 \times 10^{-5}$ . For this case the minimum helium abundance in the inhomogeneous models would increase to  $Y_p \geq 0.244$  compared to the SBB limit which would only increase to 0.242. Thus, if the  $(D+{}^3\text{He})/H$  constraint were to become more restrictive it is possible that no reduction in the primordial helium abundance could be achieved in inhomogeneous models.

Finally, the upper set of curves on Figure 1 show the upper limits to the primordial helium abundance in the inhomogeneous model for which the upper limit comes from the deuterium lower limit ( $D/H \geq 1.6 \times 10^{-5}$ ). Since for these models it may be possible to circumvent the constraint from  ${}^7\text{Li}$ , much larger helium abundances are allowed. However, since the primordial helium mass fraction inferred from observations is significantly less than this upper limit, the upper limit to the calculated helium abundance is not presently a particularly useful constraint in either the homogeneous or inhomogeneous big bang models.

#### 4. Conclusions

Several conclusions can be drawn from the present analysis. The lower limit,  $Y_p \geq 0.236$ , allowed in the standard homogeneous big bang with three neutrino flavors is only decreased to about 0.231 in the baryon inhomogeneous models. This means that a firm observational low value for  $Y_p$  ( $< 0.231$ ) would be able to falsify both the homogeneous and inhomogeneous big bang models.

However, as long as the present upper limit to the observed helium mass fraction is as high as  $Y_p \leq 0.24$ , it may be possible to increase the baryon to photon ratio as high as  $\eta_{10} \leq 23$  ( $\Omega_b \leq 0.35$ ) without violating any light element constraints if optimum conditions can occur for baryon inhomogeneities in the early universe.

If a lower helium abundance should ultimately be required from the data one must propose alternative fixes for the standard homogeneous big bang model other than the introduction of baryon inhomogeneities. These might include an unstable  $\nu_\tau$ ; (Kolb & Scherrer 1982; Scherrer & Turner 1988), non-thermal neutrino distributions or neutrino degeneracy (Olive *et al.* 1991; Kang & Steigman 1991), either of which may allow for a lower value of  $Y_p$  while satisfying the  $(D+{}^3\text{He})/H$  constraint. Late decaying particle models could also produce a low primordial helium abundance, but one must find a way to diminish the overproduction of  ${}^6\text{Li}$  (Dimopoulos *et al.* 1988). Clearly more investigation of these alternative scenarios along the lines of the present work would be warranted to see if they also could be falsifiable with a sufficiently low observed primordial helium abundance.

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### Figure Caption

Figure 1. Primordial helium abundance  $Y_p$  as a function of baryon-to-photon ratio,  $\eta_{10} = \eta \times 10^{10}$ . The central curves show the standard homogeneous big bang (SBB) as labelled. The thick lines are for a neutron half life of 888.6 s. The upper and lower thin lines show the uncertainty due to  $\pm 2\sigma$  variations of the neutron half life. The hatch marks indicate values allowed by the upper and lower limits to the light element constraints. Values for two different lower limits to the  $(D+{}^3\text{He})/H$  constraint are shown to demonstrate the sensitivity to this limit. The lowest helium abundance allowed in the inhomogeneous models is 0.231 for  $\eta_{10} = 14$ .

